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A NOTE ON ENDOGENOUS PROPAGATION IN ONE-SECTOR BUSINESS CYCLE MODELS WITH DYNAMIC COMPLEMENTARITIES

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When the production function includes dynamic complementarities and a Cobb–Douglas form, dynamic complementarities are an endogenous propagation mechanism of shocks. The proposed model explains several stylized facts of aggregate variables of interest, including (i) hump-shaped impulse response functions, (ii) positively autocorrelated growth rates of aggregate variables, and (iii) correlation coefficients of forecastable movements in aggregate variables.

Keywords: Business Cycle, Endogenous Propagation, Dynamic Complementarities, Hump-Shaped Impulse-Response Function

1. INTRODUCTION

A well-known empirical fact about business cycles is that the largest impulse response of GNP to a transitory shock will occur in the future. The evidence suggests that the GNP response to innovation is hump-shaped. In a business cycle model, the impulse response of output must be explained by input factors. The standard RBC, with one sector, perfect competition, and constant returns to scale, fails to explain the hump-shaped impulse response [see King et al. (1988a)(KPR)].

Another well-known empirical fact is that serial correlations of GNP growth are not equal to zero. The evidence suggests that output growth is persistent; that is, the first two autocorrelations of output growth are positive (i.e., equal to 0.31 and 0.24). In a business cycle model, the serial correlation of output growth must

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be explained by the serial correlation of input growth factors. However, the serial correlation of input factors in real-business-cycle (RBC) models are almost equal to zero [see King et al. (1988b); Cogley and Nason (1995)].

This paper proposes a simple one-sector model explanation of both the serial correlation of the growth of aggregates and the impulse response of aggregates. We use a dynamic complementarities (DC) model in which output depends on work hours, physical capital stock, and previous output. The production is a Cobb–Douglas function, which allows the previous output to affect the marginal productivity of input factors. These dynamic complementarities capture the aspects of learning-by-doing spillovers and changes in the stock of physical capital. The dynamic complementarities model nests the standard RBC model as a special case when the coefficient of dynamic complementarities equals zero.

In related research, Baxter and King (1991), Benhabiba and Wen (2004), and Cooper and Johri (1997) considered a representative agent model with complementarity in the production function. Baxter and King (1991) and Benhabiba and Wen (2004) considered contemporaneous complementarity (CC); this mechanism will magnify the shocks. Cooper and Johri (1997) presented an extension of Baxter and King to allow for dynamic interactions through the production function. Cooper and Johri showed that this mechanism propagates the shocks.[1] This paper extends Cooper and Johri (1997)'s model by considering an AR(1) technology shock to explain the time series of aggregate U.S. output, consumption, investment, and labor. Three propagation components persist through the transitory shocks, implying one exogenous propagation mechanism and two endogenous propagation mechanisms: technology shock, the stock of capital, and the previous output.

Most RBC models that analyze the dynamic behavior of growth rates postulate that there are multiple shocks in the economy. In particular, at least one of these shocks has a unit root and individual shocks can only partly explain the business cycle phenomena, as mentioned above.[2] This paper uses a single transitory technology shock instead of multiple shocks or permanent technology.

In a recent paper, Rotemberg and Woodford (1996) pointed out that, in the standard RBC model, the forecastable movements in output, consumption, and labor should not positively correlate with each other, but these implications contradict the real data. Therefore, this paper studies the co-movement of forecastable changes in these aggregate variables predicted by the dynamic complementary–business cycle model.

The remainder of this paper is organized as follows. Section 2 constructs a new business cycle model, in which the production function has dynamic complementarities. This section also discusses the utility function, production function, and resources constraint in detail. Section 3 then quantitatively analyzes the impulse response of various aggregate variables to technology shocks and autocorrelations for various aggregate growth rates. Section 4 shows the forecastable movements. Finally, Section 5 concludes this paper.

2. THE MODEL

The economy is populated by a large number of identical agents that are infinitely lived. The lifetime utility for the representative agent from Rogerson (1988) and Cooley and Hansen (1989) is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t - \theta_N N_t], \tag{1}$$

where C_t and N_t are, respectively, consumption and labor at time t, $\beta \in (0, 1)$ is the discount factor, and E_0 stands for the expectation conditional on the information available at time 0.

