Design of composite helicopter rotor blades to meet given cross-sectional properties

S. L. Lemanski, P. M. Weaver, G. F. J. Hill
Department of Aerospace Engineering
University of Bristol
Bristol, UK

ABSTRACT

This paper examines the design of a composite helicopter rotor blade to meet given cross-sectional properties. As with many real-world problems, the choice of objective and design variables can lead to a problem with a non-linear and/or non-convex objective function, which would require the use of stochastic optimisation methods to find an optimum. Since the objective function is evaluated from the results of a finite element analysis of the cross section, the computational expense of using stochastic methods would be prohibitive. It is shown that by choosing appropriate simplified design variables, the problem becomes convex with respect to those design variables. This allows deterministic optimisation methods to be used, which is considerably more computationally efficient than stochastic methods. It is also shown that the design variables can be chosen such that the response of each individual cross-sectional property can be closely modelled by a linear approximation, even though the response of a single objective function to many design parameters is non-linear. The design problem may therefore be reformulated into a number of simultaneous linear equations that are easily solved by matrix methods, thus allowing an optimum to be located with the minimum number of computationally expensive finite element analyses.

NOMENCLATURE

$X[i]$	general design variable
$\underline{\mathbf{X}}$	general vector of design variables
$Y[i]$	general cross-sectional target property
$\underline{\mathbf{Y}}$	general vector of cross-sectional target properties
a_{ij}	coefficient of linearity between design variable and cross-sectional property
A_{ij}	normalised coefficient of linearity between design variable and cross-sectional property
$[\mathbf{A}]$	coefficients matrix
EI_{XX}	flap-bending stiffness of section
EI_{YY}	lag-bending stiffness of section
mass	mass per unit depth of cross-section
CG	chordwise location of centre of gravity

1.0 BACKGROUND

A helicopter blade is designed to meet constraints on both inertia and stiffness properties. While the mass and chordwise location of centre of gravity (CG) of the section are important to ensure adequate blade stability, the blade stiffness properties are designed to meet target values. Geometrically, a helicopter blade is a long, slender structure and is routinely idealised as a 1D beam. From a design perspective, it is important not only that the structural properties of the cross-section can be accurately determined (usually from a finite element model), but that these properties can also be tailored to achieve desirable characteristics of the structure being designed.

In order to illustrate the scope of current work, this paper examines the design of a typical helicopter rotor blade to meet the objective that the cross-section meets given values of mass, chordwise location of centre of gravity (CG), and flap- and lag-bending stiffnesses (EI_{XX} and EI_{YY}).

The use of composite materials allows the structural designer new degrees of freedom with which to tailor the structural properties of a design. This is a good feature in itself, but it also increases the dimensionality of the design space – a negative trait from an optimisation viewpoint. Design optimisation is further complicated by the fact that a number of design variables are discrete – typically due to manufacturing considerations. Examples include ply thickness (typically 1/8th-mm increments), and ply orientation (typically 45° increments).

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Given existing computing power, it is not feasible to search the entire design space for a complex design of helicopter rotor blade. Even a simple three variable problem takes two days to run on current computing platforms (Pentium P4 1·7GHz). Problems with more variables would require the design space to be reduced in order to make the search feasible. As such, many studies 1-13 have been directed towards the optimisation of laminated composite aerospace structures. Some of these (e.g. Kameyama *et al*(1), Chattopadhyay *et al*(2) use simplified geometrical models such as modelling the wings as flat plates, or helicopter blades as a rectangular torsion boxes. Although these give important physical insight and useful results for preliminary design, the modelling simplifications made in the analysis leave the accuracy of the 'optimum' design open to question. Indeed, Chattopadhyay *et al*(2) conclude that "the results obtained must be viewed within the context of the modelling assumptions used in the analysis." While such work is of some use to the industry, further design tools are required to produce a finished detail design ready for manufacture. This work addresses that industrial need.

