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MONETARY POLICY, FACTOR SUBSTITUTION, AND CONVERGENCE

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In this paper, we examine the influence of monetary policy on the speed of convergence in a standard monetary growth model à la Sidrauski allowing for differences in the elasticity of substitution between factors of production. The respective changes in the rate of convergence and its sensitivities to the central model parameters are derived both analytically and numerically. By normalizing the constant elasticity of substitution (CES) production functions both outside the steady state and within the steady state, it is possible to distinguish between an efficiency and a distribution effect of a change in the elasticity of substitution. We show that monetary policy is the more effective, the lower is the elasticity of substitution, and that the impact of monetary policy on the speed of convergence is mainly channeled via the efficiency effect.

Keywords: Monetary Growth Models, Elasticity of Substitution, Normalized CES Production Functions, Speed of Convergence

1. INTRODUCTION

Sidrauski's (1967) model in which real cash balances enter the utility function of a Ramsey growth model is the traditional workhorse of monetary growth theory. In its steady state, the Sidrauski model generates superneutrality of money, meaning that physical capital, real production, and consumption are independent of monetary expansion and the inflation rate. Outside the steady-state monetary policy generally has real effects. Fischer (1979) and Cohen (1985) were able to show analytically how inflation can influence the speed of convergence toward the steady state by changing the time path of nominal interest rates. As long as money and consumption are not separable within the utility function monetary expansion will speed up convergence and thus decrease the time it takes for output to double. The magnitude of the real effects of monetary policy is debated, however, and merits further analyses.[1]

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Gokan (2003) provided a very careful analytical and numerical analysis of the speed of convergence in the Sidrauski model for different monetary and fiscal regimes. He could not find a strong influence of either public spending or money growth on the real economy, making the speed of convergence in the calibrated monetary growth model deviate only slightly from the values in the non-monetary Ramsey model.[2] Gokan used a simple Cobb–Douglas production function in order to model the aggregate production technology. As we will demonstrate below, this common simplification is an unnecessary restriction that prevents interesting and important effects of monetary policy on the speed of convergence and thus on real economic growth. These effects become visible when a more general constant elasticity of substitution (CES) production technology is used and the aggregate elasticity of substitution is regarded as a central parameter of the intertemporal growth model.

The impact of the aggregate elasticity of substitution on real (steady state) growth was studied theoretically by De LaGrandville (1989) and Klump and de LaGrandville (2000) using the newly developed concept of a normalized CES production function. Antras (2004) as well as Klump et al. (2007) estimated the aggregate elasticity of substitution for the United States to be significantly below one at values of about 0.7. Klump and Preissler (2000) and Turnovsky (2002) demonstrated that the elasticity of substitution is also one of the central determinants of the speed of convergence toward the steady state. Klump et al. (2012) survey the theoretical and empirical debate on normalized CES production functions.

Therefore, in this paper we investigate the monetary effects on the speed of convergence in a Sidrauski model based on a normalized CES production technology, employ different values for the aggregate elasticity of substitution that are in line with empirical results and study the interaction with other basic parameters of the model. As a general result, we find that monetary policy is the more effective in speeding up convergence, the lower is the aggregate elasticity of substitution and the higher is the aggregate preference for money. We also extend earlier work by Klump and Saam (2008) and look for a potential bias that the choice of the CES normalization parameters might have on our general result. By normalizing the CES production function both outside the steady state and within the steady state, it is possible to distinguish between an efficiency and a distribution effect of a change in the elasticity of substitution. We demonstrate that the effect of monetary policy on the speed of convergence is independent from the choice of the point of normalization and is mainly channeled via the efficiency effect.

The remainder of the paper is organized as follows: Section 2 presents the Sidrauski model with a normalized CES production technology, derives the speed of convergence, and assesses the impact of a change in the aggregate elasticity of substitution analytically. Section 3 calculates the speed of convergence for varying elasticities of substitution numerically and undertakes sensitivity analyses concerning changes of central parameters and the choice of the normalization point. The fourth section concludes.