Next, consider the production side of the economy. Let λ_t and H_t denote the technology shock on the output level and the "technology" of the representative agent, respectively. Note that H_t may also be viewed as human capital. The production function is specified as

$$Y_t = \lambda_t K_t^{\gamma_k} (H_t N_t)^{\gamma_N} \bar{y}_{t-1}^{\gamma_y}, \tag{2}$$

where γ_k, and γ_N are parameters satisfying $\gamma_k + \gamma_N = 1$ and K_t is the capital stock at the beginning of time t. The term $H_t N_t$ is referred to as the effective labor at time t. A novel feature of the production function (2) is that it exhibits dynamic complementarities. In what follows, the parameter γ_y associated with $\bar{y}_{t-1}$[3] is referred to as the coefficient of dynamic complementarity and the resulting model is called the RBC-DC model. The implications of dynamic complementarity can be seen in Appendix B.

To capture the trend growth in aggregate variables, we assume that $H_t = g_h H_{t-1}$, with the gross growth rate, g_h, being constant. The technology shock λ_t follows an AR(1) process:

$$\ln \lambda_t = \rho_\lambda \ln \lambda_{t-1} + \varepsilon_{\lambda,t},$$

where $\varepsilon_{\lambda,t} \sim$ i.i.d. $N(0, \sigma_\lambda^2)$, $|\rho_\lambda| < 1$, and $\lambda_0 = 1$. Under the assumption of stationarity, the effects of a shock gradually disappear over time. Let I_t be the amount of investment at time t. Then K_{t+1} evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t, \tag{3}$$

where δ is the depreciation rate of capital. In addition to the law of motion above, the condition of commodity market equilibrium can be expressed as

$$Y_t \geq C_t + I_t. \tag{4}$$

The representative agent chooses paths $\{C_t\}$, $\{K_{t+1}\}$, and $\{N_t\}$ to maximize his or her lifetime utility subject to the above constraints.

3. MODEL EVALUATION

Before evaluating the RBC-DC model, we must specify appropriate values for its parameters. For the parameters of the production function, we set $\gamma_N = 0.62$ and $\gamma_y = 0.37$, which are estimated in Cooper and Johri (1997). The remaining parameters can be found in most quantitative business cycle studies. This study follows the literature by assuming that $\delta = 0.025$, $\beta = 0.988$, and $g_h = 1.004$. For θ_N, we use the value in Cooley and Hansen (1989), that is, $\theta_N = 2.86$. As for the parameters in the process of technology shocks, we select the values of $\rho_\lambda = 0.6$. Because only the relative moments matter for our discussions, we arbitrarily set $\sigma_\lambda = 1$.

3.1. The Impulse Response Functions and the Autocorrelations of Aggregates

The main problem of the standard RBC model is that it is unable to explain hump-shaped impulse responses for transitory shocks. This is because it predicts that the effects of transitory shocks on aggregate variables such as output, investment, and labor are monotonically decreasing.

This paper considers a real business cycle model with dynamic complementarities. The production function considered depends on the last period's average output. Figure 1 depicts the effect of dynamic complementarities on the impulse-response function of various aggregate variables. If a technology shock happens now, it will immediately affect the economy. This shock will also affect the economy in the next period through the exogenous propagation mechanism. In this period, there exists another effect that occurs through the endogenous propagation mechanism. Thus, we will have hump-shaped impulse-response functions if changes in variables resulting from these two effects are larger than changes in variables stemming from the first period's effect.

The concept of dynamic complementarities was proposed by Cooper and Johri (1997). They found that in addition to capital, labor, and technology, the last period's average output is also an important determinant of the current period's output. To see this, we assume that a positive shock occurs in the current period. This produces an increase in the current period's output and thus increases the current period's consumption, labor, and investment. Hence, the next period's capital will increase, and, therefore, the next period's output will also increase. This is the propagation mechanism in the standard RBC model. Note that this mechanism depends on the intertemporal substitution effect and the wealth effect. By incorporation of dynamic complementarities into the technology process, there emerges another channel that allows the next period's output to increase as well. Moreover, dynamic complementarities will magnify the second period responses to the technology shock. This explains why the RBC-DC model can successfully capture the hump-shaped impulse response of aggregate variables.

