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OPTIMAL POLICY IN COLLATERAL CONSTRAINED ECONOMIES

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This paper examines optimal policy in a macroeconomic model with collateral constraints. Binding collateral constraints yield inefficient competitive equilibrium allocations because they distort the optimal utilization of real resources. I identify the set of policy instruments that can be used by a Ramsey planner to achieve the first-best and the second-best (i.e., constrained planner's) allocations. A system of distortionary taxes on capital and labor income, along with direct lump-sum transfers among borrowers and lenders replicates the first-best outcome. The tax rates correct for the marginal distortions, whereas the direct lump-sum transfers perform income redistributions among the agents. In absence of direct lump-sum transfers, the distortionary taxes have an additional role, i.e., to perform implicit income transfers, and only second-best outcomes are attainable. I also derive the optimal policy in response to real and financial shocks, and show how the policy recommendations differ depending on the set of policy instruments available.

Keywords: Collateral Constraints, Inefficiencies, Ramsey Regulation, Welfare

1. INTRODUCTION

What should optimal policy look like in models with collateral constraints? Chamley (1986) and Judd (1985) derived the theorem of a steady-state zero tax on capital and a positive tax on labor income in an economy without market distortions. This result has been invalidated in models featuring uninsurable idiosyncratic risk since agents exhibit precautionary saving motives in response to uncertainty and build up capital (savings) that is in excess of the first-best level.[1] However, not many studies have investigated the optimal policy in environments where debt is collateralized.

This type of models, nonetheless, have been extensively used to examine the amplifications of real shocks due to financial frictions [for example, Kiyotaki and Moore (1997)] or to study the effect of financial shocks, like those observed in the

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offset of the Great Recession, on real economic activity [for example, Jermann and Quadrini (2012)]. This paper contributes to the literature by identifying the distinct inefficiencies induced by collateral constraints and derives the class of optimal Ramsey regulatory tools that can be used to restore the efficient allocations. Furthermore, it distinguishes between structural policies in the long-run and cyclical policies in response to real and financial shocks.

I carry out the design of optimal policy using a model similar to the one in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012). There are two types of agents: households that are the savers (lenders) and entrepreneurs that are the borrowers in the economy. Since entrepreneurs cannot commit to repay the loan, they cannot borrow more than a fraction of their capital, which is pledged as collateral. In addition, entrepreneurs tend to be more impatient than households, which results in a binding collateral constraint.[2]

I show that binding collateral constraints yield inefficient use of the factors of production. In particular, the capital stock, in addition to its value as a productive asset, embeds a *collateral value* because an incremental unit of capital can relax the collateral constraint.[3] Moreover, the presence of a collateralized working capital loan, following the setup in Jermann and Quadrini (2012), discourages entrepreneurs from hiring labor since every additional unit of labor requires them to acquire a working capital loan, which tightens the collateral constraint. Hence, entrepreneurs may end up underutilizing labor and overutilizing capital compared to a social optimum.

The goal of the Ramsey planner is to choose policy to maximize the aggregate welfare of the economy by addressing the inefficiencies arising from binding collateral constraints. Herein, the planner does not need to finance any exogenously given government expenditure. Nevertheless, as it turns out, the Ramsey planner uses corrective (to be defined later), distortionary capital income, and payroll taxes to affect some of the structural parameters in the economy rendering a binding collateral constraint. Moreover, the policy instruments also perform implicit income transfers from one group of agents to another, governed mainly by the planner's incentive to help entrepreneurs rebuild their net-worth, as well as by its weight placed on agents' utilities in the social welfare function.

On the other hand, when direct lump-sum transfers from one class of agents to another are introduced into the tax system, the corrective taxes serve for the purpose to correct the private marginal decisions, whereas the direct lump-sum transfers perform the necessary income redistribution. The tax system with corrective taxes alone replicates the constrained planner's allocations (second-best allocations),[4] and the tax system comprising of both corrective taxes and direct lump-sum transfers replicates the first-best allocations. Note that lump-sum transfers alone cannot achieve first-best outcomes.

One may anticipate that the optimal tax on capital income would be either zero or negative in the long run such that it stimulates accumulation of capital and relaxes the collateral constraint. However, for the given calibration, the planner optimally chooses a positive tax rate in the long run regardless of whether

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lump-sum transfers are available or not. The reason is that a positive tax on capital income reduces the demand for capital and increases its return rate, bringing it closer to the socially optimal level (i.e., in equality with the lending rate). On the other hand, the payroll tax can be either positive or negative depending on the availability of lump-sum transfers. With lump-sum transfers, the payroll tax is negative (a subsidy) since entrepreneurs are compensated for the extra costs incurred by the binding collateral constraint. In absence of lump-sum transfers, the payroll tax is used to additionally implement a desirable income redistribution by affecting the wage level. The tractability of the model allows me to provide analytical solutions of the steady-state levels of all policy instruments.

Further, I show that the capital income tax can be substituted by alternative regulation. These policies should be seen as financial regulation operating either through prices, i.e., making borrowing more expensive, or through quantities, i.e., restricting further the amount of borrowing per unit of collateral. The payroll tax can be replaced by a tax on labor imposed on households. Although the levels of the tax rates of the different policy regimes differ, they all achieve the same allocations.

In the short run, I examine the responses of the Ramsey plan to a productivity and a financial shock. In both cases, policy is conducted to stimulate capital accumulation, which makes the collateral constraint more slack.

Relation to the literature—This paper focuses on the borrowers–lenders problem in presence of collateral constraints. The main contribution nests in the normative analysis of the competitive economy facing a binding collateral requirement. First, it identifies the inefficiencies in the optimality conditions induced by the binding collateral constraint; subsequently, it determines the optimal policy mix (along with the alternative policy instruments) at the steady state and in response to shocks that aim at replicating the first-best and the constrained planner's allocations. In what follows, I survey the literature relating to this topic.

A close paper to mine in terms of the normative analysis implemented is Park (2014). She studies Ramsey optimal policies in an Aiyagari (1995) type economy in which agents face exclusion from the market constraints as in, for example, Alvarez and Jermann (2000). Contrary to her, I study an economy where agents face collateralized borrowing and there is only aggregate risk. The difference in the type of the borrowing constraint (collateralized borrowing versus exclusion from the market constraint) produces distinct mechanisms and distortions in the two economies. In particular, Park finds that both the tax on capital income and labor income can be used to correct for pecuniary externalities induced by agents' optimal capital and labor decisions. Should these externalities be absent, the tax rate on capital income would be zero and the tax on labor income would serve to finance government expenditure. To the contrary, in my setup, the taxes on capital and labor income do not necessarily correct for pecuniary externalities resulting from the capital and labor decisions. But rather, they are in place to affect structural parameters that render the collateral constraint binding, as well as to implicitly redistribute income between borrowers and lenders.

Another paper related to mine is Itskhoki and Moll (2014). They study optimal Ramsey policies in a standard growth model with financial frictions. In their baseline framework, which is a one-sector small open economy, they find it is optimal to subsidize labor and place zero tax on capital income. In contrast, I consider a closed economy, in which a social planner cares about the welfare of *both* households and entrepreneurs, and I derive the set of tools necessary to replicate the first-best and the second-best equilibrium outcomes.

In the canonical borrower-saver model studied in this paper, collateral constraints always bind in the deterministic steady state and the precautionary saving motive to insure against uninsurable idiosyncratic risk is muted. Albeit in a representative agent model, Bianchi (2011) and Bianchi and Mendoza (2013) show that precautionary saving motives interact with occasionally binding constraints and the competitive equilibrium is characterized by overborrowing. Aiyagari (1995), on the other hand, shows that precautionary saving motives result in an inefficient overaccumulation of capital in an environment with uninsurable idiosyncratic uncertainty. One of the contributions of my paper is to show that capital is not efficient even in the absence of occasionally binding constraints or precautionary motives arising from uninsurable idiosyncratic risk.

Further, the paper more generally relates to the literature on Ramsey optimal policy in general equilibrium models. Some of the leading examples in this literature are Lucas and Stokey (1983) and Chari et al. (1994). However, those papers rule out direct lump-sum taxation and abstract from any market inefficiencies. I consider a Ramsey planner, who cares about redistribution of wealth among agents, and therefore, in addition to distortionary taxes, it may also have access to direct lump-sum transfers. In addition to the distortionary taxes, the direct lump-sum are crucial for the conduct of policy. In a similar fashion, Bhandari et al. (2013) let the Ramsey planner choose optimal transfers. However, in their model with heterogeneous agents, they do not allow for the transfers to depend on agents' personal identities. Consequently, those lump-sum transfers are not powerful enough to complete the markets and bring the efficiency of the competitive economy to the first-best level.

The paper also relates to the literature studying optimal capital income taxation. Chamley (1986) and Judd (1985) established the result of zero capital taxation in the long run, which rests critically on the possibility of shifting consumption across periods through perfect capital markets. In the current paper, there are borrowers and lenders; borrowing is constrained and tax policy is used solely for the purpose to correct for the inefficiency arising from financial frictions. The tax on capital income turns out to be positive in the long run for a large portion of the planner's welfare weights.

The theoretical framework I use builds on a variant of the models presented in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012). Both are dynamic stochastic general equilibrium models, populated by two types of agents, households and corporate entities (or entrepreneurs) facing a collateral constraint. In Jermann and Quadrini (2012), households are the firms' shareholders; there are two types of collateralized loans: short (working capital)- and long-term loans; and there is an additional financial friction to the model, namely dividend adjustment costs. In Kiyotaki and Moore (1997), households and entrepreneurs are separate entities in the economy and only long-term loans are collateralized. This paper uses modeling elements from both frameworks: households do not own entrepreneurs following Kiyotaki and Moore (1997); and there are two types of collateralized debt as in Jermann and Quadrini (2012). Moreover, none of the two papers studies policies aimed at alleviating the effects of collateral constraints on real allocations.

2. THE MODEL ECONOMY

This section outlines the economy featuring a collateral constraint. The model economy is populated by *households/workers* (lenders) and *entrepreneurs/firms* (borrowers), infinitely lived and of measure one. Households consume, supply labor hours, and invest in entrepreneurs' one period corporate bonds. Entrepreneurs are endowed with capital and make optimal investment, borrowing, and labor demand decisions. Further, they produce a homogeneous good by hiring labor from households and by employing the capital stock. In financing production and investment in capital, entrepreneurs borrow from households and their borrowing capacity is limited by a collateral constraint.

2.1. Households/Workers

The economy is populated by a continuum of identical households, who maximize the following sum of discounted utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - l_t\right), \tag{1}$$

where c_t is consumption, $1 - l_t$ is leisure with l_t denoting labor hours, and β is the subjective discount factor. The utility function, $u(\cdot)$, is concave and increasing in both consumption and leisure and takes the constant-relative-risk-aversion (CRRA) form.

Households take the market wage and the borrowing/lending rate as given. They can borrow/save in a one period, risk-free corporate bond, issued by entrepreneurs, which is the sole instrument available for transferring resources intertemporally. This framework leads to the following per-period budget constraint of households:

$$c_t + \frac{b^h_{t+1}}{1 + r_t} \leq b^h_t + w_t l_t, \tag{2}$$

where b^h_t is the one period bond, paying a net interest rate r_t, and w_t is the wage rate. Households choose the following sequence $\{c_t, l_t, b^h_{t+1}\}_{t=0}^{\infty}$ to maximize (1) subject to (2). The equilibrium conditions that characterize the solution to their

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problem are given by the intratemporal arbitrage condition between labor supply and consumption

$$-\frac{u_{l,t}}{u_{c,t}} = w_t, \tag{3}$$

and the Euler condition

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} (1 + r_t), \tag{4}$$

where $u_{c,t}$ and $u_{l,t}$ denote the partial derivatives at time period t of the utility function with respect to consumption and labor, respectively. I will preserve the same notation in denoting partial derivatives of functions with respect to its arguments throughout the paper, which will become clear from the context.

2.2. Entrepreneurs/Firms

There is a continuum of identical entrepreneurs, who maximize the following sum of discounted utilities:

$$E_0 \sum_{t=0}^{\infty} \gamma^t u^e \left(c_t^e\right), \tag{5}$$

where c_t^e denotes entrepreneurs' consumption and γ is their subjective discount factor. The utility function $u^e(\cdot)$ is concave and increasing in c_t^e, and it is of the CRRA form.

Entrepreneurs own the capital stock in the economy, which evolves according to the law of motion $k_{t+1} = i_t + (1 - \delta)k_t$, where i_t is investment and δ is the depreciation rate. In addition, they own a production technology that uses labor, n_t, and capital, k_t, to produce a unit of output

$$z_t F(k_t, n_t) = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{6}$$

which can immediately be turned into consumption. Here, z_t denotes the level of productivity, following a stochastic process, and α denotes the capital share used in the production of output.

Entrepreneurs face the following budget constraint:

$$c_t^e + w_t n_t + b_t^e + k_{t+1} \leq z_t F(k_t, n_t) + \frac{b_{t+1}^e}{1 + r_t} + (1 - \delta) k_t, \tag{7}$$

where w_t is the wage paid for labor and r_t is the interest rate paid on borrowing from households, denoted b_{t+1}^e.

