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Laser-induced fusion detonation wave

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Abstract

Development of a detonation wave due to α heating following short pulse laser irradiation in pre-compressed deuterium–tritium (DT) plasma is considered. The laser parameters required for development of a detonation wave are calculated. We find that a laser irradiance and energy of $I_L = 1.75 \times 10^{23}$ W/cm^2 and 12.8 kJ accordingly during 1.0 ps in a pre-compressed target at 900 g/cm^3 creates an α heating fusion detonation wave. In this case, the nuclear fusion ignition conditions for the pre-compressed DT plasma are achieved along the detonation wave orbit.

Keywords: Detonation; Fast ignition; Fusion; High-power laser

1. INTRODUCTION

The physics of inertial confinement fusion (ICF) is based on compressing and igniting the plasma fuel (Nuckolls *et al.*, 1972; Atzeni & Meyer-Ter-Vehn, 2004; Velarde & Carpintero-Santamaria, 2007). In order to ignite the fuel with less energy, it was suggested to separate the drivers that compress and ignite the target (Basov *et al.*, 1992; Tabak *et al.*, 1994). This idea is called fast ignition. Many schemes have been suggested to solve this issue (Guskov, 2013).

We suggested recently a novel shock wave ignition scheme (Eliezer *et al.*, 2014*a*; Eliezer *et al.*, 2015), where the ignition shock wave is generated in a pre-compressed target by the ponderomotive force (Hora, 1991; Eliezer, 2002) of a high-irradiance laser pulse. The shock wave velocity in this scheme is in the intermediate domain between the relativistic (Taub, 1948; Landau & Lifshitz, 1987) and non-relativistic (Zeldovich & Raizer, 1966; Fortov & Lomonosov, 2010) hydrodynamics. This shock wave is described in the literature as a "piston model" (Esirkepov *et al.*, 2004; Naumova *et al.*, 2009; Eliezer *et al.*, 2014*b*, *c*). In this domain of laser intensities, the ponderomotive force forms a double layer which acts as a piston driving a shock wave moving in the unperturbed plasma. This model is supported in the literature by particle-in-cell simulation (Esirkepov *et al.*, 2004; Naumova *et al.*, 2009) and independently by hydrodynamic two fluid simulations (Hora *et al.*, 1984; Lalousis *et al.*, 2012; Lalousis *et al.*, 2013).

Here we consider a self-sustained one-dimensional (1D) detonation wave due to heating by α-particles generated in the laser-induced ignitor. This detonation wave should sustain ignition in the remaining part of the target. The fusion energy released in the shocked material at the end of the laser pulse is calculated and compared with an analytical model of detonation. Section 2 describes the ignitor induced by the high-irradiance laser pulse, Section 3 presents the detonation wave requirements, and the conclusion with a possible application for fast ignition is described in Section 4.

2. THE IGNITOR

In this paper, the ignitor is a pre-compressed deuterium–tritium (DT) plasma mixture with equal density numbers for deuterium (n_D) and tritium (n_T). The electron density n_e and the ion density $n_i = n_D + n_T$ for $n_D = n_T$ are related to the ignitor density ρ and the proton mass m_p by

$$n_e(\text{cm}^{-3}) = n_i(\text{cm}^{-3}) = \left(\frac{\rho}{2.5 m_p}\right) = 2.39 \times 10^{23} \rho. \quad (1)$$

The α-particles (α) created in the nuclear fusion reactions

$$D + T \to n + \alpha + 17,589\,\text{keV} \quad (2)$$

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supply the ignitor energy density rate W_f

$$W_f = n_D n_T \langle \sigma v \rangle_{DT} E_\alpha = 8.07 \times 10^{40} \langle \sigma v \rangle_{DT} \rho^2 \left[\frac{\text{erg}}{\text{cm}^3 \cdot \text{s}}\right], \quad (3)$$

where σ is the DT fusion cross-section, v is the relative DT velocity, and $\langle \sigma v \rangle$ is the relevant average for the process under consideration. This velocity is achieved by a high-intensity laser-induced shock wave. $\langle \sigma v \rangle_{DT}$, the reactivity of the DT reaction fitted in the domain of ion temperatures 1 keV $< T_i <$ 100 keV, is given by (Bosch & Hale, 1992)

$$\langle \sigma v \rangle_{DT}\left[\frac{\text{cm}^3}{\text{s}}\right] = 6.4341 \times 10^{-14} \zeta^{-5/6} \left(\frac{6.661}{T_i^{1/3}}\right)^2 \times \exp\left[-19.983\left(\frac{\zeta}{T_i}\right)^{1/3}\right]$$
$$\zeta = 1 - \frac{15.136 T_i + 4.6064 T_i^2 - 0.10675 T_i^3}{1000 + 75.189 T_i + 13.5 T_i^2 + 0.01366 T_i^3}, \quad T_i \text{ in keV}. \quad (4)$$

The ignitor performance can be analyzed by an energy balance, dependent on the ions and electrons temperatures T_i and T_e (Chu, 1972; Eliezer & MartinezVal, 1998; Eliezer *et al.*, 2015)

$$\left(\frac{3}{2}\right)\frac{d}{dt}(n_e k_B T_e) = \eta_d W_d + W_{ie} - W_B + f_\alpha \eta_f W_f$$
$$\left(\frac{3}{2}\right)\frac{d}{dt}(n_i k_B T_i) = (1 - \eta_d) W_d - W_{ie} + f_\alpha (1 - \eta_f) W_f, \quad (5)$$

k_B is the Boltzmann constant. W_d [erg/(cm^3·s)] is the power density deposited by the driver (induced by the laser–piston in our case), η_d equals the fraction of the driver energy deposited in the electrons inside the shocked volume, $1 - \eta_d$ gives the fraction of the driver energy deposited in the ions inside the shocked volume. W_{ie} [erg/(cm^3·s)] is the ion–electron exchange power density, whereas W_B [erg/(cm^3·s)] describe the electron bremsstrahlung power density losses. As described above, W_f [erg/(cm^3·s)] equals the fusion power density created in the shocked volume, where f_α gives the fraction of the α-particles energy deposited inside the shocked volume with η_f equals the energy fraction that is deposited in the electrons by the α-particles and $(1 - \eta_f)$ is the energy fraction that is deposited in the ions by the α-particles.

