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Theory of Amplitude Modulation II. The Resonant Mode Interaction Model

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Abstract. With about one fourth of the RRab stars showing the Blazhko effect, amplitude modulation on long time scales is a common phenomenon in stellar pulsation. The role of nonradial oscillation modes is studied here, and it is proposed that the amplitude modulation is a result of a dynamical, nonlinear resonance between the observed radial mode and a low-degree nonradial mode. In this scenario, a nonradial mode is resonantly excited in Blazhko-type RR Lyrae stars. Degree one modes are found to have the highest probability of being excited and a crucial test of the model would be the observational determination of the degree of the proposed nonradial mode.

1. Resonances and the Blazhko Effect

The Blazhko effect refers to the pulsation amplitude modulation on long time scales observed in about 20% to 30% of the RRab stars and only marginally in RRc stars (Kovács et al. 2000). The long modulation period naturally suggests a close resonance as the cause of this phenomenon. A simple beating between two radial modes, in the case of the third overtone having twice the frequency of the fundamental (see Borkowski 1980), may appear at a first glance as very appealing, but cannot be retained for a multitude of reasons. Most compelling probably is the fact that the third overtone is simply not linearly excited, a necessary element in a linear beating scenario, but instead is heavily damped. Moreover, the closeness of the resonance needed to obtain amplitude modulation on time scales much longer than the period of the oscillation itself necessitates using a dynamical nonlinear approach for the interaction between the two resonant modes (see Buchler, Goupil, & Hansen (1997) for nonlinear effects when resonances occur among stellar pulsation modes). Interesting to note is that as early as 1936, Kluyver already started the development of a nonlinear model up to the second order, in an adiabatic approximation, to study the Blazhko effect.

A most revealing approach to study the nonlinear interaction of modes is the amplitude equation formalism, in which the full set of partial differential equations of hydrodynamics and radiative transport is reduced to a few ordinary differential equations for the long-term behaviour of the amplitudes of the dominant modes. We refer the reader to the excellent review by Buchler (1993) for a thorough description of the method. The main assumptions are that non-

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linearity, as well as nonadiabaticity are weak; these are fulfilled for the pulsations of RR Lyrae stars. In general, amplitude equations can then be expressed as

$$\frac{da_k}{dt} = (\mathrm{i}\omega_k + \kappa_k)a_k + f_k(a_j), \tag{1}$$

where $a_k(t)$ is the time-dependent amplitude of mode k, and ω_k and κ_k are the real oscillatory eigenfrequency and linear growth rate of mode k, respectively. The nonlinear term $f_k(a_j)$ consists of products of amplitudes, and its explicit form is determined by the linear spectrum, especially by the occurrence of resonances. In the amplitudes $a_k(t)$, the fast oscillatory behaviour can be separated from the time behaviour on longer time scales:

$$a_k(t) = A_k(t)\mathrm{e}^{\mathrm{i}[\omega_k t + \phi_k(t)]} \tag{2}$$

and the amplitude equations can be expressed as ordinary differential equations for the behaviour of the real amplitudes A_k and phases ϕ_k on longer time scales.

By means of amplitude equations, we investigate here the consequences of a resonance between the observed radial mode of RR Lyrae stars (fundamental or first overtone) and an as yet undetermined other mode. The idea we have in mind is that the solution of the amplitude equations which represents the steady pulsation in the observed radial mode (called the radial mode fixed point) becomes unstable due to the resonant interaction with another mode. This instability involves the resonant excitation of the other mode and in the most simple solution both modes have a fixed amplitude: a so-called double-mode solution in the two resonant modes results. No Blazhko-type behaviour can occur then, except when the other mode is of low degree and not symmetric about the rotation axis. In that case, amplitude variations can possibly be observed as a result of a changing aspect angle with rotational phase, and the modulation period is exactly equal to the rotation period. The frequencies of the two modes are expected to be phase-locked, in the sense that they exactly fulfil the resonance condition (see Buchler et al. 1997), such that only one pulsational frequency (and its multiples) is present in the frequency spectrum.

