CREDIT AND GROWTH UNDER LIMITED COMMITMENT

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We consider a linear growth model with idiosyncratic productivity shocks in which producers cannot commit to repay their loans. Borrowing constraints are determined endogenously by the borrowers' incentives to repay, assuming that defaulters lose a share of output and are excluded from future trade in the credit market. We characterize necessary and sufficient conditions for the enforceability of a first-best equilibrium growth path. Weak property rights, impatient producers, and small productivity differentials can make the efficient growth path nonenforceable and lead to an inefficient equilibrium with binding borrowing constraints. For some economies, multiple balanced growth paths coexist.

Keywords: Endogenous Growth, Limited Enforcement, Borrowing Constraints

1. INTRODUCTION

Does the Solow residual describe just the aggregate production possibility frontier of a modern economy or does it also convey information about how speedily inputs are transferred from more to less productive sectors? To understand how resource mobility influences economic growth, we examine equilibria in an environment with limited commitment to loans. This environment places bounds on how much firms can borrow and how fast capital moves from more to less productive endeavors.

Our environment provides a useful framework for studying the relationship between the development of credit markets and economic growth. The theoretical and empirical growth literature has identified a positive linkage between economic growth and financial development, as measured, for instance, by the volume of private credit relative to GDP [see, e.g., Levine (1997)]. In our model, both growth and financial development are endogenous, and they result from *economic fundamentals* such as technology and preferences and from *institutions* such as property rights and contract enforcement.

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This paper sets up in Section 2 an economy populated by infinitely lived individuals that convert capital services into a single consumption/investment good using a technology subject to uninsurable idiosyncratic shocks. Agents experiencing bad shocks wish to lend to banks, and agents with good shocks wish to borrow but cannot guarantee loan repayment. Defaulters forfeit to creditors a fraction of their current resources, as in Kiyotaki (1998), and suffer perpetual exclusion from asset trading, as in Kehoe and Levine (1993). As in Holmstrom and Tirole (1998), borrowing constraints depend positively on the borrower's net worth.

Section 3 shows how the classical first-best commitment outcome of maximal growth is sustainable as an equilibrium without commitment if property rights are strong and households greatly value the right to participate in asset markets. When the first-best cannot be implemented without commitment, producers will be bound by debt limits which are examined in Section 4, along with the adverse impact those limits have on economic growth.

Section 5 looks at a class of simple economies with two productivity states that admit up to three equilibria: The classical one with maximal growth, efficient production, and efficient distribution of the consumption good; the rationed one with lower growth, inefficient production, and inefficient distribution; and an intermediate equilibrium with inefficient distribution. Section 6 concludes.

2. THE ENVIRONMENT

Consider a stochastic growth model in discrete time $t \geq 0$ in which there are a single capital/consumption good and a continuum of agents, $i \in [0, 1]$, who act both as producers and as consumers. Agents are infinitely lived with logarithmic preferences and discount factor β. In period 0, agent i is endowed with y_0^i units of the capital/consumption good. In each period $t \geq 0$, the agent has access to a production technology transforming investment of capital k_t^i undertaken at date t into output y_{t+1}^i at date $t + 1$ according to $y_{t+1}^i = A_t^i k_t^i$. Capital productivities A_t^i are drawn from the finite set $\{A_0, \ldots, A_J\}$ according to the distribution $\pi_j = \mathrm{Prob}(A_j) > 0$. We assume that $A_0 > A_1 > \ldots > A_J > 0$. These productivity draws are independent across agents and across time. Hence, at each date t, a fraction π_j of the agents draws capital productivity A_j, and the set of these agents is partitioned into subgroups $\ell = 0, \ldots, J$ of size $\pi_j \pi_\ell$ who had access to technology A_ℓ at the previous date $t - 1$.[1]

Financial markets are incomplete; in each period agents trade in the capital market a single security whose safe rate of return between t and $t + 1$ is denoted R_t. We denote agent i's borrowing at date t by b_t^i. The timing within each period t is as follows:

1. Agents earn output $y_t^i = A_{t-1}^i k_{t-1}^i$ from investment undertaken in the previous period.
2. Agents redeem debt $R_{t-1} b_{t-1}^i$ if $b_{t-1}^i > 0$; if $b_{t-1}^i < 0$ they collect the return from security holdings.
3. The new productivity draws A_t^i are realized.

