International Journal of Microwave and Wireless Technologies, 2011, 3(1), 67–75.
doi:10.1017/S1759078711000018

Noise properties of balanced amplifier configurations

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This paper analyzes the noise properties of balanced amplifier configurations in terms of noise waves and discusses the effect of various amplifier configurations on the noise and signal parameters. It shows how the noise generated in the load on the input coupler of a balanced amplifier deteriorates the amplifiers noise resistance parameter with respect to that of its component amplifiers. The properties of two new balanced amplifier configurations will be discussed, which enable the reduction or elimination of this deterioration.

Keywords: Balanced amplifier, Microwave amplifier, Low noise amplifier, Noise parameters, Noise waves

Received 11 August 2010; Revised 4 January 2011; first published online 10 February 2011

I. INTRODUCTION

Balanced amplifiers [1, 2] have the advantage of simultaneous noise and power matching over octave bandwidth. The component amplifiers may be noise matched at the cost of power matching at their inputs. Power matching at the input of the balanced amplifier is achieved by placing two component amplifiers in a configuration with quadrature couplers, which cancels the input reflection of the component amplifiers at the balanced amplifier input, while maintaining their gain and noise properties.

For balanced amplifiers, operated at cryogenic temperatures, extremely low-noise temperatures below 5 K have been common for a long time [3, 4], e.g. in the 1–2 GHz frequency range, mainly for application in radio astronomy. At room temperature balanced amplifiers with 30–35 K noise temperature have been in use in this frequency range [4]. More recently, dual low-noise integrated amplifiers have become available for use in balanced amplifiers [5]. These room-temperature amplifiers find application in base stations, but also in radio astronomy. In particular, they may well be suited for a new large radio telescope with unprecedented sensitivity, resolution, and survey speed with a collecting area of a square kilometer: the square kilometre array (SKA, [6]). Because of the cost of the large number of required amplifiers for use in phased array feeds (hundreds per telescope, [7]) and aperture array tiles as one of the concepts for the SKA up to approximately 1 GHz (with 10 000 antenna elements and low noise amplifier (LNAs) per phased array telescope, [8]), room-temperature amplifiers are preferable over cryogenic ones. Although recently single-ended room-temperature LNAs with outstanding noise performance have been reported [9–11] and are actually being used in current prototype systems [7, 10], the excellent input matching over broad bandwidth make balanced amplifiers a serious candidate for the first-stage low-noise amplifier of these systems, despite the increased noise temperature with respect to single-ended amplifiers. Nevertheless, lowest possible noise performance is of utmost importance for the SKA and understanding and minimizing, even small, noise contributions to the system noise budget [10] is essential to obtain this goal. An example of such a noise contribution and an important issue for array systems being considered for the SKA, is the noise coupling between the antenna elements, which may result in increased system noise temperatures due to a changing effective impedance (active reflection coefficient) at the antenna ports as a function of beam steering [12, 13]. Therefore, the sensitivity of the noise temperature of the amplifiers to impedance changes at their inputs should be as small as possible. This sensitivity is described by one of the noise parameters, the noise resistance R_n.

A disadvantage of room-temperature balanced amplifiers is that the load resistor on the input quadrature coupler may contribute considerably to the amplifier input noise, in case the signal source input port is not ideally matched [14]. In view of the previous discussion it is worthwhile to study the noise parameters of the balanced amplifier configuration and the R_n-value in particular. This paper gives an analysis of the noise properties of balanced amplifier configurations, based on the noise properties of the component amplifiers. These noise properties may be described using noise waves, emanating from the input and output ports of the component amplifiers [15, 16]. In general, the noise waves are (partly) correlated. To introduce the type of analysis used in this paper, a single-ended amplifier stage as a basic element of the balanced amplifier will first be analyzed in Section II.A, leading to the noise parameters for a noise matched LNA. Then in Section II.B the noise properties and noise parameters of a balanced amplifier are analyzed.

This paper shows that the noise from the terminating resistor may be interpreted as a deterioration of R_n. However, if one replaces the resistive load by a short, the noise contribution from the load may be eliminated, but this will have adverse effects on the input reflection coefficient and the noise properties of the balanced amplifier. Results of

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the analysis of these effects will be discussed in terms of the noise and other properties of the single-ended amplifier stages. The noise properties of two new configurations of balanced amplifiers that are able to reduce the noise contribution from the input coupler termination, as well as the effect on other amplifier properties, have been analyzed and will be discussed in Section III.

II. LNA NOISE ANALYSIS

A) Single-ended LNA

In this section the noise matched single-ended LNA will be defined as a basic element of the balanced amplifier, using a description of the LNA noise properties in terms of its input and output noise waves. The main purpose is to introduce the terminology and the approach that will be used for the noise analysis in this paper.

1) NOISE ANALYSIS

Figure 1 shows a single-ended LNA of voltage gain g with its input and output noise waves b_1 and b_2. With the amplifier input connected to a source with reflection coefficient Γ_s, part of the noise wave b_1 will be reflected back into the amplifier, resulting in a total outgoing wave at the amplifier output

$$b_{out} = g\Gamma_s b_1 + b_2.$$

In the most general case that the input and output noise waves both have correlated and uncorrelated parts, the average output noise power per unit bandwidth is given by the ensemble average

$$\begin{aligned}\langle|b_{out}|^2\rangle = |g(\Gamma_s + k/g)|^2\langle|b_{1c}|^2\rangle \\ + |g\Gamma_s|^2\langle|b_{1u}|^2\rangle + \langle|b_{2u}|^2\rangle.\end{aligned} \tag{1}$$

In (1), b_{1u}, b_{2u}, and b_{1c}, b_{2c} are the uncorrelated and correlated parts of b_1, b_2, respectively. The factor k represents the relation between the correlated parts of the input and output noise waves, $b^i_{2c} = (k/g)b_{1c}$, with b^i_{2c} the correlated part of b_2, referred to the input.