The shock wave velocity (u_s) and the flow particle velocity (u_p) in this scheme are in the intermediate domain between the relativistic and non-relativistic values (see the Appendix). The shock wave driver considered here is a laser-piston, which pushes the DT plasma to move with velocity u_p. The deposition power density W_d in this piston model with a constant laser irradiance I_L[W/cm^2] and laser pulse duration τ_L is given by

$$W_d\left[\frac{\text{erg}}{\text{cm}^3 \cdot \text{s}}\right] = \frac{1}{\tau_L}\left(\frac{1}{2}\rho u_p^2\right). \quad (6)$$

The fraction of the driver energy deposition into the electrons η_d can be estimated as function of λ_i and λ_e, the appropriate mean-free paths of the ions and electrons in plasma:

$$\eta_d = \frac{\lambda_i}{\lambda_i + \lambda_e},$$
$$\lambda_i[\text{cm}] = \left(\frac{3 \times 10^{23}}{n_i}\right)\left(\frac{m_p}{m_i}\right) E_i[\text{MeV}],$$
$$\lambda_e[\text{cm}] = \left(\frac{5 \times 10^{22}}{n_e \ln \Lambda}\right) T_e[\text{keV}]^{3/2} E_i[\text{MeV}],$$
$$E_i = \frac{1}{2} m_i u_p^2 = 1250(\text{MeV})\left(\frac{u_p}{c}\right)^2, \quad (7)$$

where the ion mass in our case is $m_i = 2.5\ m_p$, the ion density n_i is defined in Eq. (1) and the plasma logarithmic term $\ln\Lambda$ is

$$\ln \Lambda = 24 - \ln\left[\frac{n_e[\text{cm}^{-3}]^{1/2}}{T_e(\text{eV})}\right]. \quad (8)$$

It is important to mention that although λ_i and λ_e can in general be larger than the time-dependent shocked domain $u_p \cdot t$, the charged particles that heat the shocked region have a velocity u_p and therefore are not moving faster than the shock wave since $u_s > u_p$. Thus, the shock wave moves into a cold domain not yet heated by the driver energy.

The ion–electron exchange power density is given by

$$W_{ie}\left[\frac{\text{erg}}{\text{cm}^3 \cdot \text{s}}\right] = \left(\frac{3}{2}\right)\frac{k_B(T_i - T_e)}{\tau_{eq}},$$
$$\tau_{eq} = \frac{3\, m_e m_i}{8\sqrt{2\pi} n_i e^4 \ln\Lambda}\left(\frac{k_B T_e}{m_e} + \frac{k_B T_i}{m_i}\right)^{3/2}. \quad (9)$$

The bremsstrahlung power density losses W_B are given by

$$W_B\left[\frac{\text{erg}}{\text{cm}^3 \cdot \text{s}}\right] = 8.58 \times 10^{21} \rho^2 T_e(\text{eV})^{0.5}\left(1 + \frac{2T_e(\text{eV})}{0.511 \times 10^6}\right). \quad (10)$$

The α fusion energy is defined in Eq. (3). However, not all of the α fusion energy is deposited into the ignitor; f_α is the fraction of the α-particles created and deposited into the ignitor domain, while $(1 - f_\alpha)$ is the escape fraction to the surrounding cold fuel. The value of f_α is (Guskov & Rozanov, 1993)

$$f_\alpha = \begin{cases} \frac{3}{2}x_\alpha - \frac{4}{5}x_\alpha^2, & x_\alpha < \frac{1}{2}, \\ 1 - \frac{1}{4x_\alpha} + \frac{1}{160 x_\alpha^3}, & x_\alpha \geq \frac{1}{2}, \end{cases}$$
$$x_\alpha(\tau) = \frac{R}{R_\alpha}. \quad (11)$$

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The ignitor dimension R in our model is taken to be the length of the shocked cylinder

$$R = l_s = (u_s - u_p)\tau_L, \tag{12}$$

and the α range R_α is approximated (Atzeni & Meyer-Ter-Vehn, 2004) by:

$$R_\alpha[\text{cm}] = \frac{1}{\kappa\rho_0}\left[\frac{1.5\times10^{-2}T_e(\text{keV})^{5/4}}{1+8.2\times10^{-3}T_e(\text{keV})^{5/4}}\right]. \tag{13}$$

The initial density of the pre-compressed target is ρ_0 and κ is the shock wave compression.

The fusion energy fraction deposited in the electrons by the α-particles is η_f and $(1-\eta_f)$ describes the energy fraction that is deposited in the ions. The function η_f for DT fusion is (Chu, 1972):

$$\eta_f = \frac{32}{32 + T_e(\text{keV})}. \tag{14}$$

The time-dependent temperatures in Eq. (5) are coupled to equations for the number densities of the ions species. For DT these equations in our case ($n_D = n_T$) are:

$$\frac{dn_D}{dt} = \frac{dn_T}{dt} = -\frac{dn_\alpha}{dt} = -n_D n_T\langle\sigma v\rangle_{DT}, \tag{15}$$

where n_D, n_T, and n_α are number densities of the deuterons, tritium, and α-particles accordingly and $\langle\sigma v\rangle_{DT}$ is given in Eq. (4).

