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MATHEMATICAL MODELING: THE CASE OF EMERGENCY DEPARTMENT WAITING TIMES

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A decision analytic model often comprises a significant part of a health technology assessment. As health technology assessment in the hospital setting evolves, there is an increased need for modeling methods that account for patient care pathways and interactions between patients and their environment. For example, an evaluation of a computed tomography (CT) scanner for a new indication would need to consider the current and increased demand of the machine and how that may affect service in other areas of the hospital. This problem solving approach views "problems" through a systems perspective.

Systems analysis techniques have been developed over decades through operations research and industrial engineering fields (19). Under systems analysis, mathematical modeling techniques involve mapping a system or process from the real world to a more simplified representation using a set of variables and equations. These models have been identified for use in health technology assessment (44), mainly because they allow decision makers to simulate hypothetical scenarios without making actual changes to the system. Such models enable the analysis of "what-if scenarios" and provide the opportunity to identify optimized solutions under constraints (e.g., resources, budget, benchmarks). Measures of systems behavior include waiting time, throughput, and resource utilization. Waiting times can be of particular interest due to adverse events and current pressure from the public to receive timely care.

The hospital emergency department (ED) is of particular importance because it is a dominant source of acute care and the main route of admission to the hospital for a large percentage of the population. Long waiting times lead to overcrowding and have been a widely documented problem in EDs (5). Overcrowding has been associated with increased risk in mortality and re-admission, higher probability of leaving without being seen, and delayed or non-receipt of antibiotics for patients with community-acquired pneumonia (3;18;39). As such, identifying causes of overcrowding is an essential step to improving safety and outcomes.

ED patient care is complex and relies on several human, physical, and organizational elements (e.g., patients and their relatives, buildings and equipment, management systems). At its most basic level, it is a system consisting of patients, resources (e.g., beds, physicians) and processes (e.g., triage). Generally, patients flow through the following order of processes: triage, registration, placement in an ED bed, clinical assessment, treatment, and/or diagnostics/laboratories followed by disposition. Waiting times in the ED exist for several reasons: capacity does not meet demand (e.g., overcrowding, insufficient number of beds), sub-optimal management of capacity or demand (e.g., scheduling, flow), significant variability over time in demand for services, and differences in patient acuity (23). The complexity of care within the ED compounded with the multifaceted issues associated with excessive waiting times lends itself well to systems analysis. Although several comprehensive reviews have outlined the application of mathematical modeling in health care (16;23;37), these reviews were not specific to the ED setting. Despite the increased pressure to reduce waiting times, there are no recent reviews analyzing the use of mathematical models for evaluating the ED. To better inform future HTA and decision making in the hospital setting, the purpose of this study was to evaluate the literature from the perspective of both the methods used (i.e., modeling techniques) and the empirical findings (i.e., study results) perspective. The specific objectives were (i) to identify recent mathematical modeling techniques that have been used to evaluate strategies for decreasing waiting times in the hospital emergency department; (ii) to compare mathematical modeling techniques; and (iii) to identify commonly modeled strategies and to summarize their impact on waiting times.

METHODS

Literature Search

A search strategy was developed to identify the published literature evaluating waiting times in a hospital ED using mathematical modeling techniques. Individualized search strategies (Appendix 1) were developed for several electronic databases using relevant subject headings supplemented by keywords. Due to the scope of this research, medical, engineering, and business (operational research) databases were searched: OVID MEDLINE and EMBASE, Engineering Village 2 Compendex and Inspec, and EBSCOhost Business Source Complete. Subject headings were derived using the thesaurus in each database and were searched individually to assess their added value to the overall strategy. This resulted in certain terms being dropped (e.g., mathematical techniques, emergency physicians). Search strategies were also developed in consultation with a health science librarian and an engineering librarian. Each strategy was limited to English, peer-reviewed journals published between January 2000 and July 2010. These dates were chosen because at the time this review was conducted, past reviews had not included studies after 2000. Additionally, there was a steady publication increase after 2000 of mathematical model healthcare applications (37). Conference proceedings from Compendex and Inspec were included because engineering conference proceedings are typically published in the format of an article with preliminary results.

Study Selection

Inclusion criteria were adapted from the study by Hoot and Aronsky (21): (i) implemented a mathematical modeling technique; (ii) analyzed data; (iii) studied waiting times from the perspective of general emergency medicine; (iv) studied waiting times with respect to typical daily arrival rates and patient demands (i.e., no catastrophic events or patient simulation studies); and (v) the primary outcome measures were waiting/process times in the ED, length of stay, or proportion of patients meeting a waiting time target in the ED. For inclusion, a study had to meet all five criteria.

Using pre-determined inclusion/exclusion criteria, two reviewers, an economist (M.L.), and an engineer (K.L.), using Reference Manager v.11 Network, screened titles and abstracts of identified studies for potential inclusion (1st level screening). The *kappa* statistic was calculated to assess reviewer agreement at this screening level. Full text versions of the published articles were obtained for those studies that met the inclusion criteria and also for those studies where suitability for the review could not be determined based on the title and abstract. One reviewer (M.L.) conducted the full-text screening (2nd level) using the same criteria as the first level screening to determine final inclusion for data abstraction and analysis. Consensus with a second reviewer (J.E.T.) was obtained when it was uncertain if a study met the inclusion criteria. The second reviewer also performed a full-text screening of a 20 percent random selection. A bibliographic search of the included studies was also completed to ensure that all relevant studies were identified.

Data Abstraction and Analysis

A data abstraction form was created to record study information. In addition to recording the mathematical modeling technique, basic study information such as country, objectives, main performance measures, and findings/conclusions were abstracted. To compare the different modeling techniques, each technique was assessed based on 10 model assumptions: analytical or simulation, deterministic or stochastic, discrete or continuous, performance measures, diagrams, capability of handling multiple resource constraints, memory, level of data abstraction, model building time, and developed software (31). Table 1 explains these concepts and their relevance to the hospital ED. Each study was also analyzed in terms of strategies used to reduce waiting times in the ED. Strategies for waiting time reduction were categorized into scheduling (staff and operational), demand management (methods to re-distribute patients), resource allocation (i.e., beds and staff), change in process times, and other. Two reviewers (M.L., J.E.T.) abstracted the data separately using a Microsoft Excel® template with predefined categories (Appendix 2).