The noise temperature associated with the noise wave is $\langle|b_{out}|^2\rangle/k_B$, where k_B is Boltzmann's constant.

When the LNA is noise matched, (1) shows a minimum and the minimum LNA noise temperature T_{min} is achieved, under the condition $\Gamma_s = \Gamma_{opt}$, defining three of the four LNA noise parameters. An expression for Γ_{opt} may be derived by differentiating (1) with respect to Γ_s and equaling the result to zero. This will lead to complex functions for Γ_{opt} and T_{min}, which are dependent on k, the phase difference between k and Γ_s, as well as the ratio of the average noise powers of the correlated and uncorrelated parts of b_1.

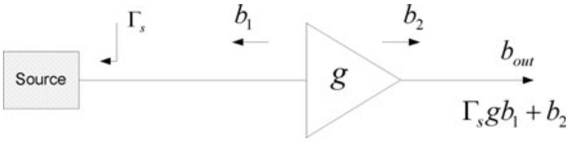

Fig. 1. LNA with input and output noise waves.

However, there are two interesting simplified cases. If $b_{1u} = 0$ (full correlation with factor k between b_1 and b_{2c}), it may be easily shown that $\Gamma_{opt} = -k/g$, for which the associated minimum noise temperature is only determined by the uncorrelated part of the output noise wave.

For the case that there is no correlation between b_1 and b_2, then $k = 0$ and (1) becomes

$$\langle|b_{out}|^2\rangle = |g\Gamma_s|^2\langle|b_{1u}|^2\rangle + \langle|b_{2u}|^2\rangle. \tag{2}$$

The average output noise power in (2) has a minimum, corresponding with T_{min}, for $\Gamma_s = \Gamma_{opt} = 0$. As in the previous case, this minimum is equal to

$$\langle|b_{out}|^2\rangle = \langle|b_{2u}|^2\rangle. \tag{3}$$

Formulas for the relation between input and output noise waves and the noise parameters Γ_{opt}, T_{min}, R_n, as will be discussed in Section II.A.2, are given in the Appendix of [16]. In terms of these noise parameters it follows from (2) that for an LNA with uncorrelated input and output noise waves $\Gamma_{opt} = 0$, giving an effective minimum input noise temperature $T_e = T_{min}$. So uncorrelated input and output noise waves lead to $\Gamma_{opt} = 0$, but the latter condition is not sufficient for the input and output noise waves to be uncorrelated. For this to be true in case $\Gamma_{opt} = 0$, an additional necessary condition is that $S_{11}^{LNA} = 0$, as may be derived from formula (34) in [16].

2) NOISE-MATCHED LNA

A general expression for the noise temperature of an LNA as a function of a commonly used set of noise parameters is given by

$$T_e = T_{min} + \frac{4R_nT_0}{Z_0}\frac{|\Gamma_s - \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2(1 - |\Gamma_s|^2)}. \tag{4}$$

In (4), Z_0 is the characteristic impedance and $T_0 = 290$ K is the, IEEE-defined, standard temperature. The LNA noise properties as a function of the input reflection coefficient Γ_s are fully described by (4).

The ultimate goal in the design of an LNA is to achieve the lowest possible noise temperature T_{min}, given by the properties of the semiconductor components in the input stage, by noise matching of the transistor. This is done by providing the optimum source impedance or reflection coefficient for minimum noise at the transistor input by transformation of the source impedance to the optimum impedance with a matching circuit. If the matching circuit is lossless, new noise parameters of the noise-matched LNA may determined by a lossless transformation at the LNA input reference plane. The resulting $T_{min}{}^1 = T_{min}$ is invariant under lossless transformation and for successful optimum noise matching, $\Gamma_{opt}{}^1 = 0$ at the input reference plane of the LNA. Herewith three of the four noise parameters of the LNA are defined. The fourth noise parameter, the noise resistance $R_n{}^1$, is derived using the invariant constant $N = R_nG_{opt}$ [17–19], with G_{opt} the real part of the optimal noise admittance.

$N = R_nG_{opt} = R_n{}^1G_{opt}{}^1$ leads to $R_n{}^1 = R_nG_{opt}Z_0$ (from $\Gamma_{opt}{}^1 = 0$ it follows that $G_{opt}{}^1 = 1/Z_0$), completing the new set of four transformed noise parameters, fully describing the LNA noise properties at its input reference plane. This is illustrated in Fig. 2, which shows the original (transistor two port) noise waves and noise parameter Γ_{opt}, as well as those for the newly defined LNA noise-matched two port.

