RESEARCH PAPER

Synthesis of hexagonal planar array using swarm-based optimization algorithms

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Nature inspired optimization algorithms, namely artificial bee colony (ABC) optimization and firefly algorithm (FA), have been applied to synthesize beam patterns of a hexagonal planar array of isotropic elements. Two different cases, comprising two different beam patterns of a pencil beam and a square footprint pattern over a bounded region with lower peak sidelobe levels are presented. The pencil beam is generated by thinning the uniformly excited array and the square footprint pattern is generated by imposing optimum amplitudes, phases, and their corresponding states ("on"/"off") to the array elements. The optimum values of the parameters for both the cases are computed using ABC and FA individually, and the superiority of FA over ABC for the proposed problem in terms of computing solutions for both the cases is established.

Keywords: Optimization algorithm, Artificial bee colony (ABC) algorithm, Firefly algorithm (FA), Hexagonal planar array, Sidelobe level (SLL), Square footprint, Thinning

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I. INTRODUCTION

Hexagonal planar arrays are capable of providing an optimum sampling strategy for signals which are band limited in a Fourier plane over a circular region [1]. Beside this, the arrays with hexagonal geometry are able to reduce the high sidelobe problems which arise in case of using circular configuration. The excellent scanning abilities of the hexagonal planar arrays along with the above-mentioned capabilities make it suitable to find applications in smart antennas for wireless communications and in satellite applications [1-5]. Synthesis of hexagonal planar array appears in the literature [1-5]. Synthesis of hexagonal planar array with the application of Gegenbauer polynomial for low sidelobe and high directivity has been reported in [2]. The radiation pattern of a hexagonal array with the elements in triangular lattice has been synthesis using a pattern of linear array [3]. Synthesis of hexagonal array for obtaining patterns having high gain and directivity is carried out by Gozasht et al. [4]. Mahmoud et al. [5] have designed hexagonal and circular array with 18 half-wave dipole elements and evaluated complex excitation amplitude and phase using particle swarm optimization (PSO).

Synthesis of uniformly excited thinned array using deterministic approach is reported in [6].

Different optimization algorithms such as genetic algorithm, PSO, etc. have been applied to a huge variety of

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electromagnetic problems especially for broadband, miniature, multiband, high-directivity antennas, and also for synthesis of different beam patterns from a variety of antenna arrays [7-11]. Apart from these population-based optimization algorithms, several other techniques [1-2, 4, 6] are reported in the literature to synthesize beam pattern from different array geometries.

In this paper, synthesis of a standard hexagonal array (SHA) [1] for generating two different beam patterns of a pencil beam and a square footprint over a bounded region is presented. Two different cases have been considered. In the first case, a pencil beam is generated by thinning [12–18] the array and in the second case a square footprint pattern [19–22] is obtained by properly modifying the array elements amplitudes, phases, and their corresponding states ("on"/ "off"). The optimum values of the parameters for the two different cases are computed individually using artificial bee colony (ABC) [23, 24] optimization and firefly algorithm (FA) [25, 26]. A comparative analysis has been carried out between ABC and FA in terms of fitness value and the superiority of FA over ABC has been established for both the cases.

II. PROBLEM FORMULATION

The positions of the array elements are assumed to be fixed and all the elements have two states either "on" or "off", depending upon whether the element is connected to the feed network or not. In the "on" state of an element, the amplitude and phase excitations are imposed through the feed network; and in the "off" state, the element can be assumed to be passively terminated to a matched load or an open circuited.

A pencil beam of the first case is obtained from the uniformly excited array by turning off some of the elements

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from the array i.e. by modifying the elements states. The process is known as array thinning and is widely employed to reduce the peak side lobe level (peak SLL) of the array. Typical applications of thinned arrays include satellite-receiving antennas that operate against a jamming environment [12], ground-based high-frequency radars [12], and design of interferometer array for radio astronomy [12]. The phase excitations of the array elements for the first case are assumed to be "zero" degree.

A footprint pattern with given specification is obtained from the array by appropriately modifying the elements amplitudes, phases, and their corresponding states. The inclusion of elements states in the synthesis method reduces the power consumption and simplifies the design process. The application of footprint patterns is involved in satellite communications, where coverage of a specified geographical region on earth surface is required while minimizing the radiation in other nearby regions.

