Laser and Particle Beams (2000), *18*, 341–345. Printed in the USA.
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The motion of a charged particle in the plane, monochromatic and linear—polarized EM waves and in the presence of a constant magnetic or electric fields

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(Received 3 March 1999; Accepted 25 June 1999)

Abstract

On the basis of the Lorentz equation the analytical solution of the equations of motion of a charged particle in the electromagnetic (EM) waves combined with constant electric or magnetic fields has been found. The possibility of controlling of the parameters of motion in velocity and spatial spaces is discussed as follows.

1. INTRODUCTION

The propagation of charged particles in the EM waves also in the presence of uniform and constant magnetic or electric fields is the problem which has occupied a great deal of attention for many years (Laird, 1968; Woolley, 1973; Kawata *et al.*, 1989; Hussein *et al.*, 1992; Quesnel *et al.*, 1997). An understanding of the conditions of the motion of a particle is interesting in a number of problems, for example, in investigation of interaction of the laser radiation with plasma (Malka *et al.*, 1997; Quesnel *et al.*, 1997), motion, and acceleration of particles in the EM waves (Hauser *et al.*, 1991; Kawata *et al.*, 1990), propagation of particles in the magnetosphere (Woolley, 1973; Wodnicka *et al.*, 1997) and others. In the majority of papers, the analysis of the problem has been made by a direct numerical integration of the equations of motion. In this paper, on the basis of a simple model and mathematical formulas obtained in an analytical way, we have been able to find a qualitative and in many cases, quantitative description of the behavior of a particle in EM waves and E, B fields. The presented results can be useful also in chemistry, biophysics, and quantum computation (Cirac *et al.*, 1995). Considerations presented in the paper regard the motion of a particle in the field of the EM wave described by the Lorentz equation of the form:

$$V\frac{dm}{dt} + m\frac{dV}{dt} = q[E + (V \times B)] + G - D, \tag{1}$$

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where

m = mass of the particle,

q = a charge,

E = electric field strength of the EM wave,

B = magnetic induction of the wave,

V = velocity of the charged particle,

G = gravity, and

D = dissipation term.

In the case of nonrelativistic motion of a particle (Meyerhofer *et al.* 1996), in the linear-polarized EM wave with components:

$$E[E_x + E_x^c, E_y + E_y^c, E_z + E_z^c],$$

$$B[B_x + B_x^c, -B_y - B_y^c, B_z + B_z^c]$$

and for $D = 0$ and $G = 0$ it is possible to obtain the differential equations of motion of a particle in the form:

$$\frac{dV_x}{dt} = \frac{q}{m}[(E_x + E_x^c) + V_y(B_z + B_z^c) - V_z(B_y + B_y^c)],$$

$$\frac{dV_y}{dt} = \frac{q}{m}[(E_y + E_y^c) + V_x(B_z + B_z^c) - V_z(B_x + B_x^c)],$$

$$\frac{dV_z}{dt} = \frac{q}{m}[(E_z + E_z^c) + V_x(B_z + B_z^c) - V_y(B_x + B_x^c)], \tag{2}$$

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where

E_x^c, E_y^c, E_z^c = components of the vector of constant electric field,

B_x^c, B_y^c, B_z^c = components of the vector of constant magnetic induction,

$-V_x, V_y, -V_z$ = components of a particle velocity,

x, y, z = spatial co-ordinates

2. ANALYSIS OF MOTION OF A PARTICLE IN THE TWO EM WAVES WITH PHASE SHIFT φ

For a plane and homogeneous EM waves with components:

$$E[E_x^1 + E_x^2, 0, 0],$$

$$B[0, -B_y^1 - B_y^2, 0],$$

where

$$E_x^1 = E_o^1 \sin\phi,$$
$$B_y^1 = B_o^1 \sin\phi,$$
$$E_x^2 = E_o^2 \sin(\phi + \varphi),$$
$$B_y^2 = B_o^2 \sin(\phi + \varphi),$$
$$B_o = \frac{E_o}{c},$$
$$\phi = \omega\left(t - \frac{z}{c}\right)$$

and

E_o, E_o = amplitudes of electric field of EM waves,

B_o, B_o = amplitudes of magnetic fields of the EM waves,

c = speed of light in vacuum,

φ = phase shift between the waves,

ω = angular frequency of the EM waves,

t = time

the equations of (2) can be reduced to the form:

$$\frac{dV_x}{dt} = \frac{qE_o}{m}\left(1 - \frac{V_z}{c}\right)[\sin\phi + \sin(\phi + \varphi)],$$

$$\frac{dV_z}{dt} = -\frac{qE_o}{m}\frac{V_x}{c}[\sin\phi + \sin(\phi + \varphi)]. \tag{3}$$

