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Warps, Waves, and Phase Spirals in the Milky Way

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Abstract. The stellar disc of the Milky Way exhibits clear departures from planarity, the most conspicuous manifestation being the Galactic Warp but also includes an apparent corrugation pattern in number counts around 15 kpc from the Galactic centre, a wave like pattern in the vertical velocities of stars as a function of guiding radius, asymmetries about the midplane in both number counts and bulk motions, and phase spirals in the $z - v_z$ projection of the local stellar distribution function. We discuss the physics of these phenomena and, in particular, suggest a possible avenue for inferring the vertical force in the Solar Neighbourhood from phase spirals. We apply Dynamic Mode Decomposition, a technique widely used in the realm of fluid mechanics, to simulations of disc galaxy simulations. This method appears to be particularly well-suited to the study of nonlinear processes such as the coupling of warps and spirals, first discussed by Masset and Tagger.

Keywords. Galaxy: kinematics and dynamics, Cosmology: dark matter

1. Introduction

In simple, highly idealized models of disc galaxies, one generally assumes that the system comprises three components, a stellar disc, a stellar bulge, and a dark halo and that the density and gravitational potential are axisymmetric, symmetric about the midplane of the disc, and time-independent. With these assumptions, one can build fully self-consistent models using either the Jeans theorem or the Jeans equations, which can serve as useful, albeit artificial, starting points for numerical N-body experiments and for interpreting observations.

Of course, real galaxies satisfy none of the above assumptions. For example, many stellar discs have bars, which can involve a significant fraction of their stars. Bars signal density perturbations in the disc plane with m-fold azimuthal symmetry where $m = 2, 4, \ldots$. In addition, discs often display prominent spiral arms.

Disc galaxies also exhibit departures from planarity. Many edge-on galaxies, for example, show prominent warps, often with the classic integral-sign shape (Binney 1992; Sellwood 2013). The Milky Way's own disc is known to be warped with a departure from planarity that extends from the edge of the disc, where its amplitude is a few kiloparsecs, to the Solar Neighbourhood. Discs also appear to have corrugations or undulations. Xu *et al.* (2015), for example, found evidence for a corrugation pattern with an amplitude of ~ 100 pc and a wavelength of 5 kpc at Galactocentric radii just beyond the Solar Circle. Similarly, wavelike features and the extension of the warp into the Solar Neighbourhood are also seen in a plot of the mean vertical velocity, v_z, as a function of in-plane angular momentum, L_z (Schönrich & Dehnen 2018). Since L_z is directly related to the guiding radii of disc stars, it can be used as a proxy for radius. The undulations have an amplitude of $\sim 1\,\mathrm{km\,s^{-1}}$. It is tempting to interpret these wavelike features as the velocity counterpart to the corrugation pattern seen in number counts, in which case, one infers a period of $\sim 600\,\mathrm{Myr}$. In any case, we are lead to the conclusion that by observing perturbations in the disc in both density and velocity, we can begin to learn something about physics.

The bulk velocity field of stars in the region of the disc near the Sun also shows what appears to be a combination of bending and breathing waves normal to the plane of the disc (Widrow *et al.* 2012; Williams *et al.* 2013; Carlin *et al.* 2013, Gaia Collaboration *et al.* 2018b; Laporte *et al.* 2019; Carrillo *et al.* 2018). In addition the number counts of stars in the Solar Neighbourhood, when plotted as a function of distance z from the midplane, show features that are asymmetric about the midplane (Widrow *et al.* 2012; Yanny & Gardner 2013; Bennett & Bovy 2019).

One of the most intriguing results from the Second Data Release of Gaia Gaia Collaboration *et al.* (2018a,b) was the discovery of phase spirals in the projection of the stellar phase space distribution function (DF) on to the $z-v_z$ plane (Antoja *et al.* 2018). The spirals are believed to arise when the stellar DF is perturbed, possibly by a passing satellite or by the buckling bar (Antoja *et al.* 2018; Laporte *et al.* 2019; Khoperskov *et al.* 2019). The main features of the spirals can be understood by considering phase mixing of the stellar DF (Antoja *et al.* 2018; Binney & Schönrich 2018; Darling & Widrow 2019a; Li & Shen 2019).