Following the above procedures, two different beam patterns of pencil beam and a square footprint pattern with lower peak SLL have been achieved. The elements excitations in terms of amplitudes and phases and their corresponding states are assumed to have a quadrantal symmetry among themselves for the two different cases. The purpose of considering the symmetric excitations is to impose the required symmetry in the desired beam patterns.

In case of SHA, the relationship between the horizontal and vertical interelement spacing of the rows is

$$d_y = \frac{\sqrt{3}}{2} d_x.$$
 (1)

Considering the horizontal interelement spacing $d_x = 0.5\lambda$, the far-field pattern of the SHA as shown in Fig. 1 on the x - y plane can be expressed as [1]

$$AF(u, v) = \sum_{m=-(N_x-1)/2}^{(N_x-1)/2} \sum_{n=0}^{N_x-|m|-1} w_{nm} \\ \times \exp\left\{\left[m\frac{\sqrt{3}}{2}v + \left(n - \frac{N_x - |m| - 1}{2}\right)u\right]\right\}, \quad (2)$$

where w_{nm} is the complex weight of the *nm*th element. N_x is the number of elements in the horizontal row passing



Fig. 1. Geometry of a hexagonal planar array with 37 isotropic elements.

through the origin (Fig. 1). The individual elements in a particular row is indexed with the variable *n*, which ranged from n = 0 to $N_r - 1$; where, N_r is the number of elements in a particular row [1] and is defined as [1] $N_r = N_x - |m|$. Each row is indexed by the variable *m*, ranges from $-(N_x - 1)/2$ to $(N_x - 1)/2$. The direction cosines are $u = \sin \theta \cos \varphi$ and $v = \sin \theta \sin \varphi$ where θ and φ represent the polar and azimuth angles, respectively.

The design objective of this paper is to minimize the fitness functions for the two different cases using ABC and FA which are defined as follows:

$$F_1 = \max_{(u,v) \in A_1} \{ AF_{dB}(u, v) \},$$
(3)

$$F_{2} = k_{1} \left(peakSLL^{d} - \max_{(u,v) \in A_{1}} \{AF_{dB}(u,v)\} \right)^{2} + k_{2} \left(r_{dB}^{d} - \left| \min_{(u,v) \in A_{2}} \{AF_{dB}(u,v)\} \right| \right)^{2}, \quad (4)$$

where F_1 is the fitness function for the pencil beam of case I and F_2 is the fitness function for the square footprint pattern of case II. *peakSLL*^d represent the desired values of the peak SLL and r_{dB}^d denotes the response ripple for square footprint pattern.

In equations (3) and (4), $\max_{(u,v)\in A_1} \{AF_{dB}(u,v)\}$ is responsible for determining peak SLL of the obtained beam pattern in the u-v plane, where A_1 represents the angular region of the sidelobes which is outside the main beam region of the pattern. The computed value of peak SLL in this manner is subtracted from the desired values of the peak SLL and finally to increase the weightage of the resulting term, its square is taken. The same procedure is adopted for determining the ripple parameter in equation (4). The term $|\min_{(u, v)\in A_2}\{A \ F_{dB}(u, v)\}|$ determines the obtained ripple for the square footprint pattern within the angular region A_2 , which is defined as $-0.12 \le u \le 0.12$ and $-0.12 \le v \le 0.12$. k_1 and k_2 are the weighting factors to give relative importance of each of the terms. The values assigned to k_1 and k_2 are 1 and 100, respectively.

III. ALGORITHM OVERVIEWS AND PARAMETRIC SETUP

A) Overview of ABC algorithm

ABC algorithm is swarm-based optimization algorithm developed by Karaboga and Basturk [23, 24]. In ABC algorithm, the solutions of an optimization problem are denoted by the positions of the food sources around a colony of artificial bees and the search ability of ABC lies on different behaviors of the three groups of bees: employed bees, onlookers, and scouts. The algorithm can be summarized in the flowchart of Fig. 2.

