Robotica (2015) volume 33, pp. 2065–2082.
doi:10.1017/S0263574714001295

Task-space bilateral teleoperation systems for heterogeneous robots with time-varying delays

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(Accepted April 24, 2014. First published online: May 22, 2014)

SUMMARY
This paper proposes control algorithms for heterogeneous teleoperation systems to guarantee stability and tracking performance in the presence of time-varying communication delays. Because robotic manipulators, in most applications of bilateral teleoperation systems, interact with a human operator and remote environment on the end-effector, the control system is developed in the task-space. When the dynamic parameters of the robots are unknown and the communication network is subject to time-varying delay, the developed controller can ensure stability and task-space position tracking. Additionally, if the robotic systems are influenced by human and environmental forces, the presented teleoperation control system is demonstrated to be stable and all signals are proven to be ultimately bounded. By employing the redundancy of the slave robot for sub-task control, the proposed teleoperation system can autonomously achieve additional missions in the remote environment. Numerical examples utilizing a redundant planar robot are addressed to validate the proposed task-space teleoperators with time-varying delay.

KEYWORDS: Teleoperation system; Time-varying delay; Heterogeneous robots; Task-space control; Adaptive control.

1. Introduction

Bilateral teleoperation systems, composed of interconnected master and slave robots, have been demonstrated to be useful in implementing tasks in remote or hazardous environments.[1,2] By exchanging information over a long distance, the slave robot can track the position of the master robot, which is manipulated by a human operator. Moreover, the master robot can convey to the human operator the contact force between the slave robot and the remote environment. The development of teleoperators could potentially contribute to a variety of applications, such as outer space manipulation, undersea exploration, and remote medical operation.[2–5] The unreliability of communication channels and the possibly kinematic dissimilarity of master and slave robots warrant the study of bilateral teleoperation with heterogeneous robots under time-varying delays.

Long-distance transmission incurs unavoidable communication delays and bandwidth limitations that can destabilize and degrade the performance of bilateral teleoperation systems.[6,7] The stability of teleoperation systems with constant delays has been extensively studied using scattering or wave-variable formulation.[8,9] By transmitting wave variables that are encoded by the passive input–output pair of a robotic system, the passivity of a delayed communication block can be guaranteed. Assuming passive human and environmental forces, the interconnected teleoperation system is passive and stable. Although this method is capable of stabilizing bilateral teleoperation with arbitrary constant delays, the issues of wave reflection[8] and position drift[10] are major impediments to system performance. To improve tracking performance, control schemes employing passivity-based synchronization,[11] PD-like control,[12] and neural networks[13] have been presented recently without utilizing wave variables.

For teleoperation systems closed by communication networks, the resulting data congestion and scarcity of transmission bandwidth may lead to time-varying delays that significantly deteriorate

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system performance and result in instability.[14–16] Time-varying delay is more difficult to compensate for than is constant delay.[17] The method of using a scattering transformation has been modified to address teleoperation systems subject to time-varying delays.[18,19] Passivity-based control algorithms have been proposed for teleoperators facing time-varying communication delays and passive external forces.[15,20] Delay-dependent[20,21] and mode-dependent[14] control schemes have been proposed to stabilize teleoperation systems with time-varying delays. Moreover, PD-like control[12] has been extended to teleoperators with variable time delays.[16]

Based on the realization that teleoperation systems mostly involve interaction between a human operator and the remote environment in the end-effector, the study of task-space teleoperators has become an emerging research topic.[22–25] Task-space teleoperation with a redundant slave robot has been developed in free motion under constant communication delays.[26] A new teleoperation framework has been addressed in ref. [23], where two master robots are utilized to control different coordinates assigned to the slave robot. Control of teleoperation systems with kinematically redundant manipulators was presented in ref. [24]. By utilizing a redundant slave robot to enhance the efficiency of complex teleoperation, a semi-autonomous teleoperation system has been developed for constant communication delay.[22] A control algorithm for task-space teleoperation with guaranteed position and orientation tracking has been proposed in ref. [25].

In this paper, we study the control problem for a task-space teleoperation system with heterogeneous robots under time-varying delay by considering dynamic uncertainties and external forces from the human operator and remote environment. We demonstrate that if the control gains of the proposed control algorithm are contingent on the communication delay, the closed-loop teleoperation system in free motion is stable and the convergence of tracking errors is guaranteed. Under external forces, the control system is shown to be stable with ultimately bounded states. Since the teleoperation is achieved in the task-space, a redundant manipulator is considered as the slave robot so that the null-space can be exploited to achieve additional missions autonomously. Simulations are introduced by considering 3-degree-of-freedom (DOF) and 5-DOF planar robots with the use of redundancy for increasing manipulability and collision avoidance.

The contributions of this paper are summarized as follows. In comparison with refs. [24]–[26], the teleoperation systems addressed here are more practical because the master and slave robots are considered to be heterogeneous owing to dissimilar kinematic structures. In contrast to refs. [22]–[24] and [26], the proposed teleoperation systems study the issue of asymmetric time-varying communication delays in the presence of external forces. In contrast to refs. [23]–[26], we propose an adaptive control algorithm to cope with parameter uncertainties in robot dynamics. By utilizing a control framework involving heterogeneous robots, the redundancy of robotic manipulators can be designed to achieve secondary tasks, while refs. [23], [25], and [26] control only the teleoperation task.

The rest of this paper is organized as follows. Section 2 presents the model of the teleoperation system and the problem description. Section 3 addresses the controller, stability analysis, and sub-task control. Section 4 contains numerical examples of a bilateral teleoperation system. Finally, Section 5 summarizes the results and discusses possible directions for future research.

2. Problem Formulation

Without loss of generality, the robotic systems in the proposed task-space teleoperator are modeled as Euler–Lagrangian systems under the assumptions that:

Assumption 1. The master and slave robots have identical dimension in the task-space.

Assumption 2. The master robot is a non-redundant manipulator, and the slave robot is a redundant manipulator.[1]

[1] A teleoperation system with redundant master robot can be accomplished directly from the control framework developed in this paper. Additionally, the assumption of redundant slave robots is necessary for the robotic manipulator to achieve an autonomous task in the remote environment.

https://doi.org/10.1017/S0263574714001295 Published online by Cambridge University Press

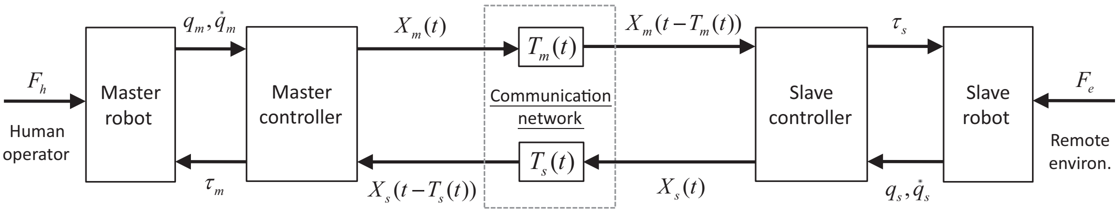

Fig. 1. Control architecture of the task-space teleoperation system for heterogeneous robots with time-varying delays.

Following ref. [27], the dynamics of the heterogeneous master and slave robots are described as

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) = \tau_m + J_m^T(q_m)F_h \\ M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) = \tau_s - J_s^T(q_s)F_e \end{cases}, \tag{1}$$

where $q_m \in R^p$ and $q_s \in R^q$ are the vector of generalized configuration coordinates, $M_m(q_m) \in R^{p\times p}$ and $M_s(q_s) \in R^{q\times q}$ are the inertia matrices, $C_m(q_m, \dot{q}_m) \in R^{p\times p}$ and $C_s(q_s, \dot{q}_s) \in R^{q\times q}$ are the vector of Coriolis/centrifugal forces, $g_m(q_m) = \partial H_m(q_m)/\partial q_m \in R^p$ and $g_s(q_s) = \partial H_s(q_s)/\partial q_s \in R^q$ are the gradient of the potential function $H_m(q_m)$ and $H_s(q_s)$, $\tau_m \in R^p$ and $\tau_s \in R^q$ are the vectors of applied torques. As the teleoperation system is implemented in the task-space, the external forces F_h and F_e exerted by the human operator and the remote environment have the same dimension such that $F_h,\ F_e \in R^p$. Therefore, the Jacobian matrices of the master and slave robots are denoted by $J_m(q_m) \in R^{p\times p}$ and $J_s(q_s) \in R^{p\times q}$, respectively. The Jacobian matrix of the slave robot J_s is a non-square matrix as the slave robot is a redundant manipulator (Assumption 2).