Figure 1 provides an illustration of how the $z-v_z$ spirals can arise. Each of the three columns corresponds to a simulation of test particles in a one-dimensional fixed potential. The top row shows the phase space frequency as a function of energy. The middle three rows show the initial conditions and two subsequent snapshots. Finally, the bottom row shows the distribution of stars in the angle-frequency plane. In the left column, the stars are initially in equilibrium and the potential is harmonic. In the middle column, the system is displaced from equilibrium by giving all stars a velocity "kick". In both cases, the cloud of stars orbits the centre of the $z-v_z$ plane. In the final column, the stars are again given an initial velocity kick, but here, the potential is anharmonic. To be precise, the potential is assumed to be given by

$$\psi(z) = K\left(\left(z^2 + D^2\right)^{1/2} - D\right) + Hz^2 \tag{1.1}$$

with $K = 800\,\mathrm{km}^2\,s^{-2}\,\mathrm{kpc}^{-1}$, $D = 0.4\,\mathrm{kpc}$, and $H = 200\,\mathrm{km}^2\,s^{-2}\,\mathrm{kpc}^{-2}$. This potential was used by Kuijken & Gilmore (1989) in their analysis of Solar Neighbourhood stars where the aim was to determine the vertical force and local density of dark matter (the so-called Oort problem). Roughly speaking, the first term corrsponds to the vertical force due to a thin disc; the second term corresponds to the force due to the dark halo. Here, the frequency decreases with increasing vertical energy and a spiral develops in phase space. Note that in the angle-frequency plane, the ridge that runs along the centre of the spiral becomes a straight-line.

The above example suggests a method for using phase spirals to infer the gravitational potential. In short, one finds the parameters (here, K, D, and H) that transforms the phase spirals into the "straightest possible lines" in the angle-frequency plot. According to the theory of angle-action variables, the slope and intercept of the line, which are two additional fit parameters, give the time since the perturbation and the phase in the $z-v_z$ plane of the perturbation. In total, the spiral is then fit by a five-parameter model.

2. Spectral Analysis and Linear Theory

In general, when an axisymmetric, equilibrium model for a disc-bulge-halo system is evolved via N-body methods, it will develop non-axisymmetric features such as bars and spiral arms. The fact that real galaxies have such features is reassuring even if

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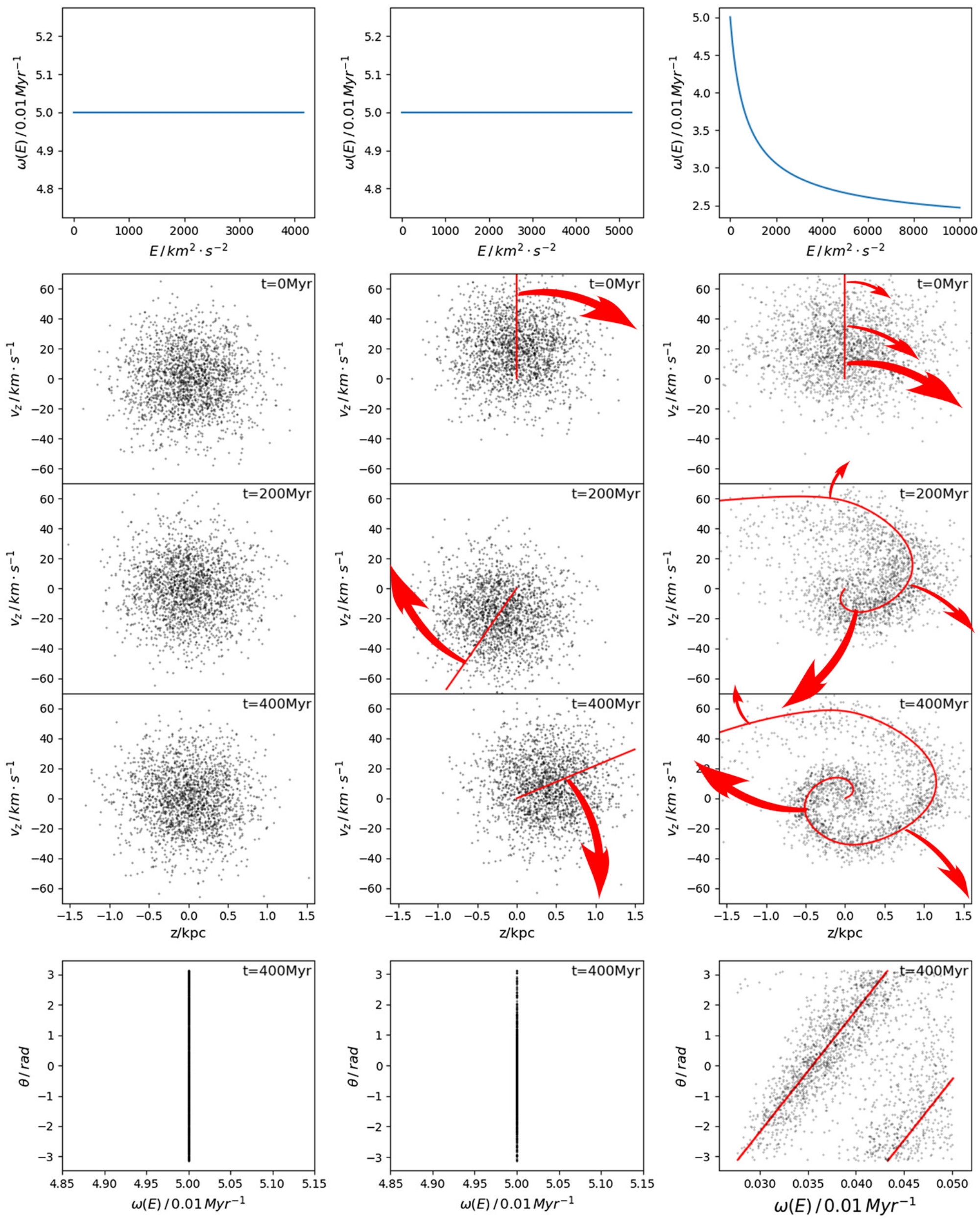