Substituting the four new noise parameters in (4), results in a simplified formula for the effective noise temperature of the noise-matched LNA, expressed in the original (transistor) noise parameters and as a function of the source reflection coefficient:

$$T_e = T_{min} + 4R_n G_{opt} T_0 \frac{|\Gamma_s|^2}{1 - |\Gamma_s|^2}. \tag{5}$$

Using [19], it may be easily derived that $4R_n G_{opt} T_0 = 4NT_0 = 2T_{min}$ in the low-frequency limit for well-behaved field effect transistor (FETs) and hetero-junction bipolar transistor (HBTs) according to the simple noise-equivalent circuits given there. With this simplification (5) may be rewritten as

$$T_e = T_{min}\left(1 + \frac{2|\Gamma_s|^2}{1 - |\Gamma_s|^2}\right), \tag{6}$$

which shows that the effective noise temperature of a noise-matched LNA in the low-frequency limit depends on the noise parameter T_{min} and the magnitude of the source reflection coefficient $|\Gamma_s|$, but is independent of the phase of Γ_s. Figure 3 shows the noise temperature, normalized to T_{min}, as a function of $|\Gamma_s|$ for an LNA according to (6), in the low-frequency limit of [19].

B) Balanced LNA

In general, the noise-matched LNA in Fig. 2 does not provide simultaneous power and noise matching at the LNA input, but may have achieved noise matching at the expense of a high input reflection coefficient ρ. The balanced amplifier [1, 2], shown in Fig. 4, provides a solution for simultaneous power and noise matching of an LNA. The quadrature input coupler directs the reflected signals from the component amplifiers (noise matched according to Fig. 2) in the two branches to the termination on the input coupler port, where they are absorbed, while the reflected signals cancel at the input port of the coupler.

According to [14], the average output noise power is given as

$$\langle |b_{out}|^2 \rangle = (|g\Gamma_s|^2 + |k|^2)\langle |b_1|^2 \rangle, \tag{7}$$

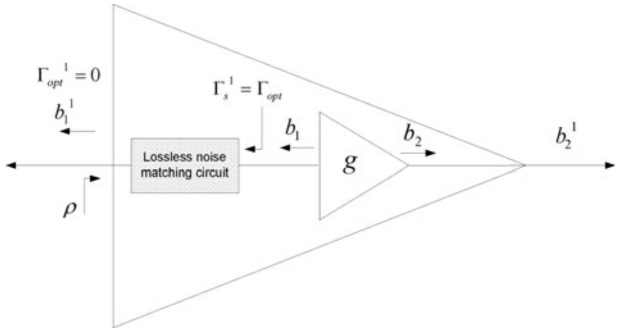

Fig. 2. Noise-matched LNA with lossless noise-matching circuit. The optimum source reflection coefficient is zero and the resulting input reflection coefficient of the noise-matched LNA is ρ.

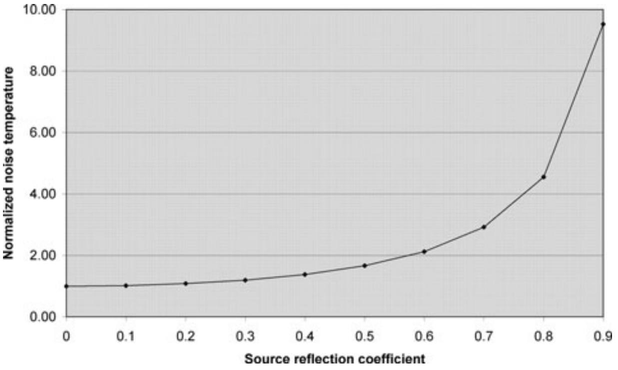

Fig. 3. LNA noise temperature, normalized to T_{min}, as a function of $|\Gamma_s|$, according to (6).

with k the correlation factor between the input and output noise waves of the component amplifiers and Γ_s the source reflection coefficient. In case of uncorrelated input and output noise waves, $k = 0$ and the average output noise power is given by (2).

Inspection of (7) leads to an interesting observation. A minimum in the output noise power of the balanced amplifier will always occur for $\Gamma_s = \Gamma_{opt} = 0$. This condition for *the location of the minimum* is independent of the properties of the noise-matching circuit of the component amplifiers. However, these properties do influence the effective values of k and b_1, and therefore the *value of the minimum*, which will depend on the transformation in the noise matching circuit in such a way that optimum noise matching leads to effective values of k and b_1, resulting in T_{min}. In case optimum noise matching has not been realized, the effective minimum noise temperature of the balanced amplifier will have a value higher than T_{min} of the component amplifiers. For an ideal balanced amplifier (ideal quadrature couplers and noise-matched component amplifiers with equal properties) its minimum noise temperature and gain are the same as those of the two component amplifiers under the same conditions ($T_e = T_{min}$ at $\Gamma_s = \Gamma_{opt} = 0$), while ideally the input reflection coefficient of the balanced amplifier $S_{11}^{\mathrm{LNA}} = 0$. As a result, (7) describes the situation that the noise waves emerging from the input and output of the balanced amplifier are uncorrelated, as may also be concluded from the discussion at the end of Section II.A.1. This means that $k = 0$ and that instead of (7), (2) correctly describes the output noise of the ideal balanced amplifier, using the effective input and output noise waves of that amplifier, determined by the properties of the component amplifiers with their associated noise-matching circuits in the balanced configuration.

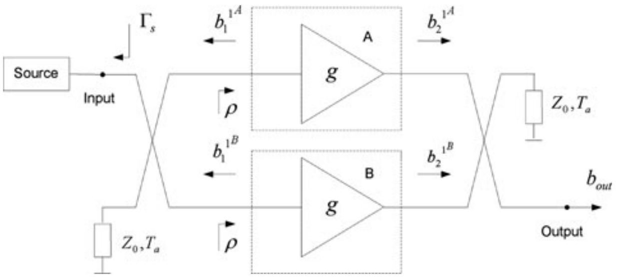

Fig. 4. Balanced amplifier with noise matched component amplifiers according to Fig. 2.