As in Kiyotaki and Moore (1997) and Iacoviello (2005), I assume there is a limit on the obligations of entrepreneurs. The maximum amount they can borrow is therefore bounded by the following collateral constraint:

$$\varepsilon_t k_{t+1} \geq z_t F(k_t, n_t) + \varepsilon_t \frac{b_{t+1}^e}{1 + r_t}, \tag{8}$$

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where ε_t is the liquidity value of entrepreneurs' capital, net of borrowing. This variable evolves stochastically and it can be considered to be a financial shock.[5]

The collateral constraint, (8), is derived in Jermann and Quadrini (2012) and Perri and Quadrini (2011) from a negotiation process between borrowers and lenders, and it takes the current form because of two crucial assumptions.[6] First, it is assumed that entrepreneurs need to acquire a working capital loan (i.e., an intraperiod loan); second, the working capital loan is also collateralized.[7]

The working capital loan is acquired to finance the flow of funds mismatch between the payments due and the realization of revenues. Since this loan is repaid within the same period, it bears no interest. It can be shown that the intraperiod loan is equal to the revenues of production, $z_t F(k_t, n_t)$, because all payments outstanding including wages, interest payments on borrowing, consumption, and investment expenditures, are assumed to be due before revenues have been realized.[8] The working capital loan is collateralized because of the timing assumption that entrepreneurs' decision to default arises *after* the realization of revenues, but *before* the working capital loan has been repaid. Therefore, since entrepreneurs can easily divert their liquidity (i.e., the revenues from production), the only asset that can be posted as collateral is the capital stock, k_{t+1}.

Finally, following Kiyotaki and Moore (1997) and Iacoviello (2005), I assume that entrepreneurs are more impatient than households, $\beta > \gamma$. This assumption yields a steady state in which entrepreneurs' return to savings is larger than the interest rate, which implies a binding collateral constraint (discussed subsequently).

Entrepreneurs choose the following sequence $\{c_t^e, n_t, b_{t+1}^e, k_{t+1}\}_{t=0}^{\infty}$ to maximize (5) subject to (7) and (8). Denoting λ_t^e to be the Lagrange multiplier on the budget constraint (7) and $\mu_t u_{c,t}^e$ to be the Lagrange multiplier (scaled by the marginal utility of entrepreneurial consumption) on the collateral constraint (8), the conditions characterizing the optimal choices of entrepreneurs are given by the optimal labor demand decision

$$(1-\mu_t)\, z_t F_{n,t} = w_t, \tag{9}$$

and the Euler conditions with respect to borrowing and capital, respectively,

$$\frac{1-\varepsilon_t\mu_t}{1+r_t} = \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}, \tag{10}$$

$$1-\varepsilon_t\mu_t = \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}\left[(1-\mu_{t+1})\, z_{t+1} F_{k,t+1} + 1 - \delta\right]. \tag{11}$$

Here, $F_{k,t}$ is the marginal product with respect to capital and $F_{l,t}$ is the one with respect to labor at t. The corresponding complementarity slackness condition is given by

$$\mu_t \left[\varepsilon_t k_{t+1} - \varepsilon_t \frac{b_{t+1}^e}{(1+r_t)} - z_t F(k_t, l_t)\right] = 0, \qquad \mu_t \geq 0. \tag{12}$$

The optimality conditions characterizing the entrepreneurs' problem in presence of a binding collateral constraint can be interpreted as follows. First, consider equations (4) and (10), i.e., the consumption-borrowing Euler equation of households and entrepreneurs, respectively. The presence of the binding collateral constraint results in a divergence between the intertemporal marginal utilities of consumption between the two types of agents.

Second, consider the consumption-capital investment Euler condition, (11). Given the formulation of the collateral constraint, (8), this intertemporal decision yields two effects. On one hand, foregoing one unit of consumption and accumulating one unit of capital today, relaxes the collateral constraint. In other words, using the terminology in Fostel and Geanakoplos (2008) and Geanakoplos and Zame (2014), the capital stock embodies a *collateral value*. Consequently, entrepreneurs would like to increase on the margin their investment in capital compared to the case when $\mu_t = 0, \forall t$. On the other hand, the new unit of capital reduces the value to entrepreneurs because the capital that was posted as collateral at t, is used in the production of the final good in the following period. The production activity requires an intraperiod loan, which tightens the collateral constraint. If the working capital loan were not collateralized, the latter effect would have been absent.

Finally, consider equation (9). Hiring one more unit of labor yields additional (to the wage) cost for entrepreneurs since, via the working capital loan, it tightens the collateral constraint. If the working capital loan were not collateralized, the optimal labor decision would not have been affected by the presence of the collateral requirement.

Overall, a binding collateral constraint of the form (8) has an effect on all marginal decisions of entrepreneurs.

2.3. Market Equilibrium

Since there are no idiosyncratic shocks, I focus on the symmetric equilibrium where all households and all entrepreneurs are alike. Then, the competitive equilibrium of the economy can be defined as follows. First, labor market clearing requires that the supply of labor by households equals the demand for labor by entrepreneurs

$$l_t = n_t, \quad \forall t. \tag{13}$$

Second, given the closed economy setup, in equilibrium, borrowing and lending is equalized

$$b^h_{t+1} = b^e_{t+1} = b_{t+1}, \quad \forall t. \tag{14}$$

Finally, combining the individual budget constraints of the two agents, the resource constraint of the economy is given by

$$c_t + c^e_t + k_{t+1} = z_t F(k_t, l_t) + (1-\delta) k_t, \ \forall t. \tag{15}$$

DEFINITION 1. *A price system is a sequence* $\{r_t, w_t\}_{t=0}^{\infty}$. *An allocation is a sequence* $\{c_t, c_t^e, l_t, n_t\}_{t=0}^{\infty}$. *An asset profile is a sequence* $\{k_{t+1}, b_{t+1}^h, b_{t+1}^e\}_{t=0}^{\infty}$.

DEFINITION 2. *For given initial values* $k_{-1}, b_{-1}^h, b_{-1}^e$, *and exogenous processes* $\{z_t, \varepsilon_t\}_{t=0}^{\infty}$, *a competitive equilibrium for the economy with a collateral constraint is a sequence of allocations* $\{c_t, c_t^e, l_t, n_t\}_{t=0}^{\infty}$, *an asset profile* $\{k_{t+1}, b_{t+1}^h, b_{t+1}^e\}_{t=0}^{\infty}$, *and a price system* $\{r_t, w_t\}_{t=0}^{\infty}$, *such that*

1. *given the price system* $\{r_t, w_t\}_{t=0}^{\infty}$, *the allocations and the asset profile solve households' and entrepreneurs' problems, and*
2. *markets clear, satisfying conditions (13)–(15).*

2.4. Steady State

It can easily be shown that the deterministic steady state of the model, where all variables are time-invariant, is unique and it can be portrayed by Proposition 1 below.

PROPOSITION 1. *The deterministic equilibrium steady state of the model can be summarized by the following equations, where the capital stock, k, and labor, l, are the solutions to*

$$1-\mu = \frac{-u_l(k,l)/u_c(k,l)}{F_l(k,l)} \tag{16}$$

$$1-\bar{\varepsilon}\mu = \gamma\left[(1-\mu)F_k(k,l)+1-\delta\right], \tag{17}$$

where $\mu = \bar{\varepsilon}^{-1}(1-\frac{\gamma}{\beta})$.

Proof. See the appendix.

COROLLARY 1. *If* $\beta > \gamma$, *then* $\mu > 0$ *and the collateral constraint binds in the steady state.*

Proof. The deterministic steady-state version of equation (10), obtained by removing the time subscripts of the variables, implies $\frac{1-\bar{\varepsilon}\mu}{1+r} = \gamma$, where $\bar{\varepsilon}$ denotes the mean value of the financial shock. Using the definition for r, obtained from the invariant deterministic steady-state version of equation (4) yields $\mu = \bar{\varepsilon}^{-1}(1-\frac{\gamma}{\beta})$. It follows that if $\beta > \gamma$, then $\mu > 0$. ■

Insofar as $\beta > \gamma$, the borrowing constraint binds in the steady state. From the demonstrated proof, its tightness in the steady state is determined by the difference in subjective discounting between the two types of agents. The more impatient the entrepreneurs are, the tighter the collateral constraint would be (i.e., the Lagrange multiplier will be larger).

3. OPTIMAL POLICY

The design of optimal policy used to correct for the inefficiencies induced by the binding collateral constraint follows the Ramsey approach. It is important to note that the planner has a single mandate, i.e., to use the policy instruments to alleviate the inefficiencies arising from the binding collateral constraint, and not to finance any exogenously given government expenditures. In particular, for the planner, I consider *corrective*, distortionary policy instruments aiming at correcting the distortions in the marginal decisions concerning the factors of production. These instruments are of corrective nature because they are imposed on one group of agents (entrepreneurs) and the revenues are returned in a lump-sum fashion back to the same agent. Further, I augment the set of instruments available to the planner by introducing *direct* lump-sum transfers from one class of agents to another aiming at performing direct income transfers. As it will become clear later, the policy conduct will be contingent on the availability of these lump-sum transfers.

The rest of the section is organized as follows. Section 3.1 presents and incorporates the policy instruments in the competitive economy with a collateral constraint. Section 3.2 outlines the Ramsey plan and discusses the properties of the optimal tax system. Section 3.3 offers alternative implementation strategies, and finally Section 3.4 shows how the Ramsey planner can implement the first-best and the second-best allocations (to be defined).

3.1. Policy Instruments

I propose the following tax system: corrective tax on capital income (τ_t^k), corrective payroll tax (τ_t^n), and direct lump-sum transfer ($\mathcal{T}_t$) from households to entrepreneurs. The policy system introduced modifies entrepreneurs' and households' budget constraints, (7) and (2), respectively, to

$$c_t^e + k_{t+1} + b_t^e + \tau_t^p w_t n_t \leq \left(1 - \tau_t^k\right)\left[z_t F\left(k_t, n_t\right) - w_t n_t\right] + \frac{b_{t+1}^e}{1 + r_t} + (1 - \delta) k_t + T_t^e + \mathcal{T}_t, \tag{18}$$

where $T_t^e = \tau_t^k[z_t F(k_t, n_t) - w_t n_t] + \tau_t^p w_t n_t$ is entrepreneurs' transfer stemming from the linear, corrective taxes, and

$$c_t + \frac{b_{t+1}^h}{1 + r_t} + T_t \leq b_t^h + w_t l_t. \tag{19}$$

With the policy system in place, the optimality conditions of households remain the same as in Section 2.1 and those of entrepreneurs change to

$$1 - \varepsilon_t \mu_t \geq \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}\left[\left(1 - \mu_{t+1} - \tau_{t+1}^k\right) z_{t+1} F_{k,t+1} + (1 - \delta)\right], \tag{20}$$

$$\left(1 - \mu_t - \tau_t^k\right) z_t F_{n,t} \geq w_t \left(1 + \tau_t^p - \tau_t^k\right), \tag{21}$$

where (20) is the capital-consumption Euler equation and (21) is the optimal labor demand decision. In what follows, I define the policy distorted competitive equilibrium (PDCE) with a collateral constraint.

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DEFINITION 3. *A PDCE, given a system of instruments* $\{\tau_t^k, \tau_t^p, T_t^e, \mathcal{T}_t\}_{t=0}^{\infty}$, *consists of a sequence of*

(i) *allocations and asset profiles for households,* $\{c_t, l_t, b_{t+1}^h\}_{t=0}^{\infty}$,
(ii) *allocations and asset profiles for entrepreneurs,* $\{c_t^e, n_t, k_{t+1}, b_{t+1}^e\}_{t=0}^{\infty}$, *and*
(iii) *prices* $\{r_t, w_t\}_{t=0}^{\infty}$, *such that,*

the allocations and the asset profiles satisfy the following conditions:

$$\varepsilon_t k_{t+1} \geq z_t F(k_t, n_t) + \varepsilon_t \frac{b_{t+1}^e}{1+r_t}, \tag{22}$$

$$\left[\varepsilon_t k_{t+1} - z_t F(k_t, n_t) - \varepsilon_t \frac{b_{t+1}^e}{1+r_t}\right] \mu_t = 0, \tag{23}$$

$$\mu_t \geq 0, \tag{24}$$

$$-\frac{u_{l,t}}{u_{c,t}} = w_t, \tag{25}$$

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} (1+r_t), \tag{26}$$

$$n_t = l_t, \tag{27}$$

$$b_{t+1}^h = b_{t+1}^e = b_{t+1}, \quad \forall t \tag{28}$$

$$\frac{1-\varepsilon_t \mu_t}{1+r_t} = \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}, \tag{29}$$

$$T_t^e = \tau_t^k [z_t F(k_t, n_t) - w_t n_t] + \tau_t^p w_t n_t, \tag{30}$$

(18)–(21).

3.2. The Ramsey Plan

Given the policy instruments, the Ramsey planner maximizes the social welfare function subject to the constraints constituting a PDCE (with a collateral constraint). Here, the objective is given by the utilitarian welfare function, which cannot be defined unanimously, given that it depends on exogenous Pareto–Negishi weights, ω for households and $(1-\omega)$ for entrepreneurs.[9] I assume that the planner has a technology to commit to the policy chosen at the beginning of time, and I formally define the Lagrangian of the Ramsey problem as follows.