Figure 1. Illustration of the origin of phase spirals, as described in the text.

axisymmetric, equilibrium initial conditions are rather contrived. Indeed, N-body simulations of isolated galaxies have been the workhorse in the study of galactic dynamics for over half a century.

Spectral analysis, first introduced by Sellwood & Athanassoula (1986), is a powerful tool for interpreting disc galaxy simulations. Originally, spectral analysis was applied to the surface density Σ, as measured across the disc, at snapshots of a simulation taken at times t_j. At each snapshot, the disc is divided into concentric rings of radius R_k and for each ring, a Fourier series in azimuthal angle ϕ is calculated. One then performs a Fourier

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transform in t. The end result is a representation of the surface density as a function of R, azimuthal mode number m, and frequency ω. Schematically, one has

$$\Sigma\,(R,\ \phi,\ t_j) \to \Sigma\,(R_k,\ m,\ t_j) \to \Sigma\,(R_k,\ m,\ \omega_j) \tag{2.1}$$

Spectral analysis is closely related to linear perturbation theory (PT). In particular, the decomposition of Σ into terms of definite ω and m can be used as a starting point for PT where the radial functions for the different modes can then be derived by techniques such as the matrix method developed by Kalnajs (1977). In general, the calculations can be quite complicated, in part because of the presence of the Lindblad resonances. In fact, when one plots the spectral density for, say the $m=2$ terms in the $R-\omega$ plane, the power is concentrated between the inner and outer Lindblad resonances, as seen in Figure 5 of Sellwood & Carlberg (2014).

Spectral analysis can be extended to bending (and presumably breathing) modes of the disc. For example, one can replace Σ by $\langle z\rangle$, the mean displacement of stars from the midplane as a function of R, ϕ, and t in the above analysis, as was done in Chequers & Widrow (2017) and Chequers *et al.* (2018). In this case the power lies outside the Lindblad resonances, as predicted by linear theory (Hunter and Toomre). These considerations illustrate an essential feature of disc dynamics, namely that the density instabilities that give rise to spiral structure and bars is complementary to bending instabilities in that the former are strongest for long wavelength modes, where self-gravity dominates over kinetic pressure while the latter are strongest for short-wavelength waves, where centripetal acceleration dominates over the gravitational restoring force (see, for example, Araki (1985); Hunter & Toomre (1969); Merritt & Sellwood (1994)).

For breathing modes, one might consider $\langle z^n\rangle$ or perhaps $P_n\,(\tanh(z))$ where P_n are the Legendre polynomials (See Araki (1985) for a discussion of this choice). The implication is that non-axisymmetric, time-dependent structures of stellar disc can be organized into a double series in azimuthal mode number m and vertical mode number n).

Attractive as this framework is, it may miss much of essential physics. For example, the phase spirals do not fit into this simple picture since a local bend of the disc quickly phase wraps, leading to a complicated mix of of bending and breathing modes. Mode coupling may also be important, as in the suggestion by Masset & Tagger (1997) that $m=2$ spiral waves inside the outer Lindblad resonance couple to $m=1$ warps outside the resonance.

