

# FOUNDATIONS OF AMBIGUITY AND ECONOMIC MODELLING

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Are foundations of models of ambiguity-sensitive preferences too flawed to be usefully applied to economic models? Al-Najjar and Weinstein (2009) say such is indeed the case. In this paper, first, we point out that many of the key arguments by Al-Najjar and Weinstein do not apply to quite a few of the ambiguity preference models of more recent vintage, and therefore to that extent do not undermine the foundational aspects or applicability of ambiguity models in general. Second, we argue the focus in that paper on Ellsberg examples is an overly narrow concern; the Ellsberg examples have their uses but they are not the best context to understand why reasonable real-world agents may find acting in an ambiguity-sensitive manner normatively or prescriptively appealing. Finally, normative considerations aside, we submit that Al-Najjar and Weinstein are unduly dismissive of the power of such preferences to provide illuminating positive analyses of economic phenomena.

Following intuitive arguments of Knight (1921) and Ellsberg (1961), and pioneering formalizations by Schmeidler (1989) and Gilboa and Schmeidler (1989), it is now customary in modern decision theory to distinguish two categories of subjectively uncertain belief: *unambiguous* and *ambiguous*. An unambiguous belief may be expressed as a probability distribution (over contingencies) and is thus akin to risk. An ambiguous belief, on the other hand, cannot be expressed using a single probability distribution. Intuitively, an event is thought to be deemed ambiguous by a decision maker (DM) if the DM's belief about the event, as revealed by

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his preferences (over acts measurable with respect to the event), cannot be expressed as a unique probability. The usual interpretation is that the DM is uncertain about the “true” probability of the ambiguous event and takes this uncertainty into account when making his choice. This uncertainty about the true probability is dubbed “ambiguity”. The nature and extent of the allowance a DM gives to this uncertainty when making his choices determines his *ambiguity attitude*. Loosely put, an ambiguity-averse DM (the ambiguity attitude commonly assumed in economic analyses) prefers acts whose outcomes are more robust to ambiguity.

Are foundations of models of ambiguity-sensitive preferences too flawed to be usefully applied to economic models? Al-Najjar and Weinstein (2009) say such is indeed the case. Their paper does a very fine job of bringing together arguments which, while may well have been articulated before, are not necessarily that well known outside the decision theory literature. However, many of the key arguments in that paper do not apply to quite a few of the ambiguity preference models, especially those of more recent vintage, and therefore to that extent do not undermine the foundational aspects or applicability of those models. Second, the focus in that paper on the Ellsberg examples is an overly narrow concern; the Ellsberg examples have their uses but they are not the best context for an illuminating discussion of the normative or prescriptive properties of ambiguity sensitive preferences. Finally, putting normative considerations to one side, I think the paper is unduly dismissive of the power of such preferences in positive analyses of economic phenomena.

Take the key argument in that paper: the (modal) Ellsberg choices are consistent with behaviour such as the sunk cost fallacy which is commonly construed as irrational; hence, the Ellsberg choices cannot be considered as rational or reasonable. The sunk cost question is a question of dynamic behaviour. Behaviour in a static/timeless model has no implication about decision with respect to a sunk cost per se. The analysis in Al-Najjar and Weinstein (2009) recognizes that, and so “extends” some of the timeless models of preferences accommodating sensitivity to ambiguity to a dynamic framework. However, the dynamic framework/axiomatics they adopt fails to respect *consequentialism*. Consequentialism entails that at any node of the relevant event/decision tree, the DM’s preferences (contingent on being at that node) over her choices only depend on payoffs at future nodes, those that may be realized *from then on*; payoffs at past nodes, realized or unrealized, do not matter. The fact that consequentialism is violated in the dynamic extensions that Al-Najjar and Weinstein build is what is responsible for the susceptibility of those preference constructs to the sunk cost fallacy, not ambiguity sensitivity per se. However, ambiguity sensitive/non-probabilistically sophisticated preferences *can* be extended to dynamic frameworks, preserving both dynamic consistency and consequentialism. For example, this is precisely