RESULTS

Literature Search

The literature search identified 1,795 unique citations following the removal of duplicates. After screening titles and abstracts, 1,712 citations were excluded, mainly because the articles evaluated (i) a simulated environment where trainees practice techniques on standardized patients or part-task trainers rather than computer simulation; (ii) evaluated catastrophic/infectious disease; (iii) did not include data analysis; or (iv) evaluated prediction scores for triaging an illness within the ED. For the first level of screening, a kappa coefficient of 0.73, reflecting good agreement, was calculated between the two reviewers. A full text review of the remaining eighty-three articles excluded fifty-four additional citations, resulting in twenty-nine studies (fifteen journal articles and fourteen conference papers). No additional articles were identified from searching the references of the included studies. For the second level of screening, a kappa coefficient of 0.88 was calculated between the two reviewers for the random 20 percent sample. Figure 1 summarizes the study selection process. Included and excluded studies from the second level of screening are in Appendix 3.

Approximately half of the journal articles were published in health science journals and half in operational research or systems management journals. The conference proceedings were all presented at the Winter Simulation Conferences (the primary international outlet for disseminating advances in the field of system simulations). The studies were set in various countries: the United Kingdom ($n = 7$), the United States ($n = 10$), Canada ($n = 3$), Finland ($n = 1$), Norway ($n = 1$), Kuwait ($n = 1$), France ($n = 1$), Taiwan ($n = 1$), Japan ($n = 1$), Trinidad and Tobago ($n = 1$), Spain ($n = 1$), and unknown ($n = 1$).

Mathematical Modeling Techniques

The included studies used four different mathematical modeling techniques: queuing analytic model ($n = 4$) (12;30;34;40), discrete event simulation ($n = 20$) (2;9;11;13–15;17;20;22;25–28;33;35;36;38;42;46;47), discrete event simulation in combination with optimization ($n = 2$) (1;51), system dynamics ($n = 2$) (29;45), and agent based modeling ($n = 2$) (30;50) (note:

Table 1. Description and Significance of Different Model Assumptions Used for Comparison

Model assumption	Description	Significance to ED modeling
Analytical or simulation	◆ Analytical solutions are mathematical models with obtainable closed-form solutions, meaning it solves in terms of common functions from a given generally accepted set. ◆ Simulation is the process of numerically exercising the model through state changes over time to see how the inputs will affect the output measures of performance.	◆ Analytical solutions are tractable when the model is relatively simple, however, a more complex (i.e. realistic) model requires the use of simulation to estimate a solution.
Deterministic or stochastic	◆ A deterministic model does not contain any probabilistic (i.e. random) components and will result in a fixed outcome given initial conditions. ◆ A stochastic model allows for random variation where inputs are estimated using probability distributions.	◆ The ED is frequently characterized by uncertainty and variability (e.g. arrival rate), requiring a stochastic approach.
Discrete or continuous	◆ Discrete models deal with variables changing at discrete points in time (i.e. countable sets that have distinct separated values such as integers). ◆ Continuous models deal with variables changing smoothly with respect to time and therefore involve differential equations.	◆ Dictates the measure of performance the model outputs: probability (what is the probability there are zero patients in the ED?), rate (what is the rate at which patients are being processed by triage?) or percentile (what is the percentile of patients who have exited the system in less than 4 hours?)
Performance measures	◆ Performance measures are based on the underlying mathematical equations. See discrete vs. continuous.	◆ Dictates choice of mathematical model in order to meet objective.
Diagrams	◆ Diagrams lay out the model logic and aid communication between the doer and the decision-makers.	◆ They provide a level of transparency and allow the decision-maker to visualize what is being modeled.
Resource utilization	◆ Individuals move through a system and utilize resources. The ability to integrate simultaneous use of multiple resources is dependent on the model type.	◆ The analyst may want to model simultaneous resource use (e.g. patient may need to meet with both a nurse and physician at the same time.)
Memory	◆ Memory describes how individual characteristics and past events can affect an individual's pathway in the model.	◆ Memory can be thought of as a patient's medical history, where past events in the model can dictate future pathways.
Level of data abstraction	◆ Individual level or aggregate level (i.e. means) are used to populate a model.	◆ Data availability can affect choice of mathematical model.
Validation	◆ Ensures the model is an accurate representation of the system under study. ◆ External validity is a non-statistical type of validity that determines if the model conceptually represents the system. ◆ Internal validity is represented by quantitative techniques that are used to test overall validity and of various components, typically accomplished with graphical plots and goodness of fit test.	◆ To inform policy, validation is essential to ensuring the model outputs will be representative of the system.
Model building time	◆ Dependent on the complexity of the system being modeled and the data requirements.	◆ May dictate the choice of mathematical model based on need for timeliness of results.
Software	◆ Availability of packaged software.	◆ May dictate the choice of mathematical model.

total adds to thirty instead of twenty-nine because one study used a queuing model and an agent based model). Table 2 presents a comparison of the mathematical modeling techniques with respect to the 10 model assumptions listed in Table 1.

Queuing models are characterized over time by an arrival process, a service process (e.g., treatment), the number of servers (e.g., doctors), a constraint on the number of patients allowed to enter the queue and a queue discipline (48). The queue discipline is the rule that a server uses to choose the next patient. Examples of queuing disciplines include: first-in, first-out (FIFO); last-in, first-out (LIFO); service in random order (SIRO); priority (PR), and general discipline (GD). One of the studies combined a queuing model with the use of fuzzy numbers which incorporates a level of uncertainty into the model.

