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Anomalous skin effect for linearly and circularly polarized intense laser light

TATSUFUMI NAKAMURA,[1] SUSUMU KATO,[2] AND TOMOKAZU KATO[1]

[1]Waseda University, Ohkubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan
[2]Electrotechnical Laboratory, Umezono 1-1-4, Tukuba 305-8568, Japan

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Abstract

Anomalous absorption of an intense short laser pulse in overdense plasmas is analyzed with a stochastic theory. A diffusion equation describing a time evolution of the electron distribution function is derived. From the equation it is shown that the electron distribution function becomes anisotropic in the momentum space, which gives rise to the absorption of the energy. The diffusion is not dominant in the p_z direction, which is longitudinal to the vacuum-plasma boundary, rather it is dominant in the p_x direction, which is transverse to the boundary. However, the diffusion in p_z enhances the absorption. Analytical expressions of the absorption coefficient and skin depth are obtained for the anomalous skin effect regime ($\omega_0^2 c^2 \ll \omega_p^2 v_e^2$), which evolves in time as the electron distribution becomes anisotropic. The asymptotic value of the absorption coefficient is proportional to $\sqrt{I}$. The temperature and density dependence of the absorption coefficient is also discussed.

Keywords: Anomalous absorption; Overdense plasma; Short intense laser pulse; Stochastic force

1. INTRODUCTION

In recent years there has been increasing interest in the interaction of ultraintense short laser pulses with overdense plasma due to the rapid improvement of the intense short laser pulse technology. The interaction of such laser pulses with plasmas are important for the fast ignition concept, the fast electron generation, X-ray sources of high brightness, and so on. The strong electric field ionizes the atoms within a time shorter than the duration of the laser radiation. Since the ions are relatively heavy, they remain immobile during such short laser pulses. The interaction of a high-intensity ultrashort laser pulse with matter can be described as that with a rigid, uniform, and overdense plasma. The characteristic laser parameters correspond to intensities above 10^{19} W/cm^2 and pulse durations of 100 fs. The absorption of the laser pulse in this regime is governed by the anomalous absorption mechanism rather than the classical absorption mechanism.

For plasmas with steep density gradient, there are well-known anomalous absorption mechanisms, as the anomalous skin effect (ASE; Weibel, 1967), sheath inverse bremsstrahlung (SIB; Catto & More, 1977), vacuum heating (Brunel effect; Brunel, 1987), and $\boldsymbol{J} \times \boldsymbol{B}$ heating (Kruer & Estabrook, 1985). In the analyses of anomalous absorption, the plasma distribution function is assumed to be in equilibrium. In the relativistic intensity regime ($a \geq 1$, where $a = eA/(mc)$ is a normalized vector potential, m is electron rest mass, and c is speed of light), however, the electron distribution function deviates from the equilibrium distribution due to the interaction between electrons and laser field, and evolves to anisotropic distribution in momentum space. This results in the anomalous energy absorption by the bulk plasma.

In this paper, we discuss the anomalous skin effect mechanism for a relativistic intensity regime with a stochastic theory, including the electrons' longitudinal motion as well as transverse motion to analyze both the linearly and circularly polarized cases. The time evolution of the electron distribution function is analytically obtained, and shown to be governed by a diffusion equation in momentum space. With the obtained distribution function, the analytic expressions of the absorption coefficient and skin depth for ASE in relativistic regime are obtained. As a result, the absorption coefficient decreases in time as the distribution function evolves to an isotropic distribution, and reaches a constant value which is determined by only diffusion coefficients D_x

Address correspondence and reprint requests to: Tatsufumi Nakamura, Department of Applied Physics, Waseda University, Ohkubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan. E-mail: tnaka@mn.waseda.ac.jp

and D_z. This asymptotic form of the absorption coefficient is given by $\eta \simeq a(c/v_{th})(\omega_0/\omega_p)^2$, where ω_0, ω_p, and v_{th} are the laser frequency, plasma frequency, and thermal velocity, respectively. The temperature and density dependence of the absorption coefficient varies in time, which is plotted for the initial, transient, and asymptotic cases. The absorption coefficient is compared between a linearly polarized case and a circularly polarized case.

2. TIME EVOLUTION OF THE ELECTRON DISTRIBUTION FUNCTION

When an intense laser light irradiates an overdense plasma, the laser field penetrates over a skin layer whose depth is much shorter than the laser wavelength in a vacuum. The laser field strongly interacts with the plasma in the thin layer. The electrons continue their periodic motion in the laser field in the underdense region. In the overdense plasma, however, the interaction takes place in the thin layer, which is localized at the surface. Thus, the periodic motion of electrons is truncated, which results in the energy transfer from the laser field to the bulk plasma.