We solve Eqs. (5) and (15) numerically and the calculated electron and ion temperatures are given in Figure 1, while the number densities of deuterium, tritium, and α are specified in Figure 2. Our input data for these calculations are:

1. A pre-compressed target of DT with initial density ρ_0 and shocked density $\rho = 3600\ \text{g/cm}^3$ and initial temperature (in energy units) of $T_e = T_i = 1$ keV.
2. The ratio of heat capacity at constant pressure to heat capacity at constant volume Γ in the laser-induced shock wave is either 3 or 5/3 and Γ = 3 for the detonation wave (Landau & Stanyukovich, 1945) as described in the next section. Therefore, the compression $\kappa = (\Gamma+1)/(\Gamma-1)$ is either 2 or 4 (see the Appendix), equivalent to pre-compressed density ρ_0, 1800 or 900 g/cm^3 accordingly. As the released fusion energy is dependent on the shocked region density, to compare the ignitor performance corresponding to the above two values of Γ, we choose similar value for the shocked density.
3. For both the values of Γ that we consider, we have a particle flow velocity u_p in the shock wave that equals to 0.010c, where c is the speed of light. As shown in the Appendix, the value of laser intensity particle I_L (W/cm^2), the initial density ρ_0 and the

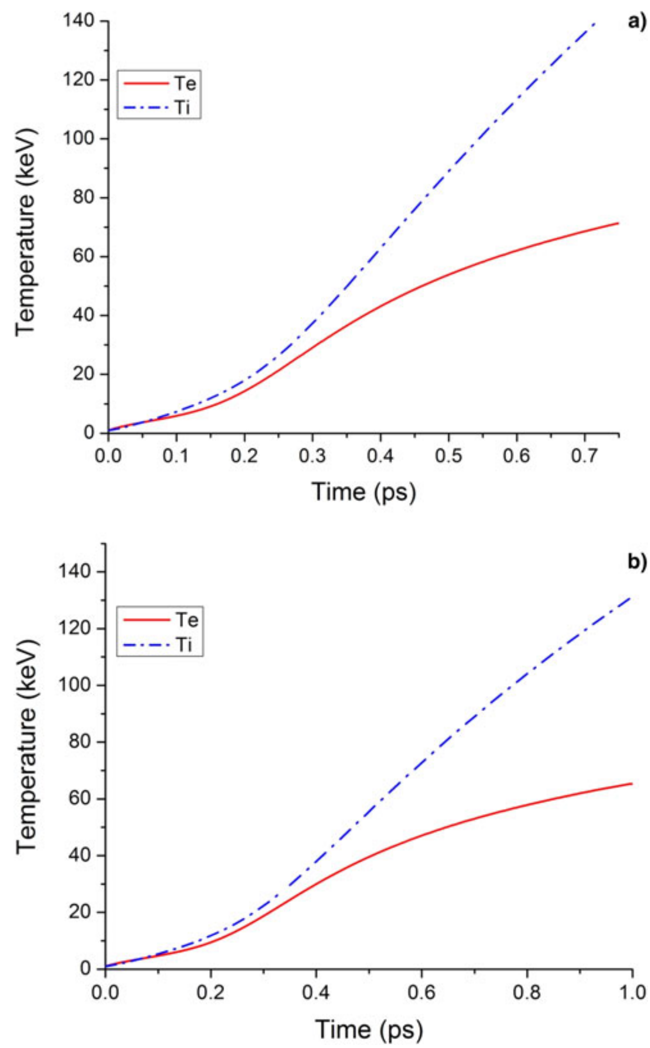

Fig. 1. Electrons and ions temperatures as a function of time for a shock wave induced by (a) a laser irradiance and energy of $I_L = 5.25\times10^{23}\ \text{W/cm}^2$ and 149 kJ accordingly during 0.75 ps assuming Γ = 3, in a pre-compressed target with density 1800 g/cm^3, (b) a laser irradiance and energy of $I_L = 1.75\times10^{23}\ \text{W/cm}^2$ and 12.8 kJ accordingly during 1.0 ps assuming Γ = 5/3, in a pre-compressed target with density 900 g/cm^3.

equation-of-state (EOS) parameter Γ determines the particle flow velocity and the shock wave velocity as well as the compression (which depends only on Γ in the domain discussed here). The laser pulse duration was adjusted to obtain the required fusion energy Q (see Section 3) for the development of detonation wave at the end of the laser pulse. Assuming a laser spot radius $R_L = 1.5\ l_s$ [see Eq. (12) and the Appendix], the required laser energy can be estimated by

$$W_L = \pi R_L^2 I_L \tau_L. \tag{16}$$

The values of the laser parameters leading to detonation development at the end of the laser pulse for the two values of Γ are given in Table 1.

From the solution of the time-dependent equations for the particles number density and temperatures, the fusion energy

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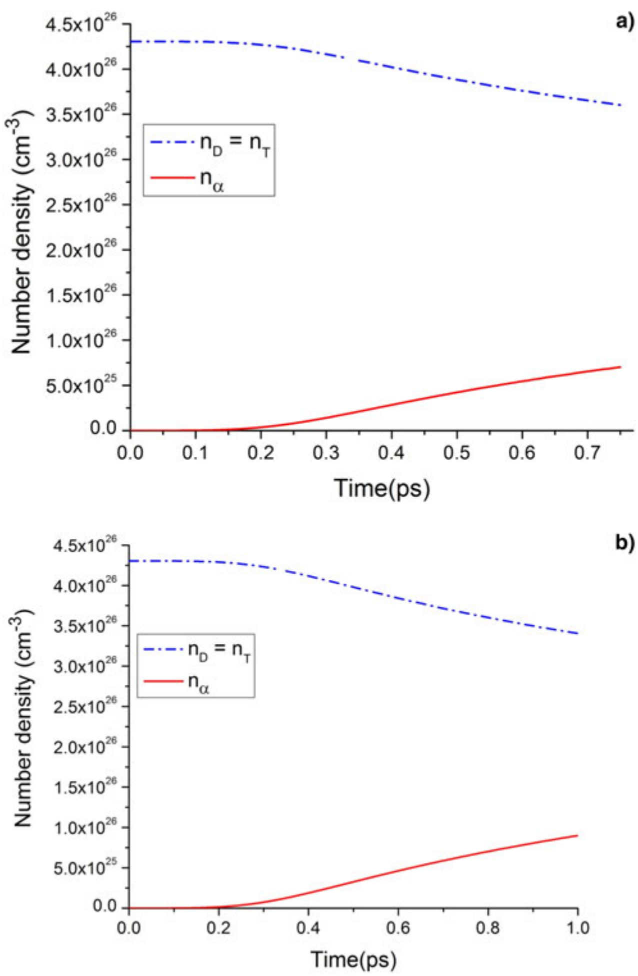

Fig. 2. The number densities of deuterium (n_D), tritium ($n_T = n_D$), and α (n_α) particles as a function of time in the shocked volume considered in Figure 1 for (a) Γ = 3, and (b) Γ = 5/3.