The robot dynamics (1) exhibit several fundamental properties resulting from the Lagrangian dynamic structure.[27]

Property 1. *The matrix $M_i(q_i)$ is symmetric and positive-definite, and there exists positive constants m_l and m_u such that $m_l I_n \le M_i(q_i) \le m_u I_n$, where I_n is an $n \times n$ identity matrix.*

Property 2. *For any differentiable vector $x_i \in R^n$, the Lagrangian dynamics are linearly parameterizable which implies $M_i(q_i)\dot{x}_i + C_i(q_i, \dot{q}_i)x_i + g_i(q_i) = Y_i(q_i, \dot{q}_i, x_i, \dot{x}_i)\Theta_i$, where $\Theta_i \in R^w$ is a constant vector of unknown parameters, and $Y_i(q_i, \dot{q}_i, x_i, \dot{x}_i) \in R^{n\times w}$ is the matrix of known functions of the generalized coordinates and their higher derivatives.*[2]

Property 3. *Under an appropriate definition of the matrix $C_i(q_i, \dot{q}_i)$, the matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric such that $x^T(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i))x = 0$ for $x \in R^n$.*

Property 4. *For $q_i, \dot{q}_i, x_i \in R^n$, there exists positive constant β_c such that the matrix of Coriolis/centrifugal torques is bounded by $\|C_i(q_i, \dot{q}_i)x_i\| \le \beta_c \|\dot{q}_i\|\|x_i\|$, where $\|\cdot\|$ denotes the Euclidean norm of the enclosed signal.*

Although the heterogeneous robots have different dimensions in the joint-space, the dimensions of the end-effector are identical (Assumption 1). Therefore, the relationships between the positions of the end-effector $X_m,\ X_s \in R^p$ and the joint-space vectors $q_m \in R^p,\ q_s \in R^q$ are described as

$$\begin{aligned} X_m &= h_m(q_m), \quad & \dot{X}_m &= J_m(q_m)\dot{q}_m, \\ X_s &= h_s(q_s), \quad & \dot{X}_s &= J_s(q_s)\dot{q}_s, \end{aligned} \tag{2}$$

where $h_m(\cdot) : R^p \to R^p$, $h_s(\cdot) : R^q \to R^p$ denote the mapping between the joint space and the task space, and $J_m(q_m) = \partial h_m(q_m)/\partial q_m$, $J_s(q_s) = \partial h_s(q_s)/\partial q_s$ are the Jacobian matrices that are assumed to be known in the control system.

Following the dynamic and kinematic model of the robotic system, the proposed control architecture for task-space teleoperation system is illustrated in Fig. 1. The master and slave robots

[2] If the dynamic parameters of a robotic manipulator are uncertain, then the constant vector Θ_i is unknown to the controller.

https://doi.org/10.1017/S0263574714001295 Published online by Cambridge University Press

exchange the positions of the end-effector X_m and X_s through the communication network. These signals are subject to *asymmetric* time-varying communication delays, which are denoted by $T_m(t)$ and $T_s(t)$. In the subsequent studies, the time-varying delays are considered under the following assumption.

Assumption 3. The time-varying delays are continuously differentiable, have known upper bounds $\bar{T}_i$, i.e. $0 \leq T_i(t) < \bar{T}_i < \infty$, and satisfy the condition that $-d_i^l \leq \dot{T}_i(t) < 1$, where $d_i^l \geq 0$ are the absolute value of the lower bounds of the derivative of time-varying delays[3].

According to the proposed control architecture (Fig. 1), the delayed task-space tracking errors between the master and slave robots are defined as

$$e_m(t) = X_s(t - T_s(t)) - X_m(t),\ e_s(t) = X_m(t - T_m(t)) - X_s(t). \quad (3)$$

Therefore, the derivative of tracking errors are given by $\dot{e}_m(t) = (1 - \dot{T}_s(t))\dot{X}_s(t - T_s(t)) - \dot{X}_m(t)$ and $\dot{e}_s(t) = (1 - \dot{T}_m(t))\dot{X}_m(t - T_m(t)) - \dot{X}_s(t)$, where $1 - \dot{T}_m(t)$ and $1 - \dot{T}_s(t)$ result from taking the time derivative of delayed position signals and are assumed to be *unknown* in the controller. Following the formulated teleoperation system, the objective of this paper is to develop a control algorithm so that the closed-loop teleoperation system under time-varying communication delays ($T_i(t)$) and dynamic uncertainties (unknown Θ_i) can be guaranteed to be stable with task-space position tracking i.e. $\lim_{t\to\infty} e_m(t) = 0$ and $\lim_{t\to\infty} e_s(t) = 0$. In the rest of this paper, the subscript $i = m$ denotes the master robot, and $i = s$ represents the slave robot.

2.1. Instrumental lemma

The following lemmas are utilized in this paper to prove stability and tracking performance of the proposed task-space teleoperation system.

Lemma 1. [7] *Given signals $x, y \in R^n$, $\forall T(t)$ such that $0 < T(t) \leq \bar{T} < \infty$ and $\alpha > 0$, the following inequality holds:*

$$-\int_0^t x^T(\sigma) \int_{-T(\sigma)}^0 y(\sigma + \theta) d\theta d\sigma \leq \frac{\alpha}{2}||x||_2^2 + \frac{\bar{T}^2}{2\alpha}||y||_2^2,$$

where $||\cdot||_2$ denotes the $\mathcal{L}_2$ norm of the enclosed signal.

Lemma 2. [17] *Given signals $x, y \in R^n$ and time-varying delays $0 \leq T(t) \leq \bar{T}$, where $\bar{T}$ is the upper bound of $T(t)$, the following inequality holds:*

$$-2x^T(t)\int_{t-T(t)}^t y(\sigma)d\sigma - \int_{t-T(t)}^t y^T(\sigma)y(\sigma)d\sigma \leq \bar{T}x^T(t)x(t).$$

3. Task-Space Teleoperator with Time-Varying Delays

3.1. Control design

To achieve task-space teleoperation under dynamic uncertainties and time-varying communication delays, the control input for the teleoperation system (1) is given as

$$\begin{aligned}\tau_i &= \hat{M}_i(q_i)\dot{\xi}_i + \hat{C}_i(q_i, \dot{q}_i)\xi_i + \hat{g}_i(q_i) - k_p p_i - k_r J_i^T r_i + k_v J_i^T \dot{e}_i \\ &= Y_i(q_i, \dot{q}_i, \xi_i, \dot{\xi}_i)\hat{\Theta}_i - k_p p_i - k_r J_i^T r_i + k_v J_i^T \dot{e}_i, \end{aligned} \quad (4)$$

[3] The necessary condition $\dot{T}_i(t) < 1$ results from the causality implications of continuous-time control system. The reader is referred to ref. [28] for more details.

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where $\hat{M}_i(q_i)$, $\hat{C}_i(q_i, \dot{q}_i)$, and $\hat{g}_i(q_i)$ denote the estimate of $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $g_i(q_i)$, which may include unknown parameters of the robotic manipulator, Property 2 is utilized for the second equality, $\hat{\Theta}_i$ is the estimate of the vector Θ_i, and $k_p,\ k_r,\ k_v \in R^+$ are constant control gains.

The control signal ξ_i in (4) is defined as $\xi_m = \lambda J_m^{-1} e_m$ for the non-redundant master robot, and $\xi_s = \lambda J_s{}^+ e_s + (I_q - J_s^+ J_s)\psi_s$ for the redundant slave robot with the use of null-space method.[22,29,30] In the definition of ξ_i, $\lambda \in R^+$ is a control constant, $J_m^{-1} \in R^{p\times p}$ is the inverse of J_m, $J_s^+ \in R^{q\times p}$ denotes the pseudoinverse of J_s, and $\psi_s \in R^p$ is the negative gradient of an appropriately defined convex function for sub-task control[4]. The pseudoinverse J_s^+, which is given by $J_s^+ = J_s^T (J_s J_s^T)^{-1}$, satisfies $J_s J_s^+ = I_q$, and has the following properties:[31,32]

$$J_s(I_q - J_s^+ J_s) = 0, \tag{5}$$

$$(I_q - J_s^+ J_s)(I_q - J_s^+ J_s) = I_q - J_s^+ J_s. \tag{6}$$

Additionally, the control signals p_i and r_i are designed as $p_i = \dot{q}_i - \xi_i$ and $r_i = J_i p_i = -\lambda e_i + \dot{X}_i$, where the robot kinematics (2) is utilized for r_i and the property of the pseudoinverse (5) is utilized for r_s.