DEFINITION 4. *Let* $\Lambda \equiv \{\lambda_t^g\}_{t=0}^{\infty}, \forall g = 1, 2, 3, \ldots$ *represent Lagrange multipliers on constraints (18)–(30), respectively. For given stochastic*

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processes $\{z_t, \varepsilon_t\}_{t=0}^{\infty}$*, plans for the control variables* $\Xi \equiv \{c_t, l_t, b_{t+1}^h, c_t^e, n_t, k_{t+1}, b_{t+1}^e, r_t, w_t, \tau_t^k, \tau_t^p, T_t^e, \mathcal{T}_t\}_{t=0}^{\infty}$ *and for the co-state variables* $\{\lambda_t^g\}_{t=0}^{\infty}$ *represent a Ramsey allocation if they solve the following maximization problem:*

$$\min_{\Lambda} \max_{\Xi} E_0 \sum_{t=0}^{\infty} \Delta^t \left[\omega u\left(c_t, l_t\right) + (1-\omega)\, u^e\left(c_t^e\right)\right], \qquad \textbf{(31)}$$

subject to conditions (18)–(30).[10] *Here,* Δ *denotes the discount factor of the planner. I follow Monacelli (2008) in setting* $\Delta \equiv \beta^{\omega}\gamma^{1-\omega}$.[11]

To deliver analytical solutions of the deterministic steady-state policy, in characterizing the Ramsey allocations, I first drop entrepreneurs' distorted optimality conditions, (20) and (21), after substituting *in* the consumption-borrowing Euler condition, (29), which holds with equality in the PDCE. Then, I show that for the allocations solving this Less Restricted Ramsey (LRR), the omitted constraints do not bind (i.e., their respective Lagrange multipliers equal zero). Intuitively, this result arises since the planner has enough degrees of freedom to choose policy instruments such that the constraints involving those instruments are not binding. Proposition 8 in the appendix establishes the equivalence result between the *more* and the *less* restricted Ramsey plans, followed by the proof.[12]

The problem of the LRR plan is given by

$$\max_{\{c_t, c_t^e, l_t, k_{t+1}, b_{t+1}, \mathcal{T}_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \Delta^t \left[\omega u\left(c_t, l_t\right) + (1-\omega)\, u^e\left(c_t^e\right)\right] \qquad \textbf{(32)}$$

$$s.t. \quad c_t + \frac{b_{t+1}}{1 + R\left(c_t, c_{t+1}\right)} + \mathcal{T}_t \leq w\left(c_t, l_t\right) l_t + b_t \qquad \textbf{(33)}$$

$$c_t^e + w\left(c_t, l_t\right) l_t + b_t + k_{t+1} \leq z_t F\left(k_t, l_t\right) + \frac{b_{t+1}}{R\left(c_t, c_{t+1}\right)} + (1-\delta)\, k_t + \mathcal{T}_t \qquad \textbf{(34)}$$

$$\varepsilon_t k_{t+1} \geq z_t F\left(k_t, l_t\right) + \varepsilon_t \frac{b_{t+1}}{R\left(c_t, c_{t+1}\right)}, \qquad \textbf{(35)}$$

where I have substituted (26) and (25) to express $R_t \equiv 1 + r_t$ and w_t, respectively, and imposed the market-clearing conditions, i.e., $b_t^e = b_t^h = b_t$ and $l_t = n_t$, $\forall t$; finally, $T_t^e = \tau_t^k[z_t F(k_t, n_t) - w_t n_t] + \tau_t^p w_t n_t$ is substituted in the budget constraint of entrepreneurs (18) to obtain (34). Then, letting $\lambda_t^{RP,h}$ denote the Lagrange multiplier on (33), $\lambda_t^{RP,e}$ on (34), and $\lambda_t^{RP,e}\mu_t^{RP}$ on (35), the first-order conditions describing the equilibrium allocations are given by

c_t :

$$\Delta^t \omega u_{c,t} - \lambda_t^{RP,h} + \left(\lambda_t^{RP,e} - \lambda_t^{RP,h}\right) \left\{ \frac{\partial \left[\frac{b_{t+1}}{R(c_t, c_{t+1})}\right]}{\partial c_t} - \frac{\partial w\left(c_t, l_t\right)}{\partial c_t} \right\}$$

$$- \lambda_t^{RP,e} \mu_t^{RP} \varepsilon_t \frac{\partial \left[\frac{b_{t+1}}{R(c_t, c_{t+1})}\right]}{\partial c_t} = 0, \qquad \textbf{(36)}$$

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c_t^e :

$$\Delta^t (1-\omega) u_{c,t}^e - \lambda_t^{RP,e} = 0, \tag{37}$$

b_{t+1} :

$$\frac{\lambda_t^{RP,e} - \lambda_t^{RP,h} - \varepsilon_t \lambda_t^{RP,e} \mu_t^{RP}}{R(c_t, c_{t+1})} - E_t \left(\lambda_{t+1}^{RP,e} - \lambda_{t+1}^{RP,h}\right) = 0, \tag{38}$$

l_t :

$$\Delta^t \omega u_{l,t} - \left(\lambda_t^{RP,e} - \lambda_t^{RP,h}\right) \left[w(c_t, l_t) + \frac{\partial w(c_t, l_t)}{\partial l_t} l_t \right] + \lambda_t^{RP,e} \left(1 - \mu_t^{RP}\right) z_t F_{l,t} = 0, \tag{39}$$

k_{t+1} :

$$\lambda_t^{RP,e} \left(1 - \varepsilon_t \mu_t^{RP}\right) - E_t \lambda_{t+1}^{RP,e} \left[\left(1 - \mu_{t+1}^{RP}\right) z_{t+1} F_{k,t+1} + 1 - \delta \right] = 0, \tag{40}$$

$\mathcal{T}_t$:

$$\lambda_t^{RP,e} = \lambda_t^{RP,h}. \tag{41}$$

Note that in the case when direct lump-sum transfers are not available ($\mathcal{T}_t = 0$), condition (41) does not need to hold.

3.3. Properties of Optimal Taxation

The Ramsey planner chooses corrective taxes with the objective to maximize the social welfare in the economy. As it turns out, policy instruments treat economic outcomes in two ways. First, they are set to affect some of the structural parameters in the economy, which in turn have an impact on the tightness of the collateral constraint as it will become clear below. In addition, in absence of direct lump-sum transfers, the policy instruments are set to perform implicit income redistributions from one group of agents to another, governed by the planner's incentive to help entrepreneurs rebuild their net-worth, as well as by the weights on each agents' utility in the social welfare function.

Tax on capital income. To derive the optimal tax on capital income, consider the (time invariant) deterministic equilibrium steady-state no-arbitrage conditions characterizing the PDCE and the Ramsey planner's equilibrium, respectively[13]

$$1 - \left(1 - \frac{\gamma}{\beta}\right) = \gamma \left\{ \left[1 - \frac{(1-\gamma/\beta)}{\bar{\varepsilon}} - \tau^k \right] F_k + 1 - \delta \right\}, \tag{42}$$

$$1 - \Phi \left(1 - \frac{\Delta}{\beta}\right) = \Delta \left\{ \left[1 - \Phi \frac{(1-\Delta/\beta)}{\bar{\varepsilon}} \right] F_k + 1 - \delta \right\}. \tag{43}$$

Here, equation (42) represents the no-arbitrage condition of the PDCE obtained by combining the consumption-capital investment and the consumption-borrowing

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Euler conditions, i.e., conditions (20) and (29) at the deterministic steady state. Equation (43) represents the no-arbitrage condition of the planner's problem derived by combining the corresponding conditions (38) and (40) at the steady state, where $\Phi \equiv \frac{\lambda^{RP,e}-\lambda^{RP,h}}{\lambda^{RP,e}}$.[14] Then, the optimal tax on capital income at the steady state can be derived by subtracting one equation from the other and by expressing τ^k.

PROPOSITION 2. *The optimal tax on capital income in the deterministic equilibrium steady-state set by a Ramsey planner, whose only instruments are corrective taxes, is given by*

$$\tau^k = \frac{1}{F_k}\left(\frac{1}{\Delta} - \frac{1}{\beta}\right) - \frac{1}{\bar{\varepsilon}}\left(1 - \frac{\gamma}{\beta}\right) - \Phi\left[\frac{1}{F_k}\left(\frac{1}{\Delta} - \frac{1}{\beta}\right) - \frac{1}{\bar{\varepsilon}}\left(1 - \frac{\Delta}{\beta}\right)\right]. \quad \textbf{(44)}$$

First, the tax rate crucially depends on some of the structural parameters in the economy, in particular, those reflecting the discounting of the future. Hence, it is set to account for the gap that exists in the discounting of the future between the private agents and the planner on one hand, and the gap in the discounting between households and entrepreneurs, which also determines the tightness of the collateral constraint in the PDCE. Second, the tax rate depends on Φ, which denotes the gap in the planner's valuation of agents' budget constraints, indicating that the planner internalizes that agents do not value an incremental unit of consumption equally. Therefore, it uses the tax rate for the purpose to implicitly redistribute income from one class of agents to another with the aim to close the gap between $\lambda^{RP,e}$ and $\lambda^{RP,h}$. In addition, the redistributional incentives of the planner are also governed by the social welfare weights, ω, as visible from equations (36) and (37).

In presence of direct lump-sum transfers, Φ equals 0,[15] implying that income redistribution is performed directly via the transfers and not implicitly via the distortionary tax rates. Then, the tax on capital income takes the following form:

$$\tau^k_{\Phi=0} = \frac{1}{F_k}\left(\frac{1}{\Delta} - \frac{1}{\beta}\right) - \frac{1}{\bar{\varepsilon}}\left(1 - \frac{\gamma}{\beta}\right). \quad \textbf{(45)}$$

In this case, the tax rate is solely used for the purpose of affecting the structural parameters in the model economy. On one hand, the difference between the discount factor of households and entrepreneurs, which drives the shadow value of the collateral constraint in the competitive economy, influences the optimal capital tax negatively. On the other hand, the difference between the discount factor of the planner and the households, which indirectly captures the efficient use of funds, influences the capital tax positively. I explore the effect of these parameters on the level of the capital tax in the calibrated economy in Section 4.2.

Payroll tax. Similarly as before, in order to derive the rate of the payroll tax, consider the deterministic steady-state optimal labor conditions characterizing the

PDCE and the one of the Ramsey planner, respectively,

$$\left(1 - \frac{1-\gamma/\beta}{\bar{\varepsilon}} - \tau^k\right) F_l = -\frac{u_l}{u_c}\left(1 + t^p - \tau^k\right), \tag{46}$$

$$\left(1 - \Phi\frac{1-\Delta/\beta}{\bar{\varepsilon}}\right) F_l = -\frac{\omega}{1-\omega}\frac{u_l}{u_c^e} + \Phi\left[w\left(c,l\right) + \frac{\partial w\left(c,l\right)}{\partial l} l\right]. \tag{47}$$

Here, (46) is obtained by substituting (10) in (9) at the steady state. Similarly, condition (47) is obtained by combining (38) and (39) at the steady state.

PROPOSITION 3. *The payroll tax in the deterministic equilibrium steady state is given by*

$$\tau^p = \left[\frac{1}{\bar{\varepsilon}}\left(1 - \frac{\gamma}{\beta}\right) + \tau^k\right] F_l \frac{u_c}{u_l} + \frac{\omega}{1-\omega}\frac{u_c}{u_c^e} + \tau^k - 1$$
$$-\Phi\frac{u_c}{u_l}\left[w\left(c,l\right) + \frac{\partial w\left(c,l\right)}{\partial l} l + \frac{1}{\bar{\varepsilon}}\left(1 - \frac{\Delta}{\beta}\right) F_l\right]. \tag{48}$$

The expression of the payroll tax suggests that the policy rate affects the tightness of the collateral constraint by affecting the gap in the future, the discounting that exists between households and entrepreneurs, as well as it implicitly transfers resources in order to bring closer to equality the planner's shadow costs of wealth, $\lambda^{RP,e}$ and $\lambda^{RP,h}$. On the other hand, when lump-sum transfers are available ($\Phi = 0$), the planner chooses the following payroll tax rate at the deterministic steady state[16]:

$$\tau^p_{\Phi=0} = -\frac{1}{\bar{\varepsilon}}\left(1 - \frac{\gamma}{\beta}\right). \tag{49}$$

This result suggests that given direct lump-sum transfers, the planner chooses the payroll tax solely to address the labor demand distortions arising from the collateral constraint, without taking care of the redistributional considerations. There is a subsidy on labor for entrepreneurs to compensate them for the costs incurred from the presence of the binding collateral constraint. Finally, it is important to note that if the intraperiod borrowing were not collateralized and the intratemporal margin were as not affected by the collateral requirement, this tax rate would be zero.

To sum up, there is a difference in the way that the planner uses linear, distortionary taxes to correct for the inefficiencies in the competitive economy when direct lump-sum transfers are available and when they are not. In the latter case, the planner faces a trade-off between using the taxes to address the distortions in the optimal conditions of the factors of production and using them to indirectly induce desirable income transfers. This trade-off is eliminated once lump-sum transfers are available, rendering corrective taxes independent from shadow prices.

Alternative policy instruments. I also exploit the possibility of using alternative policy instruments to implement the planner's allocations. In the preceding

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TABLE 1. Alternative policy systems

	RP (τ^k, τ^p)	RP (τ^b, τ^p)	RP (η, τ^p)	RP (τ^k, τ^l)
τ^k	1.35%	–	–	1.35%
τ^p	72.90%	74.82%	74.59%	–
τ^b	–	0.07%	–	–
η	–	–	0.34	–
τ^l	–	–	–	42.50%
b	3.372	3.372	3.372	3.372
k	9.816	9.816	9.816	9.816
l	0.293	0.293	0.293	0.293
F_k	0.038	0.038	0.038	0.038
F_l	2.267	2.267	2.267	2.267
Welf.Gain.	–	32.06%	32.06%	32.06%

Note: The table presents the equivalence among the different tax systems.
All variables are presented for the case when $\omega = 0.5$ and $\mathcal{T} = 0$.

sections, I focus specifically on corrective taxes, which are imposed on and distributed back to the same agent. However, one can consider alternative instruments that are not necessarily fiscal in nature or corrective per se.[17] I show analytically the equivalence between the system of capital income and payroll taxes to alternative policy systems. Table 1 confirms the results numerically for a given calibration.