Table 1. *Laser and shocked region parameters enabling the development of detonation at the end of the laser pulse for particle velocity* $u_p = 0.01c$.

Γ	κ	ρ_0 (g/cm^3)	I_L (W/cm^2)	τ_L (ps)	l_s (μm)	$2R_L$ (μm)	W_L (kJ)
3	2	1800	5.25×10^{23}	0.75	2.3	6.8	149
5/3	4	900	1.75×10^{23}	1	1.0	3.0	12.8

Q per unit mass released in the forward direction (defined by the shock front) by the end of the laser pulse duration is calculated. In our 1D piston model, f_α is the fraction of the α-particles deposited into the ignitor domain, while $(1–f_\alpha)/2$ is the escape fraction into the forward cold fuel. The α mean-free path R_α (front) in the front of the shock wave, with T_e = 1 keV from the pre-compressed conditions, is according to Eq. (13)

$$R_\alpha(\text{front}) = \frac{1.50 \times 10^{-2}}{\kappa \rho_0}[\text{cm}]. \quad (17)$$

For a pre-compressed density of 1800 g/cm^3, one gets R_α (front) = 0.08/κ μm. Since this mean-free path is extremely small, we can assume that by the end of the laser pulse duration we have in the forward direction of the ignitor a fraction $[f_\alpha + (1–f_\alpha/2)]$ of the fusion created α-particles each with an energy E_α = 3.52 MeV. Thus, the α energy per unit mass Q given by

$$\begin{aligned} Q\left[\frac{\text{J}}{\text{kg}}\right] &= \left(\frac{E_\alpha}{\rho}\right)\int_0^{t=\tau_L} dt\left(\frac{dn_\alpha}{dt}\right)\frac{1}{2}(1+f_\alpha), \\ &= \left(\frac{5.63\times10^{-10}}{\rho[\text{g/cm}^3]}\right)\int_0^{t} dt\left(\frac{dn_\alpha[\text{cm}^{-3}]}{dt}\right)\frac{1}{2}(1+f_\alpha). \end{aligned} \quad (18)$$

Our solutions for f_α and Q are given in Figures 3 and 4 accordingly.

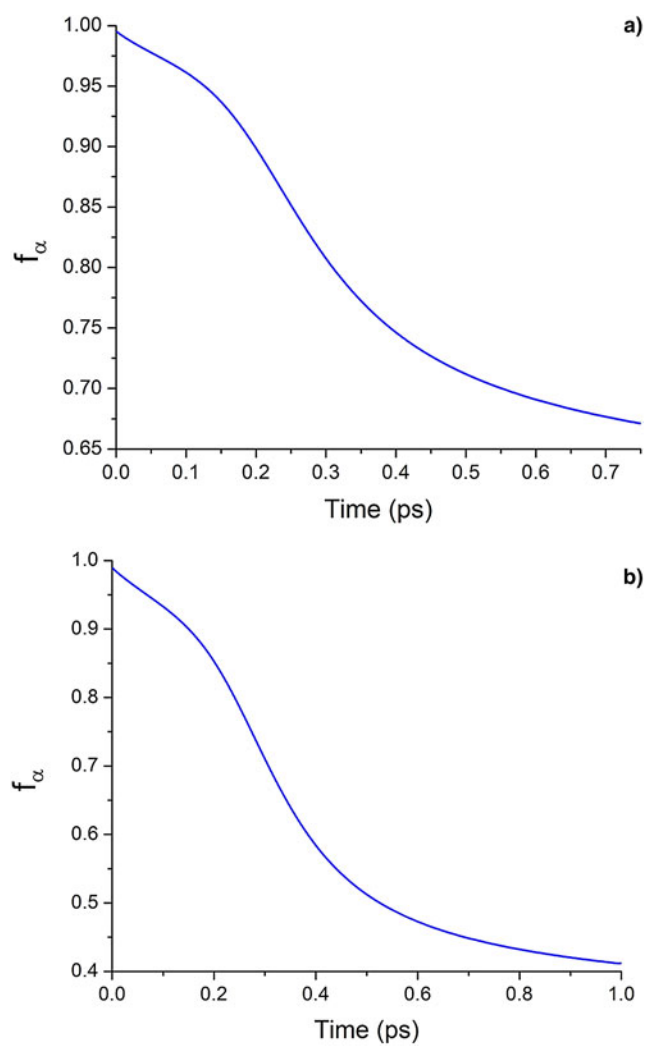

Fig. 3. The fusion energy fraction f_α deposited in the shocked volume as a function of time in the shocked volume considered in Figure 1 for (a) Γ = 3, and (b) Γ = 5/3.