Finite element analysis is an established design tool in the aerospace industry, and the use of rigorous optimisation techniques is gradually becoming more widespread. An optimisation method is therefore required that is capable of interfacing with commercially available analysis tools, thereby allowing the design to be optimised at whatever level of detail is necessary.

Although this study examines the design of a simplified helicopter blade, the finite element approach adopted may be applied directly to more complex prismatic sections. Furthermore, the lessons learned from this problem and the methods developed to solve it are also applicable to more complex structural design problems.

2.0 PROBLEM FORMULATION

The problem formulated is to design a composite helicopter rotor blade (shown in Fig. 1) to meet predetermined target values of four cross sectional properties.

The main features of this generic blade design that will be considered in optimisations studies are highlighted in Fig. 1 and are

1. nose weight
2. rear wedge
3. eight-ply composite spar wall
4. eight-ply composite blade wall
5. foam filler
6. glass fibre surface layer

The parameters to be varied (i.e. the design variables) are

$X[0]$: chordwise location of end of nose weight
$X[1]$: chordwise location of start of rear wedge
$X[2]$: thickness of each eight-ply composite

The cross-sectional properties (i.e. the target variables) are chosen as

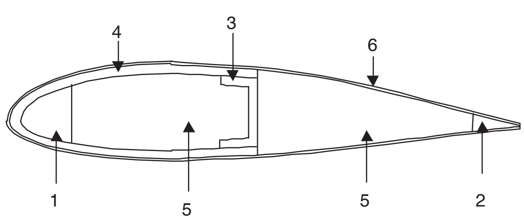

Figure 1. Generic helicopter blade design.

$Y[0]_t$: EI_{XX} $= 2{\cdot}72 \times 10^8 \text{Nm}^2$ . . . (1a)

$Y[1]_t$: EI_{YY} $= 4{\cdot}79 \times 10^9 \text{Nm}^2$ . . . (1b)

$Y[2]_t$: *mass* $= 0{\cdot}397 \text{kg/m}$ . . . (1c)

$Y[3]_t$: *CG* $= 23{\cdot}0$ % chord . . . (1d)

These target variables were chosen as they correspond to the cross sectional properties of an existing section – i.e. it is known that the solution lies within the feasible design space.

The behaviour of the structure has been assessed using the approach of Hill and Weaver(14) which is an extension of the approach presented by Bartholomew and Mercer(15). This approach uses finite element analysis of a three-dimensional slice of the cross-section of any prismatic beam with any number of materials to produce equivalent 1D beam properties, i.e. a cross-sectional stiffness matrix. It achieves this by linking the two faces of the slice model with multi-point constraint equations, which allow relative motion of the faces according to linear bending, axial displacement and torsion through 'scalar freedoms'. The individual components of the stiffness matrix are found by taking the displacements of these scalar freedoms for the four load cases of axial tension, torsion and the two bending cases. The 3D slice itself is free to deform (warp) in the plane of the section and also out-of-plane, if necessary.

It is common practice to formulate a single function that gives an objective measure of design performance. In this problem, the fitness of the design is determined by how closely the target values of cross sectional stiffness are met. The following objective function was formulated for this problem:

$$f(\underline{X}) = \left(\frac{EI_{XX\,c} - EI_{XX\,t}}{EI_{XX\,t}}\right)^2 + \left(\frac{EI_{YY\,c} - EI_{YY\,t}}{EI_{YY\,t}}\right)^2 + \left(\frac{mass_c - mass_t}{mass_t}\right)^2 + \left(\frac{CG_c - CG_t}{CG_t}\right)^2 \quad \text{. . . (2a)}$$

which may be represented in the general case as

$$f(\underline{X}) = \sum_i \left(\frac{Y[i]_c - Y[i]_t}{Y[i]_t}\right)^2 \quad \text{. . . (2b)}$$

where subscripts c and t refer to current and target values of the stiffness, respectively. It is noted that the objective function is always positive, but reduces to zero if/when all of the target values are met exactly. Each of the terms in the objective function are normalised with respect to the relevant target value, thus giving each term equal weighting.