After integrating the equations of (3) (to simplify the analysis, at initial conditions for which the integration constant is equal to zero) it is possible to obtain the solution in the form:

$$V_x = -\alpha[\cos\phi + \cos(\phi + \varphi)],$$

$$V_z = c\left\{1 - \sqrt{1 + \frac{\alpha^2}{2c^2}[\cos 2\phi + \cos 2(\phi + \varphi) + 2\cos(2\phi + \varphi)]}\right\} \tag{4a}$$

where

$$\alpha = \frac{qE_o}{m\omega}. \tag{4b}$$

For the nonrelativistic case (after approximation of the relation in Eq. (4b) by two first terms of the series of expansion of the root) one can obtain the equation in the form:

$$V_z \cong -\frac{\alpha^2}{4c}[\cos 2\phi + \cos 2(\phi + \varphi) + 2\cos(2\phi + \varphi)]. \tag{5}$$

Integrating equations (4a) and (5) it is possible to obtain the solution given by:

$$x = -\frac{\alpha}{\omega}[\sin\phi_o + \sin(\phi_o + \varphi)],$$

$$z \cong -\frac{\alpha^2}{8c\omega}[\sin\phi_o + \sin 2(\phi_o + \varphi) + 2\sin(2\phi_o + \varphi)] \tag{6}$$

$$\phi_o = \omega\left(t - \frac{z_o}{c}\right),$$

where

z_o = the parameter

The equation of (6) for several different values of the phase shift are illustrated in Figure 1. From the Figure 2 follows that in the analyzed case the particle is trapped into a closed trajectory. By changing the value of the phase shift φ, it is possible to control the parameters of motion and dimensions of the trajectory of a particle. Putting different values of frequencies into each of the waves and for different phase shift φ between them, we can obtain many other types of the trajectories and values of the parameters of motion of a particles in the controlled manner.

3. MOTION OF A CHARGED PARTICLE IN EM WAVE IN PRESENCE OF CONSTANT MAGNETIC FIELD

It can be shown that for the EM wave, combined with constant magnetic field, of the components:

$$E[E_x, 0, 0],$$

$$B[B_x^c, -B_y, 0],$$

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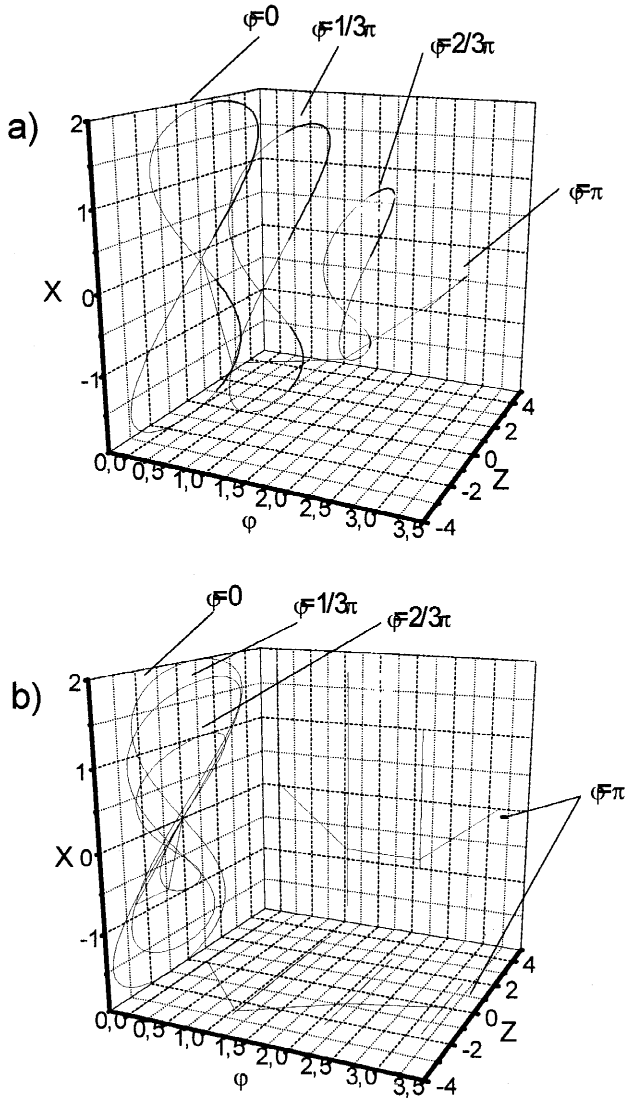