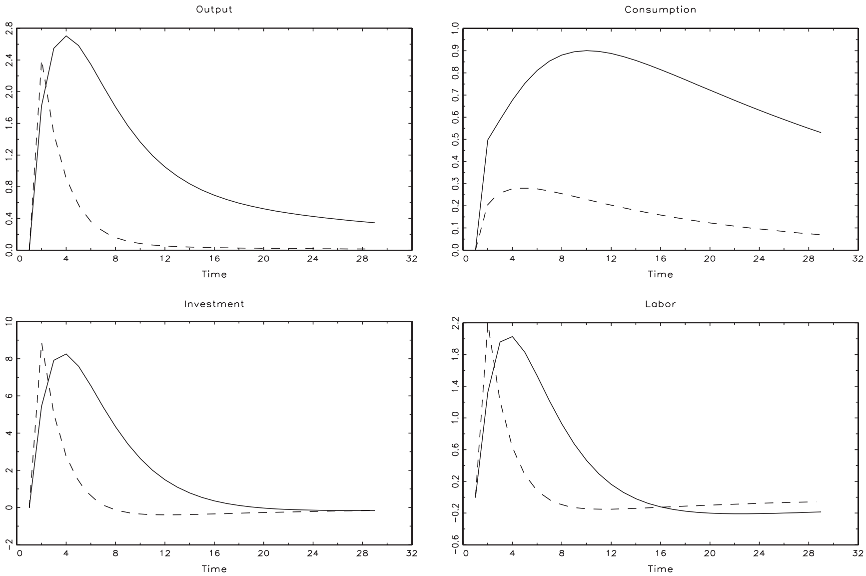

FIGURE 1. Impulse responses of output, consumption, investment, and labor to one standard deviation of technology shock and $\rho = 0.6$ (solid line: RBC-DC model with $\gamma_y = 0.37$; long dashed line: KPR model).

Another weakness of the standard RBC model is that it is unable to explain the positive correlation of growth rates of aggregate variables. We now evaluate the ability of the RBC-DC model with one technology shock to capture the stylized facts about the properties of autocorrelations, and compare the predicted measures to the U.S. estimates. The quarterly U.S. data consist of real GNP, real personal consumption, gross fixed investment, labor input, and civilian noninstitutional population from 1964:Q2 to 2002:Q2. For more detailed descriptions about these U.S. data, please see Appendix A.

Figure 2 clearly shows that the first autocorrelations of order two of output growth, consumption growth, investment growth, and labor growth are significantly positive for the U.S. data. Thus, as far as forecasting goes, we cannot ignore the property of positive autocorrelation. Using the KPR model,[4] the first autocorrelations of order two of the variables considered are predicted to be close to zero. This reveals that standard RBC models, such as the KPR model, do not have any dynamic element, which is the source of the lasting shock.

Unlike the standard RBC model, the one-shock RBC-DC model incorporates dynamic complementarities as a dynamic element. Thus, it is expected that the RBC-DC model may be able to capture the stylized fact of positive autocorrelations. Clearly, the RBC-DC model predicts that the first two autocorrelations of the variables of interest will be positive. Thus, the autocorrelations predicted by