Although the cross sectional properties of the blade cannot be accurately determined from a simple analytical model, it is relatively straightforward to obtain these results from a finite element analysis of the cross section using the method of Hill and Weaver. Many real world problems, are complicated by the fact that several design variables are discretised (e.g. ply thicknesses). It may not be possible to select the exact ply thicknesses that give the desired laminates properties, but instead must round the ply thicknesses to the nearest ply thickness (typically 0·125mm). Although this is not usually a problem for large structures with wall thicknesses of several millimetres, this may be significant for smaller structures such as that studied in this problem.

Although it is not an efficient method of approaching an optimisation problem, an exhaustive search of the solutions for different designs is first undertaken. This study will locate the discrete point that most closely matches the required solution, and the results are enlightening for, once the design space is mapped out, the nature of the design space is known and the most appropriate optimisation strategy may be chosen.

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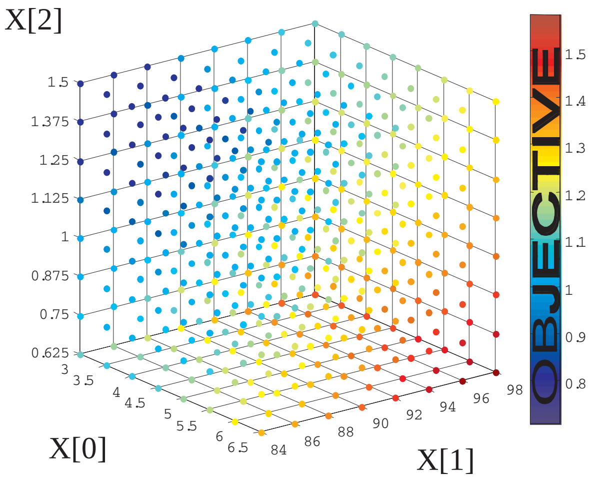

Figure 2. Variation of objective function over 3D design space.

3.0 INVESTIGATION OF FEASIBLE DESIGN SPACE

In order to gain a full understanding of the exact nature of the objective function, the design space was discretised and searched. Due to the simple (three-variable) nature of this problem, a sufficiently high-resolution search was obtained by discretising the entire design space into 512 design points. This design space required two days to exhaustively search.

Figure 2 illustrates the variation of objective function (plotted in colour) at discrete points over the three-dimensional design space. The visualisation of an objective function within a design space that has more than three dimensions is neither intuitive, nor easy to represent in a concise pictorial format. However, despite the obvious difficulties of representation, these results have significant consequences for the optimisation of helicopter blades.

Although convexity is demonstrated mathematically if the Hessian matrix is positive semi-definite, the variation of colour (i.e. value of objective function) across the design space in Fig. 2 does suggest convexity of the underlying continuous problem. Prior studies on composite cylindrical shells indicate that this design space is convex because the design variables in this problem are wall locations and ply thicknesses. It is noted that the inclusion of ply orientations or stacking sequences as design variables would lead to a non-convex objective function. If this were the case, the problem would either have to be solved using stochastic methods, or simplified by the use of lamination parameters(17-19). For the sake of simplicity, stacking sequences are ignored but their relative effects will be discussed later.