2. THE MODEL

We study a standard monetary growth model in the spirit of Sidrauski (1967) where real money balances per capita m enter the utility function of the representative household as an additional argument. The instantaneous utility function of the household is given by a general CIES function:

$$u(c_t, m_t) = \frac{\left(c^{1-\theta} m^{\theta}\right)^{1-\eta} - 1}{1-\eta}, \quad \textbf{(1)}$$

where θ denotes the weight at which money enters the utility index, ρ is the rate of time preference, and η represents the elasticity of the marginal utility. The parameter θ signals the degree of monetization of the economy. The higher the θ, the more money is needed for transacting a given volume of consumption goods, be it for objective reasons such as a higher division of labor in the economy or for subjective reasons such as a higher preference for liquidity in the composition of valuable assets. The inverse of the parameter η measures the intertemporal elasticity of substitution of the composite good *(c,m)*. A lower intertemporal elasticity of substitution indicates a higher level of risk aversion as it translates into a smaller willingness of households to deviate from consumption smoothing. Labor is supplied inelastically. Money is issued by a monetary authority and is used to pay lump-sum transfers z to each household. The growth rate of the nominal money supply ϕ is controlled by the government and thus purely exogenous. The government's budget constraint is given by $\phi m = z$.

Technology is represented by a CES production function y, which is normalized by choosing a common baseline point for the capital intensity k_0, per capita output y_0, and the marginal rate of substitution μ_0:

$$y = f(k) = C(\sigma)\left\{\alpha(\sigma) k^{\psi} - [1 - \alpha(\sigma)]\right\}^{\frac{1}{\psi}} \text{ with } \alpha(\sigma) = \frac{k_0^{1-\psi}}{k_0^{1-\psi} + \mu_0},$$

$$C(\sigma) = y_0 \left(\frac{k_0^{1-\psi} + \mu_0}{k_0 + \mu_0}\right)^{\frac{1}{\psi}}, \ \mu_0 = \frac{F_L}{F_k} = \frac{1-\alpha}{\alpha} k_0^{1-\psi} \text{ and } \psi = \frac{\sigma - 1}{\sigma}, \quad \textbf{(2)}$$

where σ denotes the aggregate elasticity of substitution, C the neutral technology parameter, and α the distribution parameter. As outlined Klump and La Grandville (2000), the normalization procedure ensures that the CES production functions differ only in their aggregate elasticity of substitution between production factors thus allowing for intra-family comparisons of a change in the elasticity of substitution. By making use of the baseline profit share $\pi_0 = \frac{k_0}{k_0 + \mu_0}$ we can reformulate the normalized CES production function in the following way:

$$y = y_0 \left[\pi_0 \left(\frac{k}{k_0}\right)^{\psi} + (1 - \pi_0)\right]^{\frac{1}{\psi}}. \quad \textbf{(3)}$$

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A higher elasticity of substitution not only increases output as long as $k \neq k_0$:

$$\frac{dy}{d\sigma} = -\frac{1}{\sigma^2}\frac{1}{\psi^2} y \left[\pi \ln\left(\frac{\pi_0}{\pi}\right) + (1-\pi)\ln\left(\frac{1-\pi_0}{1-\pi}\right)\right] > 0 \text{ with } \pi = \frac{f'(k)\,k}{y}. \quad \textbf{(4)}$$

It also increases the profit share and capital productivity as long as $k > k_0$:

$$\frac{\partial \pi}{\partial \sigma} = \frac{1}{\sigma^2}(1-\pi)\,\pi \ln\left(\frac{k}{k_0}\right) > 0 \quad \text{if} \quad k > k_0, \quad \textbf{(5)}$$

$$\frac{\partial f'}{\partial \sigma} = \frac{y}{k}\frac{\partial \pi}{\partial \sigma} + \frac{\pi}{k}\frac{\partial y}{\partial \sigma} > 0 \quad \text{if} \quad k > k_0. \quad \textbf{(6)}$$

Since k^* is uniquely determined by equation (10), we can deduce from equations (14) and (15) that the per capita capital stock in the steady state increases as the elasticity of substitution rises. Assuming is feasible when considering economies where capital is the driving force of production.