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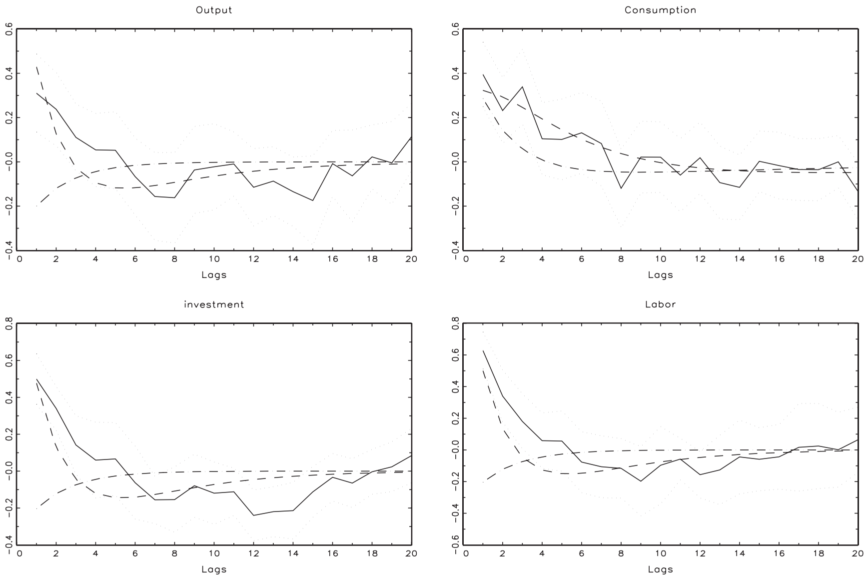

FIGURE 2. ACFs of the growth of output, consumption, investment, and labor (solid line: U.S. empirical estimates; dotted line: two standard deviations; dashed line: RBC-DC model; dashed and dotted line: KPR model).

the RBC-DC model are consistent with the U.S. data. This result also shows that dynamic elements play an important role in capturing the dynamic properties of an economic system.

It is not surprising that this model can also explain the positive autocorrelations. Cogley and Nason (1995, p. 495): state, "While the autocorrelation function provides some information about business-cycle periodicity, it also masks differences in the dynamic response of output to various kinds of shocks. This problem does not arise in one-shock RBC models." Because the RBC-DC model only contains one technology shock, the autocorrelation function provides information on the dynamic response of various aggregate variables to the shock.

The RBC-DC model also can explain the relative standard error of main aggregates and the growth rate of output and the positive correlations between growth rates of main aggregates and the growth rate of output at various leads and lags, which standard RBC can not explain. For the sake of brevity, these results are not shown.[5]

3.2. A Discussion of Recent Propagation Mechanisms

The aggregate variables have been the focus of recent research on real business cycles because of the failure of KPR to explain their growth. Table 1 is a representative

TABLE 1. A discussion of recent propagation mechanisms

Model	Propagation mechanism	Hump-shaped IRF of C,I,N,Y	Positive autocorrelation of C,I,N,Y
Benhabib, Wen (2004)	Capital utilization, output externality	Yes	Yes
Boldrin et al. (2001)	Habit, two sectors	No*	No
Chang et al. (2002)	Learning by doing	No**	No
Cooper and Johri (1997)	CC, DC	No	No
Edge (2007)	Time to build, habit	Yes	Yes
This paper	DC	Yes	Yes

Note: *means that the model cannot fit the time series of consumption. **means that the model cannot fit the time series of labor. CC is the contemporaneous complementarities.

but not comprehensive summary of the success of recent propagation mechanism models. The two criteria used in the comparison are (1) whether the model has a hump-shaped impulse response function (IRF) and (2) implied positive autocorrelation of various aggregate variables, which include output, consumption, investment, and labor.

Several papers have shown that the RBC model can explain the growth rate of aggregates if the model containing endogenous propagation mechanisms, such as learning by doing, input utilization, and habit formation in preferences [Boldrin et al. (2001), Chang et al. (2002), Benhabib and Wen (2004), and Edge (2007)]. A common failure in the RBC with the propagation mechanisms mentioned above is that they are unable to explain the aggregate variation in consumption, output, labor, and investment by one endogenous propagation mechanism. For example, Benhabib and Wen (2004) considered capital utilization with output externality, and Edge (2007) considered a time-to-build model with habit formation in preferences. Boldrin et al. (2001) considered habit formation in two sectors. Chang et al. (2002) augmented the RBC with learning by doing; this paper augments the RBC with dynamic complementarities. To the extent that dynamic complementarities are a propagation mechanism of shocks, these factors are related to the notion of endogenous propagation of shocks.

