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What is the accuracy of DGPS?

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In general, a Reference Station calculates differential corrections which are valid for that exact location (zero baseline) at that particular epoch (age of corrections zero). However, DGPS users may be located as far as 200 nm away from the Reference Station and some of the errors compensated for by the Reference Station vary with space, namely satellite ephemeris, tropospheric and ionospheric errors. Therefore, the corrections calculated at the Reference Station suffer certain accuracy degradation as the separation distance increases, because of a decreasing relevance of the Reference Station data to the user. The error growth with increasing distance to the beacon is accentuated by the inability of Reference Station and user to see the same satellites, commonly termed the lack of intervisibility. The error growth with distance is the most important factor determining DGPS accuracy, but surprisingly very little has been done to assess it. US official documents and IALA state that the achievable accuracy degrades at an approximate rate of 1 m for each 150 km (80 nm) distance from the broadcast site, but this value is based on a theoretical prediction, made back in 1993. To estimate the error growth with real data, 6 DGPS receivers were placed along the Portuguese coastline at approximately 50 nm intervals from Sagres Broadcast Station, in a South – North direction. This paper describes the results of the trial.

KEY WORDS

1. DGPS. 2. Beacons.

1. INTRODUCTION.

In general, a Reference Station calculates differential corrections which are valid for that exact location (zero baseline) at that particular epoch (age of corrections zero). The remaining errors (not corrected by DGPS) are those which are uncorrelated, i.e. errors at the Reference Station which are not correlated with errors at the user and vice-versa. These are noise and multipath. Special care must be taken with these errors at the Reference Stations, because they are directly added to the user error. Besides these uncorrelated errors, which constitute the DGPS *noise floor*, the performance of the DGPS system depends on two factors:

- *Decorrelation with time*. Corrections by the users are applied only a few seconds after their computation (but possibly minutes in case of severe interference or other problems). In that period the corrections lose part of their validity because they suffer a certain temporal decorrelation. To cope with time decorrelations, the broadcast corrections include the rate of change of corrections, allowing users to estimate the actual Pseudo-Range Correction at each particular epoch. With the end of Selective Availability (SA), the rate of change of the error sources is very slow and the use of the rate of change of corrections is able to keep corrections valid for extended periods with almost negligible accuracy degradation. To confirm this, a small trial was conducted to measure the error growth with the increasing age of corrections.

- *Error growth with distance*. Some of the errors compensated for by the Reference Station vary with space, namely satellite ephemeris, tropospheric and ionospheric errors. Therefore, the corrections calculated at the Reference Station suffer certain accuracy degradation as the separation between Reference Station and user increases, because of a decreasing relevance of the Reference Station data to the user. The error growth with increasing distance to the beacon is accentuated by the inability of Reference Station and user to see the same satellites, commonly termed the lack of intervisibility. Decorrelation with distance, is still one of the most important factors determining DGPS accuracy. Therefore, a trial was devised to evaluate, with real data, the amount of spatial decorrelation suffered by the corrections broadcast by one of the Portuguese Broadcast Stations.

Before explaining the trials which have been conducted, the theoretical background behind temporal and spatial decorrelation of DGPS errors will be discussed.

2. DECORRELATION WITH TIME.

There are four different GPS errors which decorrelate with time: satellite clock errors, ephemeris errors, tropospheric errors and ionospheric errors.

2.1. *Satellite clock errors.*

Satellite clock errors are due to differences between the satellite clock time and that predicted by the satellite data. These differences are usually accepted to be about 5 ns [Reference 1], resulting in Pseudo-Range errors of 1·5 m. The oscillator that times the satellite signal is free-running and is monitored by the GPS Control Segment stations, which establish corrections that are sent up to the satellite to set the data message. The user reads the data and adjusts the signal timing accordingly. As long as both Reference Station and user receivers are employing the same navigation message data, satellite clock errors are completely compensated by the differential technique. To achieve this, the Reference Station broadcasts a word – the Issue Of Data (IOD) – which indicates the reference time of the ephemeris and clock parameters used at the Reference Station. "*The IOD is the key to ensure that the user equipment calculations and Reference Station corrections are based on the same set of broadcast orbital and clock parameters*" [Reference 2]. This word is included in messages RTCM SC-104 type 1 and 9, so that the DGPS user equipment may compare it with the IOD of the navigation message being used. For low age corrections, satellite clock errors are entirely compensated by DGPS and – as the drift of the satellites' clocks are very slow – the corresponding decorrelation with time is almost negligible.

2.2. *Satellite ephemeris errors.* Satellite ephemeris errors are due to differences between the actual satellite location and the predicted location using the satellite orbital data. These differences are generally small (in the order of 2 m) [Reference 1] representing a positioning error to the GPS user of a few decimetres. Ephemeris errors are almost completely compensated by the differential technique as long as both Reference Station and user receivers employ the same satellite data, which is ensured by the IOD, broadcast by the Reference Station. These errors are very slowly changing and, hence, strongly correlated over many minutes.

2.3. *Tropospheric errors.* The tropospheric propagation delay is caused by the lower atmosphere and includes not only delays in the troposphere, but also in the stratosphere, mesosphere and thermosphere and should, more correctly, be designated the neutral atmosphere delay. However, as it is the troposphere which induces the largest delays, it is just known as tropospheric error. Generally, this error is decomposed into two components:

- a dry component, which is a function of surface pressure and temperature and accounts for about 80% to 90% of the total delay;
- a wet component, which is a function of the distribution of water vapour and is, therefore, harder to model, despite being responsible for only 10% to 20% of the delay.