The household maximizes expected lifetime utility according to equation (1) subject to the following equation of motion given the normalized CES production function in equation (3):

$$\dot{v} = f(k) - (\delta + n)\,k + z - (\Pi + n)\,m - c, \quad \textbf{(7)}$$

with Π denoting inflation, δ being the rate of depreciation, and n the rate of population growth. The corresponding Hamiltonian to this optimization problem is

$$H = u(c, m) + \Omega\,[f(k) - (\delta + n)k + z - (\Pi + n)\,m - c]\,, \quad \textbf{(8)}$$

Applying Pontryagin's maximum principle one receives a system of three differential equations

$$\frac{\dot{c}}{c} = \frac{[f'(k) - (n + \delta + \rho)]}{[\eta(1-\theta) + \theta]} + \frac{\theta(1-\eta)}{[\eta(1-\theta) + \theta]}\left[\phi + f'(k) - n - \delta - \frac{c}{m}\frac{\theta}{1-\theta}\right], \quad \textbf{(9)}$$

$$\frac{\dot{k}}{k} = \frac{f(k)}{k} - (\delta + n) - \frac{c}{k}, \quad \textbf{(10)}$$

$$\frac{\dot{m}}{m} = \phi - n - \Pi = \phi + f'(k) - n - \delta - \frac{c}{m}\frac{\theta}{1-\theta}. \quad \textbf{(11)}$$

It is already apparent from the modified Keynes-Ramsey rule (9) that the growth rate of the nominal money supply ϕ influences the transition toward the steady state as long as $\eta \neq 1$. If, however, $\eta = 1$ the CIES utility function given in equation (1) becomes logarithmic and money and consumption are separable within the utility

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function, thereby reducing equation (9) to the standard Keynes-Ramsey rule of the non-monetary growth model. By evaluating the differential equations (7) to (9) at the steady state $\dot{c} = \dot{k} = \dot{m} = 0$ one obtains

$$f'\left(k^*\right) = \delta + \rho + n, \tag{12}$$

$$c^* = f(k^*) - (\delta + n)k^*, \tag{13}$$

$$\phi - n = \Pi^*, \tag{14}$$

$$\frac{c^*}{m^*} = \frac{1-\theta}{\theta}(\rho + \phi). \tag{15}$$

As can be verified from equations (12)-(15), the steady-state values of the real variables of the model, c^* and k^*, are not affected by a change in the monetary growth rate. This is the well-known superneutrality result. Only the real per capita money balances m^* are influenced by a change in the nominal monetary growth rate. According to equation (15), higher monetary expansion, that is an increase in ϕ, will induce households to diminish their steady-state real money holdings.

We now turn to the derivation of the speed of convergence λ following Fischer (1979). Log-linearizing in the neighborhood of the steady state and concentrating on the one negative eigenvalue one can show that

$$\frac{d\lambda}{d\phi} > 0 \quad \text{if} \quad \eta \neq 1. \tag{16}$$

Hence, higher monetary expansion makes the system approach its steady state faster as long as the utility function (1) is non-logarithmic. Once the arguments within the utility function are additively separable, as would be the case for $\eta = 1$, the marginal utilities of money and consumption no longer depend upon another. Adjusting money balances in reaction to changes in monetary policy can then not feed back into changes in consumption and investment decisions. This can be verified from equation (7), where in the case of a logarithmic utility function the last term on the right-hand side cancels out. The feedback mechanism via the marginal utilities is, however, as shown by Cohen (1985) the driving force for the transitional effects of monetary policy.