4. FORECASTABLE MOVEMENTS

Rotemberg and Woodford (1996) argued that forecastable movements in aggregate variables constitute the essence of business cycles. On the basis of U.S. data, Rotemberg and Woodford (1996) found that forecastable movements in output, consumption, and labor are positively correlated with each other. Thus, when a shock occurs, these three variables should be expected to move in the same

TABLE 2. Correlations of various expected variables

Horizon (in quarters)	1	2	4	8	12	24
			Estimated correlations			
$\text{corr}(\triangle\widehat{c^k}_t, \triangle\widehat{y^k}_t)$	0.69	0.78	0.82	0.82	0.79	0.72
	(0.12)	(0.08)	(0.07)	(0.07)	(0.07)	(0.09)
$\text{corr}(\triangle\widehat{i^k}_t, \triangle\widehat{y^k}_t)$	0.98	0.98	0.98	0.97	0.96	0.89
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.06)
$\text{corr}(\triangle\widehat{n^k}_t, \triangle\widehat{y^k}_t)$	0.88	0.89	0.92	0.97	0.98	0.98
	(0.04)	(0.04)	(0.03)	(0.01)	(0.01)	(0.01)
			Predicted correlations			
$\text{corr}(\triangle\widehat{c^k}_t, \triangle\widehat{y^k}_t)$	0.26	0.11	−0.10	−0.06	0.15	0.54
$\text{corr}(\triangle\widehat{i^k}_t, \triangle\widehat{y^k}_t)$	0.99	0.98	0.98	0.98	0.98	0.96
$\text{corr}(\triangle\widehat{n^k}_t, \triangle\widehat{y^k}_t)$	0.98	0.97	0.94	0.93	0.93	0.88

Note: The estimated correlations are obtained from Rotemberg and Woodford (1996). The values in parentheses are estimated asymptotic standard errors, which are calculated using the Newey and West (1987) method.

direction. Unfortunately, the standard RBC model cannot account for the observed forecastable changes in these three variables; see Rotemberg and Woodford (1996) and Wen (1998).

One reason that the standard RBC model fails to account for the behavior of forecastable movements in aggregate variables may be as follows. Assuming that technology shocks are permanent, the current period's capital will be less than the long-run level when a positive shock occurs. In this situation, the labor supply will increase due to the wealth effect and the intertemporal substitution effect. However, the substitution effect will decrease the labor supply. Thus, labor may decrease although both consumption and output increase. If this is the case, the forecastable changes in output are positively correlated to forecastable changes in consumption, but negatively correlated to forecastable changes in labor.

Therefore, this section examines whether the RBC-DC model can explain the comovement of output, consumption, investment, and labor. To do this, we consider expected and actual k-quarter changes in various variables for $k = 1, 2, 4, 8, 12,$ and 24. The expected k-quarter changes in variables, say z_t, is denoted by $\triangle\widehat{z^k}_t$; it is the expectation of $z_{t+k} - z_t$ conditional on the information available at time t. Table 2 reports the estimated and predicted correlations among various expected k-quarter changes in variables. Clearly, all the estimated correlations are significantly positive. With the exception of consumption, the patterns of predicted correlation are very similar to those of estimated correlations. This may be due to the fact that once a shock occurs, output, labor, and investment reach their peaks at the same period but consumption does not; see the impulse-response functions in Figure 1. Compared with the EBC model of Schmitt-Grohé (2000), the correlations, of $\triangle\widehat{c^k}_t$ and $\triangle\widehat{y^k}_t$ predicted by the RBC-DC model are closer to the estimated correlations,

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because the EBC model predicted that $\triangle\widehat{c^k}_t$ and $\triangle\widehat{y^k}_t$ are significantly negative for all k.[6]

Another phenomenon that the standard RBC model with permanent technology shock is unable to account for is that the relative standard deviations of expected changes in output and actual changes in output are significantly different from zero; see Rotemberg and Woodford (1996). The RBC-DC model might provide appropriate approximations, in contrast to the standard RBC model, but we do not show these results, for the sake of brevity.