In what follows, I will show both analytically and numerically that the corrective tax on capital income can be substituted with either a corrective tax on borrowing or with a loan-to-value (LTV) ratio limit, optimally chosen by the planner. In addition, the payroll tax can be replaced by a tax on labor income, imposed on the side of households, but its revenues have to be distributed to entrepreneurs.

First, I consider a corrective tax on borrowing, τ_t^b, which is in place instead of the tax on capital income, τ_t^k. The borrowing tax, since corrective in nature, leaves all equilibrium conditions unchanged (as in Section 3.1 with $\mathcal{T}_t = 0$), except for the consumption-borrowing Euler condition of entrepreneurs, which becomes

$$\frac{1 - \tau_t^b - \mu_t}{1 + r_t} = \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}, \tag{50}$$

or in the deterministic steady state, it reads

$$\frac{1 - \tau^b - \mu}{1 + r} = \gamma. \tag{51}$$

PROPOSITION 4 (Equivalent Tax Systems). *Given a system, consisting of capital income and payroll taxes, $\{\tau_t^k, \tau_t^p\}_{t=0}^{\infty}$, there exists a system consisting of borrowing and payroll taxes, $\{\tau_t^b, \tilde{\tau}_t^p\}_{t=0}^{\infty}$, such that the two competitive equilibria with the corrective taxes are equivalent in both economies.*

Proof. See the appendix.

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The proof shows that given a system of prices and taxes $\{r_t, w_t, \tau_t^k, \tau_t^p\}_{t=0}^{\infty}$, one can construct a system of prices and taxes $\{\tilde{r}_t, \tilde{w}_t, \tau_t^b, \tilde{\tau}_t^p\}_{t=0}^{\infty}$, such that both economies yield the same allocations. Therefore, the planner may impose a tax on borrowing to substitute for the one on capital income and the two perform the exact same role.[18] The following Corollary reports the borrowing tax in the steady state.

COROLLARY 2. *The deterministic equilibrium steady-state value of the tax on borrowing is given by*

$$\tau^b = \frac{\gamma F_k}{1 - \frac{\gamma F_k}{\bar{\varepsilon}}} \tau^k. \tag{52}$$

Alternatively, the planner could impose quantity limits on assets instead of affecting the marginal return (cost) on investment (borrowing) thorough taxes. To examine this possibility, I consider an optimal LTV ratio limit, instead of the corrective tax on capital income or the borrowing tax, in order to replicate the planner's allocations. The LTV ratio differs from the remaining tools because it restricts the quantity of the asset that serves as collateral directly instead of affecting it indirectly via its price. Also, unlike the tax on capital income, which is a fiscal instrument, the LTV ratio is a financial policy tool. To define the LTV ratio in the model economy, I add an additional constraint to the entrepreneurs' problem, given by[19]

$$\eta_t k_{t+1} \geq \frac{b_{t+1}}{1 + r_t}, \tag{53}$$

where η_t denotes a time-varying LTV ratio. Setting $\lambda_t^e \chi_t$ to be the Lagrange multiplier on this constraint, the intertemporal optimality conditions of entrepreneurs become

$$1 - \varepsilon_t \mu_t - \eta_t \chi_t = \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e} \left[(1 - \mu_{t+1}) z_{t+1} F_{k,t+1} + 1 - \delta \right], \tag{54}$$

$$\frac{1 - \varepsilon_t \mu_t - \eta_t \chi_t}{1 + r_t} = \gamma E_t \frac{u_{c,t+1}^e}{u_{c,t}^e}. \tag{55}$$

All other equations remain the same as in Section 3.1, with $\mathcal{T}_t = 0$. At the deterministic steady state, both Lagrange multipliers (μ_t and χ_t) are positive, indicating that both debt constraints (35) and (53) bind at the same time. This result is also confirmed numerically.[20] Technically, the two debt constraints bind at the same time since they are not collinear: one limits only the long-term (i.e., per-period) borrowing, (53), and the other one limits the short-period borrowing of the entrepreneur, (35). Similarly, as with the tax on capital income, the LTV ratio is optimally chosen by the planner in order to manipulate the choice of capital and borrowing by affecting the consumption-capital Euler equation.

COROLLARY 3 (Equivalence between Fiscal and Financial Instruments). *Given a system, consisting of capital income and payroll taxes, $\{\tau_t^k, \tau_t^p\}_{t=0}^{\infty}$, there exist a system consisting of a combination of fiscal (tax) and financial (LTV)*

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policy instruments, $\{\eta_t, \tilde{\tau}_t^p\}_{t=0}^{\infty}$, *such that the two competitive equilibria with the different policy systems yield equivalent allocations.*

The proof of this Corollary can be derived following the exact same steps showing the equivalence between the taxes on capital income and borrowing, outlined above.

Therefore, both equivalence results show that the tax on capital income is essentially a financial tool. Its equivalence with the corrective tax on borrowing suggests that it can be used to affect the marginal investment/borrowing (i.e., the no-arbitrage) condition and therefore it can be used as a price-based policy tool. On the other hand, its equivalence with the LTV regulation suggests that the tax on capital income can be used to directly affect the quantities of the assets (i.e., the capital stock) and therefore acts as a quantity-based policy tool.

Finally, I consider a redistributive tax on labor income, which is imposed on households and its proceeds are redistributed to entrepreneurs. The redistributive tax on labor can be used to replicate the planner's allocations in combination with any of the policy instruments, affecting the intertemporal no-arbitrage condition, discussed above. It is important to note that the tax on labor income has to be redistributive and not corrective. The reason is that the policy tool, required to correct the intratemporal decision, would need to address the distortions present in the entrepreneurs' side of the economy. In this way, entrepreneurs' decisions would not be corrected, but they would instead be compensated by a lump-sum transfer from households, i.e., the proceeds from the labor tax.

Introducing the redistributive tax on labor income on the side of households results in the following changes of the equations in the PDCE:

$$c_t + \frac{b_{t+1}^h}{1+r_t} + T_t^h = \left(1 - \tau_t^l\right) w_t l_t + b_t^h, \tag{56}$$

$$u_{c,t}\left(1 - \tau_t^l\right) w_t = -u_{l,t}, \tag{57}$$

$$\left(1 - \mu_t - \tau_t^k\right) z_t F_{n,t} \geq w_t\left(1 - \tau_t^k\right), \tag{58}$$

$$c_t^e + w_t l_t + b_t^e + k_{t+1} = z_t F\left(k_t, l_t\right) + (1-\delta) k_t + \frac{b_{t+1}^e}{1+r_t} + T_t^h, \tag{59}$$

where $T_t^h = \tau_t^l w_t l_t$. Equation (56) is the budget constraint of households; equation (57) is the optimal labor decision of households; equation (58) is the optimal labor decision of entrepreneurs; and equation (59) is the budget constraint of entrepreneurs. All other equations remain intact, as in Section 3.1 with $\mathcal{T}_t = 0$.

PROPOSITION 5 (Equivalence between Labor and Payroll Taxes). *Given a system consisting of capital income and payroll taxes (imposed on and proceeds distributed back to entrepreneurs),* $\{\tau_t^k, \tau_t^p\}_{t=0}^{\infty}$, *there exist a system consisting of capital and labor income taxes (imposed on households and proceeds distributed to entrepreneurs),* $\{\tilde{\tau}_t^k, \tau_t^l\}_{t=0}^{\infty}$, *such that the two competitive equilibria with the different policy systems yield equivalent allocations.*[21]

The proof for this Proposition can be shown following a similar approach as the one showing the equivalence between the tax on capital income and the LTV limit or the tax on borrowing.

3.4. Implementation of Welfare Benchmarks

This section defines the first-best and the second-best allocations and outlines the policy instruments implementing those benchmarks.

The first-best allocations of this economy can be obtained as a solution to the following planning problem:

$$\max_{\{c_t, c_t^e, l_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \Delta^t \left[\omega u \left(c_t, l_t \right) + (1-\omega) u^e \left(c_t^e \right) \right] \tag{60}$$

$$s.t.\ c_t + c_t^e + k_{t+1} \leq z_t F \left(k_t, l_t \right) + (1-\delta) k_t, \tag{61}$$

where the objective function is the same utilitarian welfare function defined in Section 3.2. The capital Euler equation is as in (11) with $\mu_t = 0$, $\forall t$, and the optimal labor decision is $-\frac{u_{l,t}}{u_{c,t}} = z_t F_{l,t}$. Therefore, as $\mu_t \to 0\, \forall t$ and the collateral constraint becomes loose, the competitive economy converges to its first-best allocations (with exogenous Pareto–Negishi weights).

The first equivalence result posits that the allocations of a Ramsey planner, endowed with corrective capital income and payroll taxes, and a direct lump-sum transfer, replicates the first-best allocations. The following Proposition summarizes this equivalence result.

PROPOSITION 6. *The Ramsey planner's problem with a sequence of policies* $\{\tau_t^k, \tau_t^p, T_t^e, \mathcal{T}_t\}_{t=0}^{\infty}$ *yields equivalent optimal allocations to a social planner, who maximizes the aggregate utility function of the two agents subject to the resource constraint.*

Proof. Consider the first-order conditions of the LRR plan in Section 3.2. The presence of lump-sum transfers, $\mathcal{T}_t$, imposes an equality between the shadow costs of wealth ($\lambda_t^{RP,e} = \lambda_t^{RP,h}$ in the appendix). Honoring this equality in the remaining first-order conditions, we can derive the exact same conditions as those obtained by solving the planner's problem, outlined in this section above. ■

The second equivalence result posits that the allocations of a Ramsey planner, endowed with corrective capital income and payroll taxes replicates the allocations of a constrained social planner, as similarly formalized in Dàvila et al. (2012), and formally defined for this economy below.

DEFINITION 5. *The problem of the constrained social planner for this economy consists in maximizing the social welfare function of households and entrepreneurs subject to their individual budget constraints and the* *collateral constraint of entrepreneurs, while recognizing that* $w(c_t, l_t) = -\frac{u_{l,t}}{u_{c,t}}$ *and* $Rc_t, c_{t+1} \equiv 1 + r(c_t, c_{t+1}) = \frac{u_{c,t}}{\beta u_{c,t+1}}$.

The definition implies that the constrained planner's allocations (i.e., the second-best allocations) can be found as a solution to the LRR problem with the only difference that $\mathcal{T}_t = 0$, i.e., without direct lump-sum transfers between the two groups of agents. Given that the Ramsey problem is exactly the same as the constrained social planner's problem, then the allocations that will characterize their equilibria will be equivalent. The following Proposition summarizes this result.

PROPOSITION 7. *The Ramsey planner's problem with a sequence of policies* $\{\tau_t^k, \tau_t^p, T_t^e\}_{t=0}^{\infty}$ *yields equivalent optimal allocations to those of the constrained social planner.*

It may sound surprising that the Ramsey planner needs distortionary taxes to obtain the first-best equilibrium outcome and cannot do this with lump-transfers alone. The reason is that the collateral constraint introduces distortions in the optimality conditions of agents, which lump-sum transfers cannot fix since they do not affect marginal decisions concerning the optimal choices of the factors of production.

Finally, even though one may consider that the planner with lump-sum transfers is too powerful and would rather focus attention to second-best outcomes, omitting the lump-sum transfers poses the risk of using distortionary taxes as a tool to indirectly transfer resources among agents.[22] For the economy outlined above, by comparing the tax systems implementing the second-best versus the first-best allocations, one can disentangle the part of the tax that is being used to fix for the distortions arising from the collateral constraint and the part that is being used to implicitly transfer resources from one group of agents to another.

4. QUANTITATIVE ANALYSIS

The goal of this section is to evaluate the optimal policy *at* and outside the steady state for a calibrated version of the model economy. To perform the outside the steady-state analysis, I consider the responses of a subset of variables and the policy instruments to productivity and financial shocks. In the computations, I conjecture that the collateral constraint is always binding, which is indeed the case in the steady state.[23] In the appendix, I perform an exercise that shows that, in presence of productivity and financial shocks, the collateral constraint binds with a probability of 99%.

4.1. Calibration

Agents' preferences. The time period is a quarter. The subjective discount factor is set to $\beta = 0.99$ and $\gamma = 0.98$, respectively.[24] The discount factor of the planner is set to $\Delta = \beta^{0.5}\gamma^{0.5}$ since I take $\omega = 0.5$ in my benchmark calibration. The utility

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TABLE 2. Summary of model parameters

Description	Parameter
Discount factor	$\beta = 0.98,\ \gamma = 0.98,\ \Delta = \beta^{0.5}\gamma^{0.5}$
Utility parameter	$\psi = 2.2$
Production technology	$\alpha = 0.36$
Depreciation rate	$\delta = 0.025$
Collateral constraint parameter	$\bar{\varepsilon} = 0.16$
Std. of productivity shock	$\sigma_z = 0.0045$
Std. of financial shock	$\sigma_\varepsilon = 0.0098$
Matrix for the shock processes	$A = \begin{bmatrix} 0.9457 & -0.0091 \\ 0.0321 & 0.9703 \end{bmatrix}$
Pareto weight	$\omega = 0.5$

function of households takes the logarithmic form $u(c_t, l_t) = \log(c_t) + \psi \log(1 - l_t)$ with $\psi = 2.2$, generating a steady-state value of $l \approx 0.3$ in the competitive economy, and similarly the utility function of the entrepreneur takes the form of $u^e(c_t^e) = \log(c_t^e)$.