what is done in recursive multiple priors (Epstein and Schneider 2003), dynamic variational preferences (Maccheroni, Marinacci and Rustichini 2006) and recursive smooth ambiguity (Klibanoff, Marinacci and Mukerji 2009), papers which build dynamic extensions to the three more prominent models in the literature.<sup>1</sup> The sunk cost fallacy is not possible in any of these preference models. The sunk cost example is valid in the dynamic models constructed by Hanany and Klibanoff (2007, 2008). But that is because those models too, in general, do not respect consequentialism. The critique is misdirected in the sense that it really is a critique against relaxing consequentialism in the dynamic extensions. It is certainly not a critique that may be applied to ambiguity-sensitive static preferences, nor to dynamic extensions which respect consequentialism, as several prominent extensions of ambiguity sensitive preferences do.

A similar point can be made about the other key critique advanced in the paper, that the notion of beliefs in the models in the ambiguity literature is very different from the way the notion is understood and applied in economics customarily and that, "this is because of the lack of a useful notion of updating". First, looking at static/timeless preference models, it is true that neither the set of priors nor the non-additive measure that appears in the representation functionals of the pioneering models in the literature (maxmin expected utility preferences, Gilboa and Schmeidler (1989), and Choquet expected utility preferences, Schmeidler (1989), respectively) may be interpreted as representing pure information, strictly speaking. Taking a purist's perspective, these parameters in the representations mix, quite inseparably, both attitudes and beliefs.<sup>2</sup> However, some of the more recent (static) preference models in the literature, for instance the smooth ambiguity model (Klibanoff, Mukerji and Marinacci 2005), do not share this feature; there beliefs and (ambiguity) attitude parameters in the representation are explicitly separated. Moreover, in the smooth ambiguity representation, beliefs may be seen to have precisely the same connection to the DM's subjective information as in subjective expected utility representation and standard Bayesian theory routinely applied in economics. Second, when considering dynamic frameworks it is worth noting that for updated beliefs to be consistent with the standard notion of separation of tastes and beliefs the requirement is that the preferences satisfy a strong notion of (node-by-node)

<sup>1</sup> Al-Najjar and Weinstein point out, correctly, that the rectangularity condition (which appears as a condition in the Recursive Multiple prior model) limits some natural information environments. It is worth pointing out that the Recursive Smooth Ambiguity model does not suffer from this particular limitation.

<sup>2</sup> For a brief exposition of this point see the discussion on page 1872 in Klibanoff, Marinacci and Mukerji (2005).

monotonicity<sup>3</sup> which in turn implies consequentialism. Recursive smooth ambiguity preferences do satisfy this property and for such preferences there is as much separation of tastes and beliefs as there is in the standard Bayesian framework.<sup>4</sup> In addition, beliefs appearing in the representation of these preferences are updated by Bayes rule. Furthermore, Bayes rule emerges as the appropriate updating rule for these preferences for reasons very analogous to what leads to the rule in other preference frameworks (among others, that *ex ante* and *ex post* preferences are dynamically consistent). Klibanoff and Hanany (2008), which also obtains a dynamic extension of the smooth ambiguity model, does not generally satisfy consequentialism, as noted earlier, and hence in that extension updated beliefs typically do not maintain a separation between beliefs and attitudes (nor does it satisfy the notion of “fact based updating”).

Al-Najjar and Weinstein focus their arguments, almost exclusively, on the Ellsberg examples. The Ellsberg examples certainly have their uses: they make it crystal clear (like, perhaps, nothing else) in what precise way ambiguity-sensitive preferences depart from subjective expected utility. Also very remarkably, the examples elucidate the link between choice behaviour and the (perceived) lack of information about the relevant odds. But they are not as useful in explaining why it may be reasonable for a *real life* DM to make choices that are sensitive to ambiguity. In a real-world scenario without the pigeon-holed, structured symmetries of the Ellsberg constructs, it is a far more perplexing issue to decide what may be reasonable to assume about the relevant odds.<sup>5</sup> Despite all available data, can one really pin down, statistically, the probability distribution on the growth of GDP next period? Thinking about global warming, for instance, we know there are a variety of different models, different configurations of parameter values, that imply differing stochastic forecasts about the future paths of policy-relevant variables. But information about the correctness/reliability of particular models and parameter configurations is very sparse. When deciding on a policy alternative the DM may well want to adopt a policy that is somewhat robust *across* the available