In this section, we obtain the time evolution of the electron distribution function with a stochastic theory. In stochastic treatment of the anomalous absorption mechanism, there are three different time scales to be considered: the interaction time τ_i, the correlation time of the stochastic variable τ_i, and the laser period $\tau_L = 2\pi/\omega_0$. The interaction time τ_i is written as $\tau_i = l_s/v_z$ where l_s is the depth of the skin layer. By approximating $l_s \simeq c/\omega_p$, the interaction time is evaluated as $\tau_i \simeq c/(v_z\omega_p)$. (Since the depth of the skin layer depends on the plasma conductivity, l_s changes in time as electron distribution function evolves in time.) Over a time scale τ_i, the equation of motion of an electron is described microscopically. The plasma occupies the half-space $z \geq 0$, which is assumed to be highly overdense, $\omega_p \gg \omega_0$, and the plasma is assumed to be neutralized by the background ions. The linearly polarized laser field is normally incident on the overdense plasma from the vacuum side ($z \leq 0$),

$$E_x = -\frac{\partial A_x}{\partial t}, \tag{1}$$

$$B_y = \frac{\partial A_x}{\partial z}, \tag{2}$$

$$A_x = \Im\{A_0 \exp(ikz - i\omega_0 t)\}, \tag{3}$$

where E_x, B_y, and A_x are the electric field, magnetic field, and vector potential of the laser field with amplitude A_0, and $k = k_r + ik_i$ is a wave number with a positive imaginary part ($k_i \geq 0$). The laser intensity is in the relativistic regime. Hereafter, we omit the suffix on the electric and magnetic fields for simplicity.

The equation of motion of an electron in the laser field is given by

$$\frac{dp_x}{dt} = -eE + \frac{eB}{m\gamma} p_z, \tag{4}$$

$$\frac{dp_z}{dt} = -\frac{eB}{m\gamma} p_x. \tag{5}$$

Equations (4) and (5) are solved with an initial condition $(p_x, p_z)_{t=t_0} = (p_{x0}, p_{z0})$, where t_0 is the time when each electron enters the thin interaction layer. We assume $p_{x0} \ll p_{z0}$ because the electrons moving in the z direction with a large velocity play a dominant role in the energy transfer from the laser field to the bulk plasma (Gamaliy & Dragila, 1990). The force acting on the electron within the thin layer becomes

$$F_x(z,t) = \frac{eB(z,t)}{m\gamma_0} p_{z0} \cos(S(z,t_0,t)) - eE(z,t) + \frac{eB(z,t)}{m\gamma_0} \int_0^t eE(z,t') \sin(S(z,t',t))\, dt', \tag{6}$$

$$F_z(z,t) = -\frac{eB(z,t)}{m\gamma_0} p_{z0} \sin(S(z,t_0,t)) + \frac{eB(z,t)}{m\gamma_0} \int_0^t eE(z,t') \cos(S(z,t',t))\, dt', \tag{7}$$

$$S(z,t_0,t_1) = \frac{e}{m\gamma_0} \int_{t_0}^{t_1} B(z,\tau)\, d\tau, \tag{8}$$

$$\gamma_0 = \sqrt{1 + \frac{p_{z0}^2}{(mc)^2}}. \tag{9}$$

Since $\omega_c = eB_0/m\gamma_0 \gg \omega_0$, the electromagnetic force oscillates rapidly not to give net acceleration unless the sinusoidal function is almost zero. Thus, for an effective energy transfer to occur, $t - t_0 = t_i \simeq 0$ is required. This explains the fact that the effective interaction is limited to a short time interval and localized at the plasma surface. As the electromagnetic fields are localized in a thin layer, the magnetic field becomes dominant compared to the electric field ($B \simeq (1/l_s\omega_0)E$, where l_s is length of the thin layer). As a result, the effective force acting on each electron with the initial momentum p_{z0} in the thin layer is written as

$$F_x = \omega_c p_{z0}, \tag{10}$$

$$F_z = -\omega_c^2 \tau_i p_{z0}. \tag{11}$$

Although the phase of the laser field can be regarded as constant in time during the interaction of each electron because of $\omega_c \gg \omega_0$, the laser phase and incident electron momentum depends on time, $\omega_c(t)$ and $p_{z0}(t)$, in the time