In terms of the time decorrelation for this error, what counts is not the difficulty to model each component but their variability, which is mainly diurnal, especially temperature and humidity variations. However, the full 24-hour variations of the meteorological parameters are "*very small (…) because of the nearly constant ratio of constituents of the air, with the exception of water vapour and condensed water*" [Reference 3], which nevertheless account for a very low percentage of the delay. Therefore, this error does not suffer significant variations in timescales of a few minutes and, besides being almost entirely removed by the differential technique, has an almost negligible decorrelation with time when considering the DGPS error budget.

2.4. *Ionospheric errors.* Ionospheric errors are caused by delays in the GPS signal as it traverses the ionosphere, whose electron content is a function of the amount of incident solar radiation. Therefore, the ionospheric delay changes with time of day, season of the year and, also, following the 11·1 year solar cycle, with higher values by day (at around 14h00m local time), during the summer and at the peaks of the solar cycle. From these three periodic changes, the dominant one is diurnal variability, following the variation in incident solar radiation [Reference 4]. Superimposed on these periodic changes, severe magnetic storms occur a few times (generally not more than 4 times) during each 11·1 year solar cycle, causing extreme delays on GPS signals, in addition to amplitude fading and scintillation. These magnetic storms affect mainly the auroral latitudes (around the geomagnetic poles) and "*for some unknown reason (…) occur more frequently during the declining phase of the solar cycle*" [Reference 5].

Ionospheric propagation vertical delays are typically 20-30 m during the day and 3-6 m at night [Reference 2]. These errors are entirely compensated by DGPS and their decorrelation with time, for periods of tens of minutes, is very low, because the ionospheric delay does not change significantly on such timescales – except in the case of major magnetic storms during which the total electron content changes rapidly. Dusk and dawn are the periods when the temporal decorrelation of the ionospheric error is generally higher, because the ionosphere re-configures itself, but even then the decorrelation is not significant. Although the decorrelation of the ionospheric errors with time is a much smaller effect than its decorrelation with distance, this is the error which contributes the most to the growth of DGPS errors with age of corrections.

3. TRIAL TO EVALUATE DECORRELATION WITH TIME.

All the errors mentioned above change very slowly and therefore differential corrections are expected to remain valid several minutes after being calculated. To quantify the magnitude of the error growth with time, a small experiment was devised, using a DGPS receiver (Trimble DSM 212), with separate GPS and beacon antennas. The GPS antenna was a Trimble Combined GPS/MSK Beacon Antenna (P/N 27207), but the beacon reception was disabled, so that the equipment could receive the differential corrections from an external source, in this case a DBR IV Magellan beacon antenna. Approximately 5 minutes after having started logging NMEA messages, the beacon antenna was disconnected (thus simulating an incident that prevented the reception of the differential corrections, like for instance severe interference) and the DGPS receiver continued computing differential positions, using old corrections. The main limitation to this experiment was that, in the Trimble DSM 212, the maximum age of corrections which can be configured is 240 seconds and, if after that short period the receiver does not receive new corrections, then it reverts to stand-alone positioning. Therefore, it was not possible to test the validity of the corrections after that four minute period. Something similar occurs with most of the DGPS receivers available on the market because they were conceived in the SA era in which it was preferable to revert to stand-alone positioning after a short period without differential corrections than to continue applying old corrections for extended periods.

The experiment was conducted twice on 27 May and 4 June 2003. On 27 May, the errors remained between 0·21 and 0·53 m while the corrections were being continuously received, but after disabling the differential corrections reception at 13h22m the errors started increasing very slowly, reaching 0·75 m at 13h26m, i.e. four minutes after losing the beacon signal. On 4 June the increase in the DGPS positioning error is more pronounced: in the order of 1 m. While receiving the DGPS signal, the errors remained between 0·22 and 0·46 m and after loss of the signal they increased continuously up to 1·5 m. Curiously, the errors were already increasing slightly prior to losing the differential corrections reception. The results show that after four minutes without reception of differential corrections the accuracy remains about the same order of magnitude as it was when corrections were being received normally. Summarising, on 27 May the temporal decorrelation of the DGPS corrections caused an error increase in the order of less than 0·5 m and on 4 June the accuracy degradation after losing the differential corrections amounted to about 1 m.

These results show that after four minutes the error growth is very small and it is logical to accept that even 10 to 15 minutes after being calculated the differential corrections remain valid, although most DGPS receivers would revert to stand-alone positioning much before that. Therefore, the error growth with distance, which will

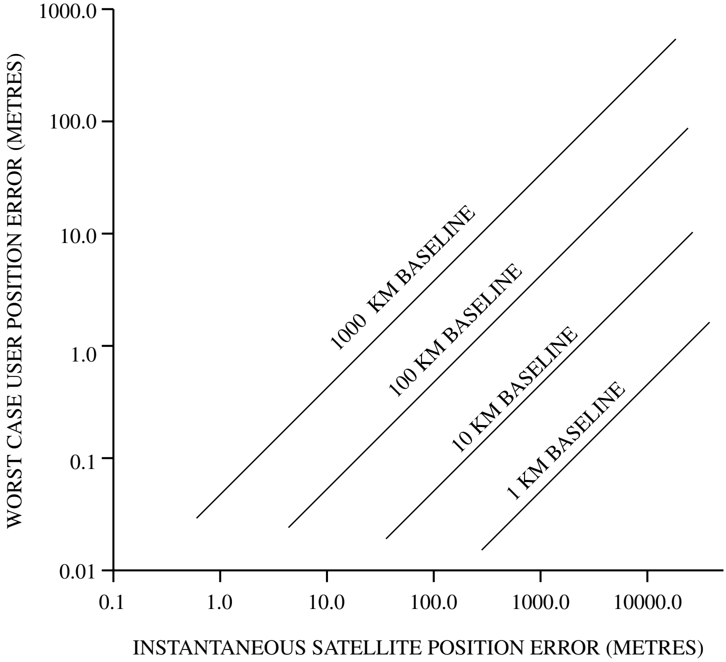

Figure 1. Worst-case DGPS errors vs satellite position errors for various separation distances [Reference 6].

be investigated below, is currently the most important factor determining DGPS accuracy.