By substituting the control input (4) into the robot dynamics (1), the closed-loop control system of the teleoperation is given as

$$\begin{cases} M_m \dot{p}_m + C_m p_m + k_p p_m = Y_m \tilde{\Theta}_m - k_r J_m^T r_m + k_v J_m^T \dot{e}_m + J_m^T F_h \\ M_s \dot{p}_s + C_s p_s + k_p p_s = Y_s \tilde{\Theta}_s - k_r J_s^T r_s + k_v J_s^T \dot{e}_s - J_s^T F_e \end{cases}, \tag{7}$$

where $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ is the estimation errors of the unknown parameters. In the proposed system, we let the uncertain dynamic parameters $\hat{\Theta}_i$ be generated by the adaption law

$$\dot{\hat{\Theta}}_i = -\Gamma_i Y_i^T p_i, \tag{8}$$

where Γ_i is a positive-definite constant matrix.

Denote by $\mathcal{C} = \mathcal{C}([-T, 0], R^n)$ the Banach space of continuous functions mapping the interval $[-T, 0]$ into R^n, with the topology of uniform convergence. Let $z = [p_m,\ p_s,\ e_m,\ e_s,\ \tilde{\Theta}_m,\ \tilde{\Theta}_s,\ \dot{X}_m,\ \dot{X}_s]$ be the state of the system and define $z_t = z(t + \phi) \in \mathcal{C}$, $-T \le \phi \le 0$ as the state of the system.[33] We assume in this paper that $z(\phi) = \eta(\phi)$, $\eta \in \mathcal{C}$, and all signals belong to $\mathcal{L}_{2e}$, the extended $\mathcal{L}_2$ space.

3.2. *Stability analysis in free motion*

By denoting the maximum round-trip delay $\bar{T} = \bar{T}_m + \bar{T}_s$, the result of task-space teleoperation system with time-varying communication delays follows.

Theorem 3.1. *Consider the closed-loop teleoperation system described by* (7) *and* (8)*. Assume that the Jacobian matrix of the non-redundant master manipulator is full rank. If the control gains satisfy the condition that*

$$\left(k_r - \frac{k_v d_s^l}{2}\right)\left(k_r - \frac{k_v d_m^l}{2}\right) > \lambda^2 k_r^2 \bar{T}^2, \tag{9}$$

then in free motion ($F_h \equiv 0,\ F_e \equiv 0$) *the task-space position tracking error e_i, the robotic velocity $\dot{X}_i$, and the control signal p_i asymptotically approach the origin in the presence of time-varying communication delays.*

[4] The details of sub-task control are discussed in Section 3.4.

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Proof. Consider the positive-definite storage functional V for the closed-loop teleoperation system:

$$V(z_t) = \frac{1}{2}\sum_{i=\{m,s\}}\left(p_i^T M_i p_i + \tilde{\Theta}_i^T \Gamma_i^{-1}\tilde{\Theta}_i + \lambda k_v e_i^T e_i + k_v \int_{t-T_i(t)}^{t} \dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma\right) + \lambda k_r (X_m - X_s)^T (X_m - X_s).$$

Taking the time-derivative of V along the trajectories of the teleoperation system with the use of the adaptive law (8), Property 3, and $r_i = J_i p_i$, the derivative becomes

$$\dot{V} = \sum_{i=\{m,s\}}\Big(-k_p p_i^T p_i - k_r r_i^T r_i + k_v r_i^T \dot{e}_i + \lambda k_v e_i^T \dot{e}_i + \frac{k_v}{2}\dot{X}_i^T\dot{X}_i - \frac{k_v}{2}(1-\dot{T}_i(t))\dot{X}_i^T(t-T_i(t))\dot{X}_i(t-T_i(t))\Big) + 2\lambda k_r (X_m - X_s)^T(\dot{X}_m - \dot{X}_s).$$

Substituting $r_i = -\lambda e_i + \dot{X}_i$, we have $-k_r r_i^T r_i + k_v r_i^T \dot{e}_i = -\lambda^2 k_r e_i^T e_i + 2\lambda k_r \dot{X}_i^T e_i - k_r \dot{X}_i^T \dot{X}_i - \lambda k_v e_i^T \dot{e}_i + k_v \dot{X}_i^T \dot{e}_i$. Thus, the derivative of the storage function can be written as

$$\dot{V} \le \sum_{i=\{m,s\}}\left(-k_p p_i^T p_i - \lambda^2 k_r e_i^T e_i - k_r \dot{X}_i^T \dot{X}_i\right) + 2\lambda k_r \dot{X}_m^T e_m + 2\lambda k_r \dot{X}_s^T e_s + 2\lambda k_r (X_m - X_s)^T(\dot{X}_m - \dot{X}_s) + \sum_{i=\{m,s\}}\left(k_v \dot{X}_i^T \dot{e}_i + \frac{k_v}{2}\dot{X}_i^T\dot{X}_i - \frac{k_v}{2}(1-\dot{T}_i(t))\dot{X}_i^T(t-T_i(t))\dot{X}_i(t-T_i(t))\right). \tag{10}$$

Substituting e_i into $2\lambda k_r \dot{X}_m^T e_m + 2\lambda k_r \dot{X}_s^T e_s$, we have the following relationship:

$$\begin{aligned} &2\lambda k_r \dot{X}_m^T e_m + 2\lambda k_r \dot{X}_s^T e_s + 2\lambda k_r (X_m - X_s)^T(\dot{X}_m - \dot{X}_s) \\ &= 2\lambda k_r\big(-\dot{X}_m(X_s - X_s(t-T_s(t))) - \dot{X}_s(X_m - X_m(t-T_m(t)))\big) \\ &= -2\lambda k_r \dot{X}_m^T \int_{-T_s(t)}^{0} \dot{X}_s(t+\sigma)d\sigma - 2\lambda k_r \dot{X}_s \int_{-T_m(t)}^{0} \dot{X}_m(t+\sigma)d\sigma. \end{aligned} \tag{11}$$

Furthermore, by expanding $\dot{e}_i$, $k_v \dot{X}_m^T \dot{e}_m$ in the second summation term of (10) can be rewritten as

$$\begin{aligned} k_v \dot{X}_m^T \dot{e}_m &= k_v \dot{X}_m^T((1-\dot{T}_s(t))\dot{X}_s(t-T_s(t)) - \dot{X}_m) \\ &= -k_v \dot{X}_m^T \dot{X}_m + k_v(1-\dot{T}_s(t))\dot{X}_m^T \dot{X}_s(t-T_s(t)). \end{aligned}$$

Similarly, we have

$$k_v \dot{X}_s^T \dot{e}_s = -k_v \dot{X}_s^T \dot{X}_s + k_v(1-\dot{T}_m(t))\dot{X}_s^T \dot{X}_m(t-T_m(t)).$$

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Therefore, the second summation term in (10) becomes

$$\sum_{i=\{m,s\}} \left(k_v \dot{X}_i^T \dot{e}_i + \frac{k_v}{2}\dot{X}_i^T \dot{X}_i - \frac{k_v}{2}(1-\dot{T}_i(t))\dot{X}_i^T(t-T_i(t))\dot{X}_i(t-T_i(t))\right)$$
$$= -\frac{k_v}{2}(1-\dot{T}_m(t))(\dot{X}_m(t-T_m(t))-\dot{X}_s)^T(\dot{X}_m(t-T_m(t))-\dot{X}_s)$$
$$-\frac{k_v}{2}(1-\dot{T}_s(t))(\dot{X}_s(t-T_s(t))-\dot{X}_m)^T(\dot{X}_s(t-T_s(t))-\dot{X}_m)$$
$$-\dot{T}_s(t)\frac{k_v}{2}\dot{X}_m^T\dot{X}_m - \dot{T}_m(t)\frac{k_v}{2}\dot{X}_s^T\dot{X}_s$$
$$\leq \frac{k_v d_s^l}{2}\dot{X}_m^T\dot{X}_m + \frac{k_v d_m^l}{2}\dot{X}_s^T\dot{X}_s, \tag{12}$$

where the inequality results from Assumption 3. By substituting (11) and (12) into (10), the derivative becomes

$$\dot{V} \leq \sum_{i=\{m,s\}}\left(-k_p p_i^T p_i - \lambda^2 k_r e_i^T e_i - k_r \dot{X}_i^T\dot{X}_i\right) + \frac{k_v d_s^l}{2}\dot{X}_m^T\dot{X}_m$$
$$+\frac{k_v d_m^l}{2}\dot{X}_s^T\dot{X}_s - 2\lambda k_r \dot{X}_m^T\int_{-T_s(t)}^{0}\dot{X}_s(t+\sigma)d\sigma$$
$$-2\lambda k_r \dot{X}_s\int_{-T_m(t)}^{0}\dot{X}_m(t+\sigma)d\sigma.$$