1) NOISE RESISTANCE OF THE BALANCED AMPLIFIER

The previous discussion shows that three of the four noise parameters of the balanced amplifiers are equal to those of the component amplifiers (T_{min}, $\Gamma_{opt} = 0$). However, this is not the case for the fourth noise parameter R_n. The reason lies in the noise generated in the termination of the input coupler (Z_0 at ambient temperature T_a), of which a part may be reflected at the input port of the balanced amplifier [14] and then will give an additional noise contribution to the effective noise temperature. Because this noise is uncorrelated with the noise of the component amplifiers, it has no effect on T_{min} and Γ_{opt}, but it does change the dependency of the balanced amplifier effective noise temperature on the source-reflection coefficient and its deviation from Γ_{opt}. Therefore, R_n^1 of the balanced amplifier including the noise contribution of the termination on the input coupler will increase with respect to R_n, the noise resistance of the balanced amplifier without taking into account the noise contribution from the termination. R_n^1 can be found after including the effect of the noise of the termination in the noise analysis.

The derivation starts with the noise from the component amplifiers referred to the input port of the balanced LNA, which is represented by the noise temperature formula (4), where we define R_n as the noise resistance of the noise-matched component amplifiers, with $\Gamma_{opt} = 0$. The noise from the termination at ambient temperature T_a on the input coupler gives rise to an increase in noise temperature, represented by $|\rho \Gamma_s g|^2 T_a$ at the output of the balanced LNA [5]. Division by the available gain $|g|^2(1 - |\Gamma_s|^2)$ gives the noise temperature increase at the input port, leading to a total effective noise temperature for a balanced amplifier

$$\begin{aligned} T_e &= T_{min} + \frac{4R_n T_0}{Z_0}\frac{|\Gamma_s|^2}{1-|\Gamma_s|^2} + T_a\frac{|\rho\Gamma_s|^2}{1-|\Gamma_s|^2} \\ &= T_{min} + \frac{4R_n^1 T_0}{Z_0}\frac{|\Gamma_s|^2}{1-|\Gamma_s|^2}. \end{aligned} \tag{8}$$

Using (8), the increase in normalized noise resistance (R_n^1/Z_0) of the balanced amplifier with respect to (R_n/Z_0) may be found as

$$\Delta r_n = \frac{T_a|\rho|^2}{4T_0}. \tag{9}$$

The increase in r_n for poorly power matched (ρ close to 1) component amplifiers of a room-temperature balanced amplifier has a maximum value of 0.25, which is significant with respect to the r_n-value of state-of-the-art transistors (e.g. 0.04 up to a few GHz for an Avago ATF54143). At cryogenic temperatures the effect of the noise from the termination is greatly reduced, resulting in much lower values for the increase in r_n, e.g. a maximum of 0.013 at an ambient temperature of 15 K. For component amplifiers with moderate input reflection coefficient ρ, Δr_n will be reduced with respect to its maximum value, proportional to $|\rho|^2$.

2) ELIMINATION OF THE NOISE CONTRIBUTION DUE TO THE LOAD AND ITS EFFECTS

Replacing the termination on the input coupler by a short circuit would effectively eliminate its noise contribution. However, the noise waves b_1^{1A} and b_1^{1B}, emerging from the inputs of the component amplifiers (see Fig. 4) will now be reflected at the short-circuit termination and will enhance the noise wave power emerging from the balanced amplifier input with a factor $1 + |\rho|^2$, making the noise behavior more sensitive to input reflections. This is derived in the Appendix, where it is also shown that for uncorrelated input and output noise waves

$$\Delta r_n = r_n|\rho|^2, \tag{10}$$

According to (10) the maximum increased r_n-value of the balanced amplifier may be two times larger than r_n of the component amplifiers.

In case the input and output noise waves are correlated, an expression for Δr_n will become more complicated and less suitable to use in a noise parameter expression, because it is no longer dependent on only the amplifier properties (also see the Appendix).

Having a short-circuit termination will also cause considerable deterioration of the input matching of the balanced amplifier, as is shown in the Appendix. Although it would result in an input reflection coefficient ρ^2 (compared to, ideally, zero input reflection coefficient for a balanced amplifier with a load on the input coupler), this still gives an improvement in return loss of a factor of two, with respect to that of the component amplifiers. In some cases the trade-off between input matching and noise performance (low r_n and low sensitivity to varying source impedance) could result in a choice for this solution.

III. OTHER BALANCED CONFIGURATIONS

The increase in r_n of a balanced amplifier due to the effect of the noise produced in the termination on the input coupler results in an increased sensitivity of the noise temperature to changes in the source reflection coefficient at the amplifier input port. This is of particular importance in low-noise receiving-antenna arrays, where mutual coupling between the array-antenna elements causes a varying source reflection as a function of scan angle [12, 13]. The result will be an increased and varying system noise temperature with respect to the minimum possible system noise temperature. It is therefore important to reduce r_n as much as possible, e.g. for application in low-noise array systems for radio astronomy. Novel balanced amplifier configurations that potentially eliminate or reduce the effect of the noise in the input coupler termination on r_n with respect to the original balanced amplifier, were studied and will be discussed in this section.