Fig. 1. Example of the trajectory of a charged particle in two EM waves for different values of the phase shift between them, where $X = (\omega/\alpha)\cdot x$; $Z = (8\omega \cdot c/\alpha^2)\cdot z$.

the equations of (2) can be reduced to the form:

$$\frac{dV_x}{dt} = \frac{qE_o}{m}\left(1 - \frac{V_z}{c}\right)\sin\phi,$$

$$\frac{dV_y}{dt} = -\frac{qB_x^c}{m}V_z,$$

$$\frac{dV_z}{dt} = \frac{q}{m}(B_o V_x \sin\phi - V_y B_x^c). \qquad (7)$$

Integrating the equations of (7), (after an approximation of V_z by two first terms of the series of expansion of the root) it is possible to obtain solution in the form:

$$V_x = -\alpha\cos\phi,$$

$$V_y = -\frac{qB_x^c}{m}z,$$

$$V_z \cong -\frac{q^2B_o^2c}{4m^2\omega^2}\cos 2\phi - \frac{q^2(B_x^c)^2}{m^2}\left(zt - \frac{z^2}{2c}\right). \qquad (8)$$

For $B_x = B_o$, after integration of the equations of (8) we can obtain the parametric equations of trajectory of a particle given by:

$$x = -\frac{\alpha}{\omega}\sin\omega(t - t_o),$$

$$y = \alpha\cdot\omega\cdot t_o\cdot t,$$

$$z \cong -\frac{\alpha^2}{8c\omega}[\sin 2\omega(t - t_o) - 4\omega^3 t_o(t_o^2 t - t_o t^2)], \qquad (9)$$

where t_o = parameter.

The trajectories of the particle described by the equations of (9) are presented in Figure 2. It results from the above that a charged particle is accelerated along an oscillating trajectory and undergoes changing from trapped state into the transporting state. The motion is characterized by a rapid fluctuation in V_z combined with two times slower one in V_x and constant in time in V_y. The parameters of the motion depend on value and orientation of a constant magnetic field. For example, when:

$$E[E_x, 0, 0],$$

$$B[0, -B_y - B_y^c, 0],$$

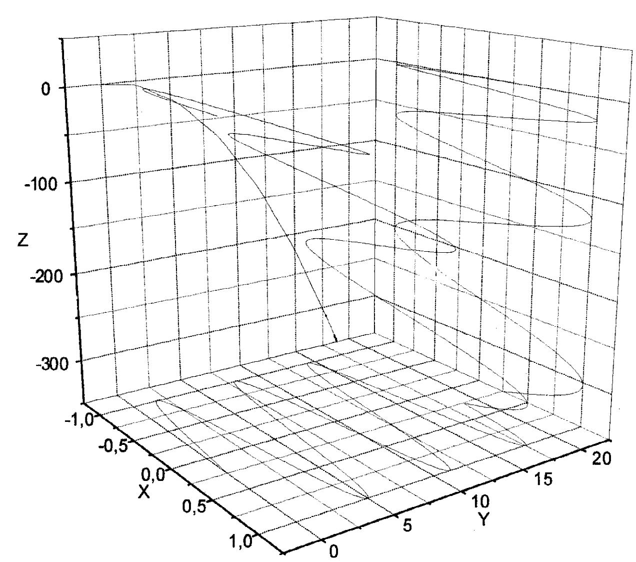

Fig. 2. Example of the trajectory of a charged particle in EM wave in presence of the B_x constant magnetic field, for $t_o = 0.25$, and $X = (\omega/\alpha)\cdot x$; $Y = (\alpha\cdot t_o)^{-1}\cdot y$; $Z = (8\omega\cdot c/\alpha^2)\cdot z$.