Furthermore, it has been investigated by Klump (2001) how a change in the elasticity of substitution affects the transitional effect of monetary policy:

$$\frac{d^2\lambda}{d\phi d\sigma} < 0 \quad \text{if} \quad \eta \neq 1 \quad \text{and} \quad k^* > k_0. \tag{17}$$

An increase in the elasticity of substitution will unambiguously diminish the positive effect of the nominal money growth rate on the speed of convergence. In another perspective, this result implies that the smaller is the aggregate elasticity of substitution the larger is the impact of monetary expansion on the transition path. If the aggregate elasticity of substitution σ is regarded as in Saam (2008) as a measure of openness of the economy this result would imply that in open economies the real effects of monetary policy in speeding up convergence are smaller than in closed economies. However, from the theoretical analysis alone it is not clear whether those effects are negligible or whether they can become a real challenge for the conduct of economic and monetary policy. A calibration of the model and a sensitivity analysis for the critical parameters are therefore the logical next steps.

3. CALIBRATION AND SENSITIVITY ANALYSES

3.1. Calibration of the Economy

Even though monetary policy does not influence the steady-state values of c^* and k^*, it is certainly effective outside the steady state. To which extent remains to be answered by numerically calculating the speed of convergence for varying parameters in both the production function and the utility function. The following numerical simulations use fairly standard values for the parameters as can be found in Turnovsky (2002) and Gokan (2003):

Benchmark values	Other parameters
Baseline points for normalization:	
$k_0 = y_0 = 1$; $\pi_0 = 1/3$	rate of time preference: $\rho = 0.04$
aggregate elasticity of substitution:	
$\sigma = 0.8$	rate of depreciation: $\delta = 0.04$
key monetary policy parameter:	
$\phi = 0.03$	rate of population growth: $n = 0.01$.
weight of money in utility: $\theta = 0.3$	
elasticity of marginal utility: $\eta = 2.5$	

Normalizing the CES production function in $k_0 = y_0 = 1$ yields the standard ACMS [Arrow et al. (1961)] form. The baseline value for the profit share is chosen at $\pi_0 = 1/3$ so that the CES production function converges to a Cobb–Douglas function with a production elasticity for capital of $1/3$ if the aggregate elasticity of substitution approaches one. We find this parameterization of the benchmark value useful since the Cobb–Douglas function with a share of capital of 1/3 is a standard assumption in numerical calculations such as Gokan (2003) if changes in substitution possibilities are not considered explicitly. The benchmark for the elasticity of substitution is selected on the grounds that empirical evidence finds most economies to exhibit an elasticity below 1, so we have taken a slightly higher value of the range of empirically sensible values for σ (which are usually in the rage of $0.6 - 0.7$ for developed countries). In addition, the benchmark values generate a capital-output coefficient in the steady state of 2.85 and a consumption-output ratio of 0.86, which are both highly plausible. As already mentioned, the distance of the point of normalization to the steady state plays a crucial role in determining the speed of convergence. The calibration of the benchmark ensures $k_0 < k^*$. This implies that (a) we are comparing economies that converge to different steady states depending on their elasticity of substitution and (b) on approaching the steady-state labor is becoming the relatively scarce factor of production.

The choice of the growth rate of the nominal money supply ϕ is derived from the steady-state condition (14) with a population growth rate of 1% and a medium to long-run target for the inflation rate of 2%. The weight of money in utility is not unreasonable high but might still be overstated. However, $\theta = 0.3$ is used by Gokan (2003), among others. The elasticity of the marginal utility is empirically within the upper range of empirical estimates. Assuming $\eta > 1$ is plausible since η reflects the relative risk aversion of the households. The smaller is the risk aversion coefficient, the more willing are households to substitute consumption today for consumption tomorrow.

3.2. Changes in the Aggregate Elasticity of Substitution

Tables 1 and 2 summarize the effects of a change in the nominal money supply on the transition path for varying elasticities of substitution given the benchmark point of normalization. Since small changes in the speed of convergence λ translate into large changes in the growth rates of capital and output, the tables also state the half time calculated from $\ln(0.5)/\lambda$.