A) Double-balanced configuration

Figure 5 shows a double-balanced amplifier configuration, i.e. two identical balanced amplifiers in a balanced configuration between quadrature couplers, with a common termination connected to their input couplers via another quadrature coupler. Using this configuration the noise from the termination shown in Fig. 5, reflected at the inputs of the component amplifiers, cancels at the input port of the double-balanced amplifier. As a result this balanced amplifier configuration would have the same r_n-value and other noise parameters as the component amplifiers, assuming equal component amplifiers and ideal couplers. This will be discussed in Section III.A.2. The analysis will also take into account in Section III.A.3 the effect of the second termination (shown as a short on Fig. 5) of the coupler with the common termination at T_a.

1) NOISE FROM THE COMPONENT AMPLIFIERS

An analysis of the effect of the noise from the component amplifiers, similar to that in [14], shows the same results as for the balanced amplifier given there. The common terminations on the input couplers of the two balanced amplifiers are assumed to both have loads with characteristic impedance Z_0, but at this stage of the analysis their noise contribution is not yet taken into account. This balanced amplifier configuration has a good input power matching and its output noise is independent of the phase of the source reflection coefficient, which means that the input and output noise waves of the balanced configuration must be uncorrelated (see also Sections II.A.1 and II.B).

2) NOISE FROM THE INPUT TERMINATION

Analysis of the noise propagating from the common termination Z_0 at ambient temperature T_a shows that the noise, reflected at the inputs from the component amplifiers of the two balanced amplifiers, is canceled at the input port of this balanced configuration (see Fig. 6) and thus does not contribute to the output noise wave. At this stage of the analysis the effective input noise temperature of the balanced LNA is only determined by the noise properties of the component amplifiers and its noise parameters would be the same as for the component amplifiers.

3) EFFECT OF TERMINATION OF OTHER COUPLER PORTS

In this section the effect of the terminations on the input coupler and on the second coupler will be discussed.

Termination on the input coupler

The termination on the input coupler has the same effect as for a balanced amplifier. The result depends on the input reflection coefficient of the amplifiers in the upper and lower branches, which in this case are balanced amplifiers with good input matching (small ρ). For a 50 Ω termination at room temperature this will result in a negligible increase in the effective input noise temperature and r_n. To reduce the effect of the termination even further, the 50 Ω load could be replaced by a short circuit, which will however introduce second-order effects in the input matching. In this double-balanced configuration with good input matching the effect will be negligible, where for the basic balanced amplifier design this would result in a strong deterioration of the input matching, because of the large reflection at the input of the component amplifiers. Therefore, a 50 Ω termination with its associated noise contribution is preferable at the input coupler of the basic balanced amplifier, but could be replaced by a short on the input coupler of the double-balanced design. Noise or signal reflected at the short circuit on the input coupler would be absorbed in the termination on the output coupler anyway and therefore will not affect the signal and noise properties of this balanced configuration.

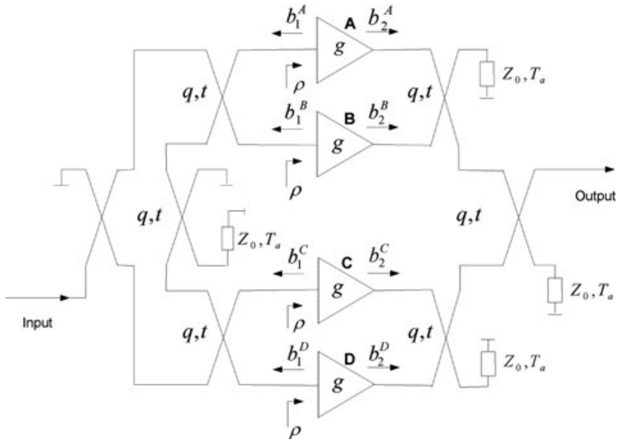

Fig. 5. Double-balanced configuration with noise waves from component amplifiers.

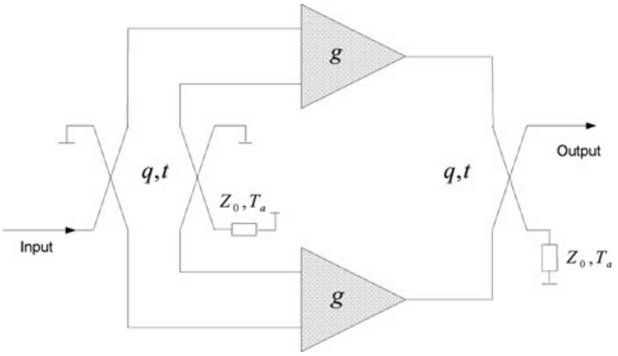

Fig. 6. Double-balanced configuration with noise components from the termination on the second quadrature coupler.

Termination on the fourth port of the second coupler

Similar to the basic balanced amplifier, noise generated in the termination on the fourth coupler port of this configuration will emerge from the signal input port and results in the same sensitivity to input impedance variations as for the basic balanced amplifier, and so nothing is gained. If one replaces the load by a short circuit, then the noise contribution disappears but the reflection coefficient at the input increases to ρ^2, identical to the behavior of the balanced amplifier with a short circuit as termination on the input coupler. From more detailed analysis of this double-balanced configuration, it appears that its behavior with respect to signal and noise properties is identical to the simpler basic balanced amplifier. This is a result of the reciprocity property of a lossless multi-port, which has the property that it cannot absorb energy at a particular port, without generating noise from a termination at that port ending up at the reciprocal port.

4) CONCLUSION

The previous analysis shows that it is not possible to use the double-balanced configuration of Figs 5 and 6 to improve the noise properties with respect to the basic balanced amplifier, without sacrificing the input power matching of that amplifier. The same behavior may be achieved with the basic balanced amplifier, and so there are no arguments to use the more complex double-balanced configuration. Nevertheless, the present study has gained more insight in the behavior of the balanced amplifier for low-noise applications.

