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The comparison of the elements for the homogenizing charged particle irradiation

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Abstract

Particle beams with uniform and well-confined intensity distributions are desirable in some high power beam applications to prolong the target lifetime or to improve the beam utilization. Three kinds of elements had been proposed for the beam homogenizing, such as octupole, pole-piece magnet, and step-like nonlinear magnets. In this paper, the new type of elements called heteromorphic quadrupole and focus sextupole are proposed. The Gaussian-like multiparticle beam redistribution by the octupole, heteromophic quadrupole, step-like nonlinear magnets, and focus sextupole has been simulated by the POISSON and LEADS code. The best redistribution result is obtained by the focus sextupole, and one of the solutions of redistributing beam with big halo can be that of using the focus sextupole and the heteromorphic quadrupole.

Keywords: Focus sextupole; Heteromorphic quadrupole; Octupole; Particle beam homogenizing; Step-like nonlinear magnet

1. INTRODUCTION

The uniform area densities of charged particle beams with large spatial dimensions at a given location are essential for many applications of high power beams, such as the spallation neutron source, the accelerator driven systems, the accelerator for the production tritium, and so on. For the electron linear collider, the nonlinear magnet is also needed for better luminosity and lower background (Pitthan, 2000; Brinkmann *et al.*, 2001). In most cases, the natural distribution of the beam coming from accelerator is Gaussian-like, and in some cases even worse. Some kinds of nonlinear elements are used for the beam redistribution such as octupole (Blind, 1991), duodecapole, step-like magnet (Tang *et al.*, 2004), and pole-piece magnet (Barlow *et al.*, 1997). Meot *et al.* (1996) gave an analytical treatment of homogenizing the transverse beam densities by the octupole and duodecapole. However, the density distribution ears always appear at the edge of the beam spot when the octupole and/or duodecapole are used (Blind, 1991; Meot *et al.*, 1996; Varentsov *et al.*, 2005). So the halo particles have to be collimated in order to cure the ears on the beam target (Kashy *et al.*, 1987). At the same time, the high field of the octupole is difficult to obtain (Pitthan, 2000). The step-like magnet and pole-piece magnet are somewhat complicate for fabrication, installation and commissioning, and they are designed for the special beam profile. When the beam profile change during the commissioning or running, they are difficult to be adjusted.

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2. THE PRINCIPLE OF REDISTRIBUTION THE GAUSS-LIKE BEAM

In theory, for any initial beam density distribution, one can make a redistribution field to obtain the uniform beam density by the special designed pole-piece magnets (Barlow *et al.*, 1997), although it is somewhat complicated and difficult. However, the most popular beam distribution is Gaussian-like, and the redistribution of this kind of beam is the most important. The Gauss function should be:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \tag{1}$$

where μ and σ is defined in the Figure 1. For the beam density curve, μ is always zero.

The curve in the Figure 1 approaches isosceles triangle curve. As we know, to homogenize the isosceles triangle beam density curve it is required to fold it from $\mathbf{a}/2$ to $\mathbf{a}$

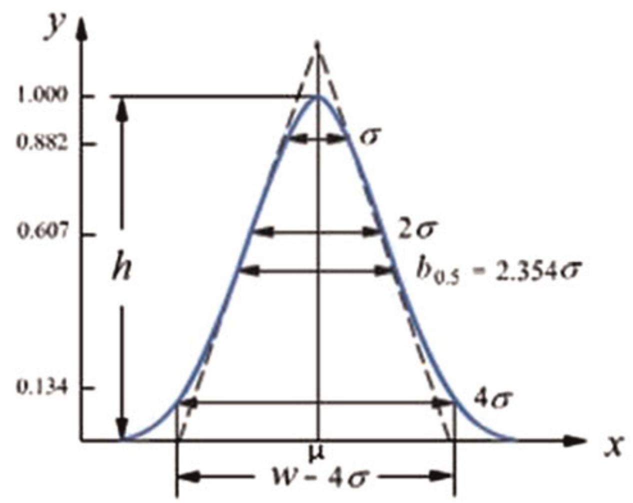

Fig. 1. (Color online) The Gauss curve and its parameter definition.

into center (where the length of the triangle bottom is 2a and the triangle is symmetric according to the *Y* axis) as shown in Figure 2. So it calls to mind that the same method can be used for the Gaussian-like. Different with the homogenizing of triangle density curve, the fold point should be at σ instead of the width of half maximum as shown in Figure 3.

To fold the Gauss curve, the special field is needed as the yellow line shown in Figure 3 (color online), whose focus coefficient is defined as:

$$k = \begin{cases} 0 & (|x| \le \sigma) \\ constant & (|x| > \sigma) \end{cases}. \tag{2}$$

Using this kind of field, the redistribution curve is shown as a pink curve, and the ratio of the maximum value between $|x| > \sigma$ and $|x| < \sigma$ is about 0.8%.