Let us first concentrate on the benchmark simulation. As can be seen from Table 1, the speed of convergence will increase if the growth rate of the nominal money supply increases. The acceleration is stronger, the more weight is attributed to money in the utility function. The reason for this is that a higher money growth rate increases the opportunity costs of holding money, so that households will reduce their money balances, thereby increasing the marginal utility of per capita money balances. This increase is higher the more money contributes to overall utility. As a result, the nominal interest rate increases (it actually overshoots) and households reallocate their assets toward physical capital.

Reduced money holdings will affect the utility derived from consumption positively since the cross-partial derivative of money and consumption is clearly negative as long as the elasticity of the marginal utility is greater than 1.[3] This implies that consumption and money balances are substitutes. The more important money is as an argument within the utility function (the higher θ) the smaller will be according to equation (11) the growth rate of per capita money balances on the transition path and, therefore, the higher will be the growth rate of consumption as given in equation (9). Consequently, the economy approaches the steady state faster. We can draw the conclusion that monetary policy is more effective in terms of accelerating the speed of convergence the higher is the degree of monetization as expressed by high values of θ.

If we now compare different weights of money at the same monetary growth rate, we notice that the speed of convergence is lower when money is more

TABLE 1. Speed of convergence with baseline points $k_0 = y_0 = 1$; $\pi_0 = 1/3$

	$\theta = 0.8$			$\theta = 0.3$	
$k_0 = y_0 = 1 \quad \pi_0 = 1/3$	λ	Half time		λ	Half time
$\sigma = 1.2$			$\pi^* = 0.43$ $c^*/y^* = 0.76$		
$\phi = 0.01$	2.75	25.19		3.27	21.21
$\phi = 0.02$	2.85	24.36		3.29	21.01
$\phi = 0.03$	2.92	23.73		3.32	20.84
$\phi = 0.05$	3.03	22.88		3.36	20.61
$\sigma = 1$			$\pi^* = 1/3$ $c^*/y^* = 0.81$		
$\phi = 0.01$	3.69	17.52		4.99	13.87
$\phi = 0.02$	4.13	16.79		5.05	13.73
$\phi = 0.03$	4.26	16.26		5.09	13.61
$\phi = 0.05$	4.47	15.52		5.16	13.44
$\sigma = 0.8$			$\pi^* = 0.26$ $c^*/y^* = 0.86$		
$\phi = 0.01$	5.39	12.86		7.30	9.49
$\phi = 0.02$	5.67	12.24		7.38	9.39
$\phi = 0.03$	5.89	11.77		7.45	9.31
$\phi = 0.05$	6.23	11.12		7.55	9.18
$\sigma = 0.6$			$\pi^* = 0.19$ $c^*/y^* = 0.89$		
$\phi = 0.01$	7.19	9.68		10.59	6.54
$\phi = 0.02$	7.59	9.13		10.69	6.48
$\phi = 0.03$	7.95	8.72		10.79	6.43
$\phi = 0.05$	8.50	8.15		10.93	6.34
$\sigma = 0.2$			$\pi^* = 0.12$ $c^*/y^* = 0.94$		
$\phi = 0.01$	13.21	5.25		26.99	2.57
$\phi = 0.02$	14.39	4.81		27.17	2.55
$\phi = 0.03$	15.40	4.5		27.34	2.53
$\phi = 0.05$	17.03	4.07		27.64	2.51

important in the utility function. Money holdings along the transition path will be larger, thus reducing funds available for capital accumulation causing the economy to converge more slowly. This can be verified from equation (9) where a higher weight of money decreases the growth rate of per capita cash balances for a given monetary growth rate. The diminished growth rate for cash balances then implies that according to equation (7) per capita consumption grows more slowly along the transition path as well.