4. ERROR GROWTH WITH DISTANCE. The growth of the DGPS error with increasing distance to the beacon is caused by two effects:

- Spatial decorrelation of the individual GPS errors, namely satellite ephemeris errors, ionospheric errors and tropospheric errors;
- Lack of intervisibility of satellites, i.e. the inability of Reference Station and user to see the same set of satellites, which is more pronounced as the distance between them increases.

4.1. *Spatial decorrelation of individual GPS errors.* There are three different GPS errors which decorrelate with displacement between Reference Station and user: satellite ephemeris errors, ionospheric errors and tropospheric errors.

4.1.1. *Satellite ephemeris errors.* Even though the Reference Station and the user employ the same ephemeris data, the compensation of this error is spatially decorrelated because the component of the ephemeris error as seen from the Reference Station and as seen from the user changes slightly at larger separations. Figure 1 shows the worst case user position error (after DGPS correction) as a function of satellite position error. This worst case is when the ephemeris error is maximum in a direction parallel to the line between the Reference Station and user. Errors of the ephemeris in a radial direction (away from the earth centre) and in a direction perpendicular to the baseline are totally compensated by the Reference Station and do not impact on DGPS user errors. Considering a 2 m satellite positioning error (which is a value consistent with the actual performance of the system) [Reference 1], a 200 nm (370 km) separation between Reference Station and user produces DGPS user errors (in the worst case) in the order of a few centimetres (less than 5 cm). This means that the spatial decorrelation of the current ephemeris errors is almost negligible for the usual ranges of maritime DGPS stations when considering the error budget of DGPS. For a separation between Reference Station and user of 100 km, the geographic decorrelation due to ephemeris errors (with broadcast ephemeris accurate to about 2 m) is less than 0·01 m.

4.1.2. *Tropospheric errors.* The compensation of the tropospheric error depends on the distance between the Reference Station and the receiver, with the user error increasing at larger separations because of:

- spatial decorrelation and
- difference in the incidence angles.

For this latter effect, while the signals from satellites at zenith experience typical delays of 2·5 m, the signals from satellites at 5° elevation suffer typical delays of nearly 30 m. However, at distances up to 200 nm (370 km), the difference in incidence angle is usually less than 2° [Reference 2] (depending on the satellite elevation and on the angle between the baseline and the satellite azimuth) and, therefore, for maritime DGPS users this effect is very small. Therefore, the decorrelation of the tropospheric delay is mainly caused by the difference on the meteorological parameters of the tropospheric volumes traversed by the signal rays to the Reference Station and to the user. However, the ratio of constituents of the air is nearly constant [Reference 3], especially over the coverage areas of DGPS beacons. Therefore, the residual position error (after DGPS correction), due to tropospheric delay, is almost always very small, unless Reference Station and user are at significantly different altitudes; this is not the case for mariners using the Portuguese Broadcast Stations sited at altitudes near to sea level.

With average conditions, the geographic decorrelation due to tropospheric errors is approximately given by the following formula (1σ estimate):

$$\text{Geographic decorrelation [m]} < 1 \text{ X } 10^{-6} \text{ X Separation [m]}$$

For a separation Reference Station to user of 100 km, the spatial decorrelation due to the tropospheric delay is less than 0·1 m (1σ estimate) [Reference 6].

4.1.3. *Ionospheric errors.* For users near the Reference Station, the respective signal paths from the satellites are sufficiently close so that compensation is almost complete. When the distance from the user to the Reference Station increases, the ionospheric paths may be sufficiently far apart to cause different delays, not only because the GPS signals received by the Reference Station and the user pass through the ionosphere at different locations, but also because the incidence angle of both signals is different.

It has already been seen that the periodic variations of the ionospheric activity are closely linked to the amount of solar radiation, but there are also spatial variations, which are caused not only by the varying incident radiation but also by the Earth's

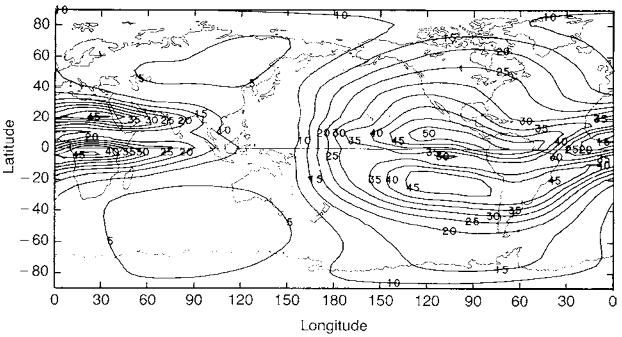

Figure 2. Worldwide contours of average ionospheric delay for March 1990 (solar maximum) – units in nanoseconds [Reference 5].

magnetic field. These spatial variations are illustrated in Figure 2, which shows that the ionospheric activity is maximum at latitudes about 20° either side of the geomagnetic equator. The spatial decorrelation of the ionospheric errors is caused not by the amount of ionospheric delay but by its variation with space. This is generally more pronounced along meridians, because it is for most of the world, the direction with higher gradient of the ionospheric delay, as seen in Figure 2. Therefore, this effect increases with the separation between Reference Station and user and, on most locations (including Portugal), it is more pronounced for users south or north of the station, because that is usually the direction with higher ionospheric activity variation rate.

