COOLING OF WHITE DWARFS
WITH ACCOUNT OF NON-EQUILIBRIUM BETA-PROCESSES

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Non-equilibrium heating of white dwarfs during two-step neutronization and formation of small but finite core of the new phase is a main source of energy in these stars when the temperature is sufficiently small and Coulomb crystal becomes degenerate.

I. Introduction

Existence of Chandrasekhar mass limit of white dwarfs $M_{ch} = 5.83/\mu_z^2$, $\mu_z = A/z$, is connected with the prevalence of the relativistic degenerate electrons in pressure. When neutronization is taken into account, the maximal value of mass is smaller and is reached at finite density (Schatzman, 1958). The central density of the white dwarf with limiting mass is larger than threshold density of neutronization. Such star has a small, but finite core of new phase (Seidov, 1967). The neutronization threshold for the iron is equal to $\rho_{co} = 1.15\ 10^9$ g/cm³. The neutronization goes through two steps

$$Fe^{56} + e^- \rightarrow Mn^{56} + \nu \ , \quad Mn^{56} + e^- \rightarrow Cr^{56} + \nu \tag{1}$$

The density on the boundary of the core of new phase is equal to $\rho_{c_1} = 1.25\ 10^9$ g/cm³ and make the jump in 26/24 times. The threshold Fermi energy of the electrons for the first reaction from (1) is equal to $\mathcal{E}_{Fe}^{(1)} = 3{,}70$ MeV, and for the second one is much smaller $\mathcal{E}_{Fe}^{(2)} = 1.61$ MeV. The second reaction goes in non-equilibrium conditions and leads to heating (Bisnovatyi-Kogan and Seidov, 1970). The amount of produced heat is equal to 476 keV on one reaction (1) taking into account the formation of excited state of Mn^{56} at $\rho > 1.24\ 10^9$ g/cm³. The models of cold iron white dwarfs with finite core of Cr^{56} are calculated by Bisnovatyi-Kogan and Seidov (1970). The parameters of the limiting mass model are:

$$M_C/M_o \approx 1.4\ 10^{-3} \ , \ \Delta\rho_c/\rho_{c_1} \approx 0.022 \ , \ \Delta M/M_o \approx 2.1\ 10^{-4} \tag{2}$$

Here M_C is the mass of the chromium core in the star of limiting mass; $M_o = 1.18\ M_\odot$ is the mass of the cold iron star at $\rho_c = \rho_{co}$ or $\rho_c = \rho_{c_1}$;

$\rho_{c_1} + \Delta\rho_c$ and $M_o + \Delta M$ are the central density and the mass of the limiting configuration. The non-equilibrium heating during the formation of new phase core leads to essential prolongation of the late stages of cooling, when the heat capacity of degenerate crystal at low temperatures is small (Mestel and Ruderman, 1967). Rough estimations of this prolongation have been done by Bisnovatyi-Kogan and Seidov (1970). Quantitative results have been obtained by Bisnovatyi-Kogan (1987).

2. White dwarf with final chromium core at nonzero temperature

When the mass of the white dwarf is equal to $M = M_o + \Delta M$, then its temperature T_f at central density ρ_{co} is determined by relation (Bisnovatyi-Kogan, 1966; Bisnovatyi-Kogan and Seidov, 1969, 1970):

$$T_f = \frac{A}{\mu_z^{4/3}} \frac{\rho_{co}^{1/3}}{1.7\,10^{-7}} \frac{\delta M}{M} = 1.24\,10^{11} \frac{\delta M}{M} = \beta \frac{\delta M}{M} \tag{3}$$

Here $M_o = M(\rho_{co}, T=0)$, $A = 56$. Cooling of the white dwarf with $M = M_o + \delta M$ leads to the formation of chromium core. The curve $M_T(\rho_c)$ has a maximum, and the curve $T_M(\rho_c)$ has a minimum, where quadratic relations are valid

$$M_T(\rho_c) = M_{max}(T) - \alpha(\rho_c - \rho_{cm})^2 \tag{4}$$

$$T_M(\rho_c) = T_{min}(M) + \gamma(\rho_c - \rho_{cm})^2 \tag{5}$$

Here α, γ, $\rho_{cm} = \rho_{c_1} + \Delta\rho_c$ are approximately constants and formally $T_{min} < 0$ for $M < M_o + \Delta M$ from (2). Taking into account that $T = T_f$ from (3) when $\rho = \rho_{c_1}$, we find T_{min} from (5) and obtain

$$\begin{aligned} T &= T_f + \gamma[(\rho_c - \rho_{cm})^2 - (\rho_{c_1} - \rho_{cm})^2] = \\ &= \beta\frac{\delta M}{M} + \gamma[(\Delta\rho_c - \delta\rho_c)^2 - \Delta\rho_c^{\,2}] \end{aligned} \tag{6}$$

