Local Rings, by Masayoshi Nagata. Interscience Tract Number 13, New York, 1962. xiii + 234 pages. $11.00.

Not the least striking thing about this book is its title, at least to anyone who has been brought up on a diet of ring theory à la Jacobson. Indeed, Jacobson's encyclopedic "Structure of Rings" does not even mention local rings as such, although they do make a fleeting appearance under the guise of "completely primary rings". The reason for such neglect is probably this: The traditional ring theorist studies rings which can be easily decomposed into their components (as a direct sum); these are the so-called "semi-simple" rings, whose radical vanishes. (If one is allowed an analogy with physics, the radical measures the interaction between the component particles; it is the glue which cements them together.) Now a local ring has a unique maximal ideal, which is its radical and vanishes only when the local ring degenerates into a field.

Local rings are indeed the building blocks from which all other commutative rings are made up, and have replaced the "subdirectly irreducible" rings that were fashionable not so long ago. Where one used to say with Birkhoff that "every ring is a subdirect product of subdirectly irreducible rings", one now chants with Grothendieck that "every commutative ring is the ring of all global sections of the sheaf of associated local rings", and never mind what this means.

Actually, the local rings studied here are a little more special, they are commutative rings with a unique maximal ideal M which satisfy the maximum condition for ideals. According to Krull, this last condition implies that the intersection of all powers of M is zero, and this assures that, if the ring is topologized in a rather natural way, all points are closed.

While one can learn something about local rings from Northcott's little book on ideal theory and from Zariski and Samuel Volume II, the present book goes beyond these and fills a gap in the literature. Not only does it offer a systematic exposition of the theory of local rings as it stands today, but the writer has gone to some trouble to incorporate a number of original results.

The style is somewhat relentless, relieved only by occasional charming nipponisms (e.g. "manywhere", "Jacobson-radical-adic"). It should be ideally suited to those who like their mathematics straight, that is undiluted by motivation. Since the motivation here comes from algebraic geometry, this may be just as well for those of us who are not so well versed in this difficult subject.

The book contains seven chapters with the following titles: "General commutative rings" (a comprehensive introduction to the whole subject); "Completions" (including a new theory of the exact tensor product); "Multiplicities"; "Theory of Syzygies" (without homological methods, hurrah); "Theory of complete local rings"; "Henselian and Weierstrass rings". There are also two appendices, one of which is a very detailed collection of historical comments.

An interesting novel feature is the "principle of idealization": Let R be any commutative ring. With every R-module M there is associated a ring R^* as follows: As an additive group, $R^* = R \oplus M$, and multiplication is defined by

$$(r+m)(r'+m') = rr' + (rm'+mr') .$$

R^* contains R as well as M, the latter as an ideal, and $M^2 = 0$. The submodules of M are nothing else than the ideal of R^* contained in M. This principle enables the author to translate statements about modules into statements about ideals.

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Geometry of Complex Numbers, by Hans Schwerdtfeger. (Mathematical Expositions No. 13) University of Toronto Press, Toronto, 1961. xi + 186 pages. $4.95.

The book under review has three chapters: I. Analytic Geometry of Circles, II. The Möbius Transformation, and III. Two-dimensional Non-Euclidean Geometries.

Chapter I treats standard topics, as indicated by the section headings: Representation of circles by Hermitian matrices, Inversion, Stereographic projection, Pencils and bundles of circles, Cross-ratio. The algebraic machinery used extensively throughout the book, especially 2×2 matrices, is introduced here. Thus, for example, inversion in a circle has a purely geometric definition which depends on the theorem that for a given circle C and point z (not on C or its center) there is exactly one other point z^* lying on all circles through z orthogonal to C. However, proof of this theorem is given in purely algebraic terms, involving the previously established equivalence between Hermitian matrices and circles, the algebraic condition for orthogonality, and - at one stage - a 3×4 complex matrix which is required to have rank two.

At the end of each section appears an extensive list of examples, some for the student to work out and some worked out in detail by the author. These are clearly intended to be an integral part of the text. For example, two pages are devoted to the problem of finding a circle with respect to which two given circles are mutually inverse, considering