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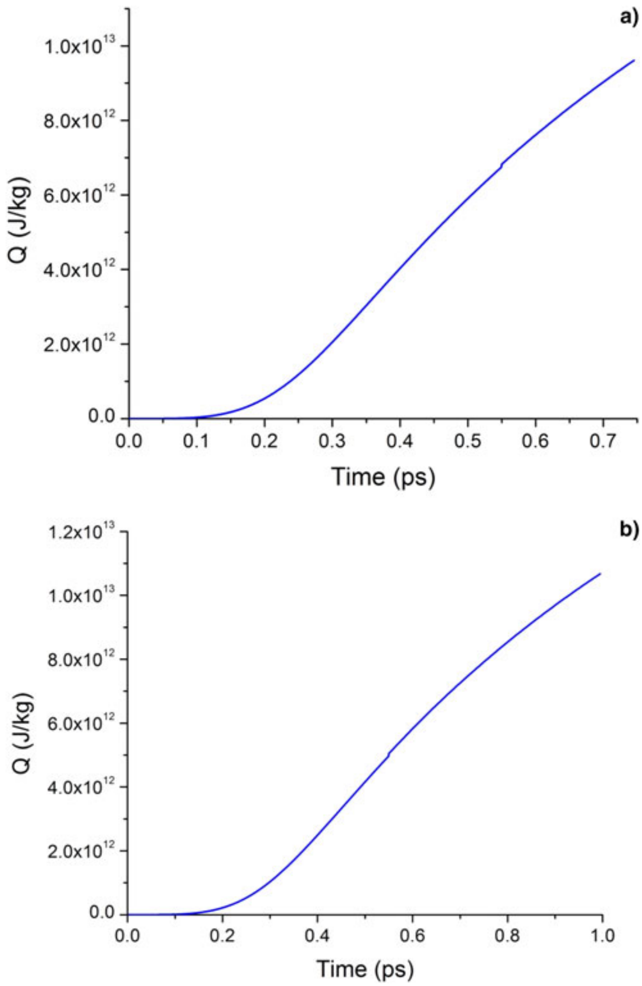

Fig. 4. The fusion energy Q per unit mass released in the shock wave forward direction as a function of time in the shocked volume considered in Figure 1 for (a) $\Gamma = 3$, and (b) $\Gamma = 5/3$.

3. THE DETONATION WAVE

At the end of the ignitor laser operation, we have chosen a particle velocity behind the shock front of 1.0% the speed of light. Now we will examine the conditions needed to support a steady detonation wave. The theoretical treatment that we consider is based on 1D plane detonation wave under Chapman–Jouguet (CJ) condition.

In the case of chemical-based detonation, the energetic material entering the shock front is compressed and thus its temperature rises. Under sufficient temperature the material transforms exothermally into gasses products releasing energy per unit mass (Q) that supports the shock. The section at which this reaction undergoes is called the reaction zone, and it is on the order of 0.1 mm for most explosives. The governing parameter effecting reaction rate in the reaction zone are the local density and temperature. Our detonation is analogous to this description where the chemical energy has been changed to nuclear fusion energy. In the DT fusion one gets 17.6 MeV fusion energy per reaction but only the 3.52 Mev of the α-particle is relevant to support the desired steady-state shock condition with a mean-free path <0.1 μm (see Eq. (17)).

The conservation laws of mass (flow of mass), momentum (sum of the flow of momenta), and energy (the sum of the flows of thermal, chemical, and kinetic energies and the work of the pressure force) are given by the following equations, respectively:

$$\begin{aligned} &(i)\ \rho(D-u) = \rho_0 D, \\ &(ii)\ P + \rho(D-u)^2 = P_0 + \rho_0 D^2, \\ &(iii)\ E_T + \frac{P}{\rho} + \frac{1}{2}(D-u)^2 = E_0 + \frac{P_0}{\rho_0} + \frac{1}{2}D^2 + Q \quad (Q>0). \end{aligned} \tag{19}$$

$P[\mathrm{erg/cm^3}]$ is the pressure, $\rho[\mathrm{g/cm^3}]$ is the density, $E_T[\mathrm{erg/g}]$ is the thermal energy, $Q[\mathrm{erg/g}]$ is the nuclear fusion energy deposited on the wave front, u is the velocity of motion of the fluid [cm/s], and D [cm/s] is the detonation wave velocity.

The detonation wave is steadily propagating with velocity D, namely all magnitudes P, ρ, u, and E_T are functions of time t and space x only in the form $x-Dt$. By using CJ formalism for the ideal gas case one can obtain from the conservation Eq. (19) some useful relations (Browne *et al.*, 2008):

$$\begin{aligned} \left(\frac{\rho}{\rho_0}\right)_{CJ} &= \frac{\Gamma+1}{\Gamma}; P_{CJ} = \frac{\rho_0 D^2}{\Gamma+1}, \\ \frac{u}{D} &= \frac{1}{\Gamma+1}; \frac{c_s}{D} = \frac{\Gamma}{\Gamma+1}, \\ \frac{Q}{D^2} &= \frac{(\Gamma-1)}{2(\Gamma+1)^2}, \end{aligned} \tag{20}$$

where c_s is the adiabatic sound velocity and $\Gamma = C_P/C_V$ is the ratio of heat capacity at constant pressure to heat capacity at constant volume. For the detonation wave, we take $\Gamma = 3$ (Landau & Stanyukovich, 1945) implying

$$\frac{u}{D} = \frac{1}{4}; \frac{c_s}{D} = \frac{3}{4}; \frac{Q}{D^2} = \frac{1}{16}. \tag{21}$$

We can see that by determining the particle velocity $u = u_p$ to equal 1.0% the speed of light we have determined the nuclear energy needed to support a steady state CJ condition, $Q = 9 \cdot 10^{12}$ J/kg. As one can see from Figure 4, this value of Q is achieved by our laser-induced detonator.

It is interesting to point out that in the detonation wave for $u/c = 0.01$ we have $D/c = 0.04$, while the α released in the DT fusion with an energy of 3.5 MeV has a velocity of $(u + v_\alpha)/c = 0.052$. This implies that some of the α-particles will reach the detonation front.