4. Agents consume c_t^i, save $s_t^i = y_t^i - R_{t-1}b_{t-1}^i - c_t^i$, borrow b_t^i, and invest $k_t^i = s_t^i + b_t^i$. Output to be earned next period is $y_{t+1}^i = A_t^i k_t^i$.

Note that between stages 3 and 4 no uncertainty is revealed. Specifically, when agents decide about consumption, saving, and investment, they know their productivity levels. We express the budget constraint of agent i in period t as follows:

$$c_t^i + s_t^i = A_{t-1}^i \left(s_{t-1}^i + b_{t-1}^i \right) - R_{t-1}b_{t-1}^i. \tag{1}$$

Let us first discuss what happens if enforcement of financial contracts is perfect, that is, all output is seizable, so that agents can always be forced to redeem debt at stage 2 above, regardless of how much they borrowed. In that case, all capital flows in each period to the group of most productive agents and the capital return is equalized to these agents' capital productivity: $R_t = A_0$ for all $t \geq 0$. Hence all agents earn the return A_0 on their savings, and because all agents save a fraction β of their wealth in each period (as we show below), the economy's growth factor is βA_0.

However, in the environment of this paper, contract enforcement is imperfect, in the sense that not all output can be seized in the event of default. Instead, the agent may keep a share $m > 0$ of his gross output (= wealth). The remaining share $1 - m$ serves as collateral. This number may be interpreted as the economy's "property rights": the better property rights are, the more of an agent's wealth is pledgeable. In addition to the seizure of a share of output, we follow Kehoe and Levine (1993) and assume that an agent defaulting on his debt is excluded from any future trade in the capital market. There is a credit authority that is able to prevent any attempts of a defaulter to obtain future credit and that can seize any security holdings. Therefore, if an agent defaults, he is left with the fraction m of output and he is forced to zero capital market trades in all future periods. Because information is perfect, and because no uncertainty is resolved during debt contracts (given the time structure above), this enforcement mechanism induces agents to trade securities only up to endogenous borrowing constraints, which are just tight enough to prevent all agents from default. Alvarez and Jermann (2000) have shown how such borrowing constraints must be specified: they should prevent default but they should not be too tight in the following sense. Whenever an agent is constrained, he should be indifferent between defaulting and not defaulting. Assuming that such indifferent agents never default, these constraints do indeed prevent default and they are not tighter than necessary to achieve just that. In our environment, it is convenient to write borrowing constraints as $b_t^i \leq \theta_t^i s_t^i$. θ_t^i is a constraint on the ratio between external funds and internal funds.

Let us now look at the consumption/savings decision of agent i. In each period t, agent i will be in one of the following two situations. First, if $A_t^i > R_t$, the capital cost is less than the return on capital. The agent would like to borrow an infinite amount, which is infeasible. Hence the borrowing limit is binding, the agent invests $k_t^i = (1 + \theta_t^i)s_t^i$, and the budget constraint (1) for period $t + 1$

becomes

$$c_{t+1}^i + s_{t+1}^i = \left[A_t^i \left(1 + \theta_t^i\right) - \theta_t^i R_t \right] s_t^i. \tag{2}$$

Second, if $A_t^i \leq R_t$, the agent is better off investing in the capital market rather than producing on his own. Hence the return on agent i's saving is R_t and the budget constraint in period t is

$$c_{t+1}^i + s_{t+1}^i = R_t s_t^i. \tag{3}$$

From (2) and (3), we can write agent i's budget constraint as

$$c_{t+1}^i + s_{t+1}^i = \tilde{R}_t^i s_t^i \, , \tag{4}$$

where $\tilde{R}_t^i = \max[R_t, A_t^i(1 + \theta_t^i) - \theta_t^i R_t]$ is the return on internal funds. Agent i maximizes expected utility $E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t^i$ subject to the sequence of stochastic constraints (4), $t \geq 0$, and the initial budget constraint $c_0^i + s_0^i = y_0^i$.