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where

$$B_y^c = a\frac{E_o}{c}$$

$$a = \text{parameter}$$

for the velocities of a particle, the analogy to the equations (8) solutions has the form:

$$V_x = -\alpha\left(a\omega\frac{z}{c} + \cos\phi\right),$$

$$V_z \cong \frac{\alpha^2 z\omega}{c^2}\left[a^2\omega\left(\frac{z}{2c} - t\right) + a\cos\phi + \frac{c}{4z\omega}\cos 2\phi\right], \qquad (10)$$

and in analogy to the equations of (9), equations for the trajectory are given by:

$$x = -\frac{\alpha}{\omega}[a\omega^2 t_o t + \sin\omega(t - t_o)],$$

$$z \cong \frac{\alpha^2}{c}\left[\frac{1}{2}a^2\omega^2(t_o^2 t - t_o t^2) + at_o\sin\omega(t - t_o) + \frac{1}{8\omega}\sin 2\omega(t - t_o)\right]. \qquad (11)$$

The calculated trajectories are presented in Figure 3. It can be noticed that orientation of the vector B^c in the analyzed cases has a strong influence on the character and direction of propagation of a particle in space.

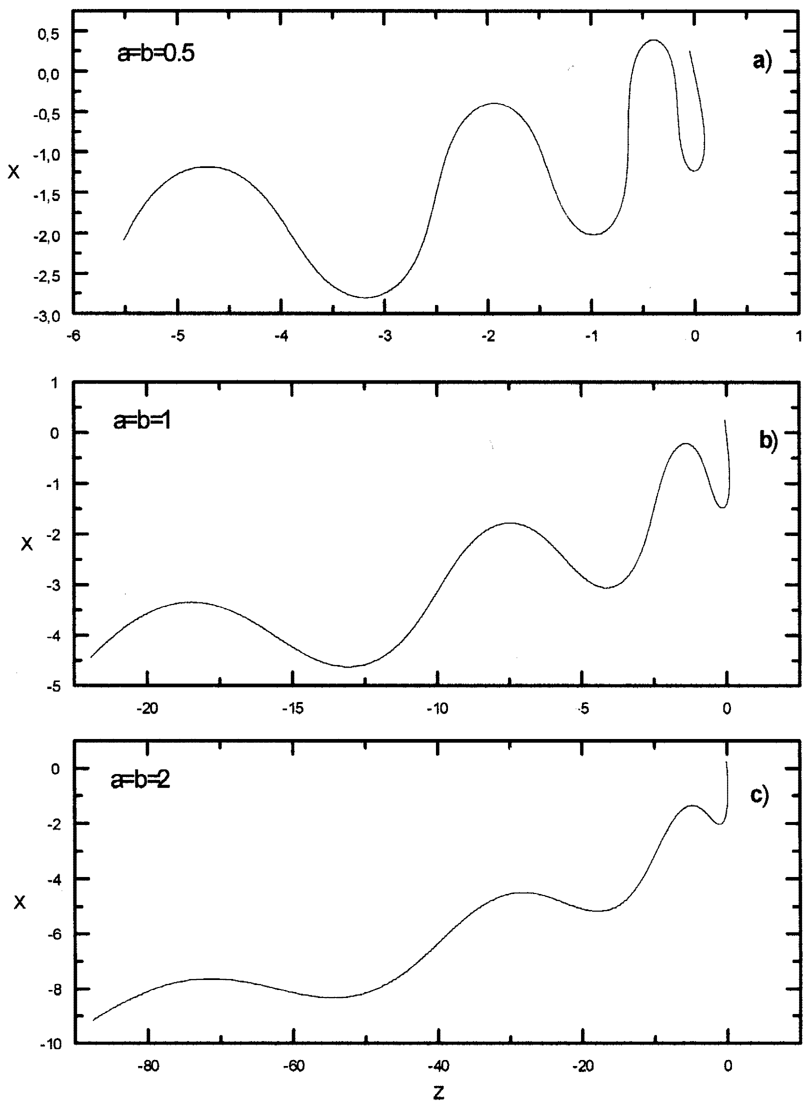

Fig. 3. Example of the trajectory of a charged particle in EM wave in presence of the B_y constant magnetic field $X = (\omega/\alpha)\cdot x$; $Z = (\omega\cdot c/\alpha^2)\cdot z$.