Despite the limitations imposed by discretisation, it is possible to meet the target values to a mean error of less than 1% at the optimum point. The optimum design values were found to be

$X[0] = 4{\cdot}0$mm
$X[1] = 98$mm
$X[2] = 1{\cdot}375$mm

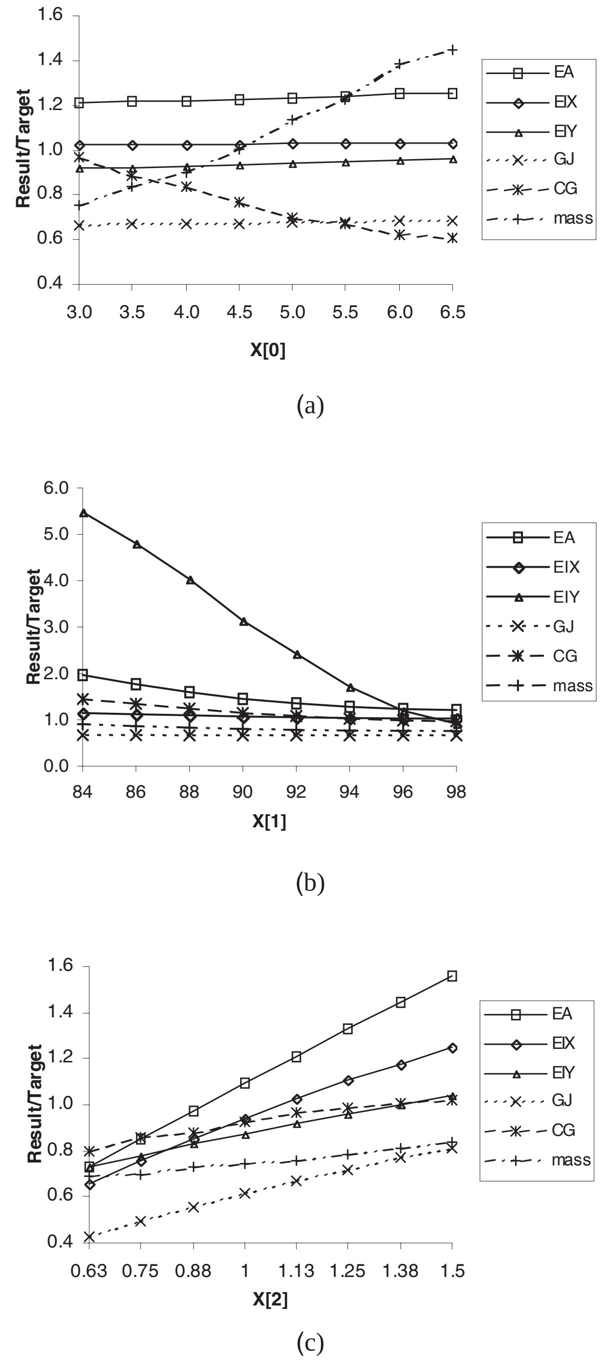

Figure 3: Variation of cross-sectional properties with design variables.

4.0 EFFICIENT SOLUTION METHODS

With the knowledge that the design space is convex, it is possible to apply sequential linear programming (SLP) techniques to solve the problem. A typical approach may use the current objective function and apply a steepest descent method with appropriate move limits – a method that reliably finds the optimum of a convex problem. Since the objective function for this problem is highly non-linear, the analysis will require several iterations – each requiring the gradients of the objective function to be evaluated via a computationally expensive finite-difference method. It is therefore desirable to minimise the number of times that such gradients are calculated. An even more efficient method linearises the variation of the target variables (EI_{XX}, EI_{YY} and CG) about an initial design point. Figures 3(a)-3(c) show the values of several cross-sectional properties (normalised to the target values) plotted against each design

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variable. The significant result is that each property varies almost linearly with the design variable over almost the entire design space. Note that only the target variables EI_X, EI_Y, *mass* and *CG* are considered in this problem, however the graphs show that other variables (*EA* and *GJ*) also vary linearly.