It is also apparent that the speed of convergence decreases if the elasticity of substitution increases and vice versa. If we compare the different speeds of convergence from our simulation results with the Mankiw et al. (1992) benchmark

TABLE 2. Speed of convergence with baseline points $k_0 = y_0 = 1$; $\pi_0 = 2/3$

	$\theta = 0.8$			$\theta = 0.3$	
$k_0 = y_0 = 1 \quad \pi_0 = 2/3$	λ	Half time		λ	Half time
$\sigma = 1$			$\pi^* = 0.23$ $c^*/y^* = 0.63$		
$\phi = 0.01$	1.50	46.04		1.67	41.48
$\phi = 0.02$	1.54	44.99		1.68	41.16
$\phi = 0.03$	1.57	44.22		1.69	40.92
$\phi = 0.05$	1.6	43.17		1.71	40.59
$\sigma = 0.8$			$\pi^* = 0.45$ $c^*/y^* = 0.75$		
$\phi = 0.01$	3.32	20.87		4.06	17.06
$\phi = 0.02$	3.45	20.09		4.10	16.89
$\phi = 0.03$	3.55	19.52		4.14	16.75
$\phi = 0.05$	3.70	18.72		4.19	16.55
$\sigma = 0.6$			$\pi^* = 0.29$ $c^*/y^* = 0.83$		
$\phi = 0.01$	5.50	12.60		7.50	9.24
$\phi = 0.02$	5.79	11.98		7.58	9.15
$\phi = 0.03$	6.02	11.51		7.64	9.07
$\phi = 0.05$	6.37	10.87		7.75	8.96
$\sigma = 0.2$			$\pi^* = 0.13$ $c^*/y^* = 0.92$		
$\phi = 0.01$	12.57	5.52		24.74	2.80
$\phi = 0.02$	13.66	5.08		24.91	2.78
$\phi = 0.03$	14.57	4.76		25.07	2.76
$\phi = 0.05$	16.06	4.32		25.35	2.73

of 2% we must acknowledge that even for $\sigma = 1.2$, the rates of convergence are 30% larger. The reference economy within the Sidrauski model converges much faster, even if we let the monetary growth rate approach zero. So in a next step, we deviate from the benchmark simulation by increasing the baseline profit share to $\pi_0 = 2/3$.[4] We can justify this by assuming that our notion of capital hitherto had been too narrow, and that capital encompasses human capital as well physical capital. Table 2 provides an extensive overview of the rates of convergence across different elasticities of substitution. Even though the half time increases tremendously with the assumption of a higher baseline profit share, we are not able to replicate the 2% benchmark result with values for the elasticity of substitution below one.

We can clearly see from Tables 1 and 2 that the elasticity of substitution affects the magnitude of the effect of monetary policy on the speed of convergence—a result we established analytically in the preceding section. For low elasticities of substitution and a low degree of monetization, capital and output double after just a few years. By checking the percentage change in the speed of convergence for a given weight of money within the utility function and by just varying the elasticity of substitution it becomes apparent that the lower the elasticity of substitution the greater the impact of monetary expansion. For $\theta = 0.8$ and $\pi_0 = 2/3$ we notice that the half time decreases roughly by 0.78 years if the monetary growth rate doubles from 1% to 2%, whereas for $\theta = 0.2$ the half time decreases by less than half a year. The percentage change, however, is more pronounced for economies exhibiting a lower aggregate elasticity of substitution (8.66% versus 3.88%). The reason for this lies in the limited substitution possibilities. At low rates for the elasticity of substitution (e.g., $\sigma = 0.2$) investment in capital becomes more attractive but the scope for capital accumulation is limited by the reduced adjustment potential in labor and capital. However, we can conclude that even for economies with a low elasticity of substitution and more reasonable values for θ (that is values lower than 0.8) the influence of monetary policy on the speed of convergence becomes negligible.

3.3. Influence of the Normalization Point

The decline in the speed of convergence for an increasing elasticity of substitution must be scrutinized further as we are comparing economies that converge to different steady states depending on their respective elasticity of substitution. A higher elasticity of substitution results in higher steady-state per capita capital stock and output as can be verified from equation (4). This means that the distance of the steady-state values k^* and y^* to the baseline points k_0 and y_0 increases. The results we have obtained so far from the numerical simulation could be influenced by the relative distance of the steady state to the point of normalization. Table 3 shows that the speed of convergence is indeed perceptible for a change in the baseline points.