The spatial decorrelation of ionospheric errors is also caused by the difference in incidence angles. For ionospheric errors, the delay at 5° elevations is about three times higher than the vertical delay [Reference 6]. Therefore, a typical day-time vertical delay of 20-30 m [Reference 2] translates into 60-90 m for observations at a 5° elevation angle and a typical night-time vertical delay of 3-6 m [Reference 2] translates into 9-18 m for satellites at 5° elevation. However, for separations up to 200 nm, the incidence angles generally vary less than 2° [Reference 2] (depending on the satellite elevation and on the angle between the baseline and the satellite azimuth), therefore not introducing significant variations in the ionospheric delay.

Spatial decorrelation is also a function of the time of day: at about 14h00m (local time) the ionospheric activity is maximum, causing larger delays and, *generally*, larger spatial decorrelation of the ionospheric errors; at night, when the ionospheric activity is minimum, the delays are lower and *usually* the spatial decorrelation caused by this errors is also lower. Nevertheless, it is important to repeat that with regard to spatial decorrelation, it is the variation of the ionospheric delay that counts, not the instantaneous magnitude of the delay. Considering average conditions (not entering with the influence of the time of day), the predicted geographic decorrelation due to ionospheric errors is approximately given by the following formula (1σ estimate):

$$\text{Geographic decorrelation [m]} < 2 \text{ X } 10^{-6} \text{ X Separation [m]}$$

For a separation Reference Station to user of 100 km, the spatial decorrelation due to the ionospheric delay is less than 0·2 m (1σ) [Reference 6].

4.1.4. *Summary of the spatial decorrelation errors.* As the three error sources which suffer a decorrelation with distance are independent, i.e. they are not correlated, and have a (approximately) normal distribution, their combined effect is estimated by their root sum square (rss). Therefore, the geographic decorrelation of the differential corrections is less than 0·22 m per 100 km separation (1σ estimate[1]). For maritime navigation it is more appropriate to use the 95th percentile, which approximates the 2σ values. Therefore, the residual Pseudo-Range error is less than 0·44 m, for every 100 km separation between Reference Station and user (95% estimate).

4.2. *Lack of intervisibility of satellites.* The estimate mentioned above considers a common view of the satellites, i.e. Reference Station and user viewing the same satellites, but in practice as the distance to the Reference Station increases the number of satellites which are visible to both the station and the user decreases. Maritime DGPS Broadcast Stations are targetted for nearby users (generally within 200 to 300 nm) and, therefore, the difference in the number of satellites is usually low. However, this effect is accentuated because DGPS receivers only use corrected satellites in their computations. This means the Reference Station computes corrections for all visible satellites, but the user only employs, from the satellites which are visible, those for which it has corrections, i.e. the user receiver discards satellites, which it is viewing, but which are not visible to the Reference Station.

In a paper presented on the GNSS 2000 Symposium, Walter Blanchard described the satellite intervisibility problem and expressed his surprise about the "*little [that had] been done to quantify it*" [Reference 7]. In that paper, the author made some geometrical considerations about intervisibility, concluding that if the Reference Station has a cut-off angle of 7·5° and users set no mask angle then a satellite being seen by the station at 7·5° is not seen by users separated more than 820 km and located on the opposite direction of the satellite. (With a Reference Station mask angle of 5°, as in Portugal, this separation reduces to approximately 550 km). However, if the user sets a cut-off angle of 5°, then he will start to lose satellites for any separation at all from the beacon, and the number of satellites lost is proportional to the distance between station and user. As a very rough approximation, for every 100 km separation an additional 1° must be added to the user mask angle to allow for lack of intervisibility [Reference 7]. Nevertheless, this applies only to satellites on azimuths along the baseline, with the effect decreasing of relevance as the angle between the azimuth and the baseline increases. When this angle reaches 90°, meaning that the satellite is at a right-angle to the baseline, then the user and the Reference Station see the satellite at about the same elevation. Therefore, the magnitude of the intervisibility problem to the user depends on three factors [Reference 7]:

- Reference Station mask angle;
- User cut-off angle;
- Azimuth of the satellite relative to the baseline.

4.3. *Current estimation of error growth with distance.* Although the accuracy of DGPS is mainly determined by the distance to the Reference Station, it is amazing the little that has been done to assess the error growth with distance. In 1993, in [Reference 8], the USCG stated that the achievable accuracy with DGPS degrades at

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[1] 1σ (or 1 standard deviation) is equal to rms only if the mean error is zero. As this is an estimate over long time intervals, the mean error may be considered zero, or very close to zero, and 1σ estimate may be considered equal to rms.

an approximate rate of 1 m for each 150 km distance from the broadcast site. This value was estimated theoretically before the USCG had declared Final Operational Capability for the DGPS network in 1999 [Reference 9], and at a time when the performance of GPS was considerably worse than now. Since then, this theoretical prediction of DGPS error growth has been included in the various editions of the US "*Federal Radionavigation Plan*" and also in several IALA documents, namely in the successive drafts of the "*Recommendation on the Performance and Monitoring of DGNSS Services in the Band 283·5–325 kHz*". However, no trial was ever conducted with the aim of evaluating with real data the amount of spatial decorrelation, especially now that the broadcast orbits are more accurate and that the number of satellites has been consistently above 24, with positive impacts on the DGPS performance.

5. THE EVALUATION OF ERROR GROWTH WITH DISTANCE.

The purpose of this trial was to evaluate the amount of error growth when applying the differential corrections broadcast by the Portuguese DGPS Station of Sagres using real data collected at different sites.