In order to decrease the coupling between *X* and *Y* plane caused by the nonlinear redistribution field, the beam is

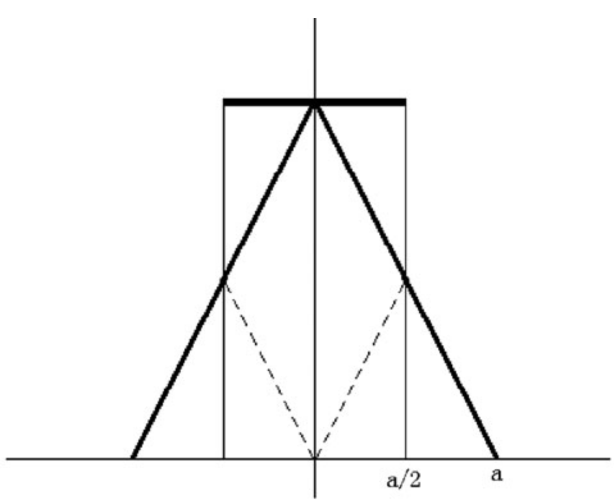

Fig. 2. The redistribution of triangle.

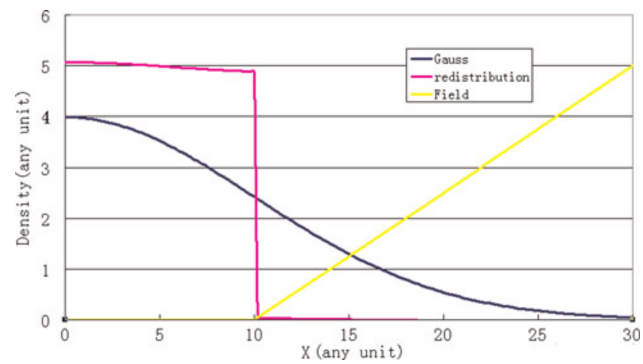

Fig. 3. (Color online) Redistribute the Gauss density curve.

always focused on a spot with a large aspect ratio at the beam redistribution element, and the phase space of the redistribution plane is represented by a line (Blind, 1991; Jason *et al.*, 1997). In this case, to simplify the analysis, the beam can be handled as a parallel beam in the redistribution plane as shown in Figure 4, and the redistribution philosophy is researched in one phase plane firstly.

3. THE PROPERTY OF THE HETEROMORPHIC QUADRUPOLE MAGNETS

The fold beam method had been proposed by Meads (1983), and the needed field should be as like Eq. (2) (Pitthan, 2000). To perform the beam fold, one used the octupole with/without duodecapole, which does not meet the needed field as shown in Section 5 of this paper. In order to obtain the field as shown in Figure 3, we designed a new type of element called heteromorphic quadrupole (HQ) as shown in Figure 5 (only a quarter of the element is calculated by POISSON (Billen *et al.*, 1993) code). The HQ is similar to the common quadrupole except that a couple of poles are inserted for shielding the magnet field. The obtained field along the *X* axis is shown in Figure 6. Different from the ideal field in Figure 3, there is an intermediate zone between the zero zone and linear focus zone.

Using the LEADS (Lu, 1995) code and the field as shown in Figure 6, the phase space and the beam distribution in the

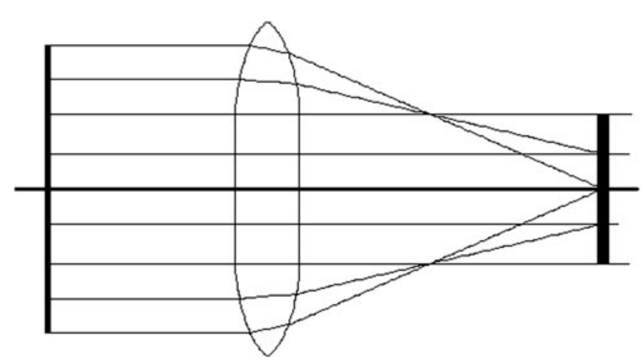

Fig. 4. The scheme of beam folding philosophy.

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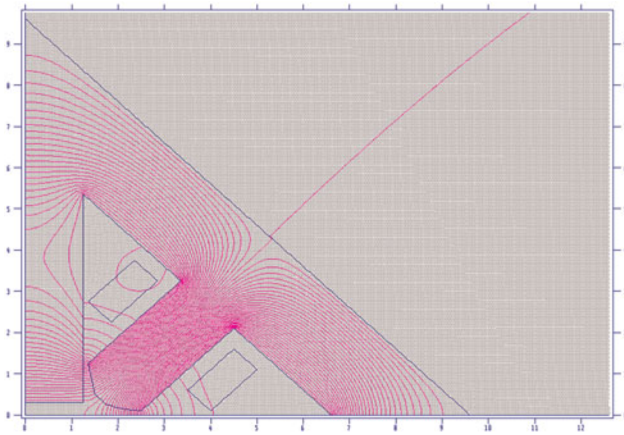

Fig. 5. (Color online) The two dimensional field of the HQ.