By integrating the above equation from 0 to t with the use of Lemma 1, we have

$$V(t)-V(0) \leq \sum_{i=\{m,s\}}\left(-k_p\|p_i\|_2^2 - \lambda^2 k_r\|e_i\|_2^2 - k_r\|\dot{X}_i\|_2^2\right)$$
$$+\frac{k_v d_s^l}{2}\|\dot{X}_m\|_2^2 + \frac{k_v d_m^l}{2}\|\dot{X}_s\|_2^2 + 2\lambda k_r\left(\frac{\alpha_1}{2}\|\dot{X}_m\|_2^2\right.$$
$$\left.+\frac{\bar{T}_s^2}{2\alpha_1}\|\dot{X}_s\|_2^2 + \frac{\alpha_2}{2}\|\dot{X}_s\|_2^2 + \frac{\bar{T}_m^2}{2\alpha_2}\|\dot{X}_m\|_2^2\right), \tag{13}$$

where $\|\cdot\|_2$ denotes the $\mathcal{L}_2$ norm of the enclosed signal. The coefficients of $\|\dot{X}_m\|_2^2$ and $\|\dot{X}_s\|_2^2$ in (13) have to be negative in order to ensure that the storage function V is a non-increasing function. Hence, we have the inequalities $k_r - \frac{k_v d_s^l}{2} - \lambda k_r(\alpha_1 + \frac{\bar{T}_m^2}{\alpha_2}) > 0$ and $k_r - \frac{k_v d_m^l}{2} - \lambda k_r(\alpha_2 + \frac{\bar{T}_s^2}{\alpha_1}) > 0$. As α_1 and α_2 are positive constants resulting from Lemma 1, the above inequalities result in the condition that

$$\left(k_r - \frac{k_v d_s^l}{2}\right)\left(k_r - \frac{k_v d_m^l}{2}\right) > \lambda^2 k_r^2(\bar{T}_m + \bar{T}_s)^2 > \lambda^2 k_r^2 \bar{T}^2. \tag{14}$$

Therefore, if control gains λ, k_r, k_v and time delays satisfy the condition (14), then $V(t) - V(0) \leq 0,\ \forall t > 0$. Hence, signals $p_i,\ e_i,\ \dot{X}_i \in \mathcal{L}_2$ from (13). In addition, we get that $p_i,\ \tilde{\Theta}_i,\ e_i$, and $X_m - X_s \in \mathcal{L}_\infty$ because V is bounded. Since $r_i = -\lambda e_i + \dot{X}_i$ and $e_i, \dot{X}_i \in \mathcal{L}_2$, we obtain $r_i \in \mathcal{L}_2$. As $\dot{X}_i(t)$ and $\dot{T}_i(t)$ are bounded, we obtain $\dot{e}_i \in \mathcal{L}_\infty$. As $e_i \in \mathcal{L}_2$ and $\dot{e}_i \in \mathcal{L}_\infty$, it is shown by invoking Barbalat's Lemma[27,34] that $\lim_{t\to\infty} e_i(t) = 0$. Therefore, the position tracking of the teleoperation system in the task space under time-varying communication delays is guaranteed.

Subsequently, we can obtain that the control input is bounded $\tau_i \in \mathcal{L}_\infty$. Hence, we get $\dot{p}_i \in \mathcal{L}_\infty$ from the closed-loop system (7) by utilizing Property 1 and Property 4. As $p_i \in \mathcal{L}_2$ and $\dot{p}_i \in \mathcal{L}_\infty$, we have $\lim_{t\to\infty} p_i(t) = 0$ by invoking Barbalat's Lemma again. Additionally, taking the time derivative

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of $r_i = J_i p_i$, which is $\dot{r}_i = \dot{J}_i p_i + J_i \dot{p}_i$, we obtain that $\dot{r}_i$ is bounded. Since $r_i \in \mathcal{L}_2$ and $\dot{r}_i \in \mathcal{L}_\infty$, we conclude that $\lim_{t\to\infty} r_i(t) = 0$. Therefore, $r_i = -\lambda e_i + \dot{X}_i$ and $\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} r_i(t) = 0$ lead to $\lim_{t\to\infty} \dot{X}_i(t) = 0$. Consequently, the proposed task-space teleoperation system is stable, and the position tracking error e_i, the robotic velocity $\dot{X}_i$, and the control signal p_i approach the origin in the presence of time-varying delays. □

We next consider the case when the external forces from the human operator and the remote environment are passive with (F_h, r_m) and (F_e, r_s) as input–output pairs, respectively. Thus, there exist constants $\beta_h, \beta_e \in R^+$ such that[12,13]

$$-\int_0^t F_h^T(\sigma) r_m(\sigma) d\sigma \geq -\beta_h, \quad \int_0^t F_e^T(\sigma) r_s(\sigma) d\sigma \geq -\beta_e. \tag{15}$$

Therefore, the task-space teleoperation system with passive external forces follows.

Corollary 3.2. *Consider the closed-loop teleoperation system described by* (7) *and* (8). *Assume that the Jacobian matrix of the non-redundant master manipulator is full rank. If the control gains satisfy condition* (9), *and the external forces are passive with* (15), *then the task-space position tracking error e_i, the robotic velocity $\dot{X}_i$, and the control signal p_i asymptotically approach the origin with time-varying communication delays.*

Proof. By considering the storage functional candidate for the teleoperation system as

$$\begin{aligned} V(z_t) = \frac{1}{2} \sum_{i=\{m,s\}} \Bigg(& p_i^T M_i p_i + \tilde{\Theta}_i^T \Gamma_i^{-1} \tilde{\Theta}_i + \lambda k_v e_i^T e_i \\ & + \frac{k_v}{2} \int_{t-T_i(t)}^t \dot{X}_i^T(\sigma) \dot{X}_i(\sigma) d\sigma \Bigg) + \lambda k_r (X_m - X_s)^T (X_m - X_s) \\ & + \int_0^t \left(-F_h^T(\sigma) r_m(\sigma) + F_e^T(\sigma) r_s(\sigma) \right) d\sigma + \beta_h + \beta_e, \end{aligned}$$

the proof can be completed by following the proof of Theorem 3.1. Therefore, under passive external forces, the closed-loop control system is stable with $\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} \dot{X}_i(t) = \lim_{t\to\infty} p_i(t) = 0$. □

Remark 1. The proposed control system in Theorem 3.1 and Corollary 3.2 is studied without the requirement of *exact* knowledge of the time-varying delay. The only required parameters are the upper bound of round-trip delay $\bar{T}$ and the absolute value of the lower bound of the time derivative of delay d_i^l. Given the control gains, which are contingent on $\bar{T}$ and d_i^l, the proposed controller can guarantee stability and tracking performance of the teleoperation, both in free motion and with passive forces. For time-varying delay with large $\bar{T}$ and d_i^l, the stability of the teleoperation system can be guaranteed by decreasing λ and k_v, and increasing k_r.

Remark 2. In the proposed control algorithm, the upper bound of communication delay $\bar{T}$ and the lower bound of time-derivative delay d_i^l are necessary for stability and tracking performance. Both of these values can be approximately obtained by transmitting a known continuous function $f_i(t)$, such as $f_i(t) = t/N$ with a positive constant N, through the communication network. Thus, the function arrives at the destination with the value of $f_i(t - T_i(t)) = (t - T_i(t))/N$. Therefore, the upper bound of time-varying delay can be obtained by comparing the received function $f_i(t - T_i(t))$ with the original function that $T_i(t) = N(f_i(t) - f_i(t - T_i(t)))$. Furthermore, by taking the time-derivative of $f_i(t - T_i(t))$, the value $-d_i^l \leq \dot{T}_i(t)$ can be computed from $\dot{T}_i(t) = 1 - N\dot{f}_i(t - T_i(t))$.

Remark 3. Since the problem of constant delay is a special case of the teleoperation system studied in Theorem 3.1 and Corollary 3.2 when $\dot{T}_i(t) \equiv 0$, by letting $d_i^l \equiv 0$ for condition (9), we can determine that the constant delay teleoperation system is stable with the convergence of tracking errors to the origin if $\lambda \bar{T} < 1$. Therefore, the values of k_v and k_r would not affect system stability or tracking performance in a constant delay problem.

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3.3. Stability analysis with non-zero external force

The stability of the task-space teleoperation system with time-varying communication delays when the external forces from the human operator and the remote environment are non-zero and non-passive is studied subsequently. The external forces are given by $F_h = k_{fh} - k_h r_m$ and $F_e = k_{fe} + k_e r_s$, where k_{fh} and k_{fe} are bounded vectors in R^p, and k_h, k_e are bounded non-negative constants. According to ref. [11], we assume that the teleoperation system exists no dynamic uncertainty, which implies $\tilde{\Theta}_i = 0$, in the case of non-passive force. Therefore, the closed-loop system of the teleoperation becomes

$$\begin{cases} M_m \dot{p}_m + C_m p_m + k_p p_m = -k_r J_m^T r_m + k_v J_m^T \dot{e}_m + J_m^T F_h \\ M_s \dot{p}_s + C_s p_s + k_p p_s = -k_r J_s^T r_s + k_v J_s^T \dot{e}_s - J_s^T F_e. \end{cases} \tag{16}$$

By defining the constant gain $\beta_k = k_r + k_f$, where $k_f = \min\{k_h, k_e\}$, and denoting $z = [p_m, p_s, e_m, e_s]$ the state of the system, the result for teleoperation system with non-passive external forces under time-varying communication delays is now presented.