Another solution which is quite common when resonances occur is a limit cycle (see Buchler et al. 1997): a solution in which the amplitudes A_k and phases ϕ_k are periodically varying. This solution can be considered as the natural representation of the observed variations in Blazhko RR Lyrae stars. It also leads to the observed multiplet structure in the frequency spectrum: Basic trigonometry shows that if, for example, the amplitude A_k of the radial mode is assumed to vary around a mean amplitude A_0 with amplitude A_m, the signal $[A_0 + A_m \cos(\omega_{\mathrm{B}} t)] \cos(\omega_k t + \phi)$ (see Eq. 2), where ω_k is the oscillation frequency and ω_{B} the Blazhko or modulation frequency, consists of the frequencies ω_k, $\omega_k - \omega_{\mathrm{B}}$ and $\omega_k + \omega_{\mathrm{B}}$. The period of the modulation is determined by the nonlinear interaction and is not related to the rotation period. Other and more complicated solutions of the amplitude equations also exist.

As RR Lyrae stars are classical radial pulsators, it is obvious to consider first a resonance between two radial modes of the type

$$n\omega_{\mathrm{F}} \approx \omega_{\mathrm{O}}, \tag{3}$$

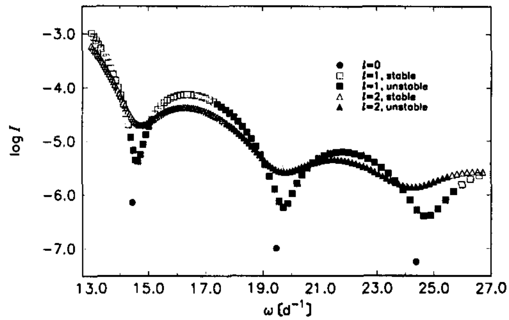

Figure 1. Inertia and stability of low-degree modes in the frequency range of the first three radial modes.

where ω_{F} is the frequency of the observed mode (fundamental or first overtone) and ω_{O} is the frequency of an overtone. Moskalik (1986) studied such a resonance, in particular $2\omega_{\mathrm{F}} \approx \omega_{\mathrm{O}}$ between the fundamental and the third overtone. It is now, however, believed that the appropriate resonances do not occur in the RR Lyrae instability strip (Kovács 1995), and, moreover, these overtones are so heavily damped that it is highly unlikely that they could disturb the steady pulsation in the observed radial mode. Therefore, nonradial modes must be involved. As shown by Buchler et al. (1997), nonradial modes cannot be resonantly excited when $n > 1$ in the resonance condition, and so only the direct 1:1 resonance $\omega_{\mathrm{F}} \approx \omega_{\mathrm{NR}}$ will be studied further.

2. Nonradial Modes in RR Lyrae Stars

In evolved stars, such as RR Lyrae stars, low-degree oscillation modes have a dual character. They behave as p modes in the outer envelope and as g modes of high radial order in the deep interior. The g-mode radial order typically is between 100 and 200 for degree 1 modes in the frequency range of the lowest radial modes. It increases with increasing degree and decreasing frequency according to $n \propto \sqrt{\ell(\ell+1)}/\omega_{n,\ell}$. The relative frequency separation between two consecutive nonradial modes $\Delta\omega/\omega_{n,\ell} \approx 1/n$ and takes on values from 10^{-2} to 10^{-3} for increasing degrees between 1 and 10. The frequency spectrum for low-degree modes is thus much denser than that of the radial modes. This is illustrated in Fig. 1, where the moments of inertia $I = \int \rho|\xi|^2 dV$ are presented as a function of frequency for a typical RR Lyrae model (for more details, see Van Hoolst et al. 1998).

As the eigenfunctions of nonradial modes, in contrast to those of the radial modes, also oscillate in the interior, their moments of inertia are larger than those of the radial modes. The moments of inertia have lowest values in the neighbourhood of the radial mode frequencies. The variations are due to differences in trapping in the outer acoustic cavity. Even for the best-trapped low-degree modes, the moment of inertia is still about one order of magnitude larger than that of radial modes.