B) Differentially balanced configuration

The discussion in Section III.A raises the question as to whether it is possible at all to define a configuration with balanced amplifiers, which enables the suppression of the noise from the common termination, necessary to absorb the energy reflected at the inputs of the component amplifiers. It has been shown in the previous section that this is not possible with a quadrature coupler, modeled as a lossless four port, which has power matching at all ports. Theoretically, an ideal three port with one common termination would be a solution, but in practice does not exist. Possibly a lossy or reflective three port in the configuration shown in Fig. 7 could provide a solution. These differentially balanced configurations will be studied in more detail in this section.

To achieve proper cancellation at the input, of the reflected signal or the internally generated noise, 180° phase difference is required between the two paths from the noise-generating termination to the input port. This may be realized with a combination of a 180° degrees coupler and an in-phase coupler which represents the differentially balanced LNA, shown in Figs 7 and 8. It provides the possibility for use in a differential configuration, omitting the input coupler, with a direct connection from the amplifier inputs to e.g. an antenna with a differential output.

1) SIGNAL ANALYSIS

Two possible implementations of a passive three-port splitter will be discussed in this section: a lossless three port with reflection at the output ports and finite isolation between them, and secondly a lossy three port, absorbing all energy at its output ports, with no reflections at the ports and finite isolation between them.

Use of a reflective three port

A lossless three port will have reflections r at its output ports, at which signal reflected at the inputs of the component amplifiers will be reflected back to those inputs and finally will cause reflected signal at the input of the balanced amplifier configuration. A second contribution to the reflected signal at the input is due to the finite isolation i between the splitter ports and finds its way through reflections at the component amplifiers of the second branch of the differentially balanced configuration. The signal reflection coefficient at the input port may be derived as

$$\rho^2(r - i), \tag{11}$$

which will be smaller than the reflection coefficient of the balanced configuration with a short-circuit termination.

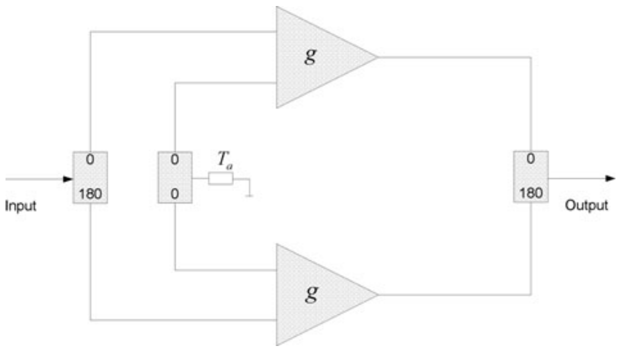

Fig. 7. Differentially balanced configuration with suppression at the amplifier input of the noise from the common termination.

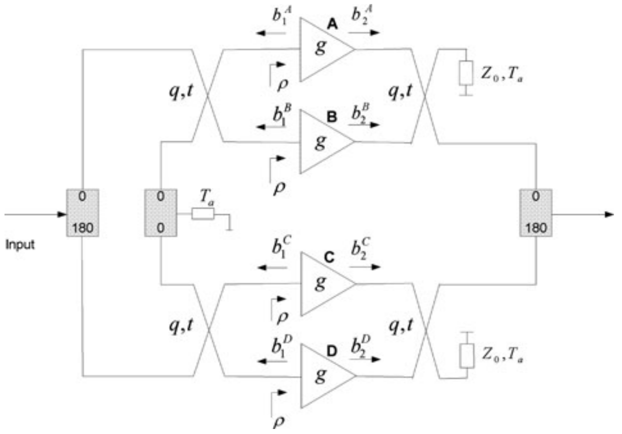

Fig. 8. Differentially balanced configuration with noise components from component amplifiers.

Use of a lossy three port

The lossy three port will not reflect any signal coming from the component amplifiers inputs. However, some signal may leak from one port to the other and will cause reflected signal at the input port of the differentially balanced configuration according to (11) with $r = 0$. This will result in an input reflection coefficient $\rho^2 i$ which will be considerably better than that of the component amplifiers, as well as the balanced amplifier with a short-circuit termination. Expressed as a return loss, the input reflection will be equal to twice the return loss of the component amplifiers, plus the isolation (dB) between the splitter ports.

2) NOISE ANALYSIS

A similar noise analysis as in Section III.A applies for the noise from the component amplifiers (see also the illustration in Fig. 8). Common (correlated) noise from the in-phase splitter propagates in a symmetric way through both input branches to the 180° input splitter and cancels at the input port. The disadvantage of the use of the in-phase splitter is that it either has non-matched ports or loss, as well as a finite isolation between the two output ports. In case of a lossy three port, uncorrelated noise from the splitter will contribute to the noise at the input port. As was shown in the previous section, both reflection and isolation of the three port will affect the input matching of this configuration and lead to a reflection coefficient $\rho^2(r - i)$. Nevertheless, this is a factor $r - i$ better than for the double-balanced amplifier with short-circuit termination. Derivation of the enhancement of the input noise wave shows the same factor to appear in the expression for the reflected input noise wave, leading to

$$\langle |b_{in}|^2 \rangle = |\Gamma_s|^2 (1 + |\rho(r - i)|^2) \langle |b_1|^2 \rangle, \tag{12}$$

giving a reduction in the enhanced noise wave, leading to a reduced sensitivity to input impedance variations with respect to the balanced amplifier (A.3). The same assumptions as in the Appendix have been used in this derivation.