<sup>3</sup> This consists of two properties: one, that preferences satisfy the usual state-by-state monotonicity applied to (information) nodes of the relevant event tree and two, that preferences are uniquely pinned down by the (conditioning) node. The former property, state-by-state monotonicity, is as follows: if an act  $f$  pays at least as much as an act  $g$  on every contingent state down the tree, then  $f$  is preferred to  $g$ . The latter property is what Al-Najjar and Weinstein call “fact based updating”, i.e. preferences only depend on events that have actually been realized and not on unrealized events.

<sup>4</sup> For a more elaborate discussion of this point see pages 944–46 in Klibanoff, Marinacci and Mukerji (2009).

<sup>5</sup> The decision making in the model in Mukerji (1997) is an example of how a procedurally rational DM aware of limitations in his knowledge of the relevant state space, may end up acting in an ambiguity-averse manner.

stochastic forecasts. Putting it another way, different priors rationalize different choices, but in many situations a DM may realize that she does not *know enough* to pin down an appropriate prior. She may thus be induced to adopt a decision rule that is robust against prior misspecification: the choice of action resulting from the decision rule does reasonably well *across* a wide range of priors. Such a rule may well violate the sure thing principle. But observing the sure thing principle would require “committing to” a particular prior and choosing a rule in accordance with that prior which may involve a welfare loss if the chosen prior were not the “right” prior. The DM is trading off a loss from violating the sure thing principle with a possible loss from selecting the wrong prior. Hence, it is argued, given natural and pervasive cognitive constraints, observing the sure thing principle is neither necessary nor sufficient for rational or at least reasonable behaviour in real-world decision making. It is very akin to the theory of the second best; it would be (first-) best if the DM had good information to settle on the appropriate prior and act in accordance with it (and the sure thing principle). But given the (perceived) lack of such information, she may well opt for a robust decision rule, one that won’t do too badly even if she were to get the prior somewhat wrong. The point being made here is not particularly original. Gilboa, Postlewaite and Schmeidler (2008), Hansen and Sargent (2008), Hansen (2007), for example, present arguments in a similar vein. The debate about the prescriptive validity of ambiguity-sensitive preferences would have been far more enriched had Al-Najjar and Weinstein opted for a broader focus, looking beyond the artificial limits of the Ellsberg examples.

Finally, I think Al-Najjar and Weinstein are unduly dismissive about the potential of ambiguity-based preference models in positive economic analyses.<sup>6</sup> This potential may be appreciated without recourse to any presumption about the prescriptive validity of ambiguity-based preference models. Take for instance, a very recent paper by Baliga, Hanany and Klibanoff (2009). This paper considers a two-period, standard model of portfolio choice with one riskless asset and one asset with uncertain payoff, where the DM gets to revise her choices made in the first period following some revelation of information in the second period. The DM has smooth ambiguity preferences and updates these preferences per the theory in Hanany and Klibanoff (2008). As was mentioned, this updating theory does not satisfy consequentialism, in general, though it does satisfy dynamic consistency. Some may argue that the theory lacks an element of prescriptive validity on account of the fact it fails to satisfy consequentialism. However, to my mind, such an argument does not detract from the principal contribution of the analysis by Baliga *et al.*

<sup>6</sup> An (inexhaustive) account of the many applications of ambiguity-based models to economic analyses may be found in Mukerji and Tallon (2004).

That contribution may be thought of as a piece of purely positive analysis. Its value lies in its ability to explain many commonly observed financial market phenomena, like under-reaction to certain kinds of information signals, on the basis of the interaction between ambiguity aversion, dynamic consistency and a kind of violation of consequentialism that an ambiguity-averse DM may be prone to. Unlike explanations obtained in some other “behavioural” models, here we get a unified explanation based on a few simple preference properties, clearly articulated in terms of axioms. These axioms may not all be prescriptive, but they present a simple enunciation of the preference properties underlying the functional form representation and the syllogism applied to explain the phenomena at hand: we get a clear and precise idea of what gives what.

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