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scale which is much larger than τ_i. As will be shown later (Eq. (39)), the correlation time τ_i is evaluated as $\tau_i \ll 1/\omega_p$. Therefore, the initial momentum is treated as a stochastic variable $\delta p_{z0}(t)$ fluctuating around the average momentum $\overline{p_{z0}}$, which will be physically explained in Section 3. The slow mode of electron distribution function is assumed to be weakly dependent on z for $z \leq l_{mfp}$, where l_{mfp} is the electron mean free path. Since $l_{mfp} \gg l_s$ is satisfied in the present situation, the electron distribution function is assumed not to depend on the z coordinate:

$$\frac{\partial f_e}{\partial t} + F_x(t)\,\frac{\partial f_e}{\partial p_x} + F_z(t)\,\frac{\partial f_e}{\partial p_z} = 0, \tag{12}$$

$$F_x(t) = \omega_c(t) p_{z0}(t), \tag{13}$$

$$F_z(t) = -\omega_c^2(t)\tau_i\, p_{z0}(t). \tag{14}$$

where γ is the Lorentz factor. Equation (12) is formally integrated, and an ensemble average is taken over the stochastic variable to give

$$\langle f_e(\boldsymbol{r},\boldsymbol{p},t)\rangle = \left\langle \exp\int_0^t \left(-F_x(t')\,\frac{\partial}{\partial p_x} - F_z(t')\,\frac{\partial}{\partial p_z}\right) dt' f_e(\boldsymbol{r},\boldsymbol{p},0)\right\rangle. \tag{15}$$

The fluctuation $\delta p_{z0}(t)$ is assumed to be Gaussian and the time correlation is assumed to obey: $\langle \delta p_{z0}(t_1)\delta p_{z0}(t_2)\rangle = G\delta(t_1 - t_2)$. To solve Eq. (15), the exponential operator is decomposed by using the following Lie algebra identity:

$$\exp(\mathcal{A} + \mathcal{B}) = \exp(\mathcal{B})\exp\left(\frac{\exp(\alpha) - 1}{\alpha}\,\mathcal{A}\right), \tag{16}$$

where the commutator of operators $\mathcal{A}$ and $\mathcal{B}$ satisfies $[\mathcal{A},\mathcal{B}] = \alpha\mathcal{A}$ (Wilcox, 1967) and α is a constant. After decomposing the exponential operator, an average is taken over the ensemble of $\delta p_{z0}(t)$ (Kubo, 1962). Riemann–Lebesgue's lemma is then used to obtain the distribution function $f_s(t)$ over the slow time scale as

$$f_s(t) = \exp\left(\frac{1}{2}\,\omega_c^2 Gt\,\frac{\partial^2}{\partial p_x^2}\right)\exp\left(\frac{3}{8}\,\omega_c^4\tau_i^2 Gt\,\frac{\partial^2}{\partial p_z^2}\right) \times \exp\left(\left[1 - \exp\left(-\frac{\omega_c^2\tau_i}{2}\,t\right)\right]\overline{p_{z0}}\,\frac{\partial}{\partial p_z}\right) f(0). \tag{17}$$

The distribution function $f_s(t)$ given by Eq. (17) satisfies the following diffusion equation in momentum space as follows:

$$\frac{\partial f_s}{\partial t} - D_x\,\frac{\partial^2 f_s}{\partial p_x^2} - C(t)\,\frac{\partial f_s}{\partial p_z} - D_z\,\frac{\partial^2 f_s}{\partial p_z^2} = 0, \tag{18}$$

where

$$D_x = \frac{\omega_c^2}{2}\,G, \tag{19}$$

$$D_z = \frac{3\omega_c^4\tau_i^2}{8}\,G, \tag{20}$$

$$C(t) = \frac{\omega_c^2\tau_i}{2}\exp\left(-\frac{\omega_c^2\tau_i}{2}\,t\right)\overline{p_{z0}}. \tag{21}$$

Here D_x and D_z are diffusion coefficients and $C(t)$ is a drift term. From this equation it is possible to estimate how the electron distribution evolves to an anisotropic distribution. The anisotropy is expressed by the diffusion coefficients, D_x and D_z. Even though D_z is much smaller than D_x, the diffusion in p_z space is not negligible and acts to enhance the energy absorption, as will be shown in Section 3.

For a circularly polarized laser pulse, the fundamental equation becomes

$$\frac{\partial f_s}{\partial t} - D_r\left[\frac{1}{p_r}\,\frac{\partial f_s}{\partial p_r} + \frac{\partial^2 f_s}{\partial p_r^2}\right] = 0, \tag{22}$$

where

$$D_r = \frac{\omega_c^2}{2}\,G, \quad p_r = \sqrt{p_x^2 + p_y^2}. \tag{23}$$

Since the electron is not accelerated longitudinally for a circularly polarized laser field, the electrons only diffuse in the transverse directions, x and y.