As the problem is essentially linear, the problem can be expressed as *N* simultaneous linear equations, where *N* is the number of target variables,

$$EI_{XXnew} = EI_{XXc} + a_{11}\Delta X[0] + a_{12}\Delta X[1] + a_{13}\Delta X[2] \quad \text{. . . (3a)}$$

$$EI_{YYnew} = EI_{YYc} + a_{21}\Delta X[0] + a_{22}\Delta X[1] + a_{23}\Delta X[2] \quad \text{. . . (3b)}$$

$$mass_{new} = mass_{c} + a_{31}\Delta X[0] + a_{32}\Delta X[1] + a_{33}\Delta X[2] \quad \text{. . . (3c)}$$

$$GC_{new} = GC_{c} + a_{41}\Delta X[0] + a_{42}\Delta X[1] + a_{43}\Delta X[2] \quad \text{. . . (3d)}$$

Since the desired new values of the target variables are the target values, so it is straightforward to solve for $\Delta X[i]$. Since the linearisation is valid across the entire design space, the change in each design variable X[*i*] (i.e. $\Delta X[i]$) does not have to be small.

These simultaneous linear equations can be conveniently represented in generalised matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} \begin{Bmatrix} \Delta X[0] \\ \Delta X[1] \\ \Delta X[2] \end{Bmatrix} = \begin{Bmatrix} Y[0]_t - Y[0]_c \\ Y[1]_t - Y[1]_c \\ Y[2]_t - Y[2]_c \\ Y[3]_t - Y[3]_c \end{Bmatrix} \quad \text{. . . (4a)}$$

which may be normalised as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ A_{41} & A_{42} & A_{43} \end{bmatrix} \begin{Bmatrix} \Delta X[0] \\ \Delta X[1] \\ \Delta X[2] \end{Bmatrix} = \begin{Bmatrix} 1-\left(Y[0]_c / Y[0]_t\right) \\ 1-\left(Y[1]_c / Y[1]_t\right) \\ 1-\left(Y[2]_c / Y[2]_t\right) \\ 1-\left(Y[3]_c / Y[3]_t\right) \end{Bmatrix} \quad \text{. . . (4b)}$$

which may be abbreviated to

$$[\mathbf{A}]\,\Delta\underline{\mathbf{X}} = \underline{\mathbf{b}} \quad \text{. . . (4c)}$$

Note that the magnitude of vector $\underline{\mathbf{b}}$ in Equation (4c) is representative of the normalised distance between the current design point and the target design point. This is equivalent to the square root of the objective function defined in Equation (2). When the magnitude of $\underline{b}$ becomes zero, the target values have all been met, and the optimal point has been found. Because the problem is not completely linear, the process must be iterated until it converges upon a solution.

There are three types of problem that arise depending on the number of target values for cross-sectional properties and the number of variables to be used in the optimisation problem. Each scenario is examined in turn.

Case 1. The number of design variables is less than the number of target variables

This three-design-variable problem requires that four target values (EI_{XX}, EI_{YY}, *mass* and *CG*) are satisfied. Although in this case, the chosen target values of cross-sectional property corresponded to an actual design, in general it is not necessarily possible to obtain an exact solution in problems where the number of target values exceeds the number of design variables. However, it is possible to obtain a least squared error solution by the pseudo-inverse method – i.e. by solving

$$[\mathbf{A}]^{\mathbf{T}}[\mathbf{A}]\Delta\underline{\mathbf{X}} = [\mathbf{A}]^{\mathbf{T}}\,\underline{\mathbf{b}} \quad \text{. . . (5a)}$$

which gives $\underline{\mathbf{X}}$ as

$$\Delta\underline{\mathbf{X}} = ([\mathbf{A}]^{\mathbf{T}}[\mathbf{A}])^{-1}[\mathbf{A}]^{\mathbf{T}}\,\underline{\mathbf{b}} \quad \text{. . . (5b)}$$

thus minimising the magnitude of $\underline{b}$ (and also the value of the objective function).