5. CONCLUSIONS

The findings of this paper suggest much empirical content in the theoretical paradigm of the RBC-DC model. The central insight of the RBC-DC is that dynamic complementarities are the relevant measure of endogenous propagation of shocks for an agent. This paper shows how dynamic complementarities, when suitably modeled, can explain why the impulse-response functions for various aggregate variables are hump-shaped. These complementarities can also explain why the growth rates of aggregate output are positively autocorrelated in the short run, but serially uncorrelated in the longer run.

The central ingredient is dynamic complementarities in the production function, where the previous output level affects input factor productivity. The impulse response has a higher value in the future when dynamic complementarities propagate the shocks. These aggregate variables must therefore have a hump-shaped impulse response. The growth rate also has a higher value when dynamic complementarities propagate the shocks persistently. The growth of these variables must therefore be positively correlated.

NOTES

1. Cooper and Johri (1997) discussed the propagation effect of dynamic complementarities, in which the transitory shock of technology follows a white noise. The propagation components in this paper are not large enough to cause the impulse response of aggregate variables to be hump-shaped.
2. Cogley and Nason (1995) assumed that technology shock is permanent and governmentexpenditure shock is transitory. Schmitt-Grohé (2000) also postulated that sunspot shocks and technology shocks exist in an economic system. According to Cogley and Nason (1995), although the autocorrelation function provides some information about business-cycle periodicity, it also masks differences in the dynamic response of output to various kinds of shocks. Benhabib and Wen (2004) analyzed various demand shocks. Cooper and Johri (2002) explained hump-shaped impulse responses using transitory shocks of technology. To explain the correlations between growth rates of aggregate variables, they still had to rely on permanent technology shocks.
3. The dynamic complementarities are defined as $\bar{Y}_{t-1}/H_{t-1}$, where $\bar{Y}_{t-1}$ is externality.
4. In the case of the KPR model, we set $\gamma_N = 0.62$, which is estimated in Cooper and Johri (1997).
5. These results will be provided upon request.
6. See Table 5 of Schmitt-Grohé (2000).

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APPENDIX A

The U.S. data are quarterly data from 1964:Q2 to 2002:Q2. Output, consumption, and investment are measured as real GNP, real personal consumption, and gross fixed investment, respectively. Note that real personal consumption includes expenditures on nondurable goods and services. These are in chain-weighted 1996 dollars, are seasonally adjusted, and are taken from the U.S. Department of Commerce Bureau of Economic Analysis. Labor input is measured by hours worked by workers in private nonfarm payrolls, which are also seasonally adjusted. Population is defined as civilian noninstitutional population. The data for both labor input and population are taken from the U.S. Department of Labor Bureau of Labor Statistics.

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APPENDIX B

The following introduces the model, including the historical technology in the RBC model. It can be shown as follows:

$$\begin{aligned} y_t &= \lambda_t F(k_t, H_t N_t) y_{t-1}^{\gamma_y} \\ &= \lambda_t F(k_t, H_t N_t)(\lambda_{t-1} F(k_{t-1}, N_{t-1}))^{\gamma_y} y_{t-2}^{\gamma_y^2} \\ &= \lambda_t F(k_t, H_t N_t)(\lambda_{t-1} F(k_{t-1}, H_{t-1} N_{t-1}))^{\gamma_y} (\lambda_{t-2} F(k_{t-2}, H_{t-2} N_{t-2}))^{\gamma_y^2} y_{t-3}^{\gamma_y^3} \\ &= \lambda_t F(k_t, N_t) \cdot \Pi_{s=1}^{n} (\lambda_{t-s} F(k_{t-s}, H_{t-s} N_{t-s}))^{\gamma_y^s} y_{t-n-1}^{\gamma_y^{n+1}}. \end{aligned} \qquad \textbf{(B.1)}$$

In equation (B.1), the current output level depends on the historical input factors, including the stock of capital, effort, and the technology shocks. In this model, the previous technology shocks will affect current technology even if the stochastic process is white noise. If the dynamic complementarity disappears, $\gamma_y = 0$, this model will reduce to the standard RBC model.

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