The higher the baseline point is in relation to the steady state the higher will be the speed of convergence. The reason for this is that an increase in the elasticity of substitution affects the rate of convergence through two channels [Xue and Yip (2012)]: an efficiency effect and a distribution effect. The efficiency effect directly raises the marginal product of capital and therefore alters the structure of the interest rates along the transition path by reducing the concavity of the production function. Recall that the savings decision in the Sidrauski model as in the standard non-monetary Ramsey model depends upon the interest rate. If markets are complete and factors are paid their marginal product $r = f'(k)$, households are willing to postpone consumption as long as $f'(k) > (n - \delta - \rho)$ holds. Since a higher elasticity of substitution increases the efficiency of capital it takes longer before the marginal product falls so that it equals the sum of population growth, depreciation, and time preference rate. This will decrease the rate of convergence toward the steady state. The distribution effect emerges from a change in the profit share of capital. According to equation (5) the profit rate increases with the elasticity of substitution as long as $k^* > k_0$. If we consider steady states to the right of the point of normalization, then we implicitly assume

TABLE 3. Speed of convergence with different baseline points

	$\theta = 0.3$	
$k_0 = 2$ $y_0 = 1$ $\pi_0 = 1/2$	λ	Half time
$\sigma = 0.8$	$k^* = 8.92$ $\pi^* = 0.41$	
$\phi = 0.01$	4.57	15.18
$\phi = 0.02$	4.61	15.03
$\phi = 0.03$	4.65	14.90
$\phi = 0.05$	4.70	14.71
$k_0 = 5$ $y_0 = 1$ $\pi_0 = 5/7$		
$\sigma = 0.8$	$k^* = 16.07$ $\pi^* = 0.65$	
$\phi = 0.01$	2.08	33.26
$\phi = 0.02$	2.10	32.98
$\phi = 0.03$	2.12	32.77
$\phi = 0.05$	2.14	32.26
$k_0 = 1$ $y_0 = 0.5$ $\pi_0 = 1/3$		
$\sigma = 0.8$	$k^* = 2.05$ $\pi^* = 0.29$	
$\phi = 0.01$	6.45	10.75
$\phi = 0.02$	6.51	10.64
$\phi = 0.03$	6.57	10.55
$\phi = 0.05$	6.67	10.41

that labor is relatively scarce. An increase in the elasticity of substitution allows households to substitute labor by capital thereby alleviating the relative scarcity of labor. More capital is employed in production, which decreases the speed of convergence.

The distribution effect can be eliminated by normalizing in the steady state itself so that $k^* = k_0$ holds. Intuitively, one is considering economies with different elasticities of substitution converging to the same steady state. Equation (5) then turns zero, and only the efficiency effect is at work.

Table 4 documents how the speed of convergence develops when we normalize with the steady-state value that the system would have for a value of $\sigma = 0.8$. When we now increase or lower the elasticity of substitution, we can study the induced efficiency effect only and can also analyze its change according to different values of monetary expansion. We can note two important results: First, it is striking that (compared to Table 1 for $\theta = 0.3$) the change in the speed of convergence resulting from a higher elasticity of substitution becomes smaller. An increase

TABLE 4. Speed of convergence with baseline point in the steady state for $\sigma = 0.8$