To attain this goal, seven DGPS receivers were placed along the Portuguese coastline at approximately 50 nm intervals from *Sagres* (in a South – North direction) and all of them tuned to this Broadcast Station (see Figure 3). The receivers were distributed in this direction because it is the direction of the highest gradients of ionospheric delay and of the highest gradients of the weather parameters, particularly temperature, which mainly vary with latitude. This is also the direction of the Portuguese occidental coast. Therefore, the trial was conducted in the most unfavourable direction. One of the receivers was the Integrity Monitor of the Broadcast Station itself, which measured the errors in a zero baseline, another was the Remote Integrity Monitor installed at the Portuguese Hydrographic Office headquarters to continuously monitor the performance of the network. Both of these Integrity Monitors comprised a Leica MX 9400N DGPS Navigator teamed with a MX 52R DGPS Beacon Receiver. The other five receivers were Trimble DSM 212 DGPS receivers connected to dedicated PC's to log the NMEA messages.

The first equipment was installed at *Sines*, which is 56·4 nm distant from *Sagres*. Another Trimble receiver was installed in the Portuguese Hydrographic Office, next to the Remote Integrity Monitor, at a distance of 104 nm from the Broadcast Station. Therefore, in Lisbon two different pieces of equipment were used, although a comparison of their performances is somewhat unfair to the Trimble receiver, which was not offered the same means of multipath mitigation in terms of the antenna site and the use of choke rings. A third Trimble receiver was installed in *Nazaré*, which is approximately 150 nm from *Sagres*, but at this site the PC *locked-up* soon after it started logging data and only the NMEA messages corresponding to the first hour were recorded. As this was a very short sample, this data was not considered in this analysis. Nevertheless, the unexpected loss of data from *Nazaré* was compensated by the data collected at two more distant sites, *Mira* and *Varzim*. *Mira* is 209·8 nm and *Varzim* is 264·7 nm from *Sagres*. On all of these sites (with the exception of *Nazaré* as mentioned above), data was recorded, at 10 second intervals, for a period of 3 days: from noon 6 June to noon 9 June 2003. The collected data was converted into *.xls files and was processed using Microsoft Excel.

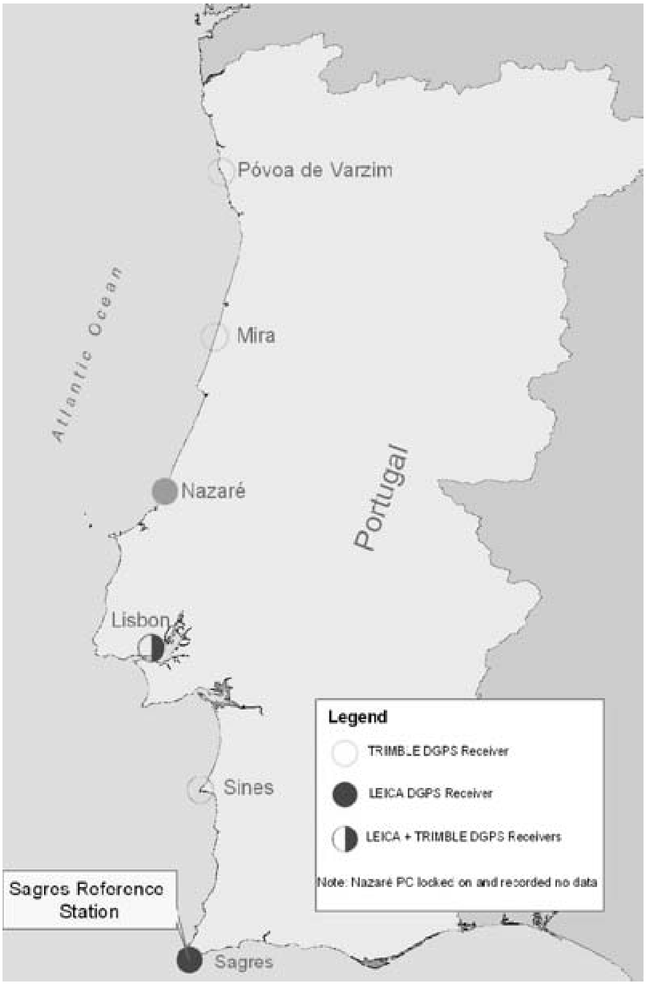

Figure 3. Location of the receivers used to evaluate the spatial decorrelation.

The 72 hours of data are considered representative of the performance of DGPS on each site, because one of the characteristics of differential positioning is its very good repeatability. Therefore, increasing the period of observations would bring negligible differences in terms of final results, but would increase significantly the size of the data files (which had already more than 60 MB), hampering their processing. All the receivers were configured with a 5° mask angle, with the exception of the Integrity Monitor receiver, at *Sagres*, which uses an all in view capability, and also of the Lisbon receivers which were configured with a cut-off angle of 7·5°, due to the intense multipath environment.

6. EMPIRICAL RESULTS OF ERROR GROWTH WITH DISTANCE.

To evaluate and quantify the error growth, it is necessary to analyse the results altogether, which can be done with the aid of Table 1.

Table 1. Summary of the results obtained to evaluate error growth with distance.