LEMMA. *In each period t, agent i saves the constant fraction β of his wealth $\tilde{R}_{t-1}^i s_{t-1}^i$.*

Proof. See the Appendix.   ∎

3. THE FIRST-BEST EQUILIBRIUM

Under what conditions does the economy with limited contract enforcement achieve the first-best growth path? To answer this question we must look at the default incentives of those agents whose productivity level is highest. Those agents are the only borrowers in the first-best equilibrium. To guarantee that debt contracts are enforceable, we must make sure that the utility of an agent who repays his debt is no less than the utility of the agent who defaults, loses a share of his output, and is excluded from future capital trades.

Consider an agent whose productivity in period t is A_0, who invests $k_t^i = s_t^i + b_t^i$ and settles his debt in $t + 1$, so that he starts period $t + 1$ with net wealth $w \equiv A_0 k_t^i - R b_t^i = A_0 s_t^i$. Because this agent does not lose any output and earns the return $R = A_0$ on all future savings, no matter what his productivity is, his utility is

$$V(w) = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1} \ln[(1-\beta)(A_0\beta)^{\tau-t-1} w]$$

$$= \frac{\ln w}{1-\beta} + \frac{\ln(1-\beta)}{1-\beta} + \frac{\beta \ln(\beta A_0)}{(1-\beta)^2}. \tag{5}$$

Consider now what happens to this agent if he defaults in period $t + 1$. In that case, the agent starts this period with net wealth $\hat{w} \equiv m A_0 k_t^i$ and the agent's return on future savings is his idiosyncratic capital productivity A_t^i, because the agent has no access to the capital market. Hence, the agent starts period $t + 1$ with larger

wealth (if m is not too small), but he forgoes higher capital returns in the future. The agent's utility $\hat{V}(\hat{w})$ solves the recursive equation

$$\hat{V}(\hat{w}) = \ln((1-\beta)\hat{w}) + \beta \sum_{j=0}^{J} \pi_j \hat{V}(A_j \beta \hat{w}),$$

whose solution takes the following form:

$$\hat{V}(\hat{w}) = \frac{\ln(\hat{w})}{1-\beta} + \frac{\ln(1-\beta)}{1-\beta} + \frac{\beta \ln(\beta A_0)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} \sum_{j=0}^{J} \pi_j \ln(A_j/A_0). \quad (6)$$

The agent decides to settle the debt instead of defaulting if, and only if, the participation constraint $V(w) \geq \hat{V}(\hat{w})$ holds. Using (5) and (6), this becomes

$$\ln(\hat{w}) - \ln(w) \leq \frac{\beta}{1-\beta} \sum_{j=0}^{J} \pi_j \ln(A_0/A_j),$$

which yields, using the definitions of w and $\hat{w}$,

$$\ln\left(1 + \frac{b_t^i}{s_t^i}\right) + \ln(m) \leq \frac{\beta}{1-\beta} \sum_{j=0}^{J} \pi_j \ln(A_0/A_j). \quad (7)$$

This inequality specifies an upper limit on the ratio between external and internal funds, b_t^i/s_t^i, that is compatible with enforceability of the first-best allocation. When Y_t denotes aggregate output in period t, the internal funds of all agents with productivity A_0 are $\pi_0 \beta Y_t$, whereas total savings of all agents with lower productivity are $(1-\pi_0)\beta Y_t$. In the first-best, all these savings are lent to the most productive agents, so the ratio of external to internal funds is $(1-\pi_0)/\pi_0$. Because this implies that the individual ratio between external and internal funds b_t^i/s_t^i must be at least $(1-\pi_0)/\pi_0$ for some borrowers, we can substitute this into the participation constraint (7) to obtain a necessary and sufficient condition for the enforceability of the first-best equilibrium:

$$\ln(m) \leq \ln(m_0) \equiv \ln(\pi_0) + \frac{\beta}{1-\beta} \left(\ln A_0 - \sum_{j=0}^{J} \pi_j \ln A_j\right). \quad (8)$$

PROPOSITION 1. *The first-best equilibrium growth path is an equilibrium under limited enforcement of financial contracts if, and only if, $m \leq m_0$ where m_0 is defined as in* (8).