$k_0 = k^*_{\sigma=0.8} = 4.409$	$\theta = 0.3$	
$y_0 = 1.547$ $\pi_0 = 0.26$	λ	Half time
$\sigma = 1.2$		
$\phi = 0.01$	5.77	12.02
$\phi = 0.02$	5.83	11.89
$\phi = 0.03$	5.88	11.79
$\phi = 0.05$	5.96	11.63
$\sigma = 0.8$		
$\phi = 0.01$	7.30	9.49
$\phi = 0.02$	7.38	9.39
$\phi = 0.03$	7.45	9.31
$\phi = 0.05$	7.55	9.18
$\sigma = 0.2$		
$\phi = 0.01$	15.64	4.43
$\phi = 0.02$	15.78	4.39
$\phi = 0.03$	15.89	4.36
$\phi = 0.05$	16.10	4.30

in the elasticity of substitution from 0.2 to 0.8 reduces λ by (for $\phi = 0.01$) by 8% points compared to about 20% points in Table 1. This means that a large proportion of the change in the speed of convergence caused by a change in σ must be attributed to the distribution effect. Second, if we analyze the percentage change in the speed of convergence for different monetary growth rates we can observe that the speed of convergence accelerates at roughly the same pace regardless of the point of normalization. By, e.g., comparing Tables 1 and 4 for $\sigma = 1.2$ a doubling of the monetary growth rate form 1% to 2% induces the half time to decrease by roughly 10% no matter if we have normalized outside or in the steady state. This implies, however, that that the elimination of the distribution effect does no significant effect and that the impact of monetary policy on the speed of convergence is largely channeled via the efficiency effect.

4. CONCLUSION

We studied the impact of monetary policy measured by an increase in the growth rate of nominal cash balances on the speed of convergence in the Sidrauski (1967) growth model, which exhibits superneutrality in the steady state. For the aggregate production technology, we employ the concept of a normalized CES production function as it was developed by Klump and de La Grandville (2000) and recently surveyed by Klump et al. (2012). As a result, we are able to derive much more pronounced and much more realistic effects of monetary expansion on real growth along the convergence path than earlier studies such as Gokan (2003) using a Cobb–Douglas technology.

Even if the 2% benchmark of Mankiw et al. (1992) cannot be replicated unless one assumes a very high elasticity of substitution between factors of production, our results for (realistic) values of the elasticity of substitution below one match the rates of convergence in more refined empirical studies. As a general result, we find that monetary policy is the more effective in speeding up convergence, the lower is the aggregate elasticity of substitution. If we follow Saam (2008) and regard openness of the economy as one determinant of a high degree of factor substitution, we predict the fastest real effects of monetary policy in rather closed economies. We can also demonstrate that a high preference for money dampens the convergence effect of monetary policy.

Finally, we show that the choice of the baseline point has a significant impact on the speed of convergence. The lower the baseline point is in relation to the steady state, the lower is the pace of adjustment. This result stems from the two different effects by which the elasticity of substitution influences the transition path, namely a distribution and an efficiency effect. For economies where labor is relatively scarce both the distribution effect and the efficiency effect have the same sign; they both decrease the speed of convergence if the elasticity of substitution increases. By normalizing within the steady state one can analytically and numerically separate the two channels. The distribution effect is the more effective of the two when it comes to a change in the elasticity of substitution, whereas the efficiency effect is the main channel through which monetary policy affects the speed of convergence.

NOTES

1. An alternative way of introducing money in a growth model is, of course, to treat real cash balances as an additional factor of production. However, as shown recently by Bencimol (2015) in a DSGE context, the quantitative effects are so small that it is much more promising to explore in more detail the interactions between money in the utility function and real growth.
2. It is common knowledge that the overall speed of convergence in Ramsey (and also Sidrauski) growth models is usually much too high compared to the 2% value calculated by Barro and Sala-i-Martin (1992) or Mankiw et al. (1992) in several cross-section studies implying a half time of 35 years—the time it takes for output to double. Later empirical growth studies, however, by addressing issues such as endogeneity [Caselli et al. (1996)], country-specific effects [Islam (1995)] and measurement errors [Temple (1998)] calculated convergence rates up to 10%. These values can match the ones obtained from intertemporal optimizing models.
3. $u_{cm} = (1-\eta)\theta(1-\theta)(c^{1-\theta}m^{\theta})^{1-\eta}/(mc) < 0 \;\; if \eta > 1$.
4. Recall that the baseline profit share is calculated according to $\pi_0 = \frac{k_0}{k_0+\mu_0}$.

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