papers, by Thibault, Sklar and Clerc and Hartmann, take us into the area of dynamical systems and consider such matters as invariant curves and flows.

Inevitably in a collection of this nature, the styles differ and the level of mathematical background required to appreciate the contents also varies considerably from author to author. A few misprints were detected and the English language takes a battering on odd occasions, notably in the appearance of words such as "modellized". In conclusion, it can be said that this is a book for dipping into, with something for everyone. If the reader's appetite for a particular topic is whetted particularly, it should be possible to delve further by using the references cited liberally at the end of many of the the contributions.

ADAM C. McBRIDE

BARNES, B. A., MURPHY, G. J., SMYTH, M. R. F. and WEST, T. T., *Riesz and Fredholm theory in Banach algebras* (Research Notes in Mathematics 67, Pitman, 1982), 123 pp, £7.95.

Of the various classes of Banach space operators which have been singled out for special study, the compact operators are arguably the most important and best understood. The classical Riesz–Schauder theory shows that their spectral structure parallels that of finite matrices while, from an algebraic point of view, they form a natural ideal in the algebra of all bounded linear operators, the corresponding quotient algebra being the so-called Calkin algebra. Closely related to the compact operators are the Fredholm operators which, although originally defined spatially, may alternatively be characterized as having invertible images in the Calkin algebra. Another cognate class of operators, less central but still of interest, is that consisting of the Riesz operators. They may be defined either as having the same spectral structure as compact operators or, equivalently, as having images in the Calkin algebra with zero spectrum.

The aim of the present monograph is to examine how the ideas of Fredholm and Riesz theory can be developed in the context of a general Banach algebra $A$. This strategy is to find appropriate definitions for Fredholm and Riesz elements of $A$ and then show that the results of operator theory have analogues in this algebraic setting. The first difficulty encountered in such a programme is that there is no satisfactory definition of a compact element in a general Banach algebra. However, by first considering invertibility modulo the socle when $A$ is semi-simple and then extending to the general case, it is possible to obtain a sensible definition of a Fredholm element of $A$. Counterparts to the main theorems of classical Fredholm theory, including the index and punctured neighbourhood theorems, can now be proved. After developing their version of algebraic Fredholm theory, the authors turn to Riesz theory in Banach algebras. Elementary results can be obtained by considering elements with zero spectrum modulo an arbitrary closed ideal $K$ but, to take things further, additional restrictions on $K$ are needed. Not surprisingly, these ideas have some special features when $A$ is a $C^*$-algebra. In that case, it is possible to give satisfactory definitions of finite rank and compact elements and, reflecting the situation for operators on Hilbert space, each Riesz element can be decomposed as the sum of a compact and a quasinilpotent element. Whether such a decomposition is always possible for Riesz operators on Banach spaces is the main problem left open in the subject at an operator-theoretic level. Unfortunately, the algebraic techniques developed here do not appear to shed much light on it.

The book has been well organized and each chapter ends with an extensive section of notes and comments. The authors have made a point of giving plenty of examples to illustrate their results, many of which appear here for the first time, and they include a chapter of applications. Finally, although the main aim is to develop the theory at an algebraic level, the book begins with an account of the necessary background material from operator theory. This in itself is a good survey, which in fact contains both classical results and some interesting new material.

T. A. GILLESPIE

SMITH, K. T., *Primer of modern analysis* (Springer-Verlag, 2nd ed. 1983), xv+446 pp. DM 97.

The original version of this book was published by Bogden & Quigley in 1971; this slightly extended version now appears in the Springer series "Undergraduate Texts in Mathematics". It is