The time evolution of electron distribution function for linearly polarized case is shown in Figure 1, where the initial distribution is chosen as an isotropic Maxwell distribution which is used in the analyses of low intensity laser case:

$$f_s(\boldsymbol{p},0) = \frac{n_0}{(2\pi\sigma^2)^{3/2}}\exp\left(-\frac{p_x^2 + p_y^2 + p_z^2}{2\sigma^2}\right). \tag{24}$$

The time evolution to an anisotropic state is obtained from Eq. (24) and using Eqs. (26)–(28):

$$f_s(\boldsymbol{p},t) = \frac{n_0}{(2\pi\sigma^2)^{3/2}}\sqrt{\frac{\sigma^4}{(2D_x t + \sigma^2)(2D_z t + \sigma^2)}} \times \exp\left(-\frac{p_x^2}{2(2D_x t + \sigma^2)} - \frac{\left(p_z + \int_0^t C(t')\,dt'\right)^2}{2(2D_z t + \sigma^2)} - \frac{p_y^2}{2\sigma^2}\right). \tag{25}$$

In Figure 1, it is shown that the electron distribution becomes anisotropic and broader in p_x than p_z, since the diffusion coefficient D_x is much larger than D_z, and the drift effect in positive p_z direction is small compared to the dif-

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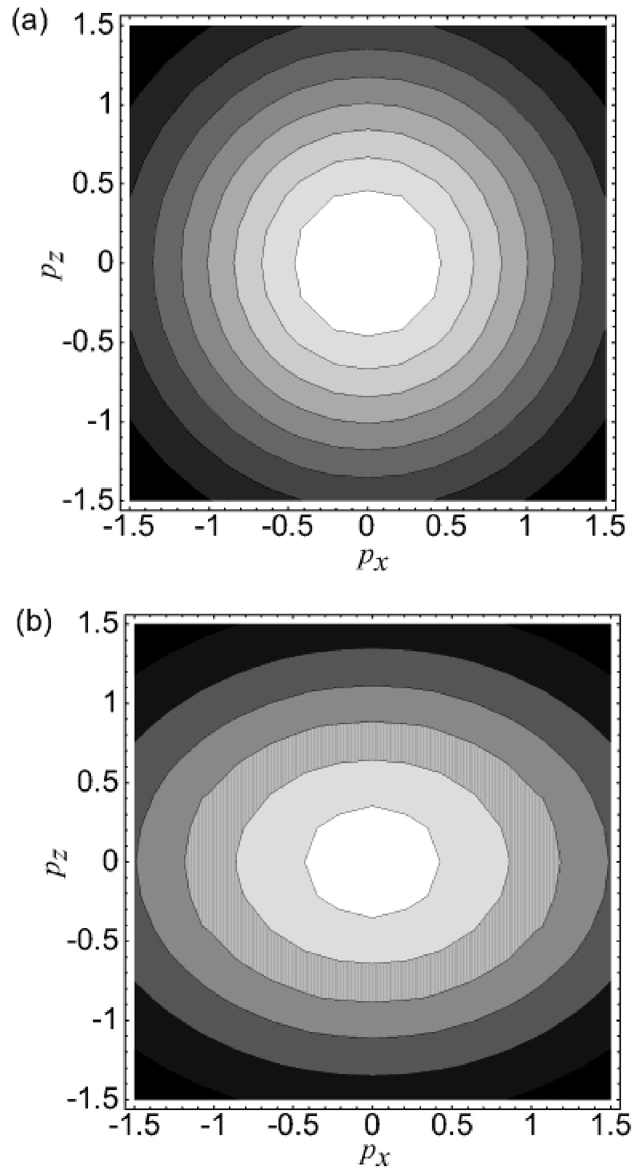

Fig. 1. (a) Isotropic electron distribution function in momentum space at $t = 0$. (b) Anisotropic electron distribution function in momentum space at $\omega_0 t = 20$.

fusion effect. However, the diffusion in p_z is not negligible in considering the energy absorption, and it enhances the energy absorption. As will be shown in the following section, the energy absorption in a circularly polarized case is smaller than in a linearly polarized case. This is because in a circularly polarized case, the momentum distribution diffuses only in the transverse direction, not in the longitudinal direction.

3. ABSORPTION COEFFICIENT AND SKIN DEPTH

In analyses of the anomalous skin effect, the high frequency surface current is calculated from the Vlasov equation. The slow mode f_s is treated as a steady-state equilibrium distribution function in the analyses of anomalous absorption mechanisms. In the high-intensity laser case, however, the electron distribution function deviates from the Maxwell distribution due to the strong interaction with the laser field, and the absorption coefficient becomes dependent on time and laser intensity.

The surface current is calculated from the fast perturbation of the electron distribution function f_f, which is obtained from the Vlasov equation:

$$\frac{\partial f_f}{\partial t} + \frac{\boldsymbol{p}}{m\gamma}\cdot\frac{\partial f_f}{\partial \boldsymbol{r}} - e\left(\boldsymbol{E} + \frac{\boldsymbol{p}}{m\gamma}\times\boldsymbol{B}\right)\cdot\frac{\partial f_s}{\partial \boldsymbol{p}} = 0. \tag{26}$$