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Many of the low-degree nonradial modes in the neighbourhood of the radial mode frequencies are unstable (Van Hoolst, Dziembowski, & Kawaler 1998). Their growth rates are typically one order of magnitude or more smaller than those of the radial modes. This can be understood quite easily by noting that the growth rate is inversely proportional to the moment of inertia.

The trapping properties depend on the width of the evanescent zone separating the two propagation zones, which changes with degree. The larger the width, the more efficient the trapping can be. It is found that modes of degree 1 can be well trapped, for degrees 2, 3 and 4 the trapping is much weaker, and pronounced minima in the moment of inertia only start to appear again from degree 5 on. For these higher-degree modes, the minima are found to be further away from the radial mode frequencies.

A small subset of modes can be studied with no quantitative information about the structure of the deep interior. These modes have been called strongly trapped unstable (STU) modes by Van Hoolst et al. (1998) and they can have linear growth rates and moments of inertia of the order of the radial modes. Very strong dissipation in the deep interior is necessary for their existence and they are only found for larger degrees (say larger than 5). Osaki (1977) has developed a scheme for calculating nonradial modes of highly condensed stars assuming that strong dissipation takes place in the deep interior. The same Osaki-method is used by Shibahashi & Takata (1995, see also Shibahashi, 2000) for calculating degree two nonradial modes of RR Lyrae stars. We note that at that degree, we didn't find any of these modes. For the resonant interaction model for the Blazhko effect, the STU modes are of no interest since they are too far from resonance with the radial modes.

3. Probability for Instability of the Observed Radial Mode Fixed Point

We consider the dynamical nonlinear interaction between the observed radial mode and one of the low-degree nonradial modes with nearly equal frequency, and study whether these nonradial modes can be nonlinearly excited due to the resonance, or, equivalently, whether the radial mode fixed point (FP) can become unstable. The FP is stable if, apart from two trivial conditions,

$$R_\ell^2 A^4 < D_\ell^2 + g_\ell^2 \tag{4}$$

(Van Hoolst et al. 1998). Here, R_ℓ represents the strength of the nonlinear resonant coupling between the two modes, A is the amplitude of the radial mode FP, D_ℓ is the nonlinear frequency difference, and g_ℓ the nonlinear growth rate of the nonradial mode. The coupling R_ℓ can be determined from nonlinear adiabatic calculations and uses the eigenfunctions and frequencies of the two resonant modes. Instability of the radial mode FP is seen to be promoted if the resonant coupling is strong and the amplitude of the radial mode FP is large; a large nonlinear frequency difference of the modes and a heavy damping of the nonradial modes tend to favour the stable FP. We therefore consider only those modes that are closest to the exact resonance, and low-degree modes which are not heavily damped.

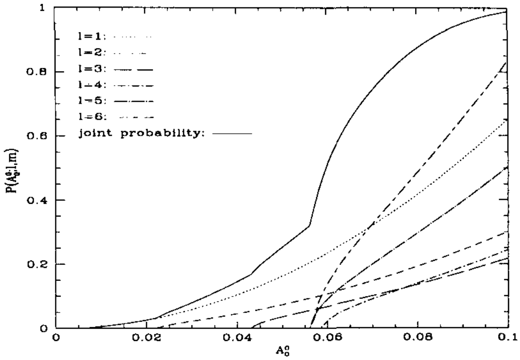

Figure 2. Probability for instability of the radial fundamental mode FP.

Because the frequency spectrum of the nonradial modes is so dense, we adopt a statistical approach for the study of the stability. From Condition (4) it follows that the radial mode FP is unstable when a nonradial mode exists with a frequency in a certain interval in frequency space close to the radial mode frequency that has a width $2(R_\ell^2 A^4 - g_\ell^2)^{1/2}$. The probability for instability can then be expressed as the ratio of this width to the frequency difference between two consecutive nonradial modes $\Delta\omega = \omega_{\ell,n} - \omega_{\ell,n+1}$. As the nonlinear growth rate g_ℓ for the lowest-degree modes is found to be much smaller than $R_\ell A^2$ for realistic amplitudes, we have