This result has a number of straightforward implications. Specifically, enforceability of the first-best equilibrium is guaranteed if one (or several) of the following requirements are met.

1. Agents are sufficiently patient. A large β makes agents suffer heavily from capital market exclusion. They do not default no matter how much output they may keep after default.
2. Property rights are strong. A sufficiently small m induces agents to settle their debt even when they do not suffer from credit market exclusion (i.e., even when $\beta = 0$).
3. A large share of agents have access to the best technology. If π_0 is big enough, the most productive agents do not need to borrow much, and hence they do not default even when property rights are weak.
4. A low mean productivity and/or a high variance of productivities lead to a smaller value of $\sum \pi_j \ln A_j$. Intuitively, a low mean productivity and/or a large variance make agents suffer more from credit market exclusion, making default less attractive.

4. CONSTRAINED EQUILIBRIUM

What happens if the requirement of Proposition 1 is not satisfied, so that the first-best is not enforceable? In this case the interest rate falls, and the most productive agents are borrowing-constrained. Consider a candidate equilibrium where the capital market return coincides with the capital productivity of agents with productivity A_ℓ, where $\ell \geq 1$. In such a situation, all agents with productivity strictly below $R = A_\ell$ do not produce, but they lend out all their savings, yielding the return R. All agents with productivity strictly above R are constrained. In a balanced growth path, the constraint on the ratio between external and internal funds is independent of time, and it turns out that it is the same for all agents of equal productivity; that is, $\theta_t^i = \theta_j$ if agent i has productivity $A_t^i = A_j$ in period t. The return on internal funds of these agents is $A_j(1+\theta_j) - \theta_j R_j$. Hence, an agent who participates in the capital market earns the return $\tilde{R}_j \equiv \max[A_j(1+\theta_j) - \theta_j R, R]$ on his savings.

How are borrowing constraints determined? Suppose agent i with productivity $A_j > R$ saves s_t^i in period t. If the agent settles debt, he starts period $t + 1$ with wealth $w = \tilde{R}_j s_t^i$, and his utility satisfies the recursive equation

$$\tilde{V}(w) = \ln((1 - \beta)w) + \beta \sum_{k=0}^{J} \pi_k \tilde{V}(\beta \tilde{R}_k w),$$

which has the solution

$$\tilde{V}(w) = \frac{\ln(w)}{1 - \beta} + \frac{\ln(1 - \beta)}{1 - \beta} + \frac{\beta \ln(\beta)}{(1 - \beta)^2} + \frac{\beta}{(1 - \beta)^2} \sum_{k=0}^{J} \pi_k \ln(\tilde{R}_k).$$

On the other hand, if the agent defaults, he enters period $t + 1$ with wealth $\hat{w} = m A_j(1+\theta_j)s_t^i$ and obtains utility $\hat{V}(\hat{w})$ as defined in (6). The borrowing constraint θ_j makes sure that the agent repays debt if the participation condition $\tilde{V}(w) \geq \hat{V}(\hat{w})$ holds. Moreover, the borrowing constraint is not too tight if this condition

is satisfied with equality. This yields

$$\ln[mA_j(1+\theta_j)] - \ln \tilde{R}_j = \frac{\beta}{1-\beta} \sum_{k=0}^{J} \pi_k (\ln \tilde{R}_k - \ln A_k).$$