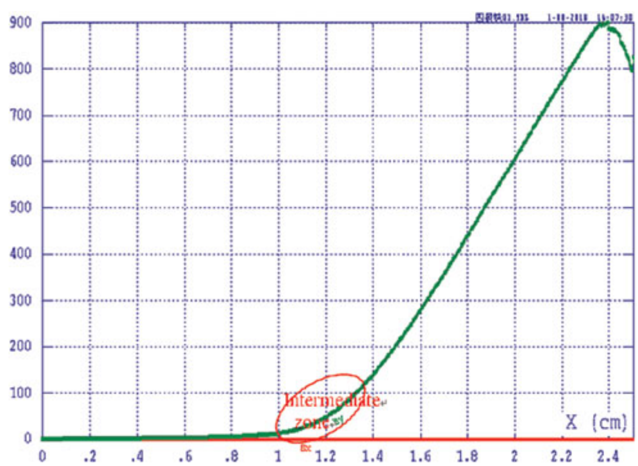

Fig. 6. (Color online) The one dimensional field of the HQ.

transverse plane can be obtained as shown in Figures 7 and 8 (Li *et al.*, 2011), where the input beam density of two transverse plane is Gauss-like. The beam density curve of the *X* plane is very different from that in Figure 3 because of the appearance of two ears. The reason is that the HQ field in the intermediate zone is greater than the ideal field in Figure 3

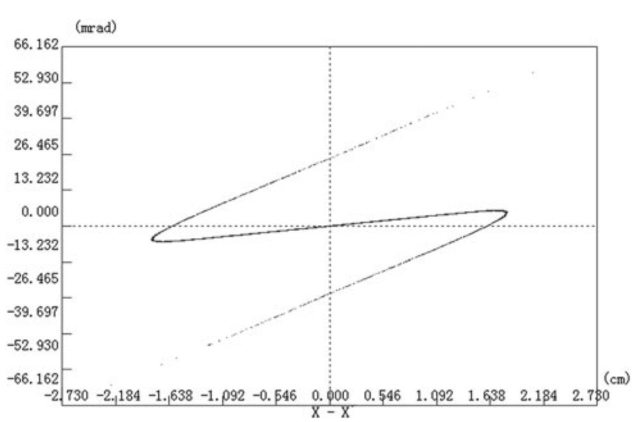

Fig. 7. The phase space after HQ and drift space.

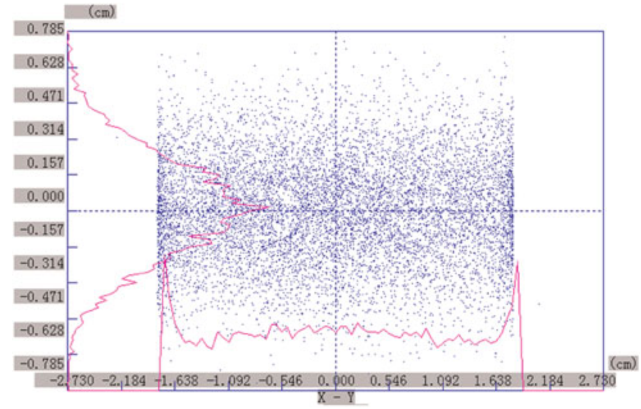

Fig. 8. (Color online) The beam distribution in the transverse plane.

and the particles experience more focus strength and collect to shape ears, which can be understood in Figure 4. However, the intermediate zone can not be cured completely because the divergence of the magnetic field is zero, and then the first order derivation of the field curve must be uninterrupted. The first order derivation of the field in Eq. (2) is interrupted at the point of $|x| = \sigma$, so the field can not be obtained in fact, although the size of the intermediate zone can be reduced by decreasing the gap of the HQ poles up to the limit of the mechanical demand. The magnitude of the ears can be changed by the space charge effect or other effects, but it can not be cancelled, because the space charge and other effects should be perturbation and can not cure the intermediate zone in Figure 6. So the HQ is not the perfect element for the beam uniformity.

4. THE PROPERTY OF THE STEP-LIKE NONLINEAR MAGNETS

Another candidate element for the beam distribution is the step-like nonlinear magnets (SNM), whose cross-section and field are shown in Figures 9 and 10 (Tang *et al.*, 2004). The redistribution philosophy is shown in Figure 11 in brief. The difference from the HQ is that the main manipulating part of the beam should take a translation movement by the force of the flat field as shown in Figure 10. According to the calculation result of LEADS code as shown in Figures 12 and 13, there are two pairs of ears on the beam density distribution, whose reason is that the phase line is folded two times, as can be shown in Figures 11 and 12. In this situation, it is difficult to obtain the better uniform beam distribution than that of using the HQ. This result is very different from that of reference (Tang *et al.*, 2004). The reason is that the approximation function is used in the calculation and the demanded beam redistribution function is parabolic. However, the real field is used in the calculation by LEADS (Lu, 1995). According to Figures 4 and 11, the property of the beam fold of HQ and SNM is linear. So we name this method the linear fold.