3. Dynamic Mode Decomposition to Stellar Dynamics

In a recent paper, we proposed that Dynamic Mode Decomposition (DMD) provides a means for analysing simulations of collisionless systems that can capture the physics of phase mixing and self-gravity (Darling & Widrow 2019b). DMD was originally developed in the field of computational fluid dynamics to study problems involving turbulent flows and jets (Mezić 2005; Rowley *et al.* 2009; Schmid 2010; Kutz *et al.* 2016). It is a model analysis algorithm similar to Principal Component Analysis, for time-series data with the aim of identifying the dominant approximate eigenfunctions of a system.

DMD proceeds as follows: At each of the equally spaced snapshots in a simulation, observation, or experiment, one measures a set of observables (say the surface density of a stellar disc at discrete points in the disc plane, or the mean displacement of the disc from the midplane across the disc) and assembles them into a single column vector $\mathbf{x}(t_j)$. In general, there is a linear map that relates the observables at snapshot j to those at snapshot $j+1$:

$$\mathbf{x}_{j+1} = \mathbf{A}\mathbf{x}_j \tag{3.1}$$

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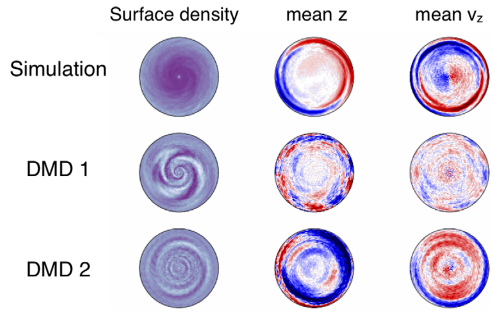

Figure 2. Results from a DMD analysis of a disc galaxy simulation as described in the text.

The idea in DMD is to approximate **A** and construct an approximation for the system's evolution in terms of its eigendecomposition. To do so, one constructs the data matrix

$$\mathbf{X} = \begin{pmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{m-1} \\ | & | & & | \end{pmatrix}, \tag{3.2}$$

and the time shifted data matrix

$$\mathbf{X}' = \begin{pmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \dots & \mathbf{x}_{m} \\ | & | & & | \end{pmatrix}. \tag{3.3}$$

Since $\mathbf{X}' \approx \mathbf{AX}$ we have

$$\mathbf{A} = \mathbf{X}'\mathbf{X}^{+} \tag{3.4}$$

where, $\mathbf{X}^{+}$ denotes the Moore-Penrose pseudo-inverse of **X**, which can be calculated via singular value decomposition. With an approximation for **A** in hand, one can then determine the (approximate) eigenvalues ω_j and eigenvectors ϕ_j by standard methods. The evolution of the system is then described by the DMD solution:

$$\mathbf{x}(t) = \sum_j b_j \phi_j e^{\omega_j t} \tag{3.5}$$

where the coefficients b_j are determined from the initial conditions. The eigenvectors **A** are referred to as the DMD modes. Significant insight into the systems behaviour can be gained from analysis of the dominant DMD modes.

In Darling & Widrow (2019b) we applied DMD to a one-dimensional, plane-symmetric system in which the force felt by each particle was due to both an external potential and the gravitational force from the other particles (self-gravity). When the external force dominates, the system quickly phase wraps, as in our example above. However, then self-gravity comes into play, phase wrapping occurs more slowly. In fact, our DMD analysis shows long-lived phase spirals. Thus, self-gravity may be important if we want to use spirals to "time" a perturbation, as in Antoja *et al.* (2018).

To further illustrate the potential power of DMD, we have applied the method to a disc galaxy simulation. Preliminary results are shown in Figure 2. The columns, from left to right, are surface density, mean displacement from the midplane, and mean velocity normal to the midplane. The top row is the simulation while the next two rows are two DMD modes. These DMD modes have the characteristics consistent with the picture suggested by Masset & Tagger (1997) in that they comprise both density waves in the inner disc (inside the outer Lindblad resonance) and bending waves in the outer disc.

The coming tsunami of data from Gaia will only intensify interest in the dynamics of the Milky Way and, in particular, the study of disc disequilibrium. More data and better simulations will require more powerful and sophisticated analysis tools. We believe that DMD may be one of these tools and potentially provide insight in the field of stellar dynamics just has it has in fluid dynamics.

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