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4. MODIFICATION OF PARAMETERS OF THE MOTION OF A CHARGED PARTICLE IN THE EM WAVE IN PRESENCE OF A STATIC ELECTRIC FIELD

For the field given by:

$$E[E_x, E_y^c, E_z^c],$$

$$B[0, -B_y, 0]$$

where:

$$E_y^c = E_z^c = -E_o$$

for the components of the velocities of a particle, the solution of the equations of (2) has a form:

$$V_x = -\alpha \cos \phi$$

$$V_y = \frac{qE_o}{m} t$$

$$V_z \cong \alpha\phi - \frac{\alpha^2}{4c} \cos 2\phi, \qquad (12)$$

and for the parameters of the trajectory are given by:

$$x = -\frac{\alpha}{\omega} \sin \phi_o,$$

$$y = \frac{1}{2} \alpha\omega \cdot t^2,$$

$$z \cong \frac{1}{2} \alpha\omega \cdot t^2 + \frac{\alpha^2}{8c\omega} \sin \phi_o.$$

The trajectory is presented in Figure 4. As we can see this trajectory has the same character as presented in Figure 2 for constant magnetic field combined with EM wave. So, from this point of view both methods of controlling of the parameters of the motion are complementary.

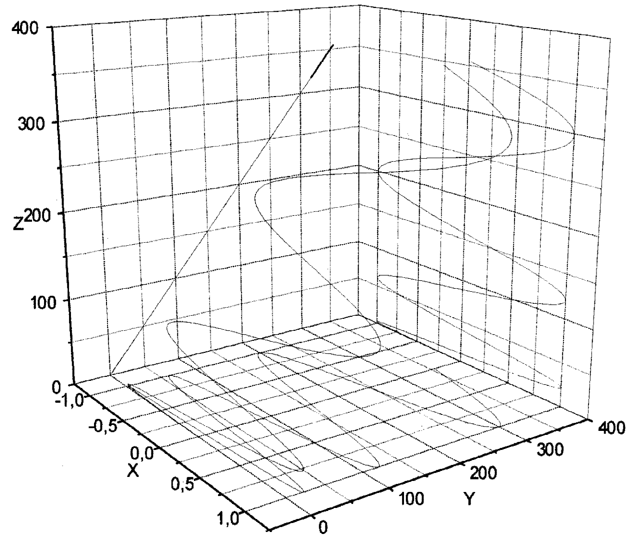

Fig. 4. Example of the trajectory of a charged particle in EM wave in presence of a static electric field $X = (\omega/\alpha)\cdot x$; $Y = (\omega/\alpha)\cdot y$; $Z = (2\omega/\alpha)\cdot z$.

5. CONCLUSIONS

It has been shown the possibility of changing and controlling the parameters of the motion of charged particles in the two EM waves which are propagating in the same direction, by changing their phase shift. By the introduction of constant magnetic or electric fields to an EM wave on a proper manner it has shown the possibility of changing the trapped state of a particle to the accelerated one. The analysis was done on the basis of the simple mathematical model and formulas which were obtained from it in the analytical way. Presented model can be also useful for evaluation of conditions of the motion of charged particles in other configurations and types of the EM waves and constant magnetic and electric fields and for other parameters of the charged particles. In particular it can be useful for evaluation of the hot electrons parameters in the presence of the magnetic field as it appeared under interaction of the laser light with matter (Kawata *et al.*, 1990), understanding of the mechanism of generation of the high energy electrons from laser plasma (Malka *et al.*, 1997; Quesnel *et al.*, 1997), investigation of the conditions of cyclotron heating (Jaeger *et al.*, 1972) and in all these problems where transport, transformation and visualization of information contained in charged particles moving in the EM fields are considered (Grover, 1997). Finally it is worth remarking that many new interesting effects can be obtained after including to equations of motion, for example, a dissipation term and for the cases when the particles are propagating in EM waves in the matter.

ACKNOWLEDGMENTS

The author is grateful to Dr. S. Jablonski for his help in preparing the figures.

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