Production. The production function of entrepreneurs is Cobb–Douglas $F(\cdot) = z_t k_t^\alpha n_t^{1-\alpha}$ with $\alpha = 0.36$. The quarterly depreciation rate is set to $\delta = 0.025$.

Financial frictions. Following Jermann and Quadrini (2012), the mean value of the financial variable $\bar{\varepsilon} = 0.16$ is chosen such that the ratio of debt to GDP replicates the one observed in the data (≈ 3).[25]

Exogenous shocks. The model economy is analyzed in response to productivity and financial shocks. The parameterization of the persistence parameters and the shock variances of these variables follows Jermann and Quadrini (2012), estimated from the following autoregressive system:

$$\begin{pmatrix} z_{t+1} \\ \varepsilon_{t+1} \end{pmatrix} = A \begin{pmatrix} z_t \\ \varepsilon_t \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^z \\ \epsilon_{t+1}^\varepsilon \end{pmatrix}, \tag{62}$$

where z_t and ε_t are log-deviations from the deterministic trend, and ϵ_{t+1}^z and $\epsilon_{t+1}^\varepsilon$ are independent and identically distributed (i.i.d.), with standard deviations σ_z and σ_ε, respectively.

The full set of parameters is summarized in Table 2.

4.2. Steady-State Results

Table 3 reports the planning outcomes and the competitive equilibrium steady-state allocations in presence and in absence of lump-sum transfers. I consider the benchmark for the planner's weights to be $\omega = 0.5$ such that both agents are equally important to the planner.

From Table 3, a few general observations can be made regarding the planning outcomes. First, the level of borrowing in the planner's economy is lower compared to the competitive economy with a binding collateral requirement,

TABLE 3. Steady-state allocations

	CE	SB	RP (SB implement.)	FB	*RP (FB implement.)*
τ^k	–	–	1.35%	–	6.46%
τ^p	–	–	72.90%	–	−6.31%
$\mathcal{T}$ % total consump.	–	–	–	–	0.36%
b	3.634	3.372	3.372	–	–
k	10.293	9.816	9.816	13.116	13.116
l	0.300	0.293	0.293	0.428	0.428
F_k	0.0375	0.038	0.038	0.040	0.040
F_l	2.285	2.267	2.267	2.195	2.195
Welf.Gain.	–	32.06%	32.06%	50.47%	50.47%

Note: The table presents the first-best (FB) (as defined in Section 3.4), the second-best (SB) (as defined in Section 3.4, Definition 5), and the competitive equilibrium allocations along with the taxes and the lump-sum transfers that implement them. RP denotes the policies set and the allocations obtained by the Ramsey planner (as defined in Section 3.2, Definition 4). The welfare gain is presented in terms of average consumption compensation of the agents. The social welfare weight is set to $\omega = 0.5$.

whereas the marginal product of capital is higher and the marginal product of labor is lower. This suggests that the private agents have an incentive to overborrow (compared to the first- and to the second-best benchmarks for the given weights), and they favor capital investment, which relaxes their collateral constraint, to labor purchases, which tightens it.

Several points are worth noting. First, the sign on the payroll tax is ambiguous depending on the availability of lump-sum transfers (see the RP column). Without lump-sum transfers, there is a positive payroll tax imposed in order to decrease labor demand by entrepreneurs since hiring more labor tightens their collateral constraint and reduces consumption. In this way, the payroll tax effectively serves to perform an implicit transfer from households to entrepreneurs since the lower equilibrium level of labor reduces wages. On the other hand, when lump-sum transfers are available, the planner sets a negative payroll tax (i.e., a subsidy). The turn in the sign of the payroll tax arises because the planner uses it to directly compensate entrepreneurs for the incurred costs from the binding collateral constraint rather than implicitly via the distortionary tax. Indeed, we can see that there is a direct lump-sum transfer going from households to entrepreneurs, amounting to 36% of aggregate consumption.

Second, the tax on capital income for the given calibration is positive regardless of whether lump-sum transfers are available or not. Nonetheless, quantitatively the tax levels differ. Without lump-sum transfers, the planner uses positive tax to bring the marginal return on capital closer to its socially optimal level, but it does not go all the way because higher taxes restrict the ability of entrepreneurs to accumulate net-worth. On the contrary, the capital tax is set at its highest level when entrepreneurs can build net-worth through lump-sum transfers by the planner. Then, a positive tax rate reduces the demand for capital and brings its marginal rate closer to the socially optimal level.

Welfare gains, measured by the compensating variation in consumption, are substantial for this set of weights even if one accounts for the transitory dynamics from one steady state to another. Figure 1 displays the change in welfare along the transition from the steady state without to the steady state with the tax regime.[26] The reason for the high levels of the welfare gain is due to the more than double increase in entrepreneurs' consumption, who have high marginal utility in the competitive equilibrium.

Since there is no consensus in the literature concerning the value of the planner's weights set in the social welfare function, I perform the analysis also by considering a grid for social welfare weights. Figure 2 presents the tax on capital income, the payroll tax, and the lump-sum transfers implementing the first- and the second-best solutions as functions of households' Pareto–Negishi weight. In particular, the tax on capital income, implementing the second-best allocations, starts from levels close to zero for a low weight on households and exhibits an inverted U-shape, while the tax implementing the first-best starts positive and decreases in ω. The planner chooses a subsidy when the weight on households' utility is high. Since at this point as ω becomes bigger, the redistributional incentives of

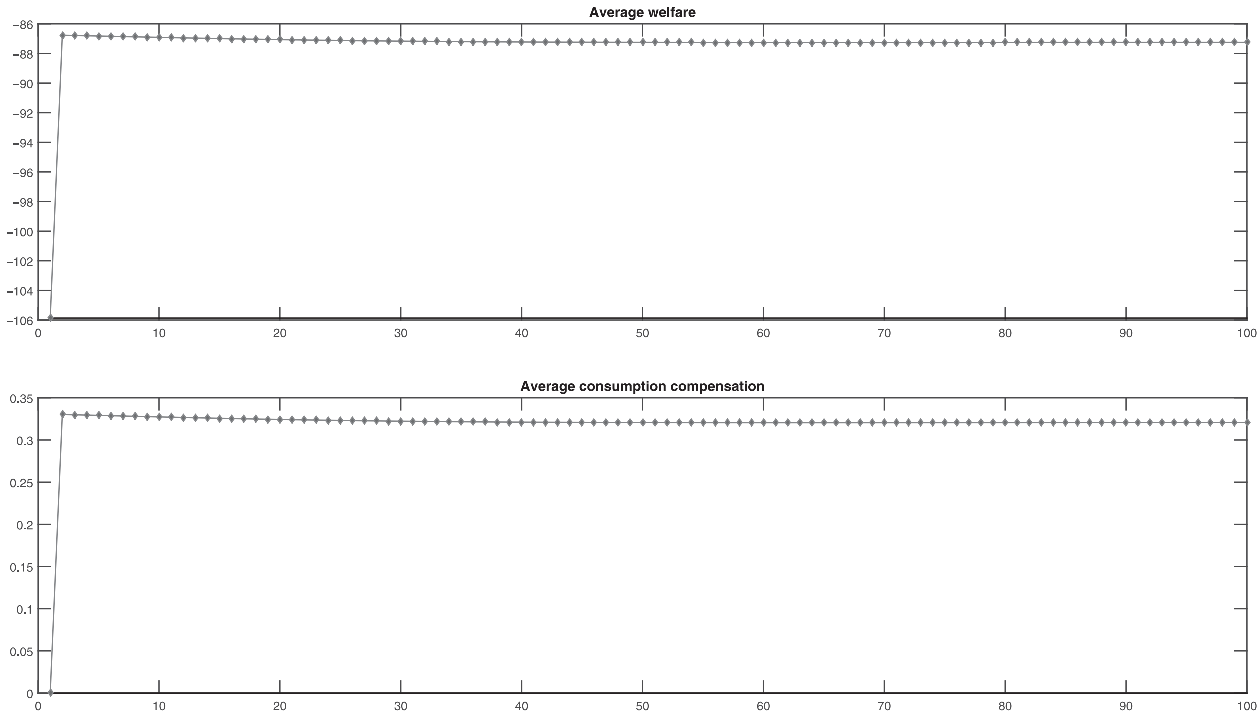

FIGURE 1. The figure presents the transition from the steady state without to the steady state with capital income and payroll taxes (no direct lump-sum transfer available). The first graph (above) shows the transition for the (average) economy-wide welfare; the second graph shows the transition of the welfare in terms of (agents' average) compensation variation.

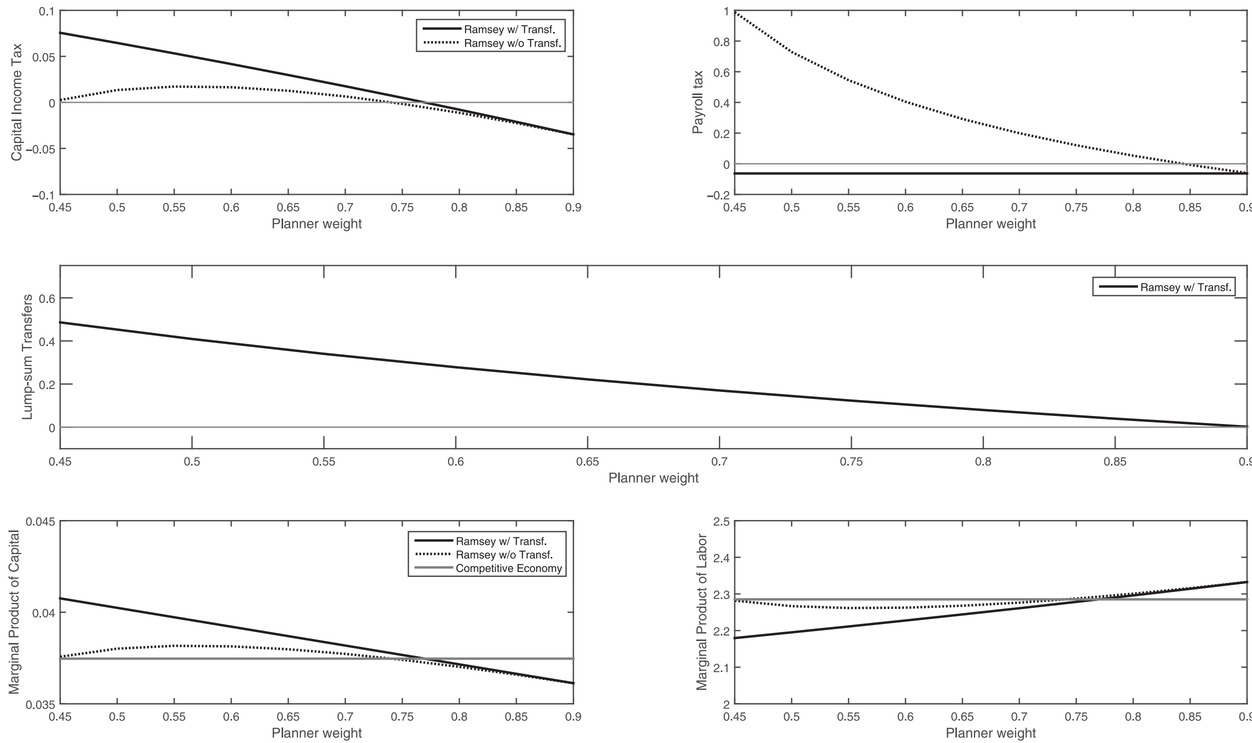

FIGURE 2. Steady-state levels of the capital income and payroll taxes for a grid of weights $\omega \in [0.45, 0.90]$, where ω denotes the weight placed on households' utility in the social welfare function. I assume this range of weights since it provides reasonable results such that the shadow values of wealth for both agents are positive. The figure compares the tax rates in the case when the Ramsey planner has access to lump-sum transfers and when it does not. Panels (4) and (5) in the bottom of the figure present the steady-state levels of the marginal products for the same grid of weights. The panels compare the values in the competitive economy, the Ramsey plan with and without lump-sum transfers.

the planner diminish (in other words, $\Phi \to 0$), and the planner becomes close to as patient as the households, $\Delta \to \beta$ (recall $\Delta = \beta^{\omega}\gamma^{1-\omega}$), it can accumulate sufficient resources to rebuild entrepreneurs' net-worth (without using households' income for redistributional purposes).[27] Similarly, the payroll tax starts from high positive levels and steadily decreases, while turning negative when it approaches its first-best (negative) level, confirming the analytical result presented earlier. Finally, in absence of lump-sum transfers, both the payroll and the capital income taxes implement an implicit income transfer from households to entrepreneurs. However, as ω becomes bigger, the redistributional incentives of the planner diminish, and the direct lump-sum transfers approach zero.