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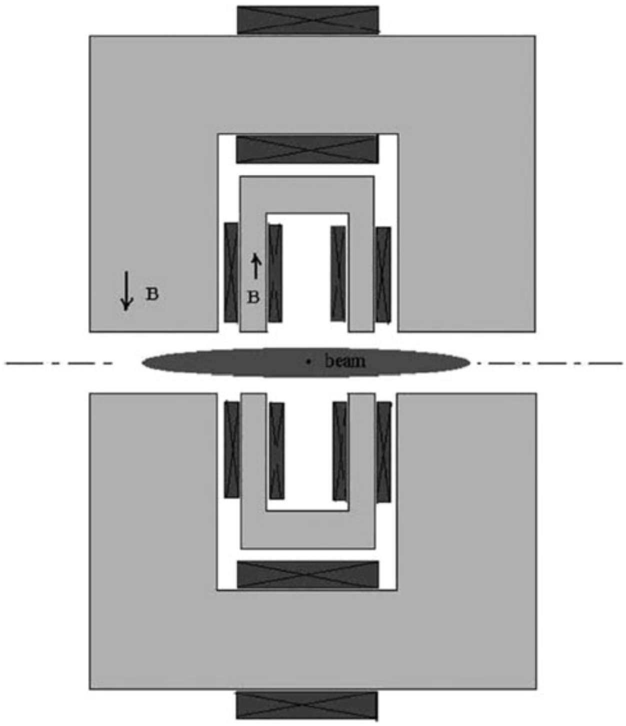

Fig. 9. The scheme of the step-like magnets.

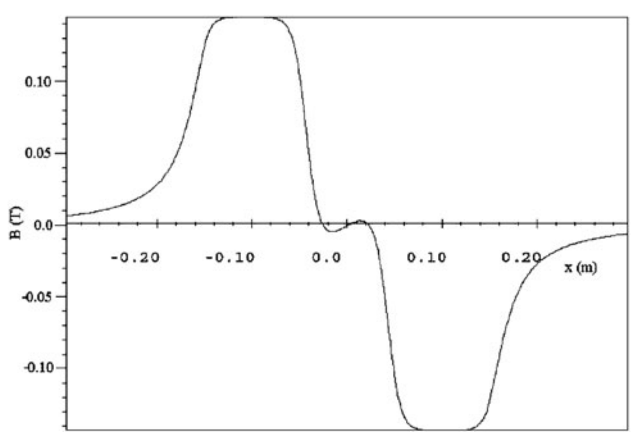

Fig. 10. Field distribution of the step-like magnets.

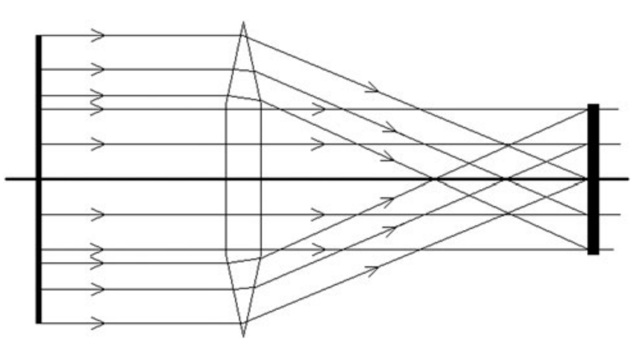

Fig. 11. The beam folding philosophy of SNM.

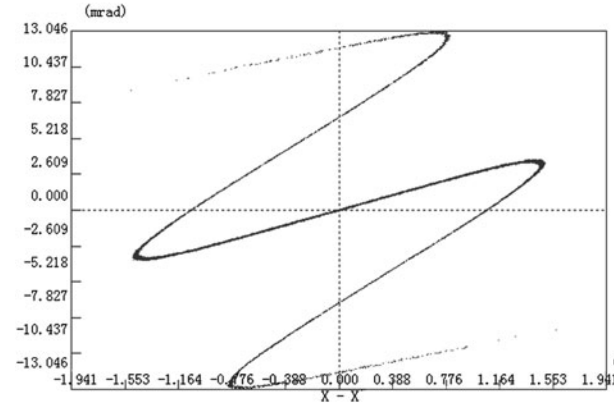

Fig. 12. The phase space after SNM and drift space.

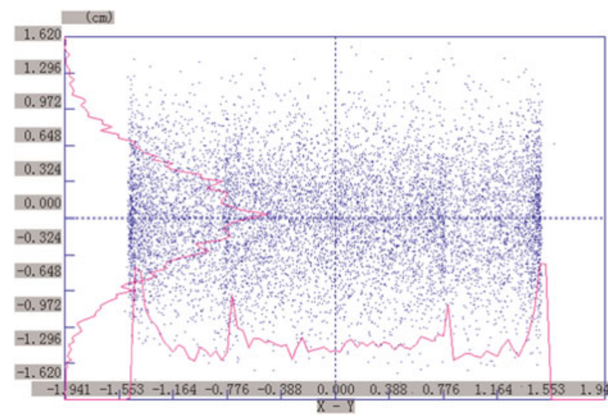

Fig. 13. (Color online) The beam distribution in the transverse plane.