$$P(A;\ell) = \frac{2R_\ell}{\Delta\omega} A^2. \tag{5}$$

Probabilities for instability of the fundamental radial mode due to resonant interaction with low-degree modes are given in Fig. 2, for the same RR Lyrae model as used in Fig. 1. Fundamental mode pulsators, or RRab stars, typically have an amplitude $A = \delta R/R = 0.075$ and probabilities are found up to 40%. First overtone pulsators, or RRc stars, have a smaller amplitude of about 0.025, and therefore the probabilities are found to be smaller, up to 20% (Van Hoolst et al. 1998). These probabilities are somewhat higher than the observed incidence of the Blazhko effect, certainly for the first overtone pulsators. It should be remembered however that instability does not necessarily imply amplitude modulation.

For increasing degrees the frequency separation decreases, but the probability for instability is generally not found to increase because either the trapping is weaker or the frequency distance between the best-trapped modes and the radial mode is larger for higher-degree modes, both resulting in a smaller coupling R_ℓ. The excitation of the degree one mode is therefore expected to be the most likely. The observational determination of the degree of the proposed nonradial component constitutes a crucial test for models for the Blazhko effect. Contrary to the proposed model, the oblique pulsator model of Shibahashi & Takata (1995, see also Shibahashi 2000) predicts a degree two component.

Dziembowski & Cassisi (1999, 2000) have shown that the instability only weakly depends on model parameters such as mass, chemical composition and evolutionary state.

The effects of rotation have been ignored here. For rotating RR Lyrae stars, the resonance between pairs of modes with $m \neq 0$ and the radial mode should also be considered. This situation is analogous to the parametric excitation of nonradial modes considered by Dziembowski, Królikowska, & Kosovitchev (1988) for δ Scuti stars, and its effect is to increase the probability of resonant instability. The rotation rate of RR Lyrae stars is unknown at present. Only an upper limit could be determined by Peterson, Carney, & Latham (1996), who found the rotation to be slow compared to that of blue horizontal branch stars (BHB). These authors also looked specifically for line broadening in Blazhko stars with short modulation period. None was found to show detectable rotation, although rotational line broadening has been detected in BHB stars with rotation periods comparable to the Blazhko modulation periods.

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References

Borkowski, K. J. 1980, Acta Astron., 30, 393

Buchler, J. R. 1993, in Non-linear Phenomena in Stellar Variability, ed. M. Takeuti & J.-R. Buchler (Dordrecht: Kluwer), 9

Buchler, J. R., Goupil, M.-J., & Hansen, C. J. 1997, A&A, 321, 159

Dziembowski, W. A. & Cassisi, S. 1999, Acta Astron., 49, 371

Dziembowski, W. A. & Cassisi, S. 2000, in these proceedings, p. 321

Dziembowski, W., Królikowska, M., & Kosovitchev, A. 1988, Acta Astron., 38, 61

Kluyver, H. A. 1936, Bull. Astron. Inst. Neth., 7, 313

Kovács, G. 1995, A&A, 295, 693

Kovács, G., Alcock, C., Allsman, R., et al. 2000, in these proceedings, p. 313

Moskalik, P. 1986, Acta Astron., 36, 333

Osaki, Y. 1977, PASP, 29, 234

Peterson, R. C., Carney, B. W., & Latham, D. W. 1996, ApJ, 465, L47

Shibahashi, H. 2000, in these proceedings, p. 299

Shibahashi, H. & Takata, M. 1995, in ASP Conf. Ser. Vol. 83, Proc. IAU Coll. No. 155, Astrophysical Applications of Stellar Pulsation, ed. R. S. Stobie & P. A. Whitelock (San Francisco: ASP), 42

Van Hoolst, T., Dziembowski, W. A., & Kawaler, S. D. 1998, MNRAS, 297, 536

Discussion

Robert Buchler: A few years ago we found that the stability of the radial modes is non-monotonic and found the 8th or 9th overtone becomes close to neutrally stable or unstable. Furthermore, these modes (which we labeled strange modes) can be in a 4:1 resonance. This is a promising scenario in the Blazhko effect and we are going to check it with hydrodynamic modeling.

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