4.3. Optimal Policy in Response to Shocks

Outside the steady state, I solve the model under the assumption that the collateral constraint is always binding. I follow the dual approach in solving the Ramsey problem, so that the planner directly chooses allocations, prices, and policies. The optimal policy in response to shocks is computed using a first-order approximation of the equilibrium conditions for the recursive Lagrangian problem, characterizing the Ramsey plan. This requires, first, computing the stationary allocations that characterize the deterministic steady state of the optimality conditions of the Ramsey plan, then computing the first-order Taylor approximation of the respective policy functions around the same steady state. Essentially, this implies that the economy has been evolving around the same steady state for a long period of time and policy has been conducted around that steady state.[28]

I compare the responses of the variables in the competitive economy and under the Ramsey plan, with and without lump-sum transfers. To avoid confusion, I will refer to the Ramsey plan with both distortionary and lump-sum transfers as the unconstrained planner (first-best economy); and to the Ramsey plan with distortionary taxes alone as the constrained planner. I choose $\omega = 0.5$.

Figure 3 displays the responses of selected variables to a one standard deviation increase in productivity. Looking at the impulse response functions, a general observation can be made: The planner's solutions with and without lump-sum transfers differ substantially, and they also differ from the competitive economy allocations.

More specifically, the optimal capital tax policy is fundamentally different depending on whether the planner can levy lump-sum transfers or not. In the case when direct lump-sum transfers are available, they are used to boost investment immediately after the shock; when they are not available, a counter-cyclical capital tax is used to bring investment gradually up (see Figure 4). Hence, when lump-sum transfers are not available, capital builds slowly, while the planner supports capital accumulation by a counter-cyclical capital income tax.

Intuitively, the constrained planner would like to directly transfer funds to entrepreneurs so that they increase their investment in capital immediately after the shock hits, and not gradually waiting for collateral to accumulate. However,

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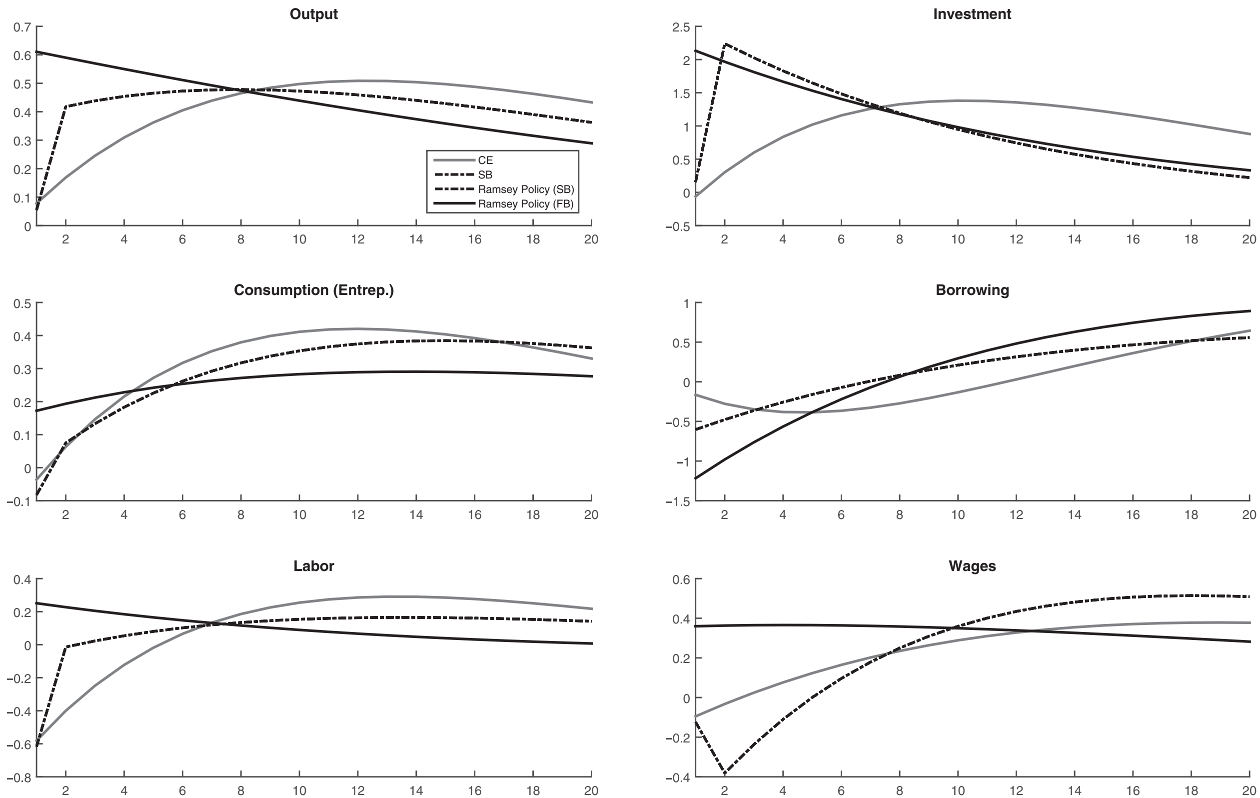

FIGURE 3. IRFs of selected variables in the competitive economy (CE) and the Ramsey plans replicating the first-best (FB) and the second-best (SB) outcomes in response to one standard deviation positive productivity shock. I compare the responses (% deviations from steady state) for a Pareto–Negishi weight set to $\omega = 0.5$.

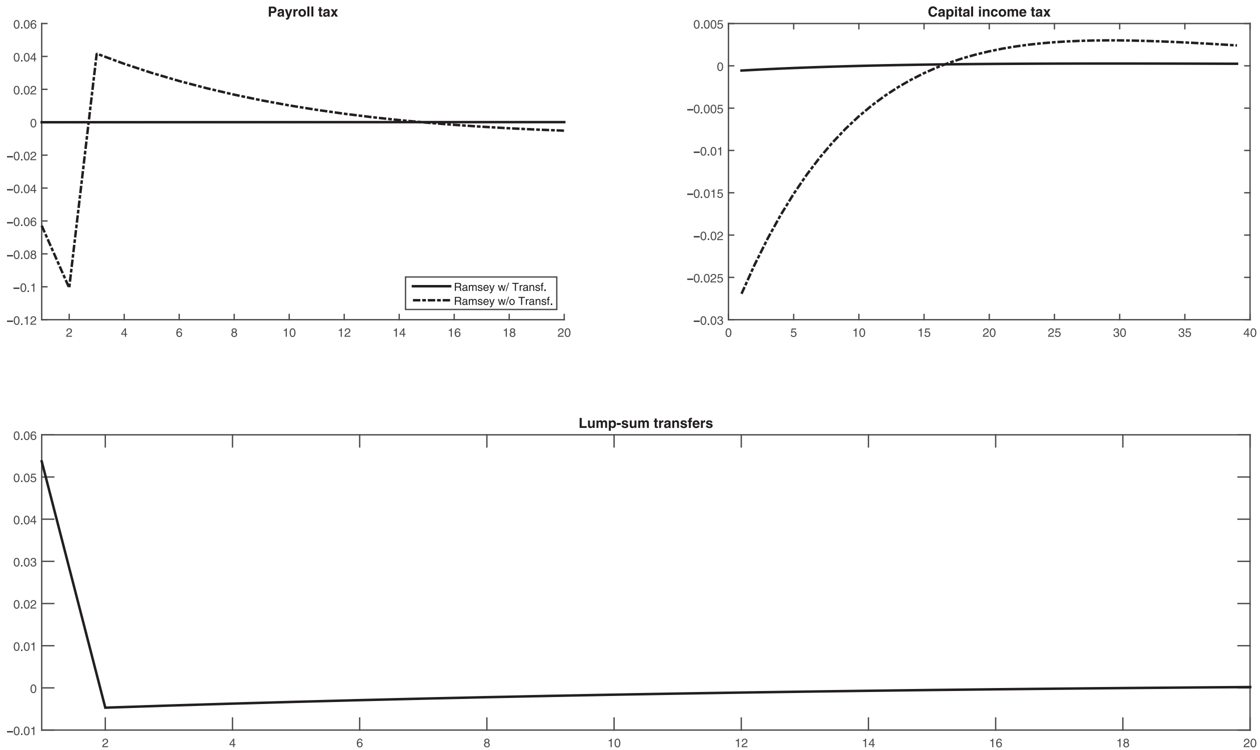

FIGURE 4. IRFs of the tax policy in response to one standard deviation positive productivity shock. I compare the responses of the policy instruments with and without lump-sum transfers for a Pareto–Negishi weight set to $\omega = 0.5$.

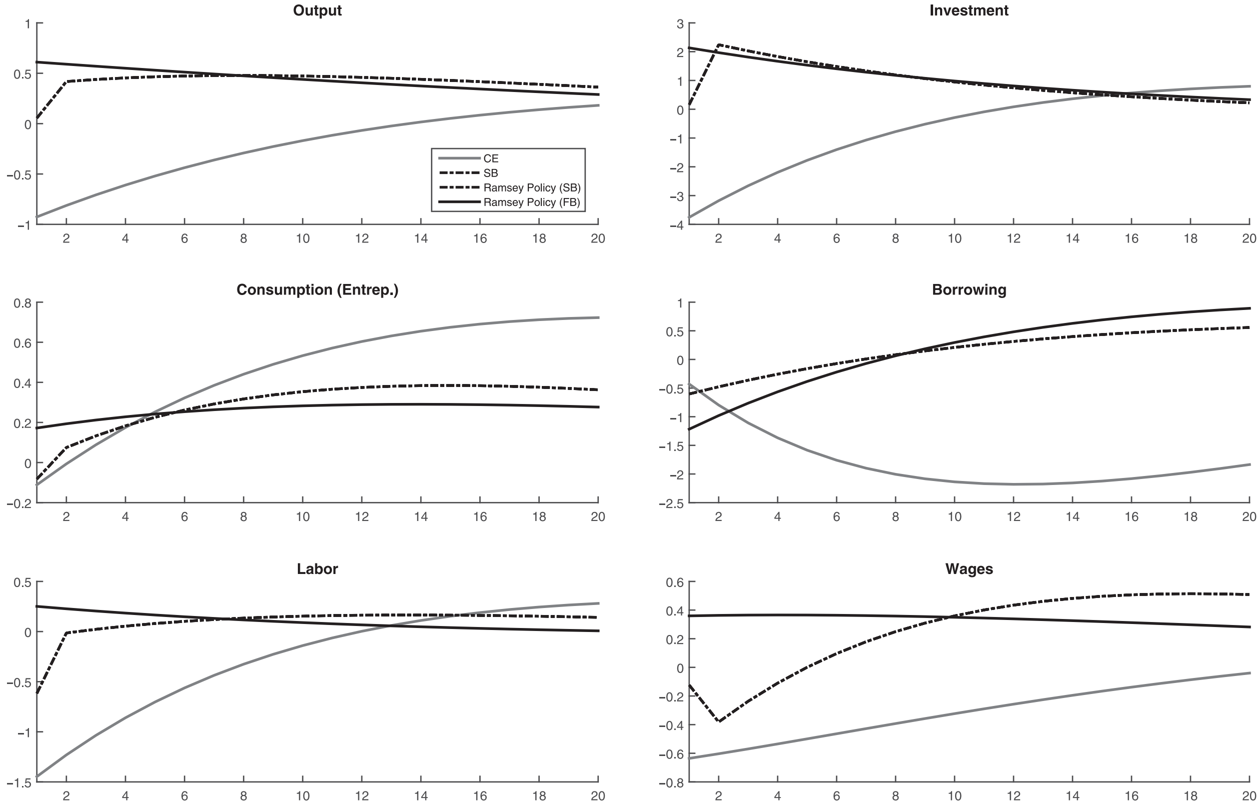

FIGURE 5. IRFs of selected variables in the competitive economy (CE) and the Ramsey plans replicating the first-best (FB) and the second-best (SB) outcomes in response to one standard deviation negative financial shock. I compare the responses (% deviations from steady state) for a Pareto–Negishi weight set to $\omega = 0.5$.