5. THE PROPERTY OF THE OCTUPOLE MAGNETS

The beam redistribution philosophy of octupole is different from that of HQ and SNM. Although, we can divide the octupole field into two parts (without considering the intermediate zone) just like the HQ, the length of the zero zone and focus zone is different as shown in Figure 14. We

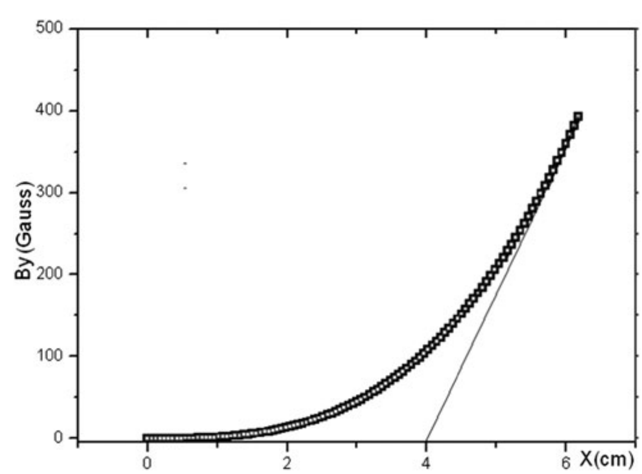

Fig. 14. The field distribution of octupole.

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suppose that the beam profile is 6 cm in the octupole, and then the cross point of the axis and the tangent line of the octupole field curve is lying at about 4 cm. The cross point position is about two-third of the beam profile. Moreover, for any point on the octupole field curve, the cross point should be lying at $2x_0/3$, which can be proved:

$$\left.\begin{array}{r} y_0 = Sx_0^3 \\ k = 3Sx_0^2 \\ y = kx + b \end{array}\right\} \Rightarrow b = -2Sx_0^3, \tag{3}$$

where S is any constant and (x_0,y_0) is the point on the octupole field curve. The cross point of the axis and the tangent line $(y = kx + b)$ is $(x,y) = (2x_0/3,\ 0)$, which means that the length ratio of the zero zone and focus zone is 2.

According to Figure 4, if the field is proper, only about one-third part of the beam is folded. Moreover, because the point (x_0,y_0) is any point on the octupole field curve, for the fixed octupole magnet, and any given beam profile, only about one-third of the beam can be folded for obtaining the good beam distribution. We name this property the nonlinear fold. In fact, the non-linear fold should be a kind of non-equidensity compression and the linear fold by HQ should be a kind of equidensity compression. This is the main difference between the HQ and octupole.

Due to the octupole property, the beam redistribution density can be obtained as shown in Figures 15 and 16. The beam density plumps near the center in Figure15 because of the shorter drift after the octupole or the weaker octupole field than the proper that. The beam density appears two ears at the edge in Figure 16 because of the longer drift or stronger field than that of Figure 15. Whatever we do, we can not obtain the beam density without the plumping and the ears at the same time.

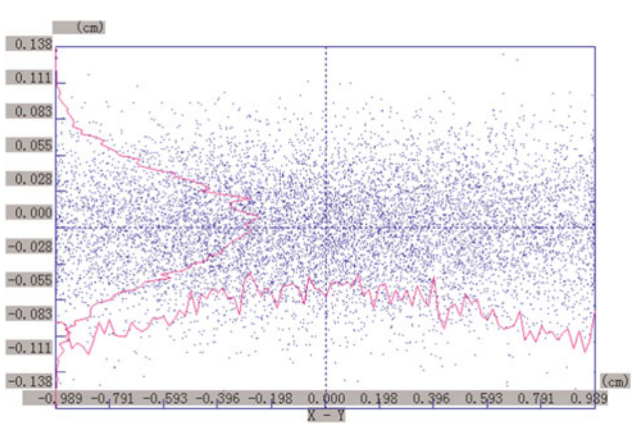

Fig. 15. (Color online) Density plumping in the center.

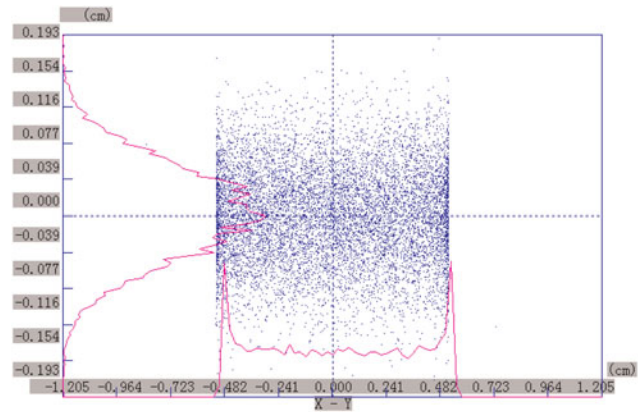

Fig. 16. (Color online) Growing ears at the edge.