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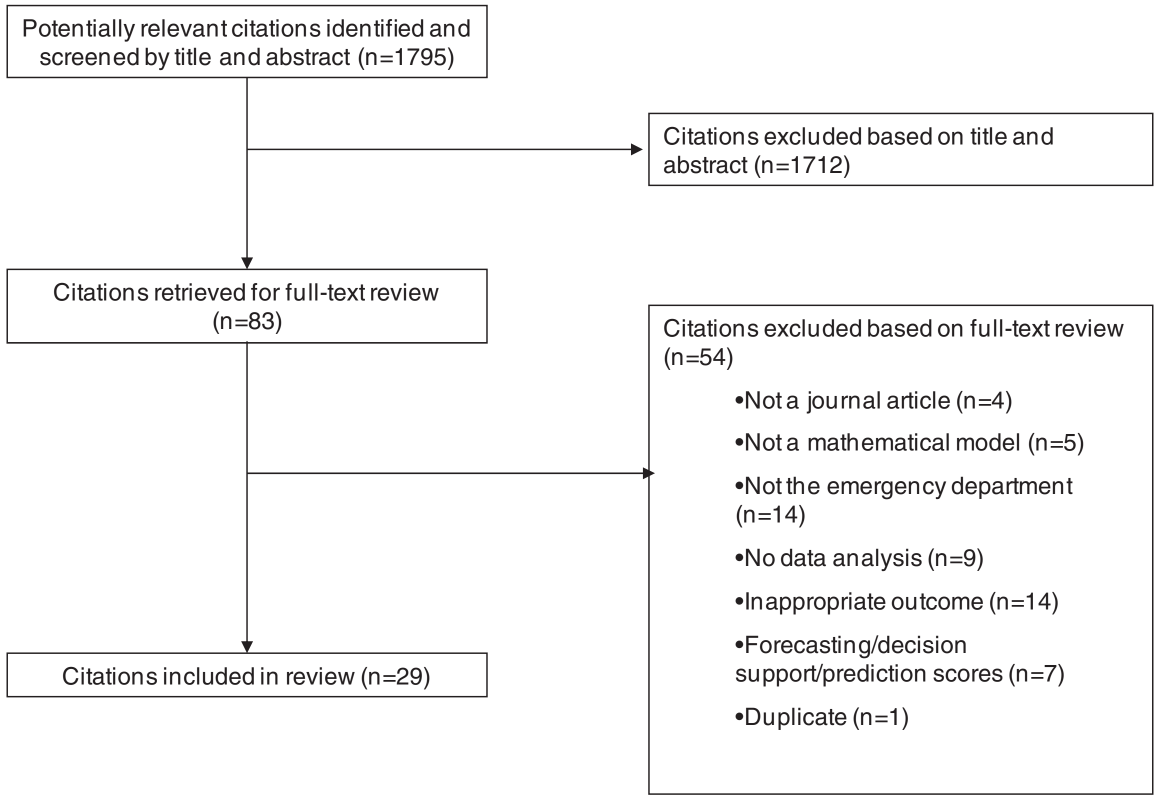

Figure 1. Diagram of included and excluded studies from the literature review.

Twenty of the studies used discrete event simulation (DES) to meet their objectives. DES is characterized by several concepts: entities that move through the model (e.g., patients), attributes that are characteristics of the entities (e.g., sex), resources that are seized by the entities (e.g., staff), queues (e.g., waiting lines), and events or processes (e.g., triage) that the entity will flow through (24). Essentially, DES represents a network of queues for services that a patient flows through where attributes determine the pathway of the patient. This technique is unique because it has a simulation clock that keeps track of the passage of time allowing analysts to control the start and end points (16).

Two studies combined DES with optimization. DES computes a set of performance measures based on defined inputs, however, combined with optimization the model can retrieve the best inputs based on an objective function. For example, the objective may be to re-allocate resources to ensure that all low acuity patients do not have a length of stay longer than 8 hours. One study specified the use of a Genetic Algorithm optimization method, which applies a class of evolutionary algorithms to derive solutions from populations (51). For instance, a new solution is taken and used to form another solution in hopes that this population will be better than the old one and eventually used to derive an optimal solution.

Two studies applied system dynamics modeling. System dynamics is composed of either a qualitative component or both a qualitative and quantitative component. The qualitative phase involves developing an understanding of the system not only by the research team but also by the stakeholders in the system (7). A causal loop diagram is developed with the aim of understanding both direct and indirect relationships between important variables within the structure of the system (8). The variables may not necessarily be quantifiable (e.g., disease advocate group pressure). The resulting causal loop diagram could be the end result of a system dynamics model, however, analysts can choose to add a quantitative component to estimate performance measures. To quantify the model, the causal loop diagram is converted into a stock and flow diagram (8). Conceptually, this diagram can closely resemble the ED process.

The remaining two studies used an agent based modeling (ABM) approach. ABM consists of a set of agents (e.g., patient, physician) where each agent is governed by a set of behaviors (e.g., treat patient), interactions (e.g., patient can interact with physician), and rules (e.g., maximize patient health) (32). Agents are autonomous in that each agent has its own decision-making process. Interactions occur within a pre-defined topographical space that includes resources. ABM is unique because it can capture emergent phenomena (e.g., collective behavior) and agents can adapt and learn (4).

Table 2. Comparison of Mathematical Modeling Techniques by Model Assumption

	Mathematical modeling technique			
	Queuing model	Discrete event simulation	System dynamics	Agent based modeling
Model Assumptions				
Analytical or simulation	Analytical	Simulation	Simulation	Simulation
Deterministic or stochastic	Stochastic	Stochastic	Deterministic	Stochastic
Discrete or continuous	Continuous	Discrete	Continuous	Discrete
Performance measures	Probabilities, average times	Percentiles, average and total times	Probabilities, average times	Percentiles, average and total times, network (cluster) identification
Diagrams	Flowchart	Influence diagram, flowchart, design layout	Causal loop, stock-flow	Flowchart, network diagram, relationship map
Resource utilization	Single	Multiple	Single	Multiple
Memory	No	Yes	No	Yes
Level of data abstraction	Low	High	Low	High
Validation	Expert opinion, GoF, historical data	Expert opinion, GoF, historical data	Expert opinion, historical data	Expert opinion, GoF, historical data
Model building time	Short	Long	Long	Long
Software	Spreadsheet	Arena, Simul8, Extend Suite V5, eM Plant, SimTalk, MedModel, Micro Saint Sharp, EdSim	STELLA (iThink), Vensim, Patient Flow Centre	Repast, NetLogo, MASON, AnyLogic

GoF, Goodness of Fit Test

Study Findings and Waiting Time Reduction Strategies

A different hospital ED was evaluated in each of the studies under review. Their individual objectives, performance measures and findings are summarized in Table 3. Below is a brief summary of study findings by common strategy used for reduction of waiting times.