Reflective three port

Equation (12) describes the reflected input noise wave for the lossless three port with reflection and finite isolation and is the same as (A.3) with ρ replaced by $\rho(r - i)$. With the same modification formula (A.7) describes the output noise for this configuration with the reflective three port. The noise from the common load on this three port does not contribute to the noise temperature, because it is canceled at the input port.

Lossy three port

For the lossy three port (A.3) and (A.7) describe the effects of the noise from the component amplifiers with ρ replaced by $i\rho$, because $r = 0$. These effects will in general be much smaller than for the reflective three port and for the balanced amplifier with short-circuit termination.

The noise from the common part of the three port and load is correlated and is thus canceled at the input port. However, there is an uncorrelated contribution from losses in the splitter, which leads to a noise power equal to

$$(1 - \alpha)T_a|\rho|^2 \leq T_a|\rho|^2, \tag{13}$$

with $\alpha \leq 1$ the loss coefficient of the splitter with respect to an ideal lossless splitter, giving rise to uncorrelated noise in both branches and T_a the temperature of the loss. This noise contribution will be smaller than for the original balanced amplifier with a load on the input coupler.

3) CONCLUSION

The results from this section show that the differentially balanced configuration with reflective three port is more favorable than the balanced amplifier with short-circuit termination, with respect to both noise and signal properties. The configuration with a lossy three port has better input matching and is comparable to the basic balanced amplifier, with less sensitivity of the noise temperature to source impedance variations.

IV. CONCLUSION

The noise properties and the input reflection coefficients have been analyzed and discussed for various configurations of balanced amplifiers. It has been shown that replacing the load on the input coupler of the basic balanced amplifier by a short circuit will reduce the sensitivity of the noise temperature to input impedance variations, but will deteriorate the input reflection coefficient. A detailed analysis of the noise from the component amplifiers is given in the Appendix. Two new balanced configurations have been proposed and analyzed. The double-balanced configuration has the same properties as the balanced amplifier and does not have the possibility to improve on them. It is more complex with twice as much component, without additional advantages and therefore is not considered as a viable alternative. The differentially balanced configuration with either a lossless or lossy three port does show advantages with respect to the balanced amplifier, with improved input reflection coefficient and reduced sensitivity of the noise temperature to input impedance variations. Therefore, it is a serious alternative to replace the balanced amplifier in situations where the sensitivity of the amplifier noise temperature to input impedance variations is of importance. A next step after the analysis of these balanced configurations is the realization of a differentially balanced amplifier in order to verify the expected performance in practice.

APPENDIX

EFFECT OF A SHORT-CIRCUIT TERMINATION ON THE INPUT COUPLER OF A BALANCED AMPLIFIER

Figure 9 shows the basic balanced amplifier with a short-circuit termination on the input coupler, instead of a load generating noise at ambient temperature. There is no direct effect of the short on the signal or noise at the output port of the balanced amplifier, as any signal or noise wave emerging from or reflected at the fourth port of the input coupler will be absorbed in the load at the output coupler or canceled at the output port. However, the effects for signal and noise components at the input port are not negligible (and as a result will appear at the amplifier output port). This will be shown in the following sections, using a noise analysis of the balanced amplifier, which is similar to that published in [14] and uses the same reasoning, also based on [15]. The same nomenclature will be used as in [14] as much as possible, to ease the comparison of results.

A) Effect on input reflection coefficient

The ideal balanced amplifier with a load on the input coupler has perfect matching at the input. Replacing the load by a short will cause the reflected signal $2tq\rho$ from the inputs of the component amplifiers to be reflected at the short, back to the amplifiers, where they are again reflected $(2t^2q + 2tq^2)\rho^2$ and coupled to the input port, giving $4t^2q^2\rho^2$. With $|t|^2 = |q|^2 = 1/2$ the reflected input signal becomes $|\rho|^2$, which leads to a degradation with respect to the perfect matching of an ideal balanced amplifier. However, an improvement in the input return loss is shown of a factor of two with respect to the input return loss of the component amplifiers.

B) Effect on input noise wave

The noise waves b_1^A and b_1^B emerging from the inputs of the component amplifiers will be coupled directly to the input

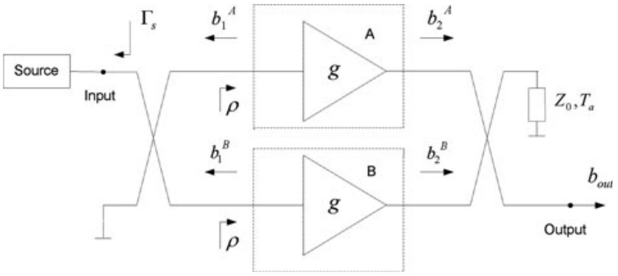

Fig. 9. Noise components for a noise-matched balanced amplifier with short-circuit termination on the input coupler.

port, but additionally a part will now be reflected at the short circuit, back into the amplifiers, where they are reflected (ρ) and also coupled to the input port. The total noise wave emerging from the input port will be reflected at the input with Γ_s and contributes to the effective amplifier noise temperature. This ingoing noise wave b_{in} is described by (see Fig. 9)

$$b_{in} = q\Gamma_s(1 + 2\rho t^2)b_1^A + t\Gamma_s(1 + 2\rho q^2)b_1^B. \tag{A.1}$$

Assuming that $\langle |b_1^A|^2\rangle = \langle |b_1^B|^2\rangle = \langle |b_1|^2\rangle$ and taking into account that $t^2 = -q^2$, the average input noise power per unit bandwidth is given by

$$\langle |b_{in}|^2\rangle = |\Gamma_s|^2 \frac{1}{2}\begin{pmatrix} |1 + |\rho|e^{j\Phi}|^2 \\ +|1 - |\rho|e^{j\Phi}|^2 \end{pmatrix}\langle |b_1|^2\rangle. \tag{A.2}$$

In (A.2) Φ is the phase of the input reflection coefficient of the component amplifiers.

