106.08 Proof without words: the Cauchy-Schwarz inequality using analytic geometry

The Cauchy-Schwarz inequality for two variables states that if a, b, c, d are real numbers, then

$$|ac + bd| \leqslant \sqrt{a^2 + b^2}\sqrt{c^2 + d^2},$$

with equality if, and only if, $bc = ad$.

If $(a, b) = (0, 0)$ inequality is true, so assume $(a, b) \neq (0, 0)$ below.

Proof:

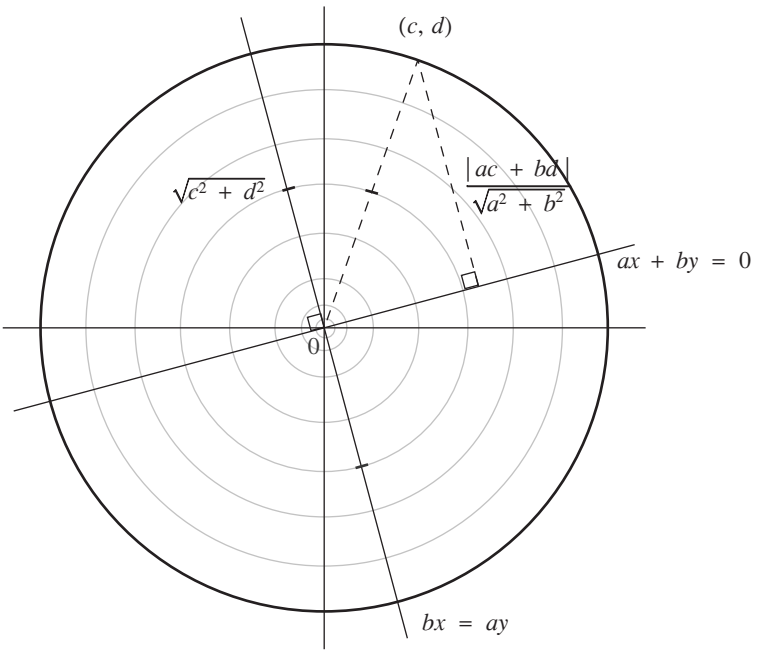

FIGURE 1

For three variables, the corresponding formula for the distance of a point to a plane may be used in a similar way to prove Cauchy-Schwarz for three variables.

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