Scheduling. Six studies, all DES, evaluated different staff shift patterns or operational hours as strategies to improve ED efficiency (1;11;14;33;36;51). All resulted in reduced patient waiting times or increased throughput.

Demand Management. Four studies found that fast-tracking low acuity patients through the ED could have both positive and negative effects (9;12;13;47). The studies indicated that any improvements for low acuity patients were at the expense of high acuity patients or it decreased door to doctor time, but only if staffing resources were concurrently re-allocated. By altering the triage process, re-allocating an extra triage nurse dependent on patient demand, using a triage team or including a physician at triage, reduced average patient throughput time (35;38;42;50). Triage to bed time decreased if a holding area, ED discharge lounge, and observation unit were added (27). Bedside registration was not found to be an effective intervention at decreasing length of stay (2).

Resource Allocation. Altering the number of staff (e.g., physician, nurse, clerks), beds, and/or rooms (14;22;28;40) showed reductions in patient waiting times, with the exception of two studies that found no change (17;26).

Process Times. Six of the studies altered inputs to the model to determine whether waiting times decreased if the proportion of patients waiting decreased. Diagnostic/laboratory process times were shortened (15;36;45;50), which was significantly associated with a decreased ED length of stay with the exception of one study that found no change (17). Increasing the rate of inpatient admission was successful in decreasing the length of stay in the ED (26).

Other. Khadem et al. (25) altered the entire layout of the hospital ED to determine the most efficient layout with respect to waiting times. Mayhew and Smith (34) evaluated whether a change in the discharge definition would decrease process completion time. They re-defined discharge as occurring when the patient is referred or becomes an inpatient as opposed to once they are transferred. Using this definition resulted in faster completion times. Takakuwa and Shiozaki (46) simulated an increase in number of patients to reallocate resources based on increased waiting times. Laskowski et al. (30) investigated the use of agent based modeling to evaluate resource optimization and workflow and an analytic queuing model to evaluate waiting times.

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Table 3. Summary of Study Characteristics, Objectives, Performance Measures, and Findings

Model technique	Study	Country	Objective	Performance measurements	Findings/conclusions
Queuing model	Puente et al. 2003	Spain	Using fuzzy numbers in a queuing model to determine optimal number of beds based on different levels of uncertainty for patient arrival rates and service rates. Secondary purpose was to compare results with and without using fuzzy numbers.	Average patient LOS, average patient waiting time, number of patients in the system and number of patients waiting in queue.	Using fuzzy numbers the model predicted an increase of two beds is needed to reduce risk of service congestion. Without fuzzy numbers the model predicted that any increase in number of beds would only result in slight improvements in the performance measurements. Found that using fuzzy numbers was more robust.
Queuing model	Mayhew et al. 2008	UK	To determine if re-designation of the discharge definition for an ED effects the percent throughput for different completion times. Re-designated patients are those that have been admitted as an inpatient but remain in the ED until their bed is available.	Percentage of patient throughput at average completion times.	The authors compared results from two models: current practice versus re-designated model. As average completion times rose, the re-designated model diverged and gains became larger.
Queuing model	Cochran et al. 2009	US	To show how fast-tracking effects staffing, utilization, and queuing time.	Expected patient wait time in queue, overflow probability, expected time from ED entry to assessment.	By fast-tracking patients, it is possible to re-allocate resources to meet targets. It also showed a decrease in door to doctor time.
Discrete event simulation	Coats et al. 2001	UK	To determine the effect of different Senior Health Officer shift patterns on waiting time.	Proportion of patients meeting waiting time targets.	By scheduling the Senior Health Officer with an earlier shift pattern resulted in a closer match to the arrival times of patients and a greater proportion of patients meeting waiting time targets.
Discrete event simulation	Mahapatra et al. 2003	US	To determine whether altering the operational hours of the fast-tracking centre will decrease average wait times.	Average total waiting time	Running the alterna care unit from 9am–9pm, rather than 11am–10pm, results in reduced average waiting times for all triage levels.
Discrete event simulation	Connelly et al. 2004	US	To compare acuity ratio triage (ART) with a fast-track approach in the reduction of waiting times. ART involves assigning a ratio of high acuity and low acuity patients to a healthcare worker.	Patient treatment time, overall patient service time	ART reduces the average waiting time by 76% and the average treatment times by 4% for high acuity patients, as well as increases service time for low acuity patients. ART reduces imaging bottlenecks relative to fast-tracking.

Table 3. Continued.