This condition states that the gain from default on the left-hand side is just equal to the loss from exclusion from capital market trade. This loss is given by a logarithmic mean of the return premia $\tilde{R}_k/A_k \geq 1$ that the agent collects by being able to borrow (if $k < \ell$) or by being able to lend (if $k > \ell$). Substituting $\tilde{R}_j$ and simplifying yields the following system of equations in the unknown debt constraints θ_j, $j = 0, \ldots, \ell - 1$:

$$\ln m + \ln(1+\theta_j) - \ln\left[1 + \theta_j\left(1 - \frac{R}{A_j}\right)\right]$$

$$= \frac{\beta}{1-\beta} \sum_{k=0}^{\ell-1} \pi_k \ln\left[1 + \theta_k\left(1 - \frac{R}{A_k}\right)\right]$$

$$+ \frac{\beta}{1-\beta} \sum_{k=\ell}^{J} \pi_k \ln\left(\frac{R}{A_k}\right), \quad j = 0, \ldots, \ell - 1. \tag{9}$$

To check whether a solution $(\theta_0, \ldots, \theta_{\ell-1})$ of (9) is indeed an equilibrium with limited enforcement, we must make sure that the capital market is in equilibrium. That amounts to showing that total borrowing by all agents with productivity $A_j > R$ does not exceed total saving by all agents with productivity $A_j \leq R$. Total borrowing by the more productive agents is

$$B_t \equiv \sum_{k=0}^{\ell-1} \pi_k \theta_k \beta Y_t,$$

whereas total saving by the less productive agents is

$$S_t \equiv \sum_{k=\ell}^{J} \pi_k \beta Y_t.$$

Now $B_t \leq S_t$ iff

$$\sum_{k=0}^{\ell-1} \pi_k (1 + \theta_k) \leq 1. \tag{10}$$

Those agents with productivity A_ℓ are indifferent between producing themselves or trading securities. Whenever total savings by the less productive agents exceed total borrowing by the more productive agents, it may happen that these indifferent agents also borrow, and if this is the case, we must make sure that they do not

default either. In the Appendix we show that these agents' participation constraint takes the following form:

$$\ln(m) \leq \ln \pi_\ell - \ln \left[1 - \sum_{k=0}^{\ell-1} \pi_k (1 + \theta_k) \right]$$

$$+ \frac{\beta}{1-\beta} \left\{ \sum_{k=0}^{\ell-1} \pi_k \ln \left[1 + \theta_k \left(1 - \frac{R}{A_k} \right) \right] + \sum_{k=\ell}^{J} \pi_k \ln \left(\frac{R}{A_k} \right) \right\}. \quad \textbf{(11)}$$

Note that this condition falls together with the enforceability condition (8) when $\ell = 0$. Summarizing, we obtain the following characterization of an equilibrium in the limited enforcement economy with constrained agents.[2]

   PROPOSITION 2. *There is a balanced growth path at which the capital return is $R = A_\ell$, $\ell > 0$, and at which all agents with productivity A_j, $j < \ell$, are borrowing-constrained, if the system of participation constraints (9) has a solution in borrowing constraints $(\theta_0, \ldots, \theta_{\ell-1})$ satisfying the capital market equilibrium condition (10) and the enforceability condition for indifferent producers (11). In this equilibrium the economy grows at a constant factor*

$$g = \beta \left[A_\ell + \sum_{k=0}^{\ell-1} \pi_k (1 + \theta_k)(A_k - A_\ell) \right], \quad \textbf{(12)}$$

which is smaller than the first-best growth factor βA_0.