6. THE IDEAL NONLINEAR FOLD FIELD FOR HOMOGENIZING

Because the field of the linear fold element is impossible to meet the ideal field as the theory required, we have to take the nonlinear fold into account for beam redistribution. So the principle of the beam redistribution introduced in Section 2 should be changed. According to Section 5, the redistribution result of octupole magnet is not better than that of the linear fold element, so we need to design a new type of element. But we should understand the beam redistribution principle of the nonlinear fold first.

According to Figure 14 and Eq. (3), the reason of bad redistribution by the octupole is that the field curve is too steep. If we suppose the field curve is $y = Sx^2$, the cross point in Figure 14 should be at $(x_0/2,0)$, and this kind of field should be the ideal redistribution field for the triangle beam density. This can be proved as follow. We can describe the function of the triangle density in the Figure 2:

$$\rho_0(x) = \begin{cases} kx + b & (0 \le x \le a) \\ b - kx & (-a \le x < 0) \end{cases}, \tag{4}$$

where, $k < 0$, $b > 0$. For the beam redistribution, the particle distribution in the phase space should be along a straight line, and then the phase space coordinate of any particle can be represent as $(x, x') = (x_0, \alpha x_0)$ when it begins to enter the redistribution element. We suppose that the field experienced by the particle is $B(x_0)$ and it can be looked as unchanged. We also suppose that: $r >> l$. Where r is the rotation radio caused by the field $B(x_0)$ and l is the length of the redistribution element. The deflection angle is about $\theta = l/r$. So the phase space coordinate after the redistribution element can be represented as $(x_0, \alpha x_0 + l/r)$. And then, after a linear system, the transformation between x and x_0 is (Jason *et al.*, 1997):

$$\begin{aligned} x = x_0R_{11} + (\alpha x_0 + l/r)R_{12} = (R_{11} + \alpha R_{12})x_0 \\ + lR_{12}B(x_0)/B\rho, \end{aligned} \tag{5}$$

where, $B\rho$ is the beam rigidity. And then, one obtains:

$$dx/dx_0 = (R_{11} + \alpha R_{12}) + lR_{12}B'(x_0)/B\rho. \quad (6)$$

It is well known that:

$$\rho(x)dx = \rho(x_0)dx_0. \quad (7)$$

According to Eqs. (6) and (7), one can obtain (Jason *et al.*, 1997):

$$B'(x_0) = [\rho_0(x_0) - (R_{11} + \alpha R_{12})\rho(x)]B\rho/[l\rho(x)R_{12}]. \quad (8)$$

For the uniform beam obtained at the target, $\rho(x)$, $B\rho$, R_{11}, R_{12}, α, and l are constant. If the initial beam density is triangle as shown in Eq. (4), the redistribution field according to Eq. (8) should be:

$$B(x_0) = S|x_0|^3/x_0. \quad (9)$$

There must be a relationship between the constants:

$$b = (R_{11} + \alpha R_{12})\rho(x). \quad (10)$$

The reason of the appearance of the absolute value in Eq. (9) is that the redistribution field should be antisymmetric respect to the beam center.

7. THE DESIGN AND SIMULATION OF THE FOCUS SEXTUPOLE MAGNETS

Because the Gauss curve approaches isosceles triangle curve, it is somewhat reasonable to use the antisymmetric quadratic field of Eq. (9) for homogenizing the beam density. The phase space and the beam density can be obtained by the ideal antisymmetric quadratic field as shown from Figures 17 to 19. The result is better than that of HQ, SNM, and octupole as shown in Figures 8, 13, 15, 16, and 19, because the ears of the beam density is almost cured.

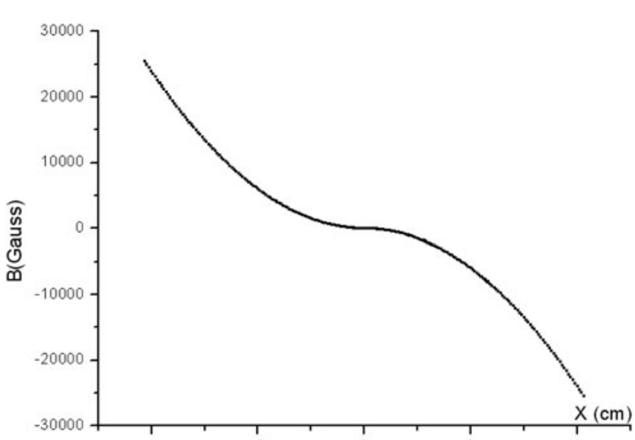

Fig. 17. The ideal quadratic field.