B) Overview of FA

FA is a swarm-based optimization algorithm developed by Xin-She Yang [25, 27]. The algorithm relates the flashing characteristics of the fireflies with the objective function and it was developed from the study of the behavior of how fireflies



Fig. 2. Flowchart of ABC algorithm [23, 24].

communicates. The algorithm considered glowing fireflies as the search agents and each firefly is characterized by two parameters: its location in the d-dimensional search space and its light intensity or brightness. Depending upon their brightness, the fireflies changes their location and the algorithm approaches toward its optimum solution.

The algorithm can be summarized in the flowchart of Fig. 3.

C) Details of parametric setup

A hexagonal planar array of total 169 isotropic elements is considered. The number of elements in the horizontal row through the origin is 15, i.e. $N_x = 15$.

For the first case of pencil beam, as the design problem is to find out an optimum set of elements state; the individual of the population for both ABC and FA is considered as follows:

$$P = [S_1 \, S_2 \dots S_X]. \tag{5}$$

For the second case of square footprint pattern, the design problem is based on finding out optimum set of elements amplitudes, optimum set of elements phases, and optimum set of the elements states; the individual of the population for both ABC and FA are considered as follows:

$$P = [I_1 I_2 \dots I_X \alpha_1 \alpha_2 \dots \alpha_X S_1 S_2 \dots S_X].$$
(6)



Fig. 3. Flowchart of FA [25-27].

As the state of a particular element is either "o" or "1", a mapping from real space to binary space for the elements states in each of the individuals for both ABC and FA has been performed using the sigmoid function restriction $Sig(S_i)$ as follows:

$$Sig(S_i) = \frac{1}{1 + \exp(-S_i)}, \quad i = 1, 2, ..., X,$$
 (7)

$$S_i = \begin{cases} 1 & \text{if } rand \leq Sig(S_i), \\ 0 & \text{otherwise,} \end{cases}$$
(8)

where *rand* is a uniformly distributed random number between [0,1] and $Sig(S_i)$ denotes the probability of bit S_i

taking "1". If S_i becomes a vector of zeros or ones, the solution is regarded as infeasible for both ABC and FA.

The limits of the variables are defined as follows:

$$0.05 \le I_m \le 1$$
 for $m = 1, 2, ..., X$, (9)

$$-\pi \le \alpha_m \le \pi \quad \text{for } m = 1, 2, \dots, X, \tag{10}$$

$$0 \le S_m \le 1$$
 for $m = 1, 2, ..., X$. (11)

The lower limits of the amplitudes are kept at 0.05 to maintain the DRR (dynamic range ratio) of the excitation amplitudes between 1 and 20. As the elements excitations and their states

ABC			FA			
Parameters	Values		Parameters	Values		
Colony size	50		Number of fireflies	50		
% Of onlooker bees	50% of the colony size		β_{o}	0.20		
% Of employed bees	50% of the colony size		γ	0.25		
Number of allowable scout bees in the population	1		α	1		
"limit" for abandonment	250		_	_		
Search space dimension	Case I	48	Search space dimension	Case I	48	
Choice of initial population	Case II Random	144	Choice of initial population	Case II Random	144	
Termination condition	Case I Case II	Maximum iteration of 700 Maximum iteration of 1500	Termination condition	Case I Case II	Maximum iteration of 700 Maximum iteration of 1500	

Table 1. Parametric setup of the ABC and FA.

 Table 2. Desired and obtained results for cases I and II computed using ABC and FA.

Design parameters		ABC		FA	
		Case I	Case II	Case I	Case II
Peak SLL (dB)	Desired	_	-15.00		-15.00
	Obtained	-19.45	-7.93	-20.48	-12.74
Ripple (dB)	Desired	—	0.60	_	0.60
	Obtained	—	0.88	—	0.87

pause a quadrantal symmetry among themselves individually, the number of different variables in each of the computed set becomes 48. So for both the cases X = 48. The search space dimension for the two cases becomes 48 and 144, respectively.

The other parametric setups of the ABC and FA for the proposed problem are set based on the guidelines provided in [23-27] and are given in Table 1. The desired specifications of the parameters required for obtaining pencil beam patterns of case I and square footprint pattern case are given in Table 2.

IV. SIMULATION RESULTS

The results presented in this section are the best set of results obtained from 20 different runs of ABC and FA while minimizing the fitness function of each individual case. The desired and obtained values of the design parameters for two different cases are listed in Table 2.