Model technique	Study	Country	Objective	Performance measurements	Findings / conclusions
Discrete event simulation	Brailsford et al. 2004*	UK	To analyze the effect of fast tracking patients with minor injuries on queuing.	Percent utilization of physicians, percent of queue less than a target waiting time.	Permanent streaming of minor injuries was an inefficient use of resources. Improvements observed for less severe cases, but at expense of higher acuity patients.
Discrete event simulation	Komashie et al. 2005	UK	To analyze how altering beds, nurses and physicians impacts key performance indicators.	Average total time and bed queuing time for minor and major patients.	Adding a nurse or physician to both minor and major patient areas resulted in the greatest reduction in total and bed queuing time for minor patients. Eliminating the admission blockage resulted in the greatest reduction for major patients.
Discrete event simulation	Ruohonen et al. 2006	Finland	To analyze if a new triage team method improves average throughput time. The new triage team consists of a receptionist, a nurse and a doctor.	Average throughput time.	Using the new triage team method reduced the average throughput time.
Discrete event simulation	Gunal et al. 2006	UK	To analyze how altering diagnostic process time, number of cubicles and experienced physicians versus junior physicians impact total time in the system.	Total time in system.	Cannot draw direct conclusions but overall the system performs better with experienced physicians.
Discrete event simulation	Duguay et al. 2007	Canada	To analyze how adding physicians, nurses, and examination rooms might effect waiting times.	Time between arrival and triage, triage and registration, registration to available examination room, first assessment to discharge.	Waiting time from registration to an examination room was the most problematic. Five alternative scenarios for adding resources were evaluated. Adding one more nurse and physician resulted in the best outcomes (reduction in waiting times). The number of examination rooms had no effect on waiting time if added without matching staff increase.
Discrete event simulation	Hung et al. 2007	Canada	To analyze how adding physicians, nurses, and volunteers might effect waiting times.	Mean pretriage wait, proportion of patients at pretriage waiting >30 min, proportion of patients at pretriage waiting >60 min, mean acute care patient time to be seen by a physician, LOS.	After running three staffing scenarios, adding a pretriage volunteer and a second triage nurse greatly reduced mean pretriage time and the percentage of patients waiting both >30 and >60 minutes. Adding an extra physician shift was optimal in the evening.

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Table 3. Continued.

Model technique	Study	Country	Objective	Performance measurements	Findings / conclusions
Discrete event simulation	Ferrin et al. 2007	US	To improve capacity and process flow by altering inpatient and resource constraints and process improvement scenarios.	Length of stay and percentage of patients who leave without being seen.	Inclusion of six inpatient beds reduced LOS by 8%. Radiology process improvement did not improve ED LOS but laboratory process improvements reduced LOS by 3–9%.
Discrete event simulation	Yeh et al. 2007	Taiwan	To use genetic algorithm optimization for scheduling nurses to determine whether waiting times could be decreased.	Queuing time and throughput	Alternate nursing schedules derived from using genetic algorithm resulted in statistically significant lower queue times compared to the current times.
Discrete event simulation	Medeiros et al. 2008	US	To test whether placing an emergency care physician at triage improves flow and average length of stay.	Average length of stay and average census.	Improvement for low acuity patients was found.
Discrete event simulation	Kolb et al. 2008	US	To test whether a patient buffer system could relieve pressure from the ED. Five different systems were tested.	Triage to bed time, diversion time, holding patient time, and buffer time.	With respect to triage to bed time, the scenario that included a holding area, ED discharge lounge and observation unit had the greatest improvement being 21.7% faster than baseline.
Discrete event simulation	Meng et al. 2008	UK	Use waiting times for consultants and labs, number of beds, 24 hour x-ray department as control variables.	Average total time by triage class and number of patients at the end of the day.	Reducing the waiting time for a consultant and increasing the number of trolley beds has the greatest impact on overall waiting time and number of patients at the end of the day.
Discrete event simulation	Khadem et al. 2008	NR	Alter layout of ED (i.e. rooms) to determine impact on waiting time.	Average time in system, average waiting time and current quantity in system.	The new layout was able to increase throughput and significantly decrease waiting times for each triage level.
Discrete event simulation	Nielsen et al. 2008	Trinidad and Tobago	To identify bottlenecks and simulate improvements.	Total ED waiting time, resource utilization.	The simulation identified a bottleneck at triage. By re-allocating the ECG nurse to help the triage nurse the simulation was able to show a 4 hour improvement in waiting time. In turn, this increased resource utilization of the clerk and physician to more efficient levels.
Discrete event simulation	Ahmed et al. 2009	Kuwait	Using optimization, evaluate the impact of an alternative staffing distribution subject to a budget constraint.	Average waiting time in system.	Derived an optimal alternative staffing distribution within the current budget that increased throughput by 28%.

Table 3. Continued.

Model technique	Study	Country	Objective	Performance measurements	Findings / conclusions
Discrete event simulation	Beck et al. 2009	US	To evaluate effect of bedside registration on length of stay.	Length of stay.	Bedside registration decreases length of stay only when a bed is available immediately after triage.
Discrete event simulation	Holm & Dahl 2009	Norway	To evaluate effect of physician triage on patient waiting time.	Average waiting time in system.	Total stay was reduced by 13 minutes but was not statistically significant.
Discrete event simulation	Khare et al. 2009	US	To evalute if altering the number of beds and increasing the rate at which patients exit the ED decreases length of stay.	Length of stay.	Increasing the number of beds did not decrease length of stay, whereas increasing the rate of departure from the ED did result in a decrease.
Discrete event simulation	Tao et al. 2009	France	To evaluate phyisican efficiency improvement (i.e. shorter consultation time by introducing computer assisted tools) and fast-tracking patients on waiting times.	Average waiting time before admission to consultation room.	Waiting time significantly decreased in both scenarios.
Discrete event simulation	Takakuwa et al. 2009	Japan	To evaluate patient waiting times when there is an increase in patient arrival due to ED expansion.	Total ED waiting time.	Under current resource allocation doubling patients dramatically increases waiting time from 8% to 59% of time in system. However, through scenario analysis, the simulation was able to provide an optimal resource allocation to decrease waiting time back to 8%.
System dynamics	Lane et al. 2000	UK	Assess changes in waiting times and other output variables when there is a change in bed capacity.	Time from registration to ED physician consult and to discharge, total waiting time, percent of elective cancellations, daily hospital occupancy, and daily ED physician utilization.	When beds were increased and decreased by 100 relative to the base case waiting times did not change. Only the average percent of elective cancellations changed. The base case has little room to increase efficiencies.
System dynamics	Storrow et al. 2008	US	Estimate the effect of decreasing laboratory turnaround times on ED diversion and ED LOS.	ED diversion, ED LOS	Point of care testing decreases turnaround time and in turn ED LOS.
Agent based modeling & Queuing model	Laskowski et al. 2009	Canada	To investigate resource optimization in the ED using agent based modeling and to evaluate waiting times using an analytic queuing model.	Number of patients in queue and waiting times.	Models are preliminary, therefore results are qualitative. Using real and simulated data, was able to show that agent based modeling was useful in predicting queue length when varying number of physicians and various patient redirection policies. This study was also able to show that an analytic queuing model was useful in predicting time spent (waiting + service) at service nodes when altering arrival rates of different patient severities.