The absorption coefficient and skin depth are calculated by following the usual treatment of the anomalous skin effect (Yang *et al.*, 1995; Ichimaru, 1973). The first term in Eq. (25) is neglected because of the condition corresponding to the anomalous skin effect, $\omega_p v_{th} \gg \omega_0 c$. The surface current is calculated by using the Fourier transform as follows:

$$\tilde{J}_x = -e\int_{-\infty}^{\infty}\frac{p_x}{m\gamma}\tilde{f}_f\,d\boldsymbol{p} \tag{27}$$

$$= -\frac{i n_0 e^2}{k}\left[\left(\frac{1}{\sqrt{2(2D_z t+\sigma^2)}} + \frac{k\xi}{m\gamma_0\omega_0}\,\frac{2D_x t+\sigma^2}{2D_z t+\sigma^2}\right)\times\sqrt{\pi}Z(\xi) - \frac{k}{m\gamma_0\omega_0}\left(\frac{2D_x t+\sigma^2}{2D_z t+\sigma^2}-1\right)\right]\tilde{E}, \tag{28}$$

where

$$Z(\xi) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{1}{z-\xi}\exp(-z^2)\,dz, \tag{29}$$

$$\xi = \frac{\int_0^t C(t')\,dt'}{\sqrt{2(2D_z t+\sigma^2)}}. \tag{30}$$

Here, $\tilde{J}_x$, $\tilde{\boldsymbol{B}}$, and $\tilde{\boldsymbol{E}}$ are the Fourier transforms of J_x, $\boldsymbol{B}$, and $\boldsymbol{E}$, respectively, and $Z(\xi)$ is the plasma dispersion function (Miyamoto, 1989). The Lorentz factor in the electromagnetic force is approximated by γ_0, and the magnetic field is eliminated by using $\tilde{B} = (k/\omega)\tilde{E}$. The electromagnetic field is extended into the half space ($z \le 0$) with $E(-z) = E(z)$ and $B(-z) = -B(z)$. The electric field is obtained as follows:

$$E(z) = \frac{-i\omega_0 c B(z=0+)}{\pi\omega_p}\int_0^{\infty}\frac{\tilde{k}\exp(i\tilde{k}z)}{\tilde{k}^3 - iK - L\tilde{k}}\,d\tilde{k}, \tag{31}$$

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where

$$K = \frac{\pi m \omega_0 c}{\omega_p} \frac{\exp(-\xi^2)}{\sqrt{2(2D_z t + \sigma^2)}}, \tag{32}$$

$$L = \frac{2D_x t + \sigma^2}{2D_z t + \sigma^2} - 1. \tag{33}$$

Here, K and L are dimensionless functions, $B(z=0+)$ is the magnetic field at the surface, and $\tilde{k} = ck/\omega_p$ is the normalized wave number. The plasma dispersion function $Z(\xi)$ is expanded using $\xi \ll 1$, which is satisfied for the case we are interested in as will be shown later:

$$Z(\xi) \simeq i\sqrt{\pi}\exp(-\xi^2) - \cdots. \tag{34}$$

The inverse transform is performed by analytically continuing the integrand into the upper half-plane of $\tilde{k}$ and the contour is closed with an upper half circle. The absorption coefficient $\eta(t)$ and skin depth $l_s(t)$ are obtained as

$$\eta(t) = \frac{2\Re(E(z=0+)B^*(z=0+))}{E_{in}^2/c}$$

$$= \frac{8}{3\sqrt{3}}\left(\frac{2}{K}\right)^{1/3} \frac{\alpha_+^2 + \alpha_-^2}{(\alpha_+ + \alpha_-)(\alpha_+^2 + \alpha_-^2 - \alpha_+\alpha_-)}, \tag{35}$$

$$l_s(t) = \frac{c}{\omega_p}(\Im(k))^{-1} = \frac{c}{\omega_p}\left(\frac{2}{K}\right)^{1/3}(\alpha_+ - \alpha_-)^{-1}, \tag{36}$$

where

$$\alpha_\pm = \left(\sqrt{\frac{4L^3}{27K^2}} \pm 1\right)^{1/3}. \tag{37}$$

Here, $E(z=0+)$ and E_{in} are the electric field at the surface and in the vacuum.

The average momentum $\overline{p_{z0}}$ is obtained by invoking pressure balance between the laser light and plasma. When the laser pressure is greater than the plasma pressure, the plasma surface is pushed inward with constant velocity as is verified by simulations (Denavit, 1992; Wilks *et al.*, 1992). By using number and momentum conservation, the moving velocity u is given by (Kruer *et al.*, 1975):

$$\frac{u}{c} = \left(\frac{n_{cr}}{2n_0}\frac{m}{M}\frac{I\lambda^2}{2.74\times 10^{18}}\right)^{1/2}, \tag{38}$$

where n_{cr}, I, λ, and M are the critical density, laser intensity, wavelength, and ion mass, respectively. Consequently, the average electron momentum is expressed as $\overline{p_{z0}} = -mu\gamma_0$.