4. CONCLUSION

In this paper, we have shown that a detonation wave can be derived by a laser-induced detonator. The energy in this detonation is supplied by the α-particles created in the fusion of

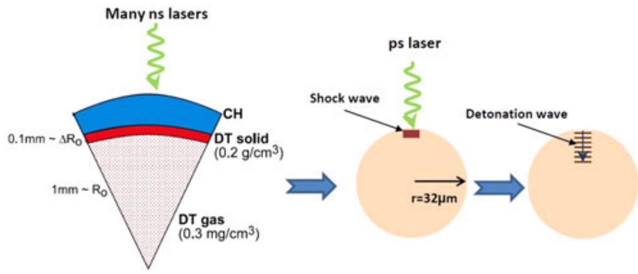

Fig. 5. A schematic view of the detonation trajectory in a pre-compressed pellet. As a numerical example an initial pellet with radius $R_0 = 1$ mm and DT fuel of density 0.2 g/cm^3 with thickness 0.1 mm (i.e., an aspect ratio of 10) is compressed to a density of $\rho_0 = 1800$ g/cm^3 by the nanosecond lasers.

DT. The fusion ignition is triggered by a laser-induced shock wave in a pre-compressed DT fuel that is able to supply the desired steady-state detonation criteria. The shock wave parameters in the detonator are in the intermediate domain between relativistic and non-relativistic hydrodynamics (see the Appendix).

The laser energy is $W_L = I_L \tau_L S_L$, where I_L, τ_L, and S_L are the laser irradiance, the pulse time duration and cross-section accordingly. The laser cross-section $S_L = \pi R_L^2$ is chosen in such a way that the 1D laser-induced shock wave is conceivable. In particular, we take $R_L = 1.5(u_s–u_p)\tau_L$. In our domain of interest, the shock wave is between the relativistic and non-relativistic hydrodynamics, where Eq. (A13) determine the shock velocity u_s and particle velocity u_p as a function of laser irradiance I_L, the initial target density ρ_0, and the EOS parameter Γ (see the Appendix).

The laser parameters creating the ignitor depend strongly on the EOS. We consider an ideal gas with $\Gamma = 5/3$ and 3. The simulation shows that for the case (a) $\Gamma = 3$ and a pre-compressed target at 1800 g/cm^3 (equal to an ion density of $n_i = 4.3 \times 10^{26}$ cm^{-3}) a laser irradiance $I_L = 5.25 \times 10^{23}$ W/cm^2 and energy 149 kJ during 0.75 ps creates an α heating fusion detonation wave; while (b) for $\Gamma = 5/3$ and a pre-compressed target 900 g/cm^3, the detonation wave requires a laser irradiance $I_L = 1.75 \times 10^{23}$ W/cm^2 and energy 13.1 kJ during 1 ps.

Although the detonator EOS may be described by $\Gamma = 5/3$ or 3 the detonation wave has $\Gamma = 3$ following the original idea of Landau (Landau & Stanyukovich, 1945). The laser-induced ignitor should supply the steady state CJ conditions as described by Eq. (21). In particular, we get that the α-particles created in the DT fusion by the end of the ignitor produce a power flux of $W_{CJ} = Q\rho_0 u = 4.86 \times 10^{25}$ W/m^2, where $Q = 9 \times 10^{12}$ J/kg.

The nuclear fusion burn is described by Figure 2. We achieve a detonation wave when the numbers of DT reactions are between 15% and 25% with an α density number of about 10^{26} cm^{-3}.

To conclude we suggest that the detonation wave created by the laser-induced shock wave can ignite a pre-compressed DT pellet. The required ignition criterion of DT ignition and burn are easily satisfied (Atzeni & Meyer-Ter-Vehn, 2004; Eliezer *et al.*, 2015) along the detonation wave trajectory.

In Figure 5, we show a schematic view of the detonation trajectory in a pre-compressed pellet. As a numerical example an initial pellet with radius $R_0 = 1$ mm and DT fuel of density 0.2 g/cm^3 with thickness 0.1 mm (i.e., an aspect ratio of 10) is compressed to a density of $\rho_0 = 900$ g/cm^3 by the nanosecond lasers. The novel aspect of this ignition is the possibility to ignite the compressed pellet from inside and not from the surface as usually suggested. The compression of a typical pellet as discussed in the literature (Eliezer *et al.*, 2007) requires between 100 and 300 kJ of energy depending on the EOS, target design, and the final required density. The fast ignition in our case needs about 13 kJ of energy. Such a laser is under development and may be available in the near future (ELI, 2015).

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APPENDIX

BETWEEN RELATIVISTIC AND NON-RELATIVISTIC SHOCK WAVES

In this appendix, we discuss the intermediate domain between relativistic and non-relativistic shock waves.

The three *relativistic shock wave* conservation laws (mass, momentum, and energy) in the laboratory frame of reference yield the following relations (Eliezer *et al*., 2014*b*):

$$\frac{u_s}{c} = \sqrt{\frac{(P_1 - P_0)(e_1 + P_0)}{(e_1 - e_0)(e_0 + P_1)}},$$
$$\frac{u_p}{c} = \sqrt{\frac{(P_1 - P_0)(e_1 - e_0)}{(e_0 + P_1)(e_1 + P_0)}},$$
$$\frac{(e_1 + P_1)^2}{{\rho_1}^2} - \frac{(e_0 + P_0)^2}{{\rho_0}^2} = (P_1 - P_0)\left[\frac{(e_0 + P_0)}{{\rho_0}^2} + \frac{(e_1 + P_1)}{{\rho_1}^2}\right], \tag{A1}$$

where P, e, and ρ are the pressure, energy density, and mass density accordingly, the subscripts 0 and 1 denote the domains before and after the shock arrival, u_s is the shock wave velocity, u_p is the particle flow velocity in the laboratory frame of reference, and c is the speed of light. We have assumed that in the laboratory the target is initially at rest, $u_{p0} = 0$. The last of Eq. (A1) is known as the Hugoniot equation. The EOS taken here in order to calculate the shock wave parameters is the ideal gas EOS

$$e_j = \rho_j c^2 + \frac{P_j}{\Gamma - 1}; \quad j = 0, 1, \tag{A2}$$

where Γ is the specific heat ratio. We have to solve Eqs. (A1) and (A2) together with our piston model equation (Esirkepov *et al*., 2004; Eliezer *et al*., 2014*b*)

$$P_1 = \frac{2I_L}{c}\left(\frac{1 - \beta}{1 + \beta}\right); \quad \beta \equiv \frac{u_p}{c}. \tag{A3}$$