Table 3. Continued.

Model technique	Study	Country	Objective	Performance measurements	Findings / conclusions
Agent based modeling	Wang 2009	US	To evaluate ED changes in triage and radiology processes.	ED length of stay, patient numbers, leaving without being seen, and waiting times.	By adding an extra triage nurse working in parallel with the current triage nurse but only when the queue exceeds 10 patients and leaves when fewer than 2 patients resulted in a statistically significant decrease in ED LOS but an increased time in waiting for radiology. Changing the radiology procedure time resulted in a statistically significant reduction in mean ED LOS, waiting time for resident and waiting time for radiology.

Acronyms: UK, United Kingdom; US, United States; NR, Not reported; ED, Emergency Department; ART, Acuity Ratio Triage; LOS, Length of Stay
*Discrete event simulation model within a system dynamics model.

DISCUSSION

Faced with the pressure to reduce resource use and improve quality of service under fixed budgets, decision makers are presented with the difficult task of reducing patient waiting times. Because of the various factors involved from both the demand and supply sides, collecting descriptive data regarding waiting times is helpful to inform whether targets are being met, but is likely to be insufficient to understand the systemic issues related to waiting times in the healthcare system ED. Mathematical modeling has an important role to play as they can consider all system components and their interactions in the same model. To address this issue we conducted a literature review to determine the use of these techniques in evaluating waiting time reduction strategies in the ED. The review revealed that twenty-two studies presented DES models (where two used optimization), two system dynamics models, four queuing analytic models and two agent based modeling. Common strategies to decrease waiting times in the ED included altering scheduling, resource utilization, and process times. Only a few studies indicated that results from the mathematical models were implemented into practice.

Selecting a mathematical modeling technique depends on several factors. The group Research Into Global Healthcare Tools (RIGHT) has recently developed a selection framework for modeling and simulation techniques (41). This framework consists of two main criteria: project life cycle stage (e.g., needs and issues identification or performance evaluation) and type of output (e.g., system interaction or comprehensive system behavior). Selection can also be characterized by the amount of time, money, knowledge and data that are available for the model.

Specifically, queuing models are more useful for modeling simple systems because as complexity is added the analytical solutions become less attainable. Arguments against queuing models focus on their theoretical assumptions. Queuing models make the assumption of Poisson distributed arrival times, exponentially distributed inter-arrival times, an infinite queue length, one server and linear relationships. Frequently, this is not the case in the hospital ED. It is possible to extend queuing models outside of these basic assumptions (e.g., multiple simultaneous servers); however, this involves the use of simulation as the analytical solutions are no longer plausible. Puente et al. (40) combined simulation with a simple queuing model.

System dynamics modeling is an attractive technique for strategic planning of large populations because of causal loop diagrams and the use of aggregate level data to populate the model (34). The causal loop diagram is flexible because both tangible (e.g., increased waiting time) and intangible (e.g., stakeholder pressure) effects can be incorporated (34). The tradeoff of using this approach is that it lacks memory and patient individuality is lost because of the indivisibility of using continuous variables (8). As such, system dynamics is not an optimal tool for understanding detailed workings of the ED. DES models may be preferred for those needing an exact or very accurate understanding of comprehensive system behavior (i.e., resource allocation, implementation, evaluation).

DES models allow modeling the hospital ED in greater detail because they are stochastic, have memory and use discrete inputs. They can also be used to identify causes of bottlenecks and queues or to simultaneously evaluate performance changes based on changes in scheduling, triage and the addition/subtraction of resources. This technique is unique because it has a simulation clock that keeps track of the passage of time allowing analysts to control the start and end points. This is important for dynamic systems like emergency departments where analysts are interested in the steady state. It also allows for analysis of a system where interest is in long-run behavior. DES is a valuable tool in modeling complex systems with non-linear patient flow, typical of the ED. It is also flexible in its ability to manage patients with numerous characteristics (e.g., patients with different acuities, illnesses, sex and age). Additionally, it is possible to model interactions between resources (e.g., physician with residents) using a form of pseudo-agent-based modeling combined with DES. Of the studies identified in the review, DES was the most frequently used technique and was used to address multiple issues simultaneously. For instance, Meng and Spedding (36) simultaneously modeled whether changes in process times, the addition of beds and a change in operational hours would reduce waiting times. The main drawback is the time required to collect appropriate and accurate data. Data requirements depend on the amount of modeling necessary to answer the proposed question. A DES model may require the following: resource shift and break schedules, time stamp data (arrival, triage, in bed, seen by nurse, seen by physician, disposition), and transfer times (e.g., how long does it take to walk between the triage and waiting rooms). It is important to assess data requirements and timelines before building a mathematical model.

Similar to DES, ABMs can be used to understand comprehensive system behavior. Agents are programmed at the micro-level (e.g., patients, physicians, organizations) to determine macro-level effects (e.g., performance measures, agent interactions) ABMs have similar modeling advantages as DES: stochastic, discrete, simulation clock and the ability to model non-linear pathways. The main difference is that there is no global system behavior in ABM. Behavior is defined at the individual level and global behavior emerges from interactions between agents and with the environment. Additionally, contrary to DES, resources such as physicians and nurses have autonomous decision-making behaviors. For instance, the physician has the ability to prioritize tasks (e.g., multi-task, interact with residents) rather than act as only a server to patients. Drawbacks are also similar to DES: large data requirements and long model building time. ABM also requires a greater understanding of the agents within the system because individual behaviors need to be programmed in the model. Additionally, there are few user friendly softwares and, therefore, computer programming skills (e.g., C++) may be required.