The source of the momentum fluctuation of incoming electrons is taken to be a thermal fluctuation. For the early stage of laser irradiation, the incoming electron distribution function can be approximated by the previously obtained f_s. Thus, the time correlation of momentum fluctuation is related to the plasma temperature and conductivity, with the aid of the fluctuation-dissipation theorem as follows:

$$G = \frac{32}{3\sqrt{3}} m^2 v_{th}^2 \frac{\omega_0}{\omega_p^2}, \tag{39}$$

where the plasma conductivity is approximated by its initial value. With these considerations, the absorption coefficient and skin depth are plotted as a function of time, intensity, temperature, and density.

The time dependence of the absorption coefficient is shown in Figure 2. Here, the plasma density is taken to be 1000 times critical density, the plasma temperature is 120 keV, and the laser intensity is chosen as $a = 5$. In this case, $\xi \ll 1$ is well satisfied. The absorption coefficient starts from the well-known absorption coefficient for the nonrelativistic case η^{non} which is written as (Rozmus & Tikhonchuk, 1996)

$$\eta^{non} = \frac{8}{3\sqrt{3}}\left(\frac{2}{\pi}\right)^{1/6}\left(\frac{v_{th}}{c}\right)^{1/3}\left(\frac{\omega_o}{\omega_p}\right)^{2/3}. \tag{40}$$

The absorption coefficient increases rapidly during the short period since the plasma surface begins to move by the laser pressure. After this rapid increase, η tends to decrease gradually and finally reaches a constant value. This is because the time evolution of the electron distribution function is mainly dominated by the drift effect at the initial stage, and the diffusion effect becomes dominant after sufficient interactions have taken place. Then gradually the diffusion effect dominates the drift effect to give a constant energy absorption coefficient. This asymptotic value η^{asy} is evaluated from Eq. (35) as

$$\eta^{asy} \simeq \frac{\omega_0}{\omega_p}(\omega_c\tau_i) \simeq a\frac{c}{v_{th}}\left(\frac{\omega_0}{\omega_p}\right)^2, \tag{41}$$

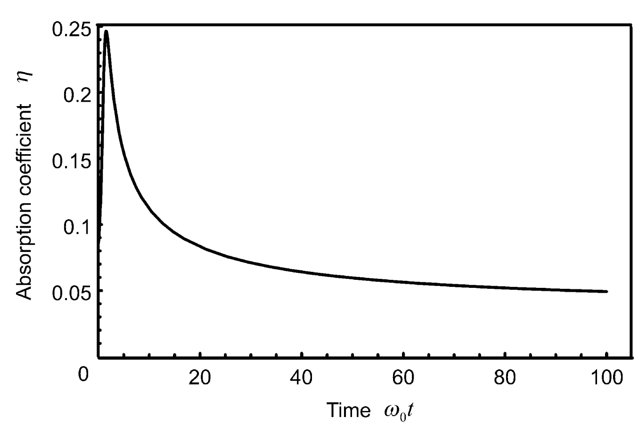

Fig. 2. Time evolution of absorption coefficient for $a = 5$, $v_{th}/c = 0.5$, $n_e/n_{cr} = 1000$. η reaches the constant value at about $\omega_0 t = 80$.

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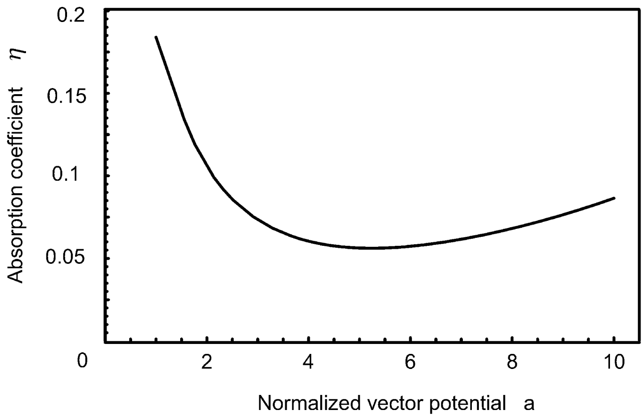

Fig. 3. Absorption coefficient at $\omega_0 t = 1000$ as a function of normalized vector potential.