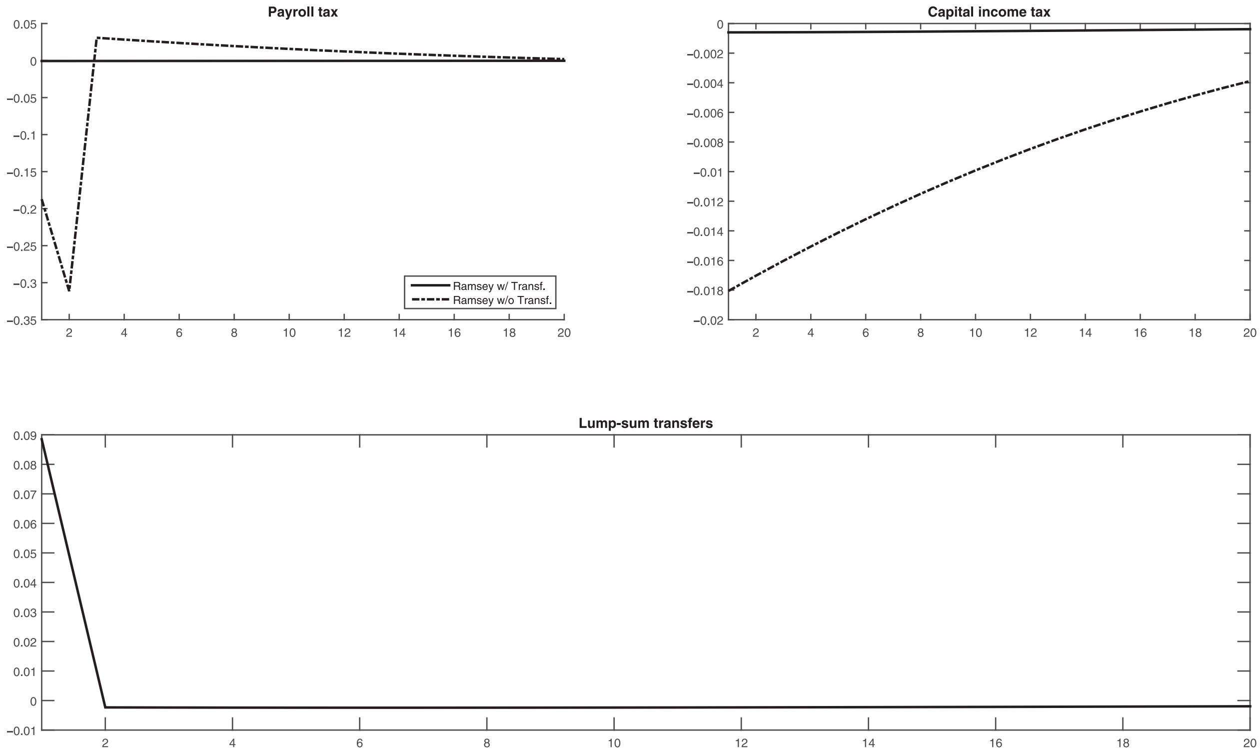

FIGURE 6. IRFs of the tax policy in response to one standard deviation negative financial shock. I compare the responses of the policy instruments with and without lump-sum transfers for a Pareto–Negishi weight set to $\omega = 0.5$.

it lacks this possibility. This motive of the planner is evident from the impulse response function when lump-sum transfers are available. Investment overshoots with the shock and gradually decreases as the shock fades out (see Figure 3).[29] Indeed, the planner transfers resources from households to entrepreneurs and implements a very small (compared to when lump-sum transfers are not available) counter-cyclical capital tax (see Figure 4).

Similarly, Figure 5 displays the responses of selected variables to a one standard deviation negative financial shock for the three economies mentioned above.[30] The general observations made for the productivity shock carry through this case, as well. However, the negative financial shock affects the economy through a different channel. Once it hits, the collateral requirement becomes tighter in the competitive economy by directly affecting the value of the collateral pledged. Consequently, output is depressed since entrepreneurs find it more difficult to acquire a loan to fund the working capital. Further, entrepreneurs also reduce their consumption and their investment in capital. The constrained planner internalizes these effects and adjusts policy in order to stimulate the economy. It reduces the tax on capital income from its steady-state level to stimulate investment. This relaxes the current and the future collateral constraints by allowing accumulation of capital. Moreover, the planner decreases the payroll tax to stimulate employment and stabilizes output in the initial periods following the shock (Figure 6).

As it was the case with the productivity shock, the planner will use lump-sum transfers to mitigate the negative effects of a more binding collateral constraint on investment and output. Indeed, it does transfer resources from households to entrepreneurs so that they can maintain a higher level of investment without the need to increase their borrowing (Figure 6). As a result, the unconstrained planner does not need to stimulate investment by substantially reducing the capital tax, which is the case for the constrained planner. The effect on the payroll tax is also mild, since lump-sum transfers can be used to mitigate the effects of the shock.

5. CONCLUSION

This paper studies the design of optimal Ramsey policy in environments with lenders and borrowers, in which the latter face collateral constraints. I consider the choice of optimal policy in the long-run (deterministic steady state) and in response to productivity and financial shocks.

The presence of binding collateral constraint results in a suboptimal use of the factors of production. In particular, agents tend to favor capital due to its ability to relax the collateral constraint, and to underemploy labor since it tightens it. The latter outcome is specific to the assumption that working capital is also collateralized.

Overall, a planner would like to correct for the distortions in the optimal utilization of the factors of production by imposing capital and labor taxes. Since the distortions are present both in the long- and in the short-run, these policies are always necessary to achieve the efficient outcomes. However, tax policy will be additionally biased by the need to perform an implicit redistribution of income when direct lump-sum transfers are not available. This highlights the political economy underlying the optimal policy choice, especially in constrained environments.

NOTES

1. See, for example, Hubbard and Judd (1986), Aiyagari (1995), and Imrohoroglu (1998). In the model in Aiyagari (1995), the positive tax rate in the long-run reduces capital accumulation and brings the pretax return on capital to equality with the rate of time preference.

2. Instead of assuming different subjective discount rates of the two groups of agents, entrepreneurs could be provided with a tax advantage on debt as in Jermann and Quadrini (2012). The two approaches yield equivalent results in terms of optimal policy since they induce a binding collateral constraint at the deterministic steady state, but the inefficiency in Kiyotaki–Moore type of economy comes from structural parameters reflecting agents' own behavior rather than policy-imposed structural parameters such as the tax benefit on debt as in Jermann and Quadrini (2012).

3. See Geanakoplos and Zame (2014) and Fostel and Geanakoplos (2008) for a discussion on the collateral premium.

4. The constrained social planner, I refer to in this paper, corresponds to a planner whose objective is to maximize agents' social welfare facing the market structure of the competitive economy. The only difference between the constrained social planner and the private agents is that the former internalizes the competitive pricing decisions [see, for example, the definition in Dávila et al. (2012)]. Throughout the paper, I will use the term constrained planner's and second-best allocations interchangeably, and I will precisely define the optimization problem.

5. Since ε_t affects the tightness of the collateral constraint and entrepreneurs' ability to borrow, in the literature it has been referred to as a "financial shock." For more detailed description of its implications, see Jermann and Quadrini (2012).

6. I refer the reader to Jermann and Quadrini (2012) and Perri and Quadrini (2011), who derive this constraint from a negotiation process in presence of limited commitment.

7. In the rest of the paper, I will refer to the working capital loan and the intraperiod loan interchangeably.

8. Using the budget constraint (7), one can show that $wkl_t = z_t F(\cdot) = c_t^e + w_t n_t + b_t^e + k_{t+1} - (1-\delta)k_t - \frac{b_{t+1}^e}{1+r_t}$, where wkl_t denotes the working capital loan. Jermann and Quadrini (2012) show that there are alternative ways of formalizing the working capital loan. A key feature of this formulation is that the working capital loan is related to the production scale.

9. Models featuring representative agents, for example, as in Bianchi (2011), do not encounter the problem of unambiguously defining a welfare criterion. In contrast, when the economy is populated by heterogeneous agents, aggregate welfare can be defined by setting Pareto–Negishi weights [see, for example, Monacelli (2006) and Bhandari et al. (2013) more recently]. Then, all allocations, prices, and policies will be contingent on the exogenously imposed weights. Hence, in the policy analysis, I consider a grid of points for the planner's weights in the social welfare function.

10. Since equation (20), (26), and (29) incorporate expectations of future variables, the problem of the planner is not intrinsically recursive. This problem can be overcome by introducing pseudostate variables, following the approach by Marcet and Marimon (2011), or by setting initial conditions of the values for the marginal utilities, following the approach by Kydland and Prescott (1980).

11. See, for example, De Bonis and Spataro (2005), which also introduces a different discount factor between the government and the private agents.

12. I refer to the more restricted Ramsey plan as the one satisfying the full set of PDCE conditions, (18)–(30); and the LRR plan as the one satisfying conditions (18), (19), (22)–(28), and (30).

13. A stepping stone in the derivation of the tax rate of capital income is determining the rate of growth of the Lagrange multipliers $\lambda_t^{RP,e}$ and $\lambda_t^{RP,h}$ at the steady state [see for discussion Reis (2012)].

The rate at which these multipliers are growing at the steady state may not be trivial at first since the private agents entertain a different discount factor from the planner. Using equations (37) and (36) at the steady state, it can be shown that both Lagrange multiplier grow at rate Δ at the steady state. Proofs are available upon request.

14. Notice that since $\lambda^{RP,e}$ and $\lambda^{RP,h}$ represent Lagrange multipliers on the budget constraints of the agents in the planner's problem, and they are always positive. Further, given that entrepreneurs are the ones facing the collateral constraint, the Lagrange multiplier on their budget constraint, as shown before, will be always higher than the one on households. It follows that $0 \leq \Phi \leq 1$. The result is confirmed numerically.

15. Recall that when direct lump-sum transfers are available, then $\lambda_t^{RP,e} = \lambda_t^{RP,h}$ (equation 41); hence, $\Phi = 0$.

16. To derive this expression, recall that when direct lump-sum transfers are available, $\Phi = 0$. Moreover, $\omega u_c = (1-\omega)u_c^e$ and $F_l = -\frac{u_l}{u_c}$. Then, it follows that $\tau^p_{\Phi=0} = -\frac{1}{\varepsilon}(1-\frac{\gamma}{\beta})$.

17. The choice of instruments, however, is constrained by two considerations. First, since both the intratemporal and the intertemporal margins are distorted, there should be at least two instruments that can affect each margin individually. Second, if the instruments considered are redistributive, distortionary taxes in particular, i.e., taxes that are imposed on one agent and their proceeds are distributed back to the other, it also matters who gets the revenues from those taxes when direct lump-sum transfers are not available. In the present model, since entrepreneurs are the ones facing the financial frictions, it is important that either their decisions are appropriately *corrected* or that funds are *distributed* to them such that the distortions are offset.

18. The tax on borrowing alone would be a sufficient policy instrument to relax the collateral constraint only in a representative agent economy, where the planner does not care about income redistribution, and where the intratemporal decision is not distorted because of collateralized working capital loan. Moreover, there should not be heterogeneity in the discounting of the future between the planner and the private agents (i.e., $\Delta = \beta$). In the current framework, the tax on borrowing has to be coupled with a payroll tax so that they correct for the distorted marginal decisions, as well as by a direct lump-sum transfer needed to perform income redistribution to close the gap between $\lambda_t^{RP,e}$ and $\lambda_t^{RP,h}$.

19. Notice that the parameter denoting the LTV ratio cannot simply be embedded in the existing collateral constraint as the constraint also contains the working capital loan. Therefore, an additional constraint has to be added to the planner's problem defining the LTV ratio.

20. One can also solve analytically for the deterministic steady-state values of the Lagrange multipliers on these constraints using the borrowing and capital Euler conditions of the PDCE. For the calibration used in the numerical analysis, they are positive along the entire grid of weights in the social welfare function as long as the planner implements a positive tax on capital income.

21. In this proposition, I consider the tax on capital income as a policy tool to affect the investment/borrowing decision. However, the corrective tax on borrowing or the LTV ratio regulation would produce the same results.

22. Consider an alternative formulation of the collateral constraint, such that working capital does not need to be collateralized: $k \geq b_{t+1}/(1+r_t)$. In this case, the payroll tax will not be used to correct for the distortions in the optimal labor conditions. Instead, it will be used solely for income redistribution between the two groups of agents in absence of lump-sum transfers. Proofs are available upon request.

23. Becker (1980) also shows analytically that the collateral constraint binds around the steady state when agents exhibit impatience rates gap á la Kiyotaki and Moore (1997).

24. See, for example, Iacoviello (2005).

25. To compute this statistics, Jermann and Quadrini (2012) use the average ratio over the period 1984Q1–2010Q2 for the nonfinancial business sector based on the data from the FoF (for debt) and the National Income and Product Accounts (for business GDP).

26. I calculate the compensating variation in average consumption following the approach in Lucas (1987). Since I consider the welfare of both agents, the consumption gain is the *average* of the consumption gains of the two agents.

27. Reis (2012) discusses how the difference in future discounting between the private agents and the planner affects the optimal tax rate on capital income. In a nutshell, as long as the planner is more patient than the private agents, it can accumulate enough resources such that it sets the tax rate to zero or to subsidize capital. On the other hand, if the planner is less patient than the private agents, it sets a positive tax on capital income.

28. For the numerical solution, see, for example, Faia (2008) among many others.

29. Notice that following a positive TFP shock, in the first-best scenario, investment reacts immediately, whereas it displays a hump-shaped response in the collateral constrained economies (both in the competitive economy and in the second-best economy). In reality, the latter response is more consistent with the data—see, for example, Smets and Wouters (2007)—suggesting that binding collateral constraint play an important role.

30. For a more detailed analysis of the effect of financial shocks, see, for example, Dellas et al. (2015), which studies the positive and normative implications of financial shocks in a standard New Keynesian model that includes banks and frictions in the market for bank capital.

31. Note that the market clearing conditions are taken into account and the collateral constraint is assumed to be binding.

32. This value is relatively small and can be found in the literature of collateral constraints. See for example, Faia and Iliopulos (2011) and Iacoviello (2005).