Our review identified two studies using optimization to model waiting times despite the fact that optimization techniques are commonly used for scheduling staff and appointments in the service sector (10;49). Optimization is a technique that can be used alone or in conjunction with simulation. It has a limited capacity to characterize complex systems (23) but is efficient because it only requires one experimental run (23). In the past, the computing complexity of hybridizing these two techniques discouraged use (23), however, standard simulation packages (e.g., Arena) are now offering options for combining optimization and simulation. This is a very fertile area of future research for waiting time targets. To further understand the utility of these models, it is important to assess implementation of recommendations derived from the models into practice. The primary purpose of five of the identified studies was to describe the development of a model rather than to analyze scenarios or policy changes, however, two studies discussed applying model recommendations. Implementing an extra staff shift resulted in reduced waiting time for a patient to be seen and expedited throughput (22). In another case, a hospital implemented split flow and achieved a 61 percent reduction in patients leaving without treatment (12). Further research needs to be conducted into this area to assess the usefulness of these modeling techniques for decision making.

Mathematical modeling techniques commonly used for economic evaluations (i.e., decision trees and Markov models) were not found in the literature review. This is a result of their limited ability to handle non-homogeneous populations and highly variable medical systems. They require more programming to include elements such as memory that are already built into other modeling techniques. The usefulness of decision trees and Markov models are limited in their capability to handle systems such as the emergency department.

A few limitations were associated with this study. First, several challenges arose when developing the literature search strategy because of the cross-disciplinary nature of systems analysis (i.e., operations research, industrial engineering, health services research). Indexing in the engineering and business databases is far less detailed than in the health literature databases and therefore their searching tools were more basic. This could have resulted in the indirect exclusion of some articles as they would not have been captured by the search terms. However, reference lists of the included studies were searched to identify additional papers. In addition, results were based on robust search strategies developed in consultation with librarians where two independent reviewers conducted the screening and abstracted the data.

Although several biases can be adjusted for in the modeling process (e.g., temporality) only two studies indicate effectiveness from implementation into practice. As such, individual study findings should be used with caution as the results were based on computer simulations. Unlike other interventional health studies (i.e., CONSORT (43) and STARD (6)), computer simulation studies do not have recommended standards of reporting. Similarly, there exist no validated quality assessment tools for such models. Finally we limited the scope of our review to publications over the last 10 years as previous simulation reviews included publications up to 1999 (16;23). Despite these limitations, this report provides evidence regarding the use of mathematical models to study waiting times in the ED. Results also call for an improvement in reporting and transparency in presenting results.

CONCLUSION

The literature search resulted in twenty-nine studies published over the last decade which use four mathematical modeling techniques to evaluate waiting time reduction strategies in the hospital ED. Although each modeling technique has strengths and weaknesses, DES was the most frequently used method because of its ability to model complex systems, staff shifts, patient history, and multiple resource constraints, its transparency for decision makers and the wealth of software available for implementation. Scheduling and altering the number of staff according to surges in patient demand showed reductions in ED waiting time. Fast-tracking low acuity patients was also found to be effective in decreasing waiting times, but only at the expense of high acuity patients or decreasing turnaround laboratory times or using point of care testing.

Ultimately, mathematical modeling is a strategy that can be used for the continuous quality improvement and safe delivery of health care without placing patients at risk. It is able to mirror, anticipate, or amplify real situations within a safe environment for healthcare practitioners. There is potential for mathematical models to be used to evaluate the cost-effectiveness of different strategies and new technologies. From the promising results found in the included studies this area of healthcare could greatly benefit from the use of mathematical modeling.

CONTACT INFORMATION

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CONFLICTS OF INTEREST

Tim Nye has not declared his conflicts of interest. The other authors report they have no potential conflicts of interest.

Appendix 1: Search Strategies of all electronic databases

COMPENDEX

(((((((({System theory} OR {Decision theory} OR {Systems analysis} OR {Scheduling}) WN CV)) AND (2000-2010 WN YR)) OR

(((({Queueing theory} OR {Operations research} OR {Queueing networks}) WN CV)) AND (2000-2010 WN YR)) OR

(((({Computer simulation} OR {Discrete event simulation} OR {Mathematical models} OR {Simulation}) WN CV)) AND (2000-2010 WN YR)) OR

((((stochastic NEAR/2 model OR process* NEAR/2 model OR theor* NEAR/2 model OR mathematical NEAR/2 model OR computer NEAR/2 model OR emergency NEAR/2 model OR triage NEAR/2 model OR queu* NEAR/2 model OR {patient flow} NEAR/2 model) WN KY)) AND (2000-2010 WN YR)) OR

((((stochastic NEAR/2 simulation OR process* NEAR/2 simulation OR theor* NEAR/2 simulation OR mathematical NEAR/2 simulation OR computer NEAR/2 simulation OR emergency NEAR/2 simulation OR triage NEAR/2 simulation OR queu* NEAR/2 simulation OR {patient flow} NEAR/2 simulation) WN KY)) AND (2000-2010 WN YR)) OR

((((model NEAR/2 simulation) WN KY)) AND (2000-2010 WN YR)) OR

(((({discrete event}) WN KY)) AND (2000-2010 WN YR)) OR

((((queu* NEAR/2 theory OR {patient flow} NEAR/2 theory) WN KY)) AND (2000-2010 WN YR))))) AND

((((((({Emergency rooms}) WN CV)) AND (2000-2010 WN YR)) OR

((($triage) WN KY) AND (2000-2010 WN YR)) OR

((((((Health) OR (Health Care)) WN CV) AND emergency WN KY)) AND (2000-2010 WN YR)) OR

((((emergency NEAR/2 room OR emergency NEAR/2 department OR emergency NEAR/2 ward OR emergency NEAR/2 unit OR emergency NEAR/2 triage) WN KY)) AND (2000-2010 WN YR))))))

INSPEC

(((((((((((({digital simulation} OR {simulation} OR {discrete event simulation}) WN CV)) AND (2000-2010 WN YR)) OR