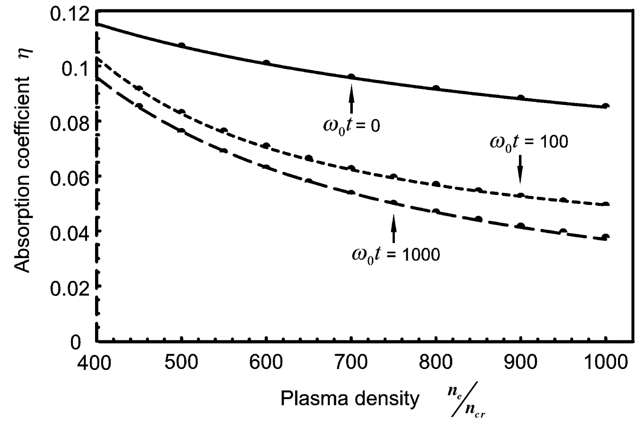

Fig. 5. The density dependence of the absorption coefficient at $\omega_0 t = 0$, $\omega_0 t = 100$, and $\omega_0 t = 1000$. The solid line (for $\omega_0 t = 0$), dashed line (for $\omega_0 t = 100$), and dotted line (for $\omega_0 t = 1000$) correspond to $(n_e/n_{cr})^{-1/3}$, $(n_e/n_{cr})^{-3/2}$, and $(n_e/n_{cr})^{-1}$, respectively. The parameters are chosen as $a = 5$, $v_{th}/c = 0.5$.

which is valid for $a \geq 1$. The vector potential dependence of η is shown in Figure 3. In the range where a is less than about 5, the absorption coefficient decreases with increasing intensity, which is consistent with the simulation result (Denavit, 1992). When the laser intensity becomes even higher, the absorption coefficient increases with increasing intensity as $\eta \propto \sqrt{I}$, as is clear from Eq. (41). The temperature dependence and density dependence of η is clearly given by Eq. (40) and Eq. (41) for the nonrelativistic case and the asymptotic case, respectively. In the transient stage, however, the temperature and density dependence of η change as the distribution function evolves. This is numerically shown in Figures 4 and 5. In Figure 4, the temperature dependence is plotted for the nonrelativistic case ($\omega_0 t = 0$), the transient case ($\omega_0 t \simeq 50$), and the asymptotic case ($\omega_0 t \simeq 200$). The solid line, dashed line, and dotted line

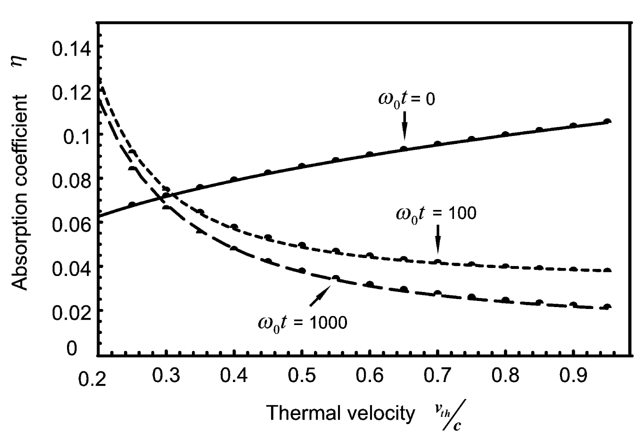

Fig. 4. The temperature dependence of the absorption coefficient at $\omega_0 t = 0$, $\omega_0 t = 100$, and $\omega_0 t = 1000$. The solid line (for $\omega_0 t = 0$), dotted line (for $\omega_0 t = 100$), and dashed line (for $\omega_0 t = 1000$) correspond to $(v_{th}/c)^{1/3}$, $(v_{th}/c)^{-2}$, and $(v_{th}/c)^{-1}$, respectively. The parameters are chosen as $a = 5$, $n_e/n_{cr} = 1000$.

represent $\eta \propto (v_{th}/c)^{1/3}, (v_{th}/c)^{-2}$, and $(v_{th}/c)^{-1}$, respectively. From the figure it is shown that $\eta \propto (v_{th}/c)^{-2}$ for the transient range. The density dependence is plotted in Figure 5, where the solid line, dashed line, and dotted line represent $\eta \propto (n_e/n_{cr})^{-1/3}, (n_e/n_{cr})^{-3/2}$, and $(n_e/n_{cr})^{-1}$, respectively. It is shown that $\eta \propto (\omega_0/\omega_p)^3$. Therefore, the absorption coefficient in the transient range η^{tr} is written as

$$\eta^{tr} \propto \left(\frac{c}{v_{th}}\right)^2 \left(\frac{\omega_0}{\omega_p}\right)^3. \tag{42}$$

As a result, in the nonrelativistic regime the density dependence is weak as $\eta \propto (n_e/n_{cr})^{-1/3}$. In the relativistic regime, the density dependence is strong at the asymptotic stage as $\propto (n_e/n_{cr})^{-1}$ and $\propto (n_e/n_{cr})^{-2}$ during the transient stage. The temperature dependence is opposite for the nonrelativistic regime and the relativistic regime. In the nonrelativistic regime, the absorption coefficient increases with increasing temperature as $\eta \propto (v_{th}/c)^{1/3}$. In the relativistic regime, the absorption coefficient decreases with increasing temperature as $\eta \propto (v_{th}/c)^{-1}$ at the asymptotic range and $\eta \propto (v_{th}/c)^{-2}$ at the transient range.