The calculations are conveniently done in the dimensionless units defined by

$$\Pi_L \equiv \frac{I_L}{\rho_0 c^3}; \; \kappa \equiv \frac{\rho_1}{\rho_0}; \; \kappa_0 \equiv \frac{\Gamma + 1}{\Gamma - 1}; \; \Pi = \frac{P_1}{\rho_0 c^2}; \; \Pi_0 = \frac{P_0}{\rho_0 c^2}. \tag{A4}$$

It is important to emphasize that if we take $P_0 = 0$, then we get only the $\kappa > 4$ solutions (Eliezer *et al*., 2014*a*); therefore, in order to see the behavior at the transition between relativistic and nonrelativistic domains one has to take $P_0 \neq 0$! In this case, we get from Eqs. (A1) and (A2) the following equation relating dimensionless shock wave pressure Π with the compression κ, namely the relativistic Hugoniot equation for an ideal gas EOS is

$$\begin{cases} \Pi^2 + B\Pi + C = 0, \\ \kappa \equiv \dfrac{\rho_1}{\rho_0} \geq 1, \end{cases}$$
$$\Pi = \left(\frac{1}{2}\right)(-B \pm \sqrt{B^2 - 4C}),$$
$$B = \frac{(\Gamma - 1)^2}{\Gamma}(\kappa_0\kappa - \kappa^2) + \Pi_0(\Gamma - 1)(1 - \kappa^2), \tag{A5}$$
$$C = \frac{(\Gamma - 1)^2}{\Gamma}(\kappa - \kappa_0\kappa^2)\Pi_0 - \kappa^2{\Pi_0}^2.$$

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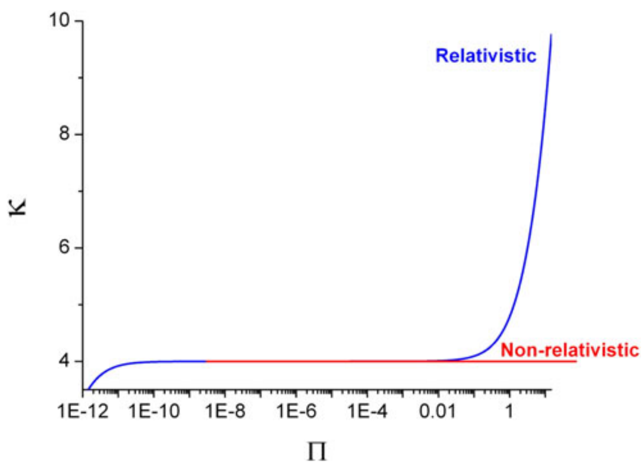

Fig. 6. The shock wave compression $\kappa = \rho/\rho_0$ as a function of the dimensionless shock wave pressure $\Pi = P/\rho_0 c^2$ for $\Gamma = 5/3$.

The solution of this equation is shown in Figure 6 for $\Gamma = 5/3$. As one can see from Figure 6, the compression $\kappa = \rho/\rho_0$ is constant in a large domain of the dimensionless pressure $\Pi = P/(\rho_0 c^2)$, where P is the shock wave pressure. Therefore, for the intermediate relativistic shock wave, we have to a good (as desired) approximation the following relation:

$$\kappa \equiv \frac{\rho_1}{\rho_0} = \kappa_0 = \frac{\Gamma + 1}{\Gamma - 1}. \tag{A6}$$

The *non-relativistic* shock wave equations are obtained from the relativistic Eq. (A1) using $e = \rho c^2 + \rho E$, P, and ρE are much smaller than ρc^2 and $u/c \ll 1$, where u stands for the velocities under consideration. The appropriate non-relativistic equations describing the shock wave in the laboratory frame of reference are (Zeldovich & Raizer, 1966).

$$\begin{aligned}
&(i)\ \ u_p = (P_1 - P_0)^{1/2}(1/\rho_0 - 1/\rho_1)^{1/2},\\
&(ii)\ \ u_s = (1/\rho_0)(P_1 - P_0)^{1/2}(1/\rho_0 - 1/\rho_1)^{-1/2},\\
&(iii)\ E_1 - E_0 = \left(\frac{1}{2}\right)(P_1 + P_0)(1/\rho_0 - 1/\rho_1),\\
&\left.\begin{matrix}(iv)\\(v)\end{matrix}\right\} E_j = \left(\frac{1}{\Gamma - 1}\right)\left(\frac{P_j}{\rho_j}\right),\ \text{for } j = 0, 1.
\end{aligned} \tag{A7}$$

The relativistic Hugoniot equation for an ideal gas EOS is given by Eq. (A5), while the non-relativistic Hugoniot equation for an ideal gas EOS is [from (A7)]

$$\Pi = \left(\frac{\kappa\kappa_0 - 1}{\kappa_0 - \kappa}\right)\Pi_0. \tag{A8}$$

Figure 6 describes the transition between the relativistic and non-relativistic Hugoniot, namely the transition between Eqs. (A5) and (A8). In this transition domain, between relativistic and non-relativistic shock waves, we have

$$10^{-9} \le \Pi \le 10^{-3} \Leftrightarrow \kappa = \frac{\rho}{\rho_0} = 4.00\ (\text{for } \Gamma = 5/3). \tag{A9}$$