Theorem 3.3. *Consider the closed-loop teleoperation system described by* (16). *Assume that the Jacobian matrix of the non-redundant master manipulator is full rank, and the external forces are given by $F_h = k_{fh} - k_h r_m$ and $F_e = k_{fe} + k_e r_s$. If the control gains and delays satisfy the condition that*

$$2\beta_k(1 - \lambda\bar{T}) > \max\left\{k_v d_s^l + 1, k_v d_m^l + 1\right\}, \tag{17}$$

then all signals in the control system are ultimately bounded.

Proof. Consider the positive-definite storage functional as

$$\begin{aligned} V(z_t) = \frac{1}{2}\sum_{i=\{m,s\}}\Bigg(& p_i^T M_i p_i + \lambda k_v e_i^T e_i + k_v \int_{t-T_i(t)}^{t} \dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma \\ &+ 2\lambda\beta_k \int_{t-\bar{T}_i}^{t} (\sigma - t + \bar{T}_i)\dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma\Bigg) \\ &+ \lambda\beta_k (X_m - X_s)^T (X_m - X_s). \end{aligned} \tag{18}$$

Taking the time-derivative of the storage functional and following the proof of Theorem 3.1 with the substituting of F_h and F_e, we get

$$\begin{aligned} \dot{V} = \sum_{i=\{m,s\}} \Bigg(& -k_p p_i^T p_i - k_r r_i^T r_i + k_v r_i^T \dot{e}_i + \lambda k_v e_i^T \dot{e}_i + \frac{k_v}{2}\dot{X}_i^T \dot{X}_i \\ & - \frac{k_v}{2}(1 - \dot{T}_i(t))\dot{X}_i^T(t - T_i(t))\dot{X}_i(t - T_i(t)) + \lambda\beta_k \bar{T}_i \dot{X}_i^T \dot{X}_i \\ & - \lambda\beta_k \int_{t-\bar{T}_i}^{t} \dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma\Bigg) + 2\lambda\beta_k (X_m - X_s)^T (\dot{X}_m - \dot{X}_s) \\ & + k_{fh}^T r_m - k_{fe}^T r_s - k_h r_m^T r_m - k_e r_s^T r_s. \end{aligned} \tag{19}$$

Since the last two terms are less than $-k_f r_m^T r_m - k_f r_s^T r_s$, by substituting $r_i = -\lambda e_i + \dot{X}_i$, we have

$$\begin{aligned} & -k_r r_i^T r_i + k_v r_i^T \dot{e}_i - k_f r_i^T r_i \\ & \quad = -\lambda^2 \beta_k e_i^T e_i + 2\lambda\beta_k \dot{X}_i^T e_i - \beta_k \dot{X}_i^T \dot{X}_i - \lambda k_v e_i^T \dot{e}_i + k_v \dot{X}_i^T \dot{e}_i. \end{aligned}$$

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In addition, we have the relationships that $k_{fh}^T r_m \leq k_{fh}^T k_{fh} + \frac{\lambda^2}{2} e_m^T e_m + \frac{1}{2}\dot{X}_m^T \dot{X}_m$, $-k_{fe}^T r_s \leq k_{fe}^T k_{fe} + \frac{\lambda^2}{2} e_s^T e_s + \frac{1}{2}\dot{X}_s^T \dot{X}_s$ and

$$-\lambda\beta_k \int_{t-\bar{T}_i}^{t} \dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma \leq -\lambda\beta_k \int_{t-T_i(t)}^{t} \dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma.$$

Utilizing the aforementioned mathematical manipulations, the derivative becomes

$$\begin{aligned}\dot{V} \leq & \sum_{i=\{m,s\}} \Bigg(-k_p p_i^T p_i - \lambda^2\beta_k e_i^T e_i - \beta_k \dot{X}_i^T \dot{X}_i + k_v \dot{X}_i^T \dot{e}_i \\ & + \frac{k_v}{2}\dot{X}_i^T \dot{X}_i - \frac{k_v}{2}(1-\dot{T}_i(t))\dot{X}_i^T(t-T_i(t))\dot{X}_i(t-T_i(t)) \\ & + \lambda\beta_k \bar{T}_i \dot{X}_i^T \dot{X}_i - \lambda\beta_k \int_{t-T_i(t)}^{t} \dot{X}_i^T(\sigma)\dot{X}_i(\sigma)d\sigma \Bigg) + 2\lambda\beta_k \dot{X}_m^T e_m \\ & + 2\lambda\beta_k \dot{X}_s^T e_s + 2\lambda\beta_k (X_m - X_s)^T(\dot{X}_m - \dot{X}_s) + k_{fh}^T k_{fh} \\ & + k_{fe}^T k_{fe} + \sum_{i=\{m,s\}} \left(\frac{\lambda^2}{2} e_i^T e_i + \frac{1}{2}\dot{X}_i^T \dot{X}_i \right). \end{aligned} \tag{20}$$

By employing the relationships in (11) and (12), $\dot{V}$ can be written as

$$\begin{aligned}\dot{V} \leq & \sum_{i=\{m,s\}} \left(-k_p p_i^T p_i - \lambda^2\beta_k e_i^T e_i - \beta_k \dot{X}_i^T \dot{X}_i + \frac{\lambda^2}{2} e_i^T e_i + \frac{1}{2}\dot{X}_i^T \dot{X}_i \right) \\ & + \frac{k_v d_s^l}{2}\dot{X}_m^T \dot{X}_m + \frac{k_v d_m^l}{2}\dot{X}_s^T \dot{X}_s + \lambda\beta_k \bar{T}_m \dot{X}_m^T \dot{X}_m + \lambda\beta_k \bar{T}_s \dot{X}_s^T \dot{X}_s \\ & - 2\lambda\beta_k \dot{X}_m^T \int_{-T_s(t)}^{0} \dot{X}_s(t+\sigma)d\sigma - 2\lambda\beta_k \dot{X}_s \int_{-T_m(t)}^{0} \dot{X}_m(t+\sigma)d\sigma \\ & - \lambda\beta_k \int_{t-T_m(t)}^{t} \dot{X}_m^T(\sigma)\dot{X}_m(\sigma)d\sigma - \lambda\beta_k \int_{t-T_s(t)}^{t} \dot{X}_s^T(\sigma)\dot{X}_s(\sigma)d\sigma \\ & + k_{fh}^T k_{fh} + k_{fe}^T k_{fe}. \end{aligned} \tag{21}$$

By exploiting Lemma 2 for the integral terms in (21), we get

$$\begin{aligned}\dot{V} \leq & \sum_{i=\{m,s\}} \Bigg(-k_p p_i^T p_i - \lambda^2\beta_k e_i^T e_i - \beta_k \dot{X}_i^T \dot{X}_i + \lambda\bar{T}\beta_k \dot{X}_i^T \dot{X}_i \\ & + \frac{\lambda^2}{2} e_i^T e_i + \frac{1}{2}\dot{X}_i^T \dot{X}_i \Bigg) + \frac{k_v d_s^l}{2}\dot{X}_m^T \dot{X}_m + \frac{k_v d_m^l}{2}\dot{X}_s^T \dot{X}_s + k_{fh}^T k_{fh} \\ & + k_{fe}^T k_{fe}, \end{aligned} \tag{22}$$

where $\lambda\bar{T}\beta_k \dot{X}_m^T \dot{X}_m$ is the sum of $\lambda\bar{T}_m\beta_k \dot{X}_m^T \dot{X}_m$ in (21) and $\lambda\bar{T}_s\beta_k \dot{X}_m^T \dot{X}_m$ resulting from Lemma 2. By following similar argument, we obtain $\lambda\bar{T}\beta_k \dot{X}_s^T \dot{X}_s$ in (22). Therefore, if the control gains and delays

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satisfy condition (17), we get

$$\dot{V} \leq -k_p p_m^T p_m - k_p p_p^T p_s - \left(\lambda^2 \beta_k - \frac{\lambda^2}{2}\right) e_m^T e_m$$
$$-\left(\lambda^2 \beta_k - \frac{\lambda^2}{2}\right) e_s^T e_s + k_{fh}^T k_{fh} + k_{fe}^T k_{fe}$$
$$\leq -\beta_{\min}(1-\rho)\|z\|^2 - \beta_{\min}\rho\|z\|^2 + k_{fh}^T k_{fh} + k_{fe}^T k_{fe}$$
$$\leq -\beta_{\min}(1-\rho)\|z\|^2 \quad \forall \|z\| \geq \sqrt{\frac{k_{fh}^T k_{fh} + k_{fe}^T k_{fe}}{\beta_{\min}\rho}}, \tag{23}$$

where $0 < \rho < 1$ and $\beta_{\min} = \min\left\{k_p, \lambda^2\beta_k - \frac{\lambda^2}{2}\right\} > 0$ as $\beta_k > 1/2$. Considering that $\beta_{\min}$ and ρ are positive and bounded away from zero, and k_{fh}, k_{fe} are assumed to be bounded constant, we have that $\dot{V} < 0$, $\forall z(t) \neq 0$.[11] Consequently, we conclude that the signals of the teleoperation system are ultimately bounded. □

Remark 4. By observing conditions (9) and (17), the control gain λ and upper bound of round-trip delay $\bar{T}$ have to satisfy $\lambda\bar{T} < 1$ for the existence of at least one solution to the control gains. Therefore, the value of λ can be selected from the estimation of $\bar{T}$ in the communication network by $\lambda < 1/\bar{T}$. Consequently, the control gains k_r and k_v can be decided according to d_m^l and d_s^l by adhering to conditions (9) and (17).