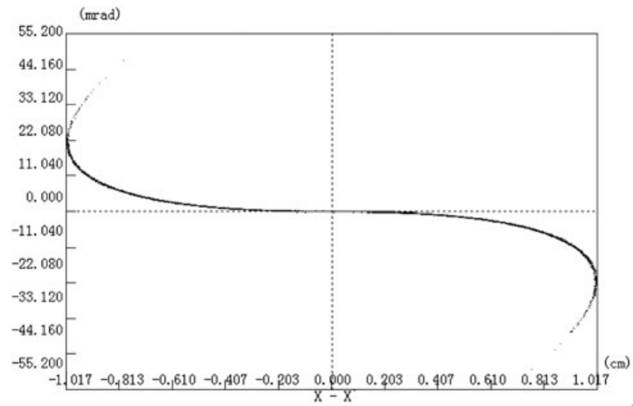

Fig. 18. After the ideal quadratic field and drift space.

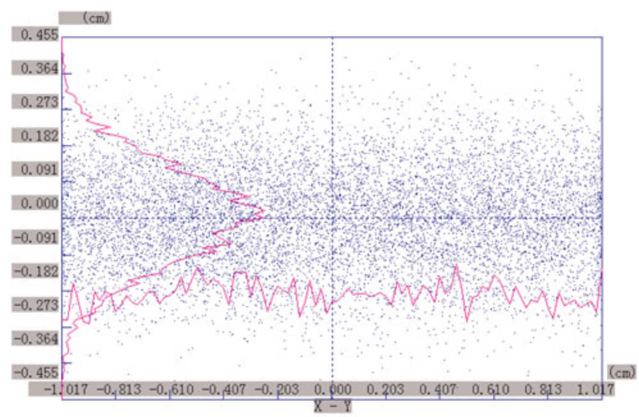

Fig. 19. (Color online) The beam distribution in the transverse plane.

As we know, the field of the sextupole is quadratic, but it is an even function, and the redistribution needs an odd function as shown in Figure 17. So we change the current direction of one pair of coils and delete the pair of coils of the up and down pole as shown in Figure 20, and then the obtained field compare with the ideal field as shown in Figure 21. We name this kind of element the focus sextupole (FS). The obtained beam redistribution in Figure 22 is worse than that in Figure 19, which is because the field of FS near the beam center is greater than that of the ideal field. So we decrease the distance of the up and down pole and the obtained field approaches the ideal field very much as shown in Figures 23 and 24, and then the beam redistribution as shown in Figure 19 can be obtained.

Obviously, the structure of the FS is simpler than that of SNM, octupole, and duodecapole. Because of the simpler structure and the quadratic field, it is easier to obtain the higher field near the beam center than octupole and duodecapole. Moreover, the beam redistribution performance of FS is not affected by the beam profile, which is the same as that of octupole and duodecapole introduced in Section 5. That is, for the fixed FS magnet and any given beam profile, the beam can be redistributed as the similar density curve when its field is proper. But, the beam redistribution

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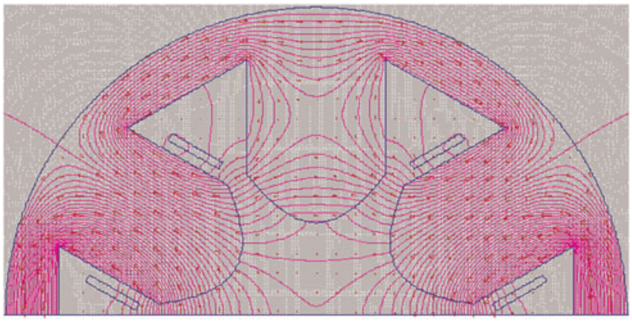

Fig. 20. (Color online) The field distribution of the focus sextupole.

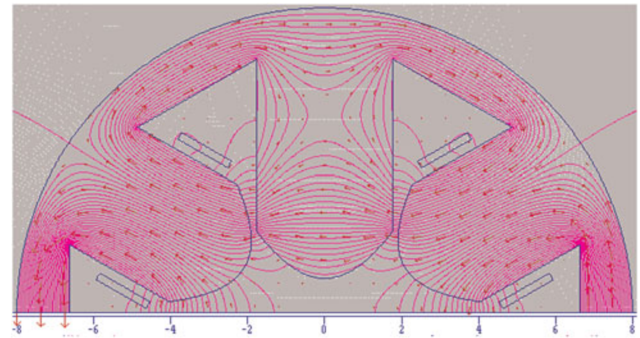

Fig. 23. (Color online) The field distribution of the modified FS.

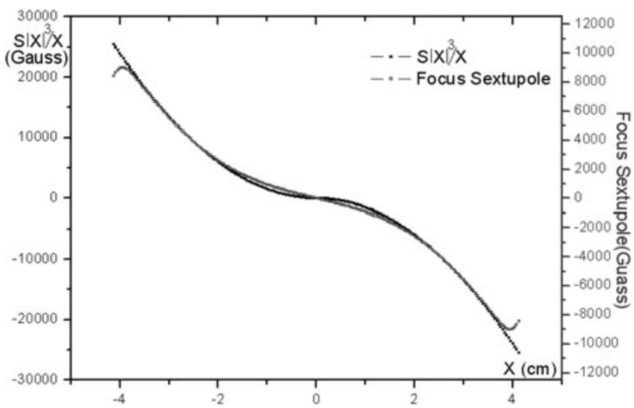

Fig. 21. The one dimensional field of the FS.