The beam pattern of the uniformly excited array and zero phases among the elements are shown in Fig. 4. The array with uniform excitation among the elements and constant phase gives peak SLL of 16.47 dB.

It can be observed from Table 2 that the obtained values of peak SLL for the pencil beam pattern of case I using ABC and FA are -19.45 and -20.48 dB. Figures 5 and 6 present the thinned array of best design using ABC and FA and their corresponding beam patterns are shown in Figs 7 and 8, respectively.

The obtained values of peak SLL for the square footprint patterns using ABC and FA are -7.93 and -12.74 dB corresponding to a desired value of -15 dB. The obtained response ripple using ABC and FA are 0.88 and 0.87 dB corresponding to their desire value of 0.60 dB. The state of array elements for



Fig. 4. Far field pattern of a uniformly excited planar hexagonal array of 169 numbers of isotropic elements.





Fig. 5. Thinned array using ABC (case I).

Fig. 6. Thinned array using FA (case I).



Fig. 7. Far field pattern of the thinned array using ABC (case I).



Fig. 8. Far field pattern of the thinned array using FA (case I).



Fig. 9. State of the array elements computed using ABC for obtaining square footprint pattern (case II).



Fig. 10. State of the array elements computed using FA for obtaining square footprint pattern (case II).

the footprint patterns is shown in Figs 9 and 10 and their corresponding beam patterns are shown in Figs 11 and 12, respectively.



Fig. 11. Far field pattern of the optimized array with a target of a square footprint within the range of $-0.12 \le u \le 0.12$ and $-0.12 \le v \le 0.12$ using ABC.



Fig. 12. Far field pattern of the optimized array with a target of a square footprint within the range of $-0.12 \le u \le 0.12$ and $-0.12 \le v \le 0.12$ using FA.

The optimum excitations of the switched-on elements in terms of amplitudes and phases for the rectangular footprint pattern of case II using ABC and FA are shown in Figs 13 and 14, respectively (Table 3).



Fig. 13. Excitations of the switched-on elements computed using ABC for generating square footprint pattern ("case II"): (a) amplitude distribution and (b) phase distribution.



Fig. 14. Excitations of the switched-on elements computed using FA for generating square footprint pattern ("case II"): (a) amplitude distribution and (b) phase distribution.

Table 3. Total switched-on elements and DRR for cases I and II.

Different cases	ABC			FA		
	No. of elements in the "on" state	% Of switched on elements	DRR	No. of elements in the "on" state	% Of switched-on elements	DRR
Case I	101	59.76	_	91	53.85	_
Case II	78	46.15	11.76	90	53.25	16.92



Fig. 15. Convergence characteristics of ABC and FA for case I.

The convergence characteristics of the ABC and FA presented in this paper for all the cases are in terms of best fitness value versus generations for the best run of each algorithm (best out of 20 different runs).

Figure 15 shows the convergence of ABC and FA for case I and Fig. 16 shows the convergence of ABC and FA for case II. From Figs 15 and 16, it can be seen that the performance of FA over ABC is much better for the minimization of the fitness functions for both the cases (Table 4).

V. CONCLUSIONS

Two different swarm-based global optimization techniques have been applied to synthesize beam patterns of a hexagonal



Fig. 16. Convergence characteristics of ABC and FA for case II.

Table 4. Comparative performance of the FA and ABC.

Different cases	Algorithms	Best fitness value
Case I	ABC	-19.45
	FA	-20.48
Case II	ABC	57.70
	FA	12.39

planar array of isotropic elements. The proposed method is capable of producing beam patterns that meet the desired specification to construct a pencil beam by thinning the uniformly excited array and a square footprint pattern over a bounded region by appropriately modifying the elements amplitudes, phases, and their corresponding states. The performances of the two algorithms are also compared in terms of minimizing the fitness functions for the two different cases and the superiority of FA over ABC is established. The fitness functions are formed in such a manner to get satisfactory output. The termination conditions of FA and ABC are chosen by specifying a maximum number of iterations after which the probability of getting improved solutions is negligible.

Finally, the inclusion of elements states improve the overall power consumption of the system and the low value of DRR reduces the design complexity of the feed network.

The proposed technique can also be extended to generate other types of footprint from hexagonal planar array like rectangular, circular, and U-shaped footprints which often required in satellite communication applications.

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