Remark 5. Although for simplicity the control gains λ, k_p, k_r, and k_v are assumed to be constant, the proposed teleoperation system can be easily extended to accommodate control gains in matrix forms.

3.4. Sub-task control

The advantage of controlling teleoperation system in task-space is that the null space resulting from the redundancy can be considered to achieve various additional missions to improve teleoperation performance. Since the slave robot is assumed to be a redundant manipulator, where the null space of the Jacobian matrix has a minimum dimension of $q - p$, the redundancy can be employed to achieve several sub-tasks in addition to following the task-space position of the master robot. In this section, we briefly introduce the null-space control of the redundant slave robot that is utilized in the numerical examples. The reader is referred to refs. [24] and [35] for more details.

According to ref. [35], the sub-task control error for the slave robot can be defined as $e_{st} = (I_q - J_s^+ J_s)(\dot{q}_s - \psi_s)$, where $(I_q - J_s^+ J_s)\psi_s$ is utilized to control the slave robot in the null space of J_s.[36] Thus, premultiplying p_s by $(I_q - J_s^+ J_s)$ with the use of the properties of pseudoinverse (6), we have that $(I_q - J_s^+ J_s)p_s = (I_q - J_s^+ J_s)(\dot{q}_s - \psi_s) = e_{st}$.[30,32] Consequently, the convergence of signal p_s would also imply the convergence of the sub-task tracking error e_{st}. Thus, the proposed controller for the task-space teleoperation can guarantee that the sub-task tracking error goes to the origin because $\lim_{t\to\infty} p_s(t) = 0$ in free motion (Theorem 3.1) or in the presence of passive external forces (Corollary 3.2). Moreover, if the external forces are non-passive, then the sub-task tracking error is bounded resulting from bounded p_s (Theorem 3.3).

In the design of sub-task control, the vector ψ_s in ξ_s is considered as a negative gradient of an auxiliary function $f_s(q_s)$ such that $\psi_s = -\frac{\partial}{\partial q_s} f_s(q_s)$.[29,30,32] The auxiliary function $f_s(q_s)$ can be designed for various sub-task controls, where the lower value corresponds to more desirable configurations. Therefore, we can control the null space of the slave robot by designing appropriate function for $f_s(q_s)$ to achieve additional task. Various functions can be considered as the auxiliary functions for the redundant slave robot to avoid singularities, limit joint angles, and avoid collisions.[22,30,31,37,38] Since the development of sub-task control is not the main contribution of this paper and the null-space control is only utilized to demonstrate the benefits of using task-space teleoperation, readers are referred to null-space control in the literature.[29,36] In the next section, the sub-task control for increasing manipulability[24,38] and collision avoidance[22] are considered for the proposed task-space teleoperation system.

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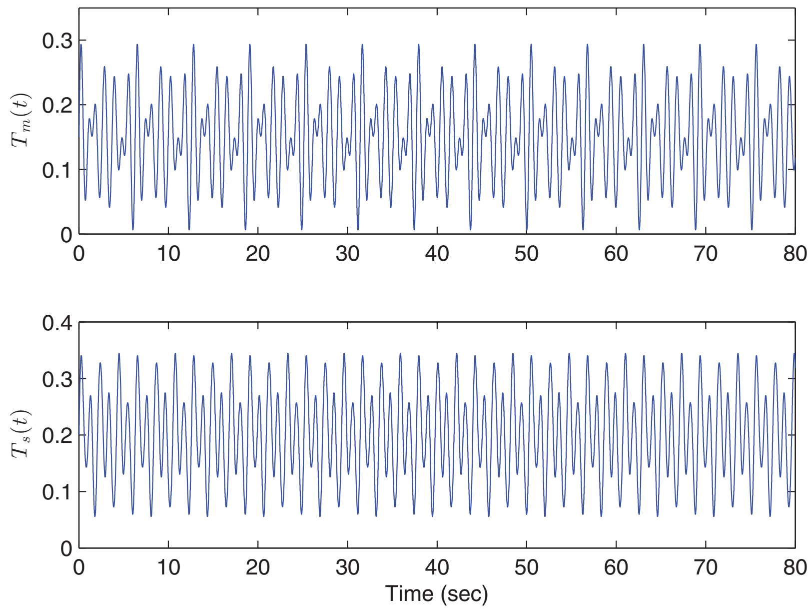

Fig. 2. Asymmetric time-varying communication delays in the simulations.

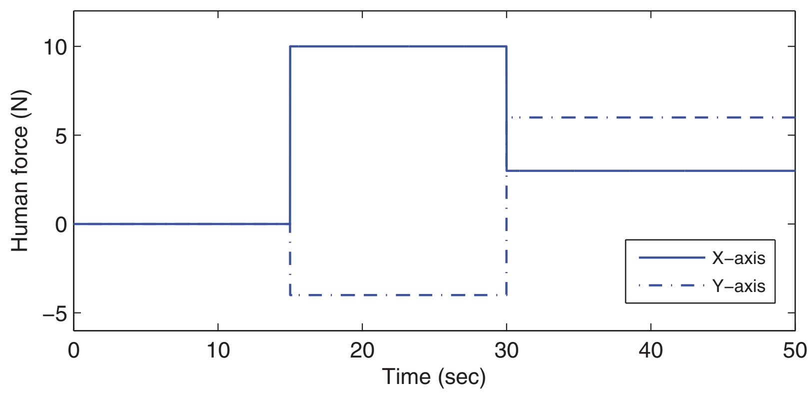

Fig. 3. Generalized human input force.

4. Simulation Results

Numerical examples are addressed in this section to demonstrate the efficacy of the proposed task-space teleoperation system under time-varying communication delays and dynamic uncertainties. In the following simulations, the time-varying delays are selected as $T_m(t) = 0.15 + 0.02\sin(8t) + 0.06\sin(7t) + 0.07\sin(5t)$ and $T_s(t) = 0.2 + 0.01\sin(10t) + 0.1\sin(6t) + 0.05\sin(3t)$, which are illustrated in Fig. 2. Based on the given communication delays and their derivative, we have $\bar{T} = 0.65$ s, $d_m^l = 0.80$, and $d_s^l = 0.71$. Moreover, both delays satisfy the causality condition that $\dot{T}_m(t),\ \dot{T}_s(t) < 1$.

In the first case, we consider a 2-DOF planar manipulator for the non-redundant master robot, and a 3-DOF planar manipulator for the redundant slave robot. The robot dynamics are referred to ref. [27] where the physical parameters of the robots are given by $m_m = [3.14,\ 2.26]$ kg, $I_m = [0.16,\ 0.07]$ kgm^2, $L_m = [1.04,\ 0.96]$ m for the master robot, $m_s = [3.12,\ 1.85,\ 1.02]$ kg, $I_s = [0.12,\ 0.07,\ 0.04]$ kgm^2, $L_s = [0.74,\ 0.72,\ 0.64]$ m for the slave robot, and $g = 9.8$ m/s^2. The initial conditions of the robotic systems are $q_m(0) = [0.2,\ 0.5]$ rad and $q_s(0) = [-0.6,\ 0.3,\ 0.4]$ rad with zero initial angular velocities. Additionally, the adaptive control gains are given as $\Gamma_m = 0.2I_5$, $\Gamma_s = 0.1I_9$, $\hat{\Theta}_m(0) = [4,\ 1,\ 0.6,\ 4,\ 1]^T$, and $\hat{\Theta}_s(0) = [3.5,\ 1.5,\ 0.5,\ 1.5,\ 0.5,\ 0.5,\ 43.5,\ 19.5,\ 6.5]^T$.