Site	Sagres	Sines	Lisbon	Lisbon	Mira	Varzim
Baseline	0 nm	56·4 nm	104 nm	104 nm	209·8 nm	264·7 nm
Receiver	Leica MX 9320	Trimble DSM 212	Leica MX 9320	Trimble DSM 212	Trimble DSM 212	Trimble DSM 212
Horiz. Error (95%)	0·41 m	0·93 m	0·81 m	1·20 m	1·48 m	1·84 m
Average N° of SV	7·58	7·44	7·36	7·12	7·07	6·94
Average HDOP	1·14	1·16	1·17	1·22	1·25	1·28

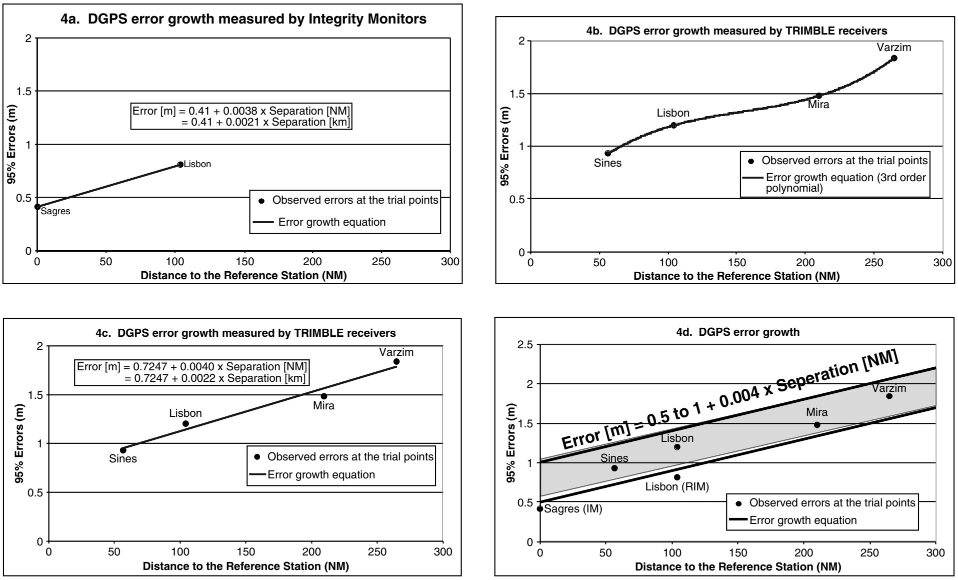

Figure 4. a. Error growth equation for the Integrity Monitors. b. Error growth equation (3rd order polynomial) for Trimble DSM 212 receivers. c. Error growth equation (linear function) for Trimble DSM 212 receivers. d. DGPS error growth.

The table shows clearly that the accuracy of differential positioning is inversely proportional to baseline length, but to help analysing the error growth, those results will be presented graphically, first using only the data collected by the Integrity Monitors, then considering only the data from the Trimble receivers and finally using all data gathered by the six receivers, so as to derive a "final" error growth equation. Figure 4a illustrates the growth of the DGPS errors observed by the two Integrity Monitors, the local one and the remote one. As there are only two points, the function that represents the errors measured by the Integrity Monitors is a linear equation according to which:

$$95\%\ \text{DGPS error}\ [m] = 0{\cdot}41 + 0{\cdot}0038S$$

where S is the separation distance in nm. This means that the 95th percentile DGPS error is approximately equal to 0·4 m plus 0·4 m for each 100 nm distance from the Reference Station (or plus 0·2 m for each 100 km distance from the Reference Station). When using only the data collected with the Trimble receivers the equation is a little different. In this case as there are 4 points it is necessary to use a 3rd order polynomial (see Figure 4b) to describe perfectly the errors (in m) as function of the separation between Reference Station and user (measured in nm):

$$95\%\ DGPS\ error\ [m] = 0{\cdot}234 + 0{\cdot}0172S - 1 \times 10^{-4} S^2 + 2 \times 10^{-7} S^3$$

This equation gives the error measured by a Trimble receiver with 100% certainty, but it is possible to use a much simpler function (linear equation) to estimate the error in a certain site, as function of the distance, with a good degree of certainty. The linear equation which best fits the actual data is:

$$95\%\ DGPS\ error\ [m] = 0{\cdot}7247 + 0{\cdot}0040S$$

which gives the 95% errors with a 98·4% certainty[2].

This means that the 95th percentile DGPS error obtained in a Trimble DSM 212 receiver is approximately equal to 0·7 m plus 0·4 m for each 100 nm distance from the Reference Station (or plus 0·2 m for each 100 km distance from the Reference Station).

The slope of this equation is very similar to the slope of the equation obtained for the Leica Integrity Monitors, meaning that the rate of accuracy degradation is very similar with both equipment: in the order of 0·4 m per 100 nm separation. This is not a surprising result because the growth of the DGPS error is caused by the decorrelation of the individual GPS errors and by the lack of intervisibility and not by the receivers' performance. Where the distinct performances of different receivers are revealed is on the intercept values (i.e. errors on a zero baseline). For the two receivers used in this trial the intercept values were 0·41 nm (measured by the Integrity Monitor) and 0·72 m (extrapolated from the results of the four Trimble DSM 212 receivers). The Integrity Monitor is an unusual equipment, because it is a high-quality receiver that uses very good antennas with choke rings, which are very carefully sited to properly suppress multipath. The Trimble DSM 212s are also high quality receivers, but their antennas, in this trial, were not equipped with choke rings or groundplanes and the choice of the siting did not follow the same care as for the Integrity Monitors antennas. This is the usual case with shipboard antennas, which generally do not make use of multipath mitigation devices. Therefore, considering that the Integrity Monitor results are exceptionally good, it is possible to state with a very good degree of certainty that the accuracy near the Reference Station can vary from 0·5 m to 1 m, depending on the type of receiver.