Equation (A.2) may be easily rewritten as

$$\langle |b_{in}|^2\rangle = |\Gamma_s|^2(1 + |\rho|^2)\langle |b_1|^2\rangle, \tag{A.3}$$

which shows that the average noise power of the input noise wave is enhanced with a factor $1 + |\rho|^2$ with respect to the original balanced amplifier. For uncorrelated input and output noise waves (or the uncorrelated parts of these noise waves) this leads to an easy to calculate effective increase of r_n in the standard noise formula, because for this case the increase with respect to T_{min} in exclusively determined by the reflected input noise wave of (A.3). Inspection of (4) with $\Gamma_{opt} = 0$ shows that the enhanced value of $\langle |b_{in}|^2\rangle$ can be incorporated in the noise formula with an increase in r_n of

$$\Delta r_n = r_n|\rho|^2. \tag{A.4}$$

C) Full correlation between input and output noise waves

In case the input and output noise waves are (partly) correlated, the expression for the output noise wave becomes more complicated and is not suitable to model the larger sensitivity of the effective input noise temperature to input impedance variations as an increase in r_n, except for the uncorrelated part that may be modeled as described in the previous section. Doing so for the fully correlated parts will make r_n dependent on the source reflection coefficient Γ_s, which would be undesirable.

The general expression for the output noise wave, in case of correlated input and output noise waves, is given by (see Fig. 9)

$$\begin{aligned} b_{out} &= [2g\Gamma_s t q^2(1 + 2\rho t^2)b_1^A + tb_2^A] \\ &\quad + [2g\Gamma_s t^2 q(1 + 2\rho q^2)b_1^B + qb_2^B]. \end{aligned} \tag{A.5}$$

If we assume that the input and output noise waves of each amplifier are fully correlated, with $b_2^A = kb_1^A$ and $b_2^B = kb_1^B$, then

$$\begin{aligned} b_{out} &= t[2g\Gamma_s q^2(1 + 2\rho t^2) + k]b_1^A \\ &\quad + q[2g\Gamma_s t^2(1 + 2\rho q^2) + k]b_1^B. \end{aligned} \tag{A.6}$$

Assuming that $\langle |b_1^A|^2\rangle = \langle |b_1^B|^2\rangle = \langle |b_1|^2\rangle$ and with $t = 1/\sqrt{2}e^{j\Theta/2}$, $\rho = |\rho|e^{j(\Phi-\Theta)}$, $g\Gamma_s = |g\Gamma_s|e^{j\Delta}$, $k = |k|e^{j(\Psi+\Theta+\Delta)}$ and $q/t = -j$, the average noise output power per unit bandwidth is, after some manipulations, given by

$$\langle |b_{out}|^2\rangle = \begin{bmatrix} |g\Gamma_s|^2(1 + |\rho|^2) + |k|^2 \\ -2|g\Gamma_s\rho k|\cos(\Phi - \Psi) \end{bmatrix}\langle |b_1|^2\rangle. \tag{A.7}$$

(A.7) shows that the average output noise power has now become dependent on the phase of the input reflection coefficient Γ_s (incorporated in Ψ) and the amplitude and phase of the input reflection coefficient ρ of the component amplifiers, as well as the phase of k. This is contrary to the basic balanced amplifier, where these dependencies do not exist (see (7) in Section II.B, as well as [14]).

In case $\Gamma_s = 0$ the output noise and the equivalent input noise temperature are determined only by the original output noise waves $b_2 = kb_1$ or b_2 if there is no correlation between the original input and output noise waves ($k = 0$). This is consistent with the results in Sections II.A.1 and II.B, also for the original balanced amplifier with a load on the input coupler, for which terms with ρ in (A.7) will disappear and lead to (7).

For both, uncorrelated and fully correlated input and output noise waves of the component amplifiers, the input noise wave of the balanced amplifier is enhanced with a factor $1 + |\rho|^2$, which may be incorporated in r_n in the same way as (A.4) in the previous section. The term $|k|^2\langle |b_1|^2\rangle$ represents the output noise wave b_2 and the minimum noise temperature of the component amplifiers, as well as of the basic balanced amplifier. T_{min} is achieved for $\Gamma_s = \Gamma_{opt} = 0$, for both uncorrelated and fully correlated noise waves.

However, for the most general case with fully correlated waves, (A.7) shows an additional factor $2|g\Gamma_s\rho k|\cos(\Phi - \Psi)$, which also depends on the source reflection coefficient and is due to the correlation between the input and output noise waves of the component amplifiers. This makes it unsuitable for incorporation in r_n and the noise temperature formula (4), because that would make r_n dependent on an external property, where it should be an independent noise parameter.

The result of the dependency of the output noise power on the phase of the input reflection coefficient is that the resulting input and output noise waves of the balanced amplifier with short circuit on the input coupler are no longer uncorrelated, contrary to the basic balanced amplifier with load on the input coupler.

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