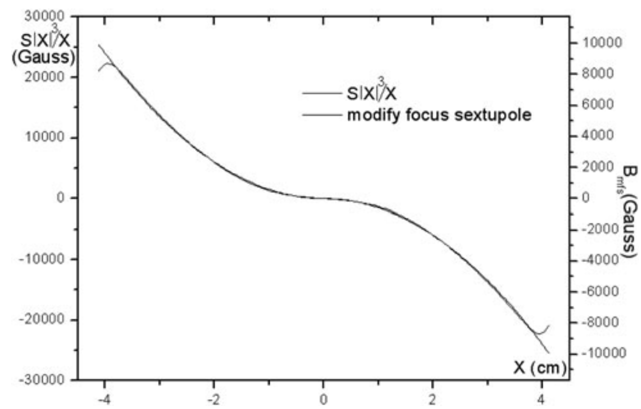

Fig. 24. The one dimensional field of the modified FS.

performance of the HQ and SNM is different and difficult to be cured when the beam profile is changed, because their fold point is fixed.

An example of two phase plane homogenizing is also shown in Figures 25 and 26. In Figure 25, Q means quadruple and *S* means FS. The beam transport system is similar to that of Kashy *et al.* (1987), but there is no beam collimator. The result means that the more uniform beam can be obtained by the FS.

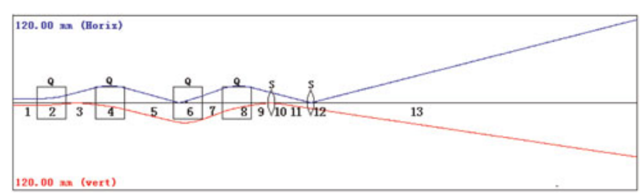

Fig. 25. (Color online) The envelope of the beam homogenizing.

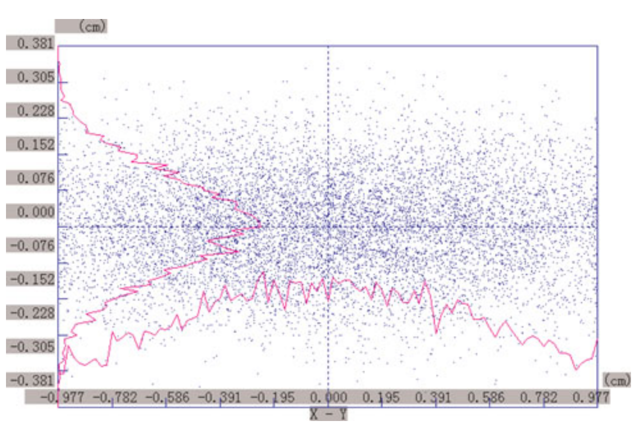

Fig. 22. (Color online) The beam distribution after the FS.

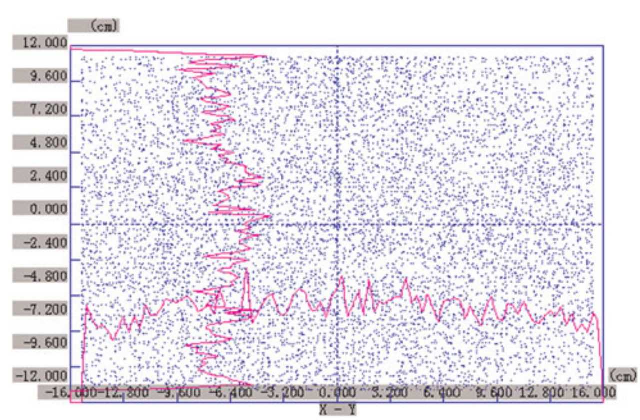

Fig. 26. (Color online) The beam distribution in the two phase plane.

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8. CONCLUSION

According to the calculation result, the modified focus sextupole is the simplest and most effective element for the beam homogenizing than the other type of elements. It can cure the ears and easily obtain high field near the beam center. The better homogenizing result can be obtained without collimator. Also, the performance of beam redistribution is not affected by the beam profile. For convenience, we can also name the modified focus sextupole the focus sextupole. Although the result of Figure 19 is not the perfect one, it can meet the most of applications.

In some fields of high energy and high current beam application, the beam halo is very big and it is difficult to clean. One of the solutions is to use the HQ and FS, because the HQ has a very good zero zone, which can not change the state of the particle in the zero zone, and the focus zone can fold the halo into the beam center. If the focus strength is not enough, the pole of quadrupole can be replaced by that of sextupole, octupole, and so on, which depends on the actual design. However, it must be remembered that the FS is only effective for the Gauss-like distribution and quasi-triangle distribution.

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