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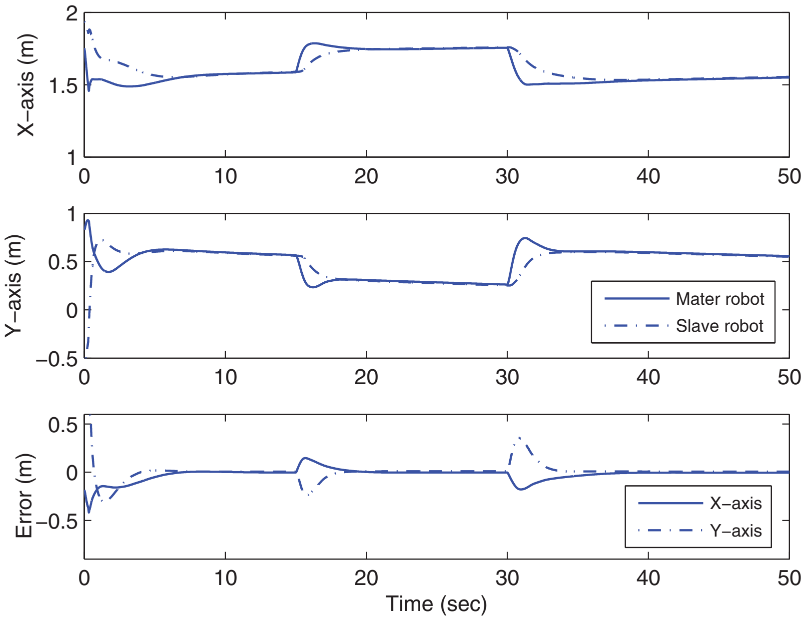

Fig. 4. Position configurations and tracking errors of the teleoperation system with a 3-DOF slave robot.

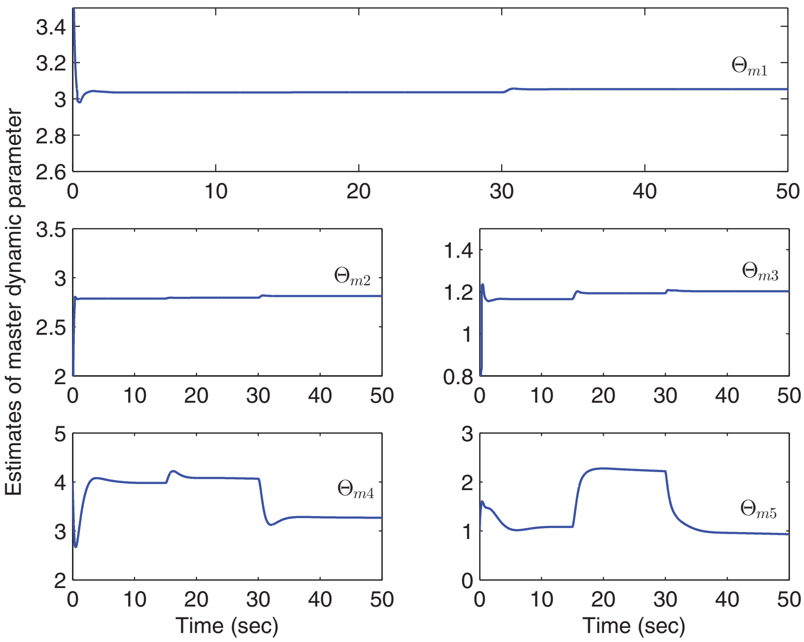

Fig. 5. Estimates of the dynamic uncertainties of the non-redundant master robot.

The control gains, which are considered to be identical throughout this section, are given as $\lambda = 1$, $k_r = 4$, $k_p = 3$, and $k_v = 0.1$. In the simulation, the human operator is assumed to exert the generalized forces shown in Fig. 3 on the end-effector of the master robot, whereas there is no external force applied on the slave robot. Moreover, the null space of the redundant slave robot is utilized to avoid singularity by increasing the manipulability.[31,38] The simulation results are shown in Figs. 4–6. It can be observed from Fig. 4 that the closed-loop teleoperation system with time-varying delays is stable and the task-space position tracking is guaranteed. Furthermore, the estimates of uncertain dynamic parameters are bounded and converge to constant values as seen in Figs. 5 and 6. The

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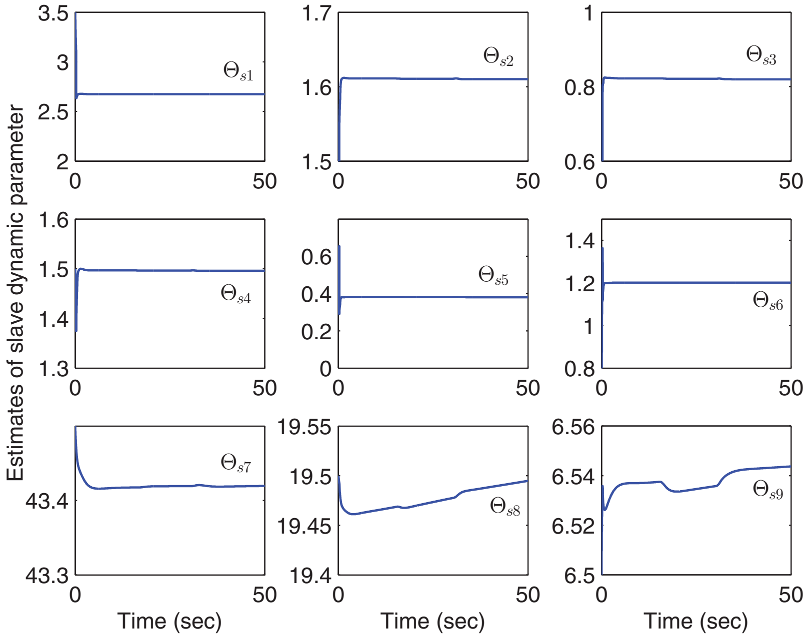

Fig. 6. Estimates of the dynamic uncertainties of the redundant slave robot.

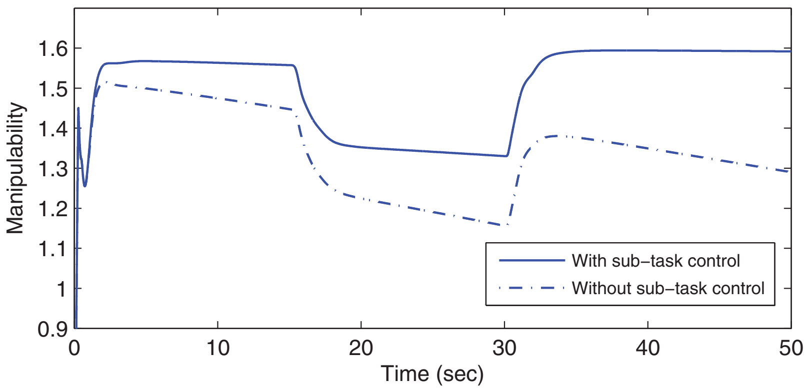

Fig. 7. The manipulability of the slave robot with and without utilizing sub-task control.

manipulability of the slave robot with and without utilizing sub-task control is shown in Fig. 7, which demonstrates that the redundancy can be exploited to increase manipulability of the slave robot in the remote environment.

The next simulation illustrates the performance of the proposed teleoperation systems when the slave robot contacts the remote environment with the consideration of collision avoidance. The master robot is the same as in the previous case, and the slave robot is considered as a 5-DOF planar manipulator. The physical parameters of the slave robot are given by $m_s = [3.12,\ 1.85,\ 1.02,\ 0.8,\ 0.7]\,\text{kg}$, $I_s = [0.12,\ 0.07,\ 0.04,\ 0.02,\ 0.01]\,\text{kgm}^2$, and $L_s = [0.4,\ 0.4,\ 0.4,\ 0.4,\ 0.4]\,\text{m}$. The initial conditions of the master and slave robots are selected as $q_m(0) = [2\pi/3,\ \pi/6]\text{rad}$ and $q_s(0) = [\pi/2,\ -\pi/6,\ -\pi/6,\ -\pi/6,\ -\pi/6]\text{rad}$ with zero angular initial velocities. In this case, we assume that there is no dynamic uncertainty in the robotic systems as considered in Theorem 3.3. Moreover, the control gains and communication delays are considered identical to the previous case.