In the domain defined by Eq. (A6) (yielding (A9) for $\Gamma = 5/3$), we can use the first two equations of (A7) for $u_p/c < 0.03$ in order to get

$$\begin{aligned}
\frac{u_p}{c} &= \sqrt{\frac{2\Pi}{\Gamma + 1}};\\
\frac{u_s}{c} &= \sqrt{\frac{(\Gamma + 1)\Pi}{2}}.
\end{aligned} \tag{A10}$$

Using now our piston model Eq. (A3) together with Eq. (A10), we obtain

$$\beta = \frac{u_p}{c} = \frac{-\Pi_L + \sqrt{\left(\frac{\Gamma + 1}{4}\right)\Pi_L - {\Pi_L}^2}}{\left(\frac{\Gamma + 1}{4}\right) - 2\Pi_L}, \tag{A11}$$

$$\Pi = 2\Pi_L\left[\frac{\left(\frac{\Gamma + 1}{4}\right) - \Pi_L - \sqrt{\left(\frac{\Gamma + 1}{4}\right)\Pi_L - {\Pi_L}^2}}{\left(\frac{\Gamma + 1}{4}\right) - 3\Pi_L + \sqrt{\left(\frac{\Gamma + 1}{4}\right)\Pi_L - {\Pi_L}^2}}\right]. \tag{A12}$$

In Figures 7 and 8, we describe appropriately the dimensionless particle shock velocity $\beta = u_p/c$ and the dimensionless pressure $\Pi = P/(\rho_0 c^2)$ as a function of the dimensionless laser intensity $\Pi_L = I_L/(\rho_0 c^3)$ for the two cases $\Gamma = 3$ and $5/3$. An identical graph for Π as a function of Π_L is derived for $\Gamma = 5/3$, since the Γ dependence in this domain is negligible.

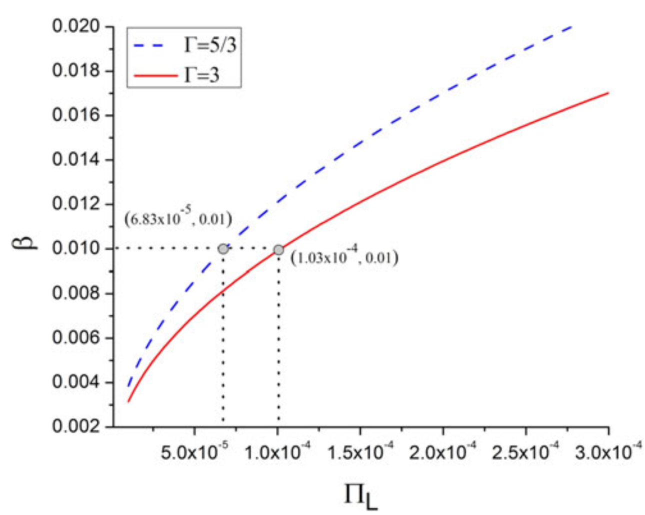

Fig. 7. The dimensionless particle shock velocity $\beta = u_p/c$ as a function of the dimensionless laser intensity $\Pi_L = I_L/(\rho_0 c^3)$ for the two cases $\Gamma = 3$ and $\Gamma = 5/3$.

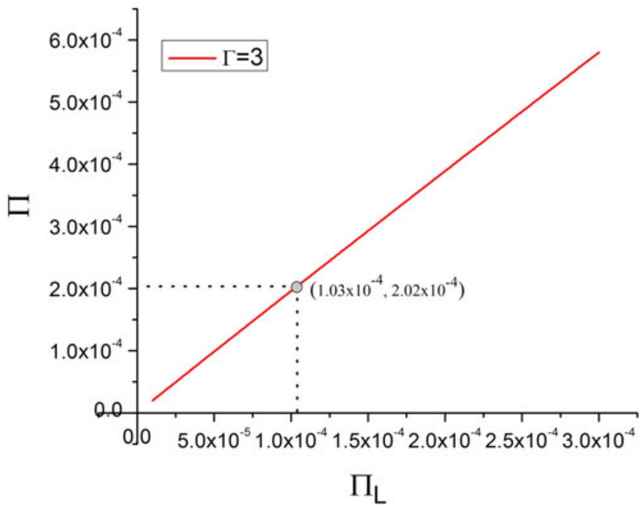

Fig. 8. The dimensionless shock wave pressure $\Pi = P/(\rho_o c^2)$ as a function of the dimensionless laser intensity $\Pi_L = I_L/(\rho_0 c^3)$ for $\Gamma = 3$. An identical graph is derived for $\Gamma = 5/3$, since the Γ dependence in this domain is negligible.

Since $\Pi_L < 10^{-2}$, we have to a good approximation in the domain between relativistic and non-relativistic shock waves the following approximations:

$$\begin{aligned} \Pi &\approx 2\Pi_L, \\ \frac{u_p}{c} &\approx 2\sqrt{\frac{\Pi_L}{\Gamma+1}} = 2\sqrt{\frac{I_L}{(\Gamma+1)\rho_0 c^3}}, \\ \frac{u_s}{c} &\approx \sqrt{(\Gamma+1)\Pi_L} = \sqrt{\frac{I_L(\Gamma+1)}{\rho_0 c^3}}. \end{aligned} \tag{A13}$$

Under this approximation, the shock wave length l_s by the end of the laser pulse is

$$l_s = (u_s - u_p)\tau_L = \left(\frac{\Gamma - 1}{(\Gamma+1)^{1/2}}\right)\sqrt{\frac{I_L {\tau_L}^2}{\rho_0 c}}. \tag{A14}$$

The laser cross-section $S_L = \pi R_L^2$ is chosen $R_L = 1.5(u_s–u_p)\tau_L$ in order that the 1D laser-induced shock wave is conceivable. Therefore for a constant laser irradiation I_L, we need a laser energy W_L given by

$$W_L = I_L S_L \tau_L = 2.25\pi\left[\frac{(\Gamma-1)^2}{\Gamma+1}\right]\left(\frac{{I_L}^2 {\tau_L}^3}{\rho_0 c}\right). \tag{A15}$$

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