The time evolution of the skin depth is shown in Figure 6, where the skin depth is normalized by the nonrelativistic skin depth c/ω_p. After the laser pulse irradiates the plasma, the skin depth increases, which results in the increase of the absorption coefficient. As the diffusion effect becomes dominant, the skin depth increases more slowly.

The absorption coefficient is plotted for both a circularly and linearly polarized laser pulse in Figure 7. Since the electrons are accelerated periodically with $2\omega_0$ in the longitudinal direction for a linearly polarized case, the absorption coefficient is larger in the linearly polarized case than in the circularly polarized case. Thus, the longitudinal diffusion D_z is negligible for a circularly polarized case; however, it

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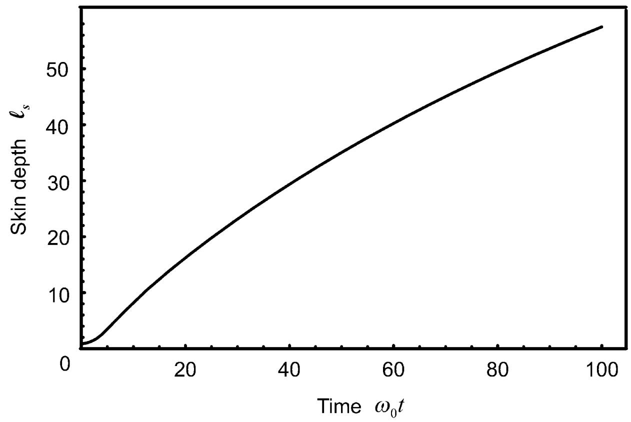

Fig. 6. Time evolution of the skin depth for $a = 5$, $v_{th}/c = 0.5$, and $n_e/n_{cr} = 1000$.

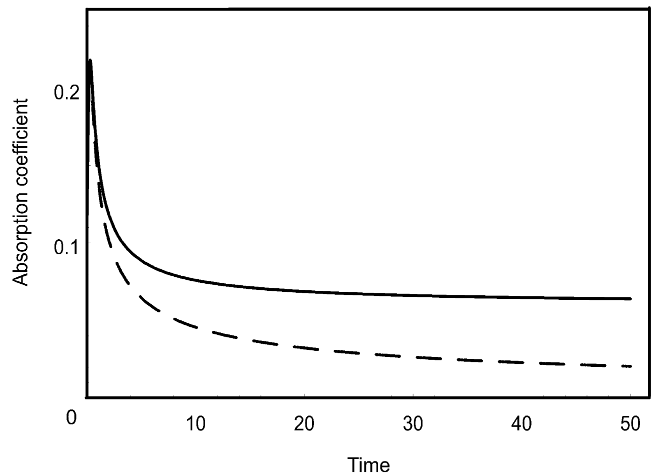

Fig. 7. The absorption coefficient for linearly (solid line) and circularly (dashed line) polarized laser light.

plays an important role in energy absorption for a linearly polarized case.

4. DISCUSSION

The anomalous absorption effect for an intense laser pulse is analyzed with a stochastic theory. In relativistic intensity case, the electron distribution function evolves in time due to a strong interaction between electrons and the laser field. Since the interaction is limited in a short time interval, the electromagnetic force is treated as a stochastic force ($\tau_c \ll \tau_i$). By a stochastic analysis, it is shown that the time evolution of the electron distribution function is governed by a diffusion equation in the momentum space. Even though the transverse diffusion coefficient D_r is much smaller than longitudinal coefficient D_z, the diffusion in p_z is not negligible and enhances the energy absorption. With the obtained electron distribution function, the analytical expression of absorption coefficient and skin depth are obtained. The absorption coefficient has an constant asymptotic form as $\eta^{asy} \simeq (\omega_0/\omega_p)(\omega_c \tau_i) \simeq a(c/v_{th})(\omega_0/\omega_p)^2$. In the relativistic regime $a \geq 1$, the absorption coefficient decreases with increasing intensity, and in the even higher intensity regime, the absorption coefficient increases with increasing intensity as $\eta \simeq \sqrt{I}$. The temperature and density dependence of the absorption coefficient is numerically obtained as $\eta^{tr} \simeq (c/v_{th})^2(\omega_0/\omega_p)^3$ for the transient stage.

The applicability of this theory is given from the condition $\omega_c \tau_i \ll 1$. By using $\omega_c \simeq a\omega_0$, the condition is evaluated as $\omega_c \tau_i \simeq a(c\omega_0/v_{th}\omega_p)$. Because $(c\omega_0/v_{th}\omega_p) \ll 1$ is assumed and satisfied in the situation of the anomalous skin effect, $\omega_c \tau_c \ll 1$ is not a very strict condition. But this gives us the upper limit for the normalized vector potential.

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