Given this, we propose the following DGPS error growth equation: ***the DGPS error [95%] is equal to 0·5 m to 1 m near the Reference Station plus 0·4 m for each***

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[2] This certainty degree corresponds to the square root of the R-squared value. The R-squared value (also known as the coefficient of determination) is automatically calculated by Microsoft Excel to reveal how closely the estimated values for the equation correspond to the actual data. The R-squared value can assume values from 0 to 1, so that the equation is more reliable when the corresponding R-squared value is at or near 1.

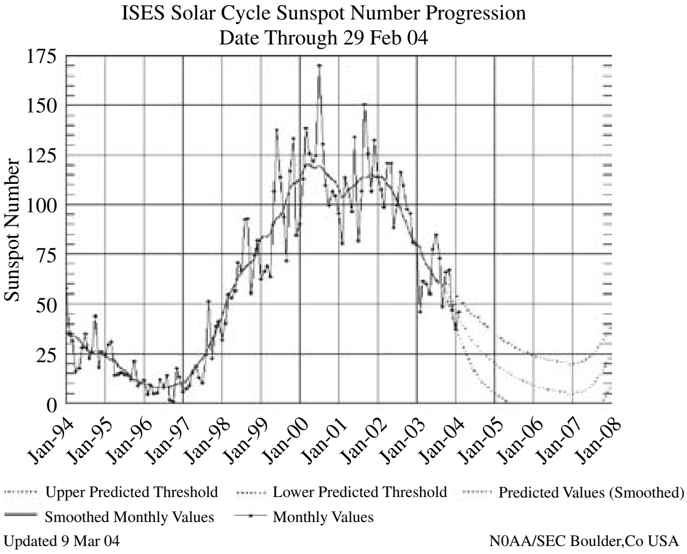

Figure 5. Solar cycle [Reference 10].

100 nm distance from the Reference Station (or plus approximately 0·2 m for each 100 km distance from the Reference Station).

This formula is illustrated by the shadowed area on Figure 4d, which accommodates all the values measured by the Trimble DSM 212 receivers. The errors measured by the Integrity Monitor (of *Sagres*) and by the Remote Integrity Monitor (sited at Lisbon) are a little bit below the formula prediction, but, as said previously those equipments have unique characteristics, in the ability of their antennas to mitigate multipath.

It is important to stress that this trial was conducted on a period of moderate to high ionospheric activity, for both the seasonal variation and variations that follow the 11·1-year solar cycle. It was conducted at summertime and near the peak of a solar cycle. According to Figure 5, the solar cycle peaked during 2000 and 2001, but at the time of this trial (middle of 2003) solar activity reached a high level.

Therefore, excluding magnetic storms (which affect mainly auroral/polar cap latitudes), the ionospheric activity which could be anticipated during the trial period was moderate to high, with expected high ionospheric errors and higher spatial decorrelation of these errors. This was confirmed by the space weather reports issued by the Space Environment Center (National Oceanic and Atmospheric Administration) on their website. The most used index to give a measure of the ionospheric activity is the estimated planetary K-index: kp. The kp index gives a measure of the amount of protons and electrons emitted by the Sun, which cause increased ionisation in the ionosphere. This is a global index derived from observations at all longitudes and which varies from 0 to a maximum of 9. "*K-indices of 5 or greater indicate storm-level geomagnetic activity*" [Reference 11].

As shown in Figure 6 the ionospheric activity on the trial period was moderate, with two periods of slightly high activity, with kp reaching 5. Furthermore, the observations were made on a South – North direction, which is the direction with

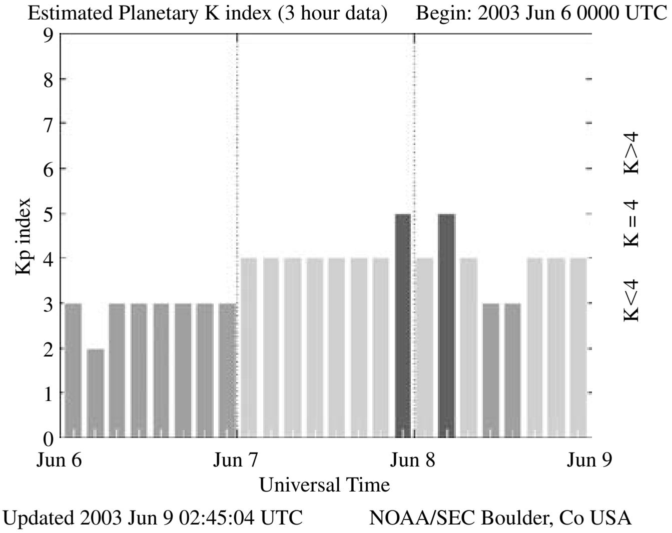

Figure 6. Kp index plot for the trial period: 6–9 June 2003 [Reference 12].

higher rate of change of ionospheric errors. Therefore, this trial was conducted on a period of moderate to high ionospheric activity for mid-latitudes and along the most unfavourable direction. This means that the derived formula is expected to allow a reasonable estimate of the DGPS error growth on mid-latitudes because on most occasions the ionospheric activity will be similar or lower and also because users on azimuths other than North or South relative to a beacon are expected to have slightly lower decorrelation of the ionospheric delay. Nevertheless, on locations with higher ionospheric activity (namely around the geomagnetic equator) the actual errors may exceed the formula prediction; this may also occur during magnetic storms.