In this simulation, the redundancy of the slave robot is utilized to avoid colliding obstacles in the remote environment, as seen in Fig. 9, where two circular obstacles are located at $X = [0.3,\ 0.1]\,\text{m}$ and $X = [1,\ 0.3]\,\text{m}$. The collision avoidance auxiliary function proposed in ref. [22] is adopted in

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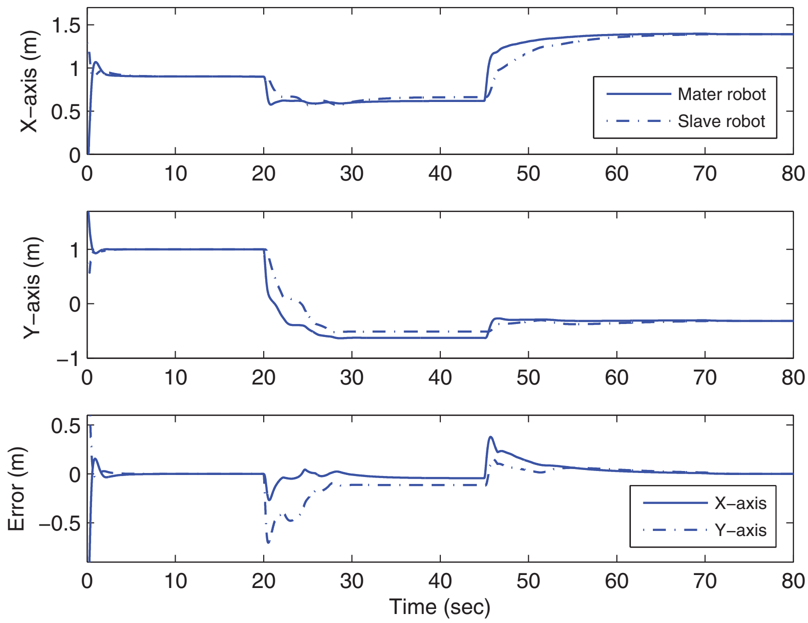
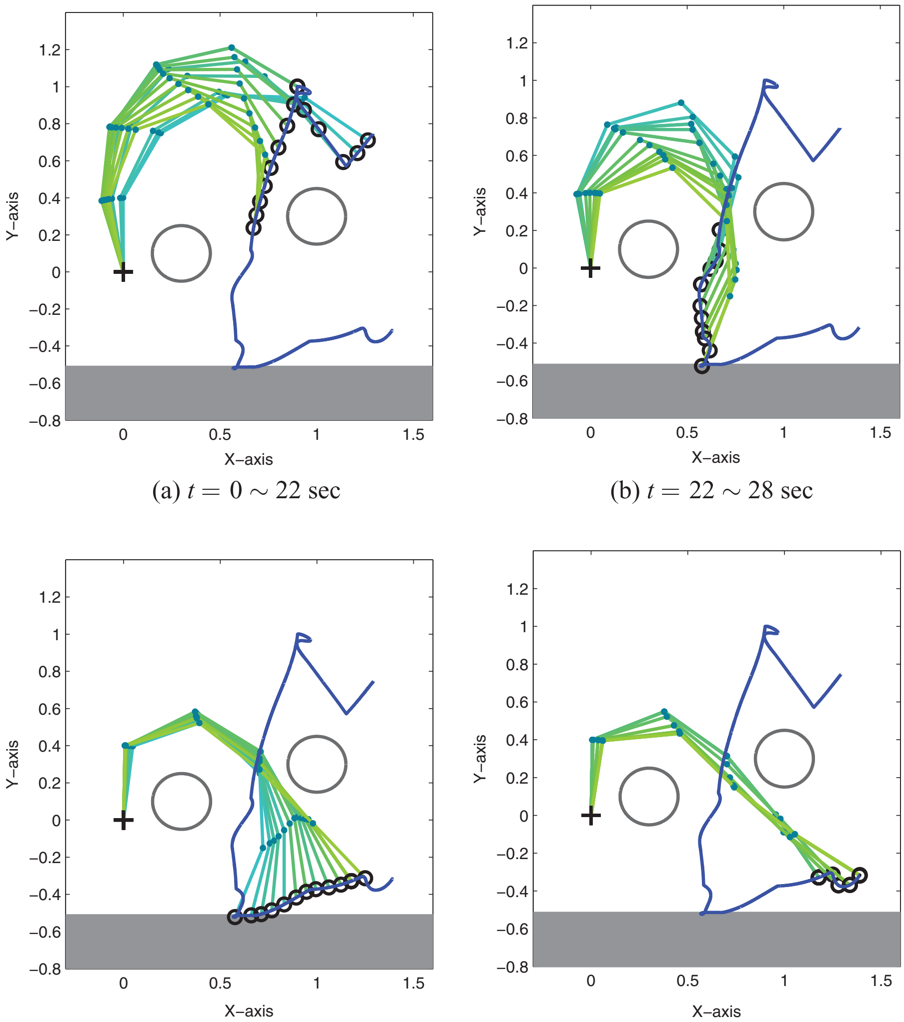


Fig. 8. Position configurations and tracking errors of the teleoperation system with a 5-DOF slave robot.

this case with avoidance distance $R = 0.3$ m and safe distance $r = 0.15$ m. The 2nd to 5th joints are considered as the collision-free points. Therefore, the slave robot will start avoiding the obstacles when the distance from these joints to an obstacle is less than 0.3 m, and the distances between obstacles and 2nd to 5th joints will remain greater than 0.15 m. In addition to collision avoidance, the slave robot will contact a wall in the remote environment at $y = -0.5$ m, which is the gray area in Fig. 9. In this simulation, the external forces from the human operator and the remote environment are modeled as a spring-damper force.[12,39] The spring and damping gains are given as 12 N/m and 1 N s/m for the human operator and 80 N/m and 1 N s/m for the remote environment. The simulation results are illustrated in Figs. 8–10. Figure 8 demonstrates that the control system is stable with time-varying communication delays. Moreover, the task-space position tracking between the robots is guaranteed except for the slave robot is in contact with the remote environment at around $t = 28 - 45$ s. Figure 9 illustrates that the redundant slave robot is able to regulate its configuration to avoid obstacles in the remote environment with guaranteed position tracking in the task space. The external forces from the human and the environment are shown in Fig. 10.

5. Conclusion and Future Work

The development of task-space teleoperation is significant because robots generally interact with human operators and remote environments on the end-effector. In this paper, we proposed a control algorithm to guarantee stability and task-space position tracking when teleoperation systems are subject to time-varying delay. Provided that the control gains are contingent on communication delay, an adaptive control algorithm was proposed to ensure stability and position tracking of the teleoperation system under dynamic uncertainties. For a teleoperation system influenced by non-passive external forces, ultimate boundedness of all signals in this system was also studied. Furthermore, the redundant slave robot with the use of the developed control algorithm, in addition to tracking the end-effector of the master robot, could regulate its configuration to achieve additional tasks autonomously. Simulation results showed the efficacy of the proposed control system and the efficiency of sub-task control for the slave robots. Future work will encompass not only developing control algorithms for task-space teleoperation systems under multiple robotic manipulators and kinematic uncertainties, but studying the control schemes without requiring knowledge of time delay.



(a) $t = 0 \sim 22$ sec (b) $t = 22 \sim 28$ sec

(c) $t = 28 \sim 50$ sec (d) $t = 50 \sim 80$ sec

Fig. 9. Configurations of the slave robot in the presence of obstacles and a wall in the remote environment. Two circles indicate the obstacles, and the gray color denotes the area of the wall. The blue solid-line denotes the trajectory of the end-effector of the slave robot.

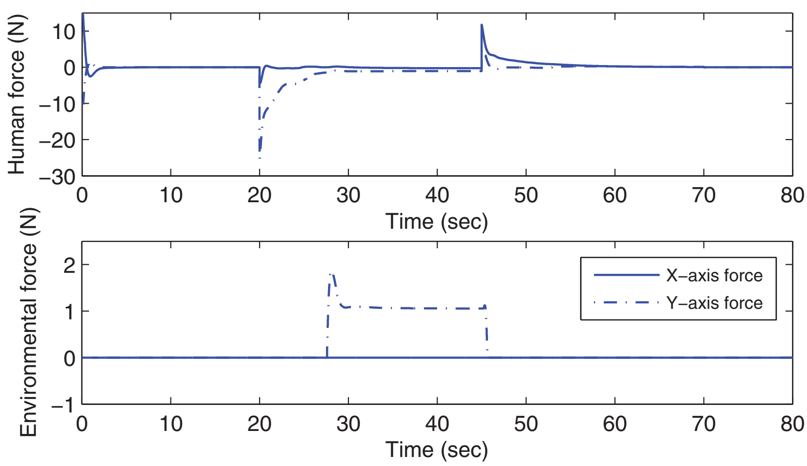


Fig. 10. External forces from the human operator and remote environment.

https://doi.org/10.1017/S0263574714001295 Published online by Cambridge University Press

Acknowledgments

The author would like to thank Prof. Nikhil Chopra for useful discussions on this topic, and the anonymous reviewers for their valuable comments and suggestions. This work was supported by the National Science Council, Taiwan, under grant NSC 102-2221-E-006-265.

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