5. EXISTENCE AND MULTIPLICITY

In the preceding section we characterized features of equilibrium growth paths where the most productive agents are borrowing-constrained, but we did not establish conditions on the economic fundamentals that guarantee existence of these equilibria. To derive such conditions generally is difficult because it involves keeping track of all solutions to the equation systems (9) for arbitrary values of ℓ, satisfying the two restrictions (10) and (11). We can, however, characterize constrained equilibria in the special case $J = 1$ in which there are only two productivity levels. In this simpler environment, any constrained equilibrium of Proposition 2 has capital return $R = A_1$, and the borrowing constraint θ_0 on productive agents solves the participation constraint

$$\ln m + \ln(1 + \theta_0) = \frac{1 - \beta + \beta \pi_0}{1-\beta} \ln \left(1 + \theta_0 - \theta_0 \frac{A_1}{A_0} \right). \quad \textbf{(13)}$$

The participation constraint (11) on less productive agents is automatically satisfied because these agents do not borrow. The capital market equilibrium condition is $\pi_0(1 + \theta_0) \leq 1$.

Both the left-hand side and the right-hand side of (13) are strictly increasing in θ_0. At $\theta_0 = 0$, the left-hand side is less than the right-hand side (since $m < 1$). There will thus be a positive solution compatible with the capital-market equilibrium condition $\theta_0 \leq \theta_0^* = (1 - \pi_0)/\pi_0$ if the left-hand side is not less than the right-hand side at $\theta_0 = \theta_0^*$. This requirement amounts to

$$\ln m \geq \ln m_1 \equiv \ln \pi_0 + \frac{1 - \beta + \beta\pi_0}{1 - \beta} \ln \left[1 + \frac{1 - \pi_0}{\pi_0} \left(1 - \frac{A_1}{A_0} \right) \right]. \qquad \textbf{(14)}$$

If this inequality is strict, there is a constrained equilibrium with $\theta_0 < \theta_0^*$ at which production is inefficient: not all funds of the less productive agents are lent to the most productive agents.

PROPOSITION 3. *Let* $J = 1$, *so that there are only two productivity levels,* $A_0 > A_1$. *Then there exists an equilibrium in which the most productive agents are constrained at the interest rate* $R = A_1$ *if* $m \geq m_1$. *The equilibrium is production-inefficient if* $m > m_1$.

Conditions that favor a constrained equilibrium with inefficient production are just the opposite of the conditions leading to enforceability of the first-best equilibrium: weak property rights, impatient agents, a low share of productive agents, or small productivity differences may all induce an equilibrium with inefficient production and binding borrowing constraints.

What is more surprising is that enforceability of the first-best equilibrium (which is guaranteed by $m \leq m_0$) does not automatically preclude the existence of another production inefficient equilibrium. Indeed, Propositions 1 and 3 imply that there are multiple equilibria for some values of m whenever $m_1 < m_0$, which is equivalent to

$$\beta(1 - \pi_0) \ln \left(\frac{A_0}{A_1} \right) > (1 - \beta + \beta\pi_0) \ln \left[1 + \frac{1 - \pi_0}{\pi_0} \left(1 - \frac{A_1}{A_0} \right) \right]. \qquad \textbf{(15)}$$

This inequality is certainly satisfied if the productivity differential A_0/A_1 is sufficiently large for any $\beta < 1$. The inequality is also satisfied if β is sufficiently close to unity for any given value of $A_0/A_1 > 1$.

PROPOSITION 4. *Let* $J = 1$ *and suppose that* (15) *holds* (*e.g., the productivity differential is large or the discount factor is high*). *Then there are values of* m *for which there exist the following two coexisting equilibria in the economy with limited enforcement:*

(a) *The first-best equilibrium with growth factor* βA_0.
(b) *A production-inefficient equilibrium with growth factor*

$$\beta A_1 + \beta\pi_0(1 + \theta_0)(A_0 - A_1) < \beta A_0 \quad \text{for some} \quad \theta_0 \in (0, (1 - \pi_0)/\pi_0).$$

We remark that in generic circumstances with two equilibria, there will also be a third equilibrium "in between" the two equilibria (a) and (b). The third

equilibrium may be another production-inefficient equilibrium, but it may also be production-efficient with an interest rate $R \in (A_1, A_0)$. In such an equilibrium, all funds flow to the most productive agents, so that the growth rate is as in the first-best equilibrium; however, producers are still constrained, so that individual consumption is not smooth: the economy is *production-efficient* and *consumption-inefficient*.