Using relations $\delta M = \Delta M$ when $T = 0$ and $\delta\rho_c = \Delta\rho_c$ from (2) we find γ and finally obtain the relation between $\delta\rho_c$ and T for given δM in the form:

$$\delta\rho_c = \Delta\rho_c\left[1 - \left(1 + \frac{T}{\beta}\frac{M_o}{\Delta M} - \frac{\delta M}{\Delta M}\right)^{1/2}\right] \tag{7}$$

Using (2) and quadratic dependence of $\rho(r)$ near the center (Chandrasekhar, 1957) we find the connection between the mass of chromium core

m_c and central density $\rho_c = \rho_{c_1} + \delta\rho_c$ of the star:

$$\frac{m_c}{M_0} = \frac{M_c}{M_0}\left(\frac{\delta\rho_c}{\Delta\rho_c}\right)^{3/2} \approx 1.1\ 10^{-14}(\delta\rho_c)^{3/2} \tag{8}$$

The white dwarf with $\delta M < \Delta M$ is stable at $T = 0$ and has chromium core with the mass $m_c < M_c$. When $\delta M > \Delta M$ the loss of stability occurs at the finite temperature $T_c > T_{min}$.

3. Evolution with account of non-equilibrium heating

The approximate theory of white dwarf cooling leads to relation between luminosity L and the temperature of isothermal core T (Schwarzschild, 1958)

$$L \approx 5.75\ 10^5 \frac{\mu T^{3.5}}{\mu_z^2}\frac{M/M_\odot}{x_z(1+x_H)} \approx 2.0\ 10^6 \frac{M}{M_\odot} T^{3.5}\ \frac{\text{ergs}}{\text{sec}} \tag{9}$$

Here in the envelope we take $x_z = 0.1$, $x_H = 0$, $\mu = 1.38$, $\mu_z = 2$ and Krammers opacity have been used. The energy losses due to heat capacity source of nondegenerate nuclei $\dot{Q}_T$ and degenerate crystal $\dot{Q}_D$ are (Shapiro and Teukolsky, 1983)

$$\dot{Q}_T = \frac{3}{2}\frac{kM}{Am_p}\dot{T}\ , \quad \dot{Q}_D = \frac{16\pi^4}{5}\frac{kM}{Am_p}\left(\frac{T}{\theta_{DC}}\right)^3\dot{T} \tag{10}$$

The losses due to nonequilibrium neutronization source are

$$\dot{Q}_\nu = -(476\,\text{keV})\frac{\dot{m}_c}{Am_p} \tag{11}$$

The equation

$$\dot{Q} = -L \tag{12}$$

determines the evolution of the white dwarf where $\dot{Q} = \dot{Q}_T + \dot{Q}_\nu$ for $T > 0.1\,\theta_{DC}$ and $\dot{Q} = \dot{Q}_D + \dot{Q}_\nu$ for $T < 0.1\,\theta_{DC}$. Here θ_{DC} is the central value of $\theta_D = \hbar\omega_i/k \approx 1.6\ 10^3\sqrt{\rho}$ = Debye temperature for the iron. Solving equation (12) with account of (7)-(11) for $M = M_0 + \Delta M$ we obtain the following solutions (Bisnovatyi-Kogan, 1987):

$$\tau \approx 2.8\cdot 10^{15}\left[T_7^{-2.5} - T_{7,0}^{-2.5} + 0.2\left(T_7^{-3} - T_{7,0}^{-3}\right)\right]\ \text{sec} \tag{13}$$

for nondegenerate case with $\dot{Q}_T$ and

$$\tau \approx 2.8\ 10^{15}\left[\left(\frac{T_0}{0.1\theta_{DC}}\right)^3 T_{7,0}^{-2.5} - \left(\frac{T}{0.1\theta_{DC}}\right)^3 T_7^{-2.5}\right. + \tag{14}$$

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$$+ 0.2\,(T_7^{-3} - T_{7,0}^{-3})\Big]\ \mathrm{sec}$$

for degenerate case with $\dot{Q}_D$. Here τ is the cooling time from initial temperature T_o up to temperature T, $T_7 = T/10^7$K, $0.1\ Q_{DC} \approx 5.5\ 10^6$K. Cooling to the temperature $T_7 = 0.55$ is $\sim$27% longer with account of nonequilibrium heating according to (13). Cooling up to almost zero temperature from $T_{7,0} = 0.55$ without $\dot{Q}_\nu$ last $\sim 4\ 10^8$ years and with account of $\dot{Q}_\nu$ after cosmological time $\tau_c = 2\ 10^{10}$ years the temperature reaches $T = T_* \approx 10^6$ K. Stars with $M > M_o + \Delta M$ collapse after cooling and because of $\dot{Q}_\nu$ the temperature at the beginning of collapse is always greater than T_*.

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