(((({queueing theory}) WN CV)) AND (2000-2010 WN YR)) OR

(((({operations research} OR {systems analysis} OR {system theory}) WN CV)) AND (2000-2010 WN YR)) OR

(((({scheduling}) WN CV)) AND (2000-2010 WN YR)) OR

(((((stochastic NEAR/2 model OR process NEAR/2 model OR mathematical NEAR/2 model OR computer NEAR/2 model OR emergency NEAR/2 model OR triage NEAR/2 model OR queueing NEAR/2 model OR {patient NEAR/2 flow} NEAR/2 model) WN KY))) AND (2000-2010 WN YR)) OR

(((((stochastic NEAR/2 simulation OR process NEAR/2 simulation OR mathematical NEAR/2 simulation OR computer NEAR/2 simulation OR emergency NEAR/2 simulation OR triage NEAR/2 simulation OR queueing NEAR/2 simulation OR {patient NEAR/2 flow} NEAR/2 simulation OR dynamic NEAR/2 simulation OR discrete NEAR/2 simulation) WN KY))) AND (2000-2010 WN YR)) OR

(((({operations research}) WN KY) AND (2000-2010 WN YR)) OR

(((({discrete event}) WN KY) AND (2000-2010 WN YR)) OR

(((((queueing NEAR/2 theory OR {patient NEAR/2 flow} NEAR/2 theory) WN KY))) AND (2000-2010 WN YR)) OR

((((model NEAR/2 simulation) WN KY)) AND (2000-2010 WN YR)) OR

(((({queueing networks}) WN KY) AND (2000-2010 WN YR))))) OR

(((({mathematical programming}) WN CV)) AND (2000-2010 WN YR))))) AND

((((((((health care) WN CV) AND (emergency) WN KY)) AND (2000-2010 WN YR)) OR

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(((((emergency NEAR/2 room OR emergency NEAR/2 department OR emergency NEAR/2 ward OR emergency NEAR/2 unit OR emergency NEAR/2 triage) WN KY))) AND (2000-2010 WN YR)) OR
((((triage) WN KY)) AND (2000-2010 WN YR))))))

BUSINESS SOURCE COMPLETE

1. ((DE "DECISION theory" or DE "DISCRETE choice models" or DE "MANAGEMENT science" or DE "OPERATIONS research") OR (DE "SYSTEM theory" or DE "MATHEMATICAL optimization" or DE "PROGRAMMING (Mathematics)" or DE "SIMULATION methods" or DE "QUEUING theory")) OR (DE "MATHEMATICAL models" or DE "SIMULATION models")
2. TX stochastic N2 simulation OR process N2 simulation OR mathematical N2 simulation OR computer N2 simulation OR emergency N2 simulation OR triage N2 simulation OR queueing N2 simulation OR patient N2 flow N2 simulation OR dynamic N2 simulation OR discrete N2 simulation
3. TX emergency N2 room OR emergency N2 department OR emergency N2 ward OR emergency N2 unit OR emergency N2 triage OR triage
4. (1 OR 2) AND 3

MEDLINE

1. exp decision theory/ or exp operations research/
2. mathematical computing/ or exp computer simulation/ or exp probability/
3. ((stochastic or process* or theor* or math* or comput* or emergency or triage or queu* or (patient adj2 flow)) adj2 (model* or simulation* or microsimulation*)).ti,ab.
4. (model* adj2 simulation*).ti,ab.
5. ((queu* or patient flow) adj2 theor*).ti,ab.
6. "discrete event".ti,ab.
7. or/1-6
8. exp Emergency Service, Hospital/
9. (emergency adj2 (department* or ward* or room* or triage)).ti,ab.
10. 8 or 9
11. 7 and 10

EMBASE

1. exp decision theory/ or exp system analysis/
2. process model/ or exp theoretical model/ or exp mathematical model/ or exp simulation/ or exp probability/
3. exp mathematical computing/
4. ((stochastic or process* or theor* or math* or comput* or emergency or triage or queu* or (patient adj2 flow)) adj2 (model* or simulation* or microsimulation*)).ti,ab.
5. (model* adj2 simulation*).ti,ab.
6. ((queu* or patient flow) adj2 theor*).ti,ab.
7. "discrete event".ti,ab.
8. or/1-7
9. exp Emergency Ward/
10. (emergency adj2 (department* or ward* or room* or triage)).ti,ab.
11. exp Emergency Care/
12. or/9-11
13. 8 and 12

Appendix 2. Data Abstraction Form

		Example
Ref ID		123
Year		2009
Author		Lim
OR/ENG Journal	**Y/N**	Y
Country		Canada
Model type		Discrete event simulation
Objective(s)		To analyze how adding physicians, nurses, and examination rooms might effect wait times.
Performance Measure		Time between arrival and triage, triage and registration, registration to available examination room, first assessment to discharge.
Findings/Conclusions		Waiting time from registration to an examination room was the most problematic. Five alternative scenarios for adding resources were evaluated. Adding one more nurse and physician resulted in the best outcomes (reduction in wait times). The number of examination rooms had no effect on waiting time if added without matching staff increase.
Data Source	**Primary/Secondary/Not Reported**	Not reported
Acuity Grouping	**Y/N**	Y
Arrival Process	**Hourly/weekday/weekly**	Weekdays
Processes		Triage, registration, waiting, assessment, tests
Variables/Parameters		Time between: arrival and triage, triage and registration, registration to available exam room, first assessment to discharge. Process time: lab test, physician assessment, exam room ready.

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Appendix 2. Continued.

Resources		rooms, physicians, nurses, lab tests
Staff Shifts	Y/N	Y
Distributions Indicated	Y/N	Y
Optimization	Y/N/NA	NA
Model Validation	Y/N	Y
Need calibration?	Y/N	Y
Calibration	Y/N	N
Goodness of Fit Test Performed	Y/N	N
Diagram	Y/N	Y
Software		Arena
Implemented in practice	Y/N	N

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