6. CONCLUSIONS

In this essay, we have analyzed a growth model with limited commitment to loan repayment. We showed how weak property rights, impatient producers, and a low variability of productivity shocks can make the first-best growth path nonenforceable and establish a low-growth equilibrium with underdeveloped credit markets. Thereby our model accounts for a positive linkage between credit market development and economic growth.

The multiplicity result of Section 5 shows that fundamental and institutional parameters are not sufficient statistics for growth: economies with similar fundamentals and institutions may end up with quite different patterns of credit and growth. Put differently, even small changes in institutions may have a big impact on growth and credit market development.

NOTES

1. The assumption that productivity draws are uncorrelated across time can be relaxed, and the results turn out to be very similar. They are available on request from the authors.

2. There are also equilibria where the interest rate lies strictly in between two neighboring capital productivities A_ℓ and $A_{\ell+1}$. In such equilibria, no agent is indifferent between investing and security trading.

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APPENDIX

Proof of the Lemma. Because $\tilde{R}_t^i$ is certain at the beginning of period t, agent i's Euler condition is

$$c_{t+1}^i = \tilde{R}_t^i \beta c_t^i.$$

Let $w_t^i = \tilde{R}_{t-1}^i s_{t-1}^i$ be the wealth at the beginning of period t, and let λ_t denote agent i's savings rate out of wealth so that $c_t^i = (1 - \lambda_t)w_t^i$. Substituting this and $w_{t+1}^i = \tilde{R}_t^i \lambda_t w_t^i$ into the Euler condition shows that interest factors and wealth levels cancel out, and we are left with

$$\lambda_{t+1} = 1 - \frac{\beta(1 - \lambda_t)}{\lambda_t}.$$

The only solution to this equation that is compatible with the transversality condition is the stationary solution $\lambda_t = \beta$.   ∎

Proof of Proposition 2. What remains is to show the enforceability condition (11) for indifferent agents and to establish the formula for the growth factor. Indifferent agents with productivity A_ℓ invest $K_\ell \equiv S_t - B_t = \beta Y_t [1 - \sum_0^{\ell-1} \pi_k(1+\theta_k)]$, and they save $S_\ell \equiv \pi_\ell \beta Y_t$. Hence these agents borrow

$$K_\ell - S_\ell = \beta Y_t \left[1 - \pi_\ell - \sum_0^{\ell-1} \pi_k(1 + \theta_k) \right].$$

Note that this expression may also be negative, in which case these agents are net savers. The ratio between external and internal funds for these agents is thus

$$\theta_\ell = \frac{1 - \pi_\ell - \sum_0^{\ell-1} \pi_k(1 + \theta_k)}{\pi_\ell},$$

and by reasoning similar to the derivation of equations (9), their participation constraint is

$$\ln m + \ln(1 + \theta_\ell) \le \frac{\beta}{1 - \beta} \left\{ \sum_{k=0}^{\ell-1} \pi_k \ln\left[1 + \theta_k \left(1 - \frac{R}{A_k} \right) \right] + \sum_{k=\ell+1}^{J} \pi_k \ln(R/A_k) \right\}.$$

After substitution of θ_ℓ, this becomes inequality (11).

Consider now the growth factor. If Y_t is aggregate output in t, aggregate output in $t+1$ is the sum of output of all agents with productivities $A_j \ge A_\ell$:

$$Y_{t+1} = \sum_{k=0}^{\ell-1} \pi_k A_k(1 + \theta_k)\beta Y_t + A_\ell(S_t - B_t)$$

$$= \beta Y_t \left[A_\ell + \sum_{k=0}^{\ell-1} \pi_k(1 + \theta_k)(A_k - A_\ell) \right].$$

This establishes (12). The growth factor is below βA_0 because g is a convex combination of $\beta A_k \le \beta A_0$, $k = 0, \dots, \ell$, with strict inequality for $k > 0$.   ∎