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APPENDIX

A.1. STEADY STATE OF THE COMPETITIVE ECONOMY (PROOF OF PROPOSITION 1)

In the long run, the model is characterized by the following set of equations, which represent the steady-state conditions satisfying the competitive equilibrium:

$$1 = \beta (1 + r), \tag{A.1}$$

$$F_l (1 - \mu) = -u_l / u_c, \tag{A.2}$$

$$\bar{\varepsilon} k = \frac{\bar{\varepsilon} b}{1 + r} + F(\cdot), \tag{A.3}$$

$$c^e + k + wl + b = (1 - \delta) k + F(\cdot) + \frac{b}{(1 + r)}, \tag{A.4}$$

$$1 - \bar{\varepsilon}\mu = \gamma [(1 - \mu) F_k + 1 - \delta], \tag{A.5}$$

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$$\frac{1-\bar{\varepsilon}\mu}{1+r}=\gamma, \tag{A.6}$$

$$c+\frac{b}{1+r}=wl+b. \tag{A.7}$$

Using (A.1), yields $1+r=\frac{1}{\beta}$. The Lagrange multiplier can be obtained using (A.6), $\mu=\bar{\varepsilon}^{-1}(1-\frac{\gamma}{\beta})$. The steady-state level of borrowing, as a function of capital and labor, can be obtained using the borrowing constraint, which holds with equality, $b(k,l)=\frac{1}{\bar{\varepsilon}\beta}[\bar{\varepsilon}k-F(\cdot)]$. Households' consumption can be obtained from (A.7), and similarly entrepreneurial consumption can be obtained from (A.4). Then, all variables depend on the steady-state levels of capital and labor. The steady-state level of the model can then be summarized by two equations with two unknowns, k and l

$$1-\mu=\frac{-u_l(k,l)/u_c(k,l)}{F_l(k,l)} \tag{A.8}$$

$$1-\bar{\varepsilon}\mu=\gamma\left[(1-\mu)F_k(k,l)+1-\delta\right], \tag{A.9}$$

where (A.8) is obtained by substituting (A.7) in (A.2). This completes the proof of Proposition 1. ■

A.2. THE RAMSEY PLAN AND THE LESS RESTRICTED RAMSEY (LRR)

PROPOSITION 8. *Ramsey plans that satisfy the constraints of the LRR plan also satisfy the conditions describing the PDCE and vice versa.*

Essentially, the proof below shows that the two plans yield the same allocations because the entrepreneurs' optimality conditions containing the tax rates, (20) and (21), which represent constraints in the Ramsey plan, turn out not to be binding.

Proof. With the taxes on capital and labor income, the planner cannot affect the borrowing Euler condition, (29), directly (the policy instruments are not embedded in this equation). Since this is a necessary condition for a competitive equilibrium, the planner has to honor it. Thus, using equation (29), I solve for μ as a function of the other variables $\mu(\cdot)=\frac{1}{\varepsilon_t}[1-\gamma E_t\frac{u^e_{c,t+1}}{u^e_{c,t}}(1+r_t)]$ and substitute it in all the places of the planner's Lagrangian where it appears

$$\begin{aligned}\mathcal{L}=&\,E_t\sum_{t=0}^{\infty}\delta^t\left[\omega u\left(c_t,l_t\right)+(1-\omega)\,u\left(c^e_t\right)\right]\\&+\lambda_t^{RP,e}\left[z_tF\left(k_t,l_t\right)+\frac{b_{t+1}}{R\left(c_t,c_{t+1}\right)}+(1-\delta)\,k_t+\mathcal{T}_t-c^e_t-w\left(c_t,l_t\right)-b_t-k_{t+1}\right]\\&+\lambda_t^{RP,h}\left[w\left(c_t,l_t\right)+b_t-c_t-\frac{b_{t+1}}{R\left(c_t,c_{t+1}\right)}-\mathcal{T}_t\right]\\&+\lambda_t^{RP,e}\mu_t^{RP}\left[\varepsilon_tk_{t+1}-\frac{\varepsilon_tb_{t+1}}{R\left(c_t,c_{t+1}\right)}-z_tF\left(k_t,l_t\right)\right]\end{aligned}$$

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$$+\mathcal{F}_t^1\left\{\gamma E_t \frac{u^e_{c,t+1}}{u^e_{c,t}} R\left(c_t, c_{t+1}\right)\right.$$
$$\left.-\gamma E_t \frac{u^e_{c,t+1}}{u^e_{c,t}}\left[\left(1+\gamma E_{t+1} \frac{1}{\varepsilon_{t+1}} \frac{u^e_{c,t+2}}{u^e_{c,t+1}}-\frac{1}{\varepsilon_{t+1}}-\tau^k_{t+1}\right) z_{t+1} F_{k,t+1}+1-\delta\right]\right\}$$
$$+\mathcal{F}_t^2\left\{\left[1+\frac{\gamma}{\varepsilon_t} E_t \frac{u^e_{c,t+1}}{u^e_{c,t}}-\frac{1}{\varepsilon_t}-\tau^k_t\right] z_t F_{l,t}-w\left(c_t, l_t\right)\left(1+\tau^l_t-\tau^k_t\right)\right\},$$

where I have also substituted (26) and (25) to express $R_t \equiv 1+r_t$ and w_t, respectively, and imposed the market-clearing conditions, i.e., $b^e_t = b^h_t = b_t$ and $l_t = n_t$, $\forall t$; finally, $T^e_t = \tau^k_t[z_t F(k_t, n_t) - w_t n_t] + \tau^p_t w_t n_t$ is substituted in the budget constraint of entrepreneurs. Taking first-order conditions with respect to τ^k_t and τ^l_t, shows that $\mathcal{F}^1_t = \mathcal{F}^2_t = 0$. Hence, the more restricted plan discussed in the main text reduces to the LRR plan and the two yield the same allocations.

To reiterate, since the policy instruments are of corrective nature, they give the planner an additional degree of freedom. To the contrary, if the instruments were used for the purpose to finance, for example, government expenditure, then they would also appear in the budget set of the planner and they may not be able to fully relax the Euler condition with respect to capital and the labor decision for any set of allocations chosen in the LRR problem. ■

A.3. ALTERNATIVE POLICY INSTRUMENTS (PROOF OF PROPOSITION 4)

The proof consists in showing that given an allocation $\{\Theta\}_{t=0}^{\infty}$ and a system of prices and taxes $\{r_t, w_t, \tau^k_t, \tau^p_t\}_{t=0}^{\infty}$, one can construct prices and taxes $\{\tilde{r}_t, \tilde{w}_t, \tau^b_t, \tilde{\tau}^p_t\}_{t=0}^{\infty}$ such that both tax systems support the given allocation. The taxes and prices, supporting the allocation, can be constructed by equating the equilibrium conditions characterizing the two model economies.[31]

1. The system of equations in economy I, given $\{\tau^k_t, \tau^p_t\}_{t=0}^{\infty}$, are as follows:

$$c_t + \frac{b_{t+1}}{1+r_t} + \mathcal{T}_t = w_t l_t + b_t, \tag{A.10}$$

$$c^e_t + b_t + k_{t+1} + w_t l_t = z_t F\left(k_t, l_t\right) + (1-\delta) k_t + \frac{b_{t+1}}{1+r_t} + \mathcal{T}_t, \tag{A.11}$$

$$\varepsilon_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right) \geq z_t F\left(k_t, l_t\right), \tag{A.12}$$

$$u_{c,t} = \beta E_t u_{c,t+1}\left(1+r_t\right), \tag{A.13}$$

$$w_t u_{c,t} = -u_{l,t}, \tag{A.14}$$

$$\left(1-\mu_t-\tau^k_t\right) z_t F_{l,t} = \left(1+\tau^p_t-\tau^k_t\right) w_t, \tag{A.15}$$

$$\frac{1-\varepsilon_t \mu_t}{1+r_t} = \gamma E_t \frac{u^e_{c,t+1}}{u^e_{c,t}}, \tag{A.16}$$

$$1-\varepsilon_t \mu_t = \gamma E_t \frac{u^e_{c,t+1}}{u^e_{c,t}}\left[\left(1-\mu_{t+1}-\tau^k_{t+1}\right) z_{t+1} F_{k,t+1} + 1 - \delta\right]. \tag{A.17}$$

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2. The system of equations in economy II, given $\{\tau_t^b, \tilde{\tau}_t^p\}_{t=0}^{\infty}$, are as follows:

$$\tilde{c}_t + \frac{\tilde{b}_{t+1}}{1+\tilde{r}_t} + \tilde{\mathcal{T}}_t = \tilde{w}_t \tilde{l}_t + \tilde{b}_t, \tag{A.18}$$

$$\tilde{c}_t + \tilde{b}_t + \tilde{k}_{t+1} + \tilde{w}_t \tilde{l}_t = z_t F\left(\tilde{k}_t, \tilde{l}_t\right) + (1-\delta)\tilde{k}_t + \frac{\tilde{b}_{t+1}}{1+\tilde{r}_t} + \tilde{\mathcal{T}}_t, \tag{A.19}$$

$$\varepsilon_t \left(\tilde{k}_{t+1} - \frac{\tilde{b}_{t+1}}{1+\tilde{r}_t}\right) \geq z_t F\left(\tilde{k}_t, \tilde{l}_t\right), \tag{A.20}$$

$$u_{\tilde{c},t} = \beta E_t u_{\tilde{c},t+1}(1+\tilde{r}_t), \tag{A.21}$$

$$\tilde{w}_t u_{\tilde{c},t} = -u_{\tilde{l},t}, \tag{A.22}$$

$$(1-\tilde{\mu}_t) z_t F_{\tilde{l},t} = \left(1+\tilde{\tau}_t^p\right)\tilde{w}_t, \tag{A.23}$$

$$\frac{1-\tau_t^b - \varepsilon_t \tilde{\mu}_t}{1+\tilde{r}_t} = \gamma E_t \frac{u^e_{\tilde{c},t+1}}{u^e_{\tilde{c},t}}, \tag{A.24}$$

$$1 - \varepsilon_t \tilde{\mu}_t = \gamma E_t \frac{u^e_{\tilde{c},t+1}}{u^e_{\tilde{c},t}} \left[(1-\tilde{\mu}_{t+1}) z_{t+1} F_{\tilde{k},t+1} + 1 - \delta\right]. \tag{A.25}$$

Equating the allocations in the two economies, requires setting

$$\tilde{r}_t = r_t, \tag{A.26}$$

$$\tilde{w}_t = w_t. \tag{A.27}$$

In addition, note that the Lagrange multipliers on the collateral constraints in the two economies are not equal. Namely, from (A.24), we can express $\tilde{\mu}_t = \mu_t - \frac{\tau_t^b}{\varepsilon_t}$. Then, to derive the borrowing rate as a function of the given allocations, prices and taxes, substitute the derived expression for $\tilde{\mu}_t$ and equate (A.17) and (A.25). This generates a difference equation in τ_t^b, which can be solved by using an iterative solution method. Here, for simplicity, I solve for the steady-state result instead, which is given by

$$\tau^b = \frac{\gamma F_k}{1 - \frac{\gamma F_k}{\bar{\varepsilon}}} \tau^k. \tag{A.28}$$

To solve for the payroll tax in the second economy, follow a similar procedure, i.e., substitute the expression for $\tilde{\mu}_t$ and equate (A.15) and (A.23). Then, express the payroll tax in the second economy

$$\tilde{\tau}_t^p = \frac{\left(1+\tau_t^p - \tau_t^k\right)\left(1 - \mu_t - \frac{\tau_t^b}{\varepsilon_t}\right)}{1 - \mu_t - \tau_t^k} - 1, \tag{A.29}$$

as a function of the prices and taxes in the first economy. ■

A.4. COLLATERAL CONSTRAINT IN THE COMPETITIVE ECONOMY OUTSIDE THE STEADY STATE

The equilibrium allocations (at the deterministic steady state) and in responses to shocks (outside the steady state) are solved under the assumption that the collateral constraint always binds. Thus, I have omitted the possibility that outside the steady state the collateral

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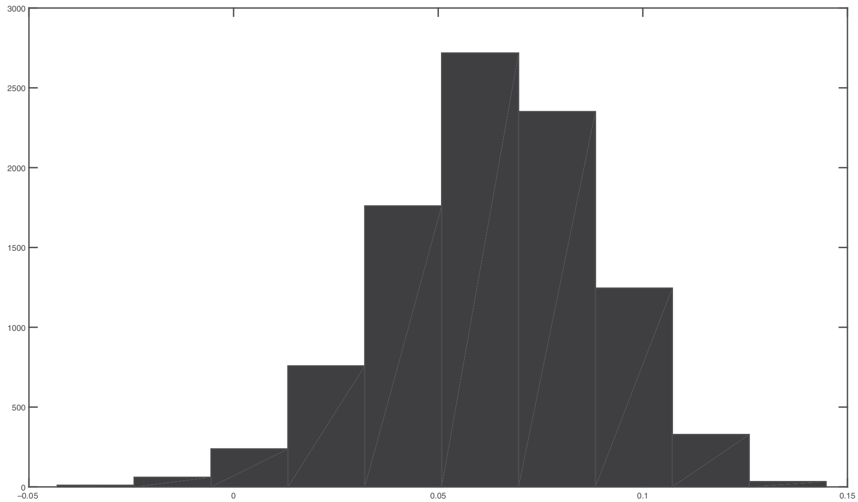

FIGURE A.1. Distribution of the Lagrange multiplier in the competitive economy associated with the collateral constraint, μ_t.

constraint, in fact, may not bind. To check for this possibility, following Faia and Iliopulos (2011), I perform the following exercise. I let the competitive economy be hit by a suite of stochastic shocks (productivity and financial) during 10,000 periods and store the values of the Lagrange multiplier associated with the collateral constraint. Figure A.1 displays the distribution of the multiplier on the constraint. It appears to be distributed normally and it is centered around its steady-state value. The probability that the collateral constraint does not bind coincides with the cumulative distribution for which the multiplier is smaller or equal to zero, i.e., $\mu_t \leq 0$. I find that the constraint does not bind, i.e., that the collateral constrained has surpassed zero, only in 1% of the cases.[32] This result allows me to rule out the possibility of a nonbinding collateral constraint also outside the steady state.

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