Case 2. The number of design variables is equal to the number of target variables

If the problem considered here had only required that EI_X, EI_Y and *CG* values are matched, the number of target values is equal to the number of design variables. In this case there is one unique optimum solution, which may be found directly from

$$\Delta\underline{\mathbf{X}} = [\mathbf{A}]^{-1}\,\underline{\mathbf{b}} \quad \text{. . . (6)}$$

Case 3. The number of design variables is greater than the number of target variables

In this case, there is a range of solutions. This is analogous to redundancy in a structure. The appropriate number of design variables may therefore be fixed, until the remaining number of design variables is equal to the number of target variables. A unique solution may then be obtained from Equation (6) above.

In the Discussion section relative advantages and disadvantages of the proposed method are presented.

5.0 DISCUSSION

The optimisation method presented works well because the design space is well-understood. Indeed, the fact that optimal solutions are relatively close in some sense to the initial values, as borne out by the linear perturbations of optimal variables from their starting values, is indicative of a good initial design. This is a reflection on the mature state of blade design for current configurations. The great merit of the current approach is that optimisation of blade sections becomes very quick and routine such that optimisation of an entire blade may take place by a succession of two-dimensional optimisation analyses along the blade. The fact that blades are mostly prismatic in nature implies geometry and material do not vary much along the blade length giving support this approach. Despite the obvious advantages of the proposed method it does have disadvantages and these are discussed in the following two sections.

5.1 The effect of discretisation

The analysis presented here does not consider the effect of discretisation. This problem is additionally constrained by the discretisation of individual ply thicknesses to 0·125-mm increments.

For this problem, the simplest solution is to evaluate the discrete designs either side of the continuous optimum and re-optimise any remaining continuous variables for each point. Each solution can be obtained analytically by solving the matrix equation above, using the same values of A_{ij} and searching through all the appropriate discretised combinations of ΔX[*i*]. Depending on the accuracy of the linearisation and the degree of discretisation, the calculated discrete optimum may then be verified using finite element analysis.

In the general case for convex problems (i.e. with more than just one discretised variable) the discretised solution will be one of the design points that immediately surround the continuous solution in *M*-dimensional design space, where *M* is the number of discretised variables.

This will give 2^M possible discretised solutions. For each discretised solution, optimisation must be carried out with the remaining continuous design variables. The best of these optimised discrete solutions will be the global discretised optimum.

5.2 Effect of ignoring stacking sequences

The effect of ignoring stacking sequences significantly simplifies the problem from one that is non-convex to one that is. Neglecting such effects implies that in-plane stiffness effects of laminates greatly outweigh flexural stiffness contributions. This is a feature of thin-walled closed section structures, in which buckling and vibration effects are discounted, and as such, is a feature of efficient structures. Flexural stiffness effects are expected to become significant for ratios of $t / r < 10$ where t represents a wall thickness and r is the smallest outer dimension of the section. For a blade r is half the section depth. However, it is noted that not all rotor blades are thin-walled. Under these circumstances it is recommended that the current approach is adopted to find proportions of plies with specified fibre angles from which a subsequent optimisation analysis taking into account stacking sequences may be effected. A possible means of achieving this is via use of lamination parameters. Finally, it is re-emphasised that for thin-walled closed section structures stacking sequences are expected to have minimal effects.

6.0 CONCLUSIONS

It is well known that while stochastic methods are useful for locating several near optimal points in a non-convex design space, they are relatively poor at locating a single best point. Conversely, deterministic methods efficiently locate an optimum point, but do not guarantee global optimum for non-convex problems. Since the design space of the helicopter rotor blade problem is convex, the optimum point can be located using computationally efficient, deterministic methods. By linearising the target variables in terms of the design variables about a given design point, the problem is solved by matrix methods.

It is a straightforward procedure to program this method and interface with finite element analysis packages to reliably and efficiently design complex closed-section composite structures to meet given structural properties.

In order to ensure that the design space is convex, lay-up and stacking sequence have been fixed. Existing composite helicopter blades are not always thin-walled, so these design freedoms could affect the design envelope significantly. However, it is noted that designs found by the proposed method may be used in a subsequent optimisation analysis using stacking sequence as a variable.

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