7. QUANTIFICATION OF THE INFLUENCE OF THE LACK OF INTERVISIBILITY. The formula derived in the previous Section, encompasses the added effects of the spatial decorrelation of the individual GPS errors and of the forced selection of satellites. Generally, within the advertised areas of coverage, there is very little difference between the satellites visible at the Reference Station and at the user. According to Walter Blanchard "*there is virtually no difference between what the DGPS station and the user see even at the maximum range of the IALA beacons*" [Reference 13]. However, the beacons can be received at considerably long ranges when reception conditions are favourable and then the number of simultaneously visible satellites begins to fall. In this trial, Table 1 shows a close relationship between decreasing number of satellites and increasing errors. As the distance to the Reference Station increases, the average number of satellites decreases (with corresponding increase in HDOP values) because, on some periods, satellites which are visible to the Reference Station may not be above the mask angle of the user receivers (particularly those located farthest away) and DGPS receivers only employ satellites for which they received corrections. But does

Table 2. Results as a function of the difference between the number of satellites employed by the Reference Station and the number of satellites employed by the user.

		Same n.° of SV at RS and at user		Difference of SV at RS and at user equal to 1		Difference of SV at RS and at user ≥2	
Site	Error (m)	Error (m)	% of time	Error (m)	% of time	Error (m)	% of time
Sines	0·93	0·82	88 %	1·30	11 %	1·45	1 %
Lisbon	1·20	0·95	63 %	1·34	30 %	1·80	7 %
Mira	1·48	1·30	60 %	1·52	31 %	2·37	9 %
Varzim	1·84	1·62	51 %	1·76	36 %	2·77	13 %

this have a significant impact on the final user accuracy or, is the performance mainly determined by the spatial decorrelation of the atmospheric errors? That is investigated in this Section.

Using the data gathered in this trial, it was possible to calculate, at each epoch, the difference between the number of satellites used at the Reference Station and the number of satellites employed by each of the four Trimble receivers. The error was then calculated for each site corresponding to all the positions computed with the same number of satellites viewed at the Reference Station, the error for the positions which were calculated with one satellite less and, finally, the error of all solutions which used two or more satellites less than the Reference Station. The results are summarised in Table 2.

According to these results, the influence of the lack of intervisibility is not very significant. The values of the 3rd column are the 95% errors measured during the periods in which all satellites were simultaneously visible at the Reference Station and at the four sites. Those errors are only determined by the spatial decorrelation of the individual GPS errors (mainly ionospheric and tropospheric delays). For those periods, the errors are a little bit lower than during the whole trial period, but the difference is not significant: 2 decimetres, on average. This means that the errors caused only by the DGPS *noise floor* (multipath plus receiver's noise) and by the spatial decorrelation of GPS errors are approximately 2 decimetres lower than the errors caused by those sources plus the lack of intervisibility.

This conclusion is illustrated by Figure 7a, which shows that the error growth equation for the periods of optimum intervisibility runs almost parallel to the error growth equation derived for the totality of the trial period, but approximately 0·2 m below the latter. This plot alone could lead to the conclusion that the lack of intervisibility effect is approximately the same regardless of the distance to the beacon, which would be a precipitated conclusion, because the depicted errors must be analysed in conjunction with the percentages of time that each receiver was able to see all satellites viewed at the Reference Station. For instance, on *Sines* the error with all satellites simultaneously visible was calculated with almost 90% of the positions, but on *Varzim* the corresponding value was obtained using only half of the positions, because for the other half of the time the receiver was not able to see all satellites for which corrections were broadcast.

However, in Figure 7a, the equation fitting to the errors measured during the periods of optimum intervisibility (the lower linear function) is very revealing and meaningful. For this equation, the intercept value corresponds to the DGPS *noise*

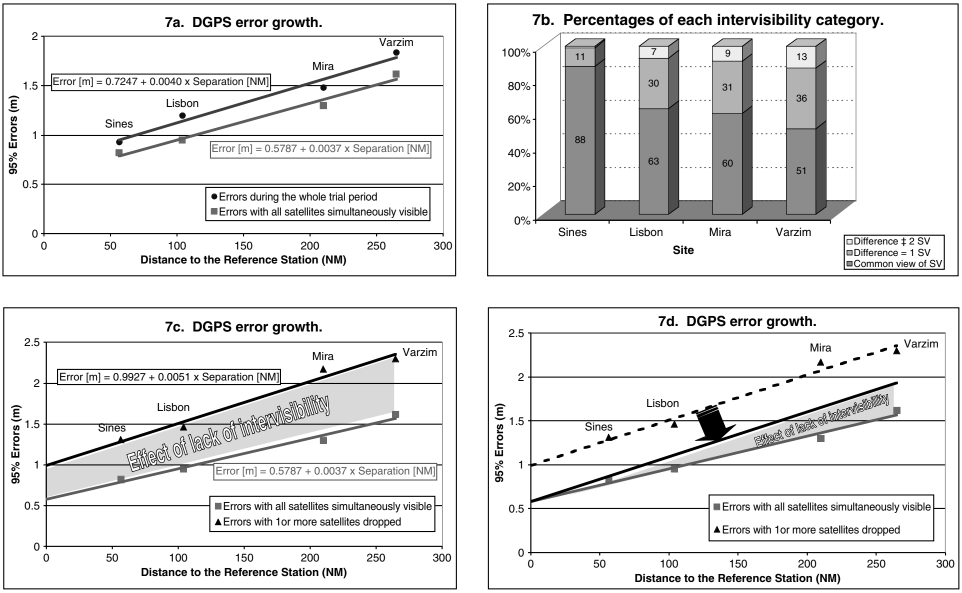

Figure 7. a. Comparison between errors during the whole trial period and errors during periods of common visibility. b. Percentages of each intervisibility category. c. Effect of lack of intervisibility on this trial. d. Effect of lack of intervisibility on ideally sited receivers.

floor (0·5787 m) and the slope of the function represents the decorrelation of the individual GPS errors (0·0037 × Separation [nm]): 0·37 m per each 100 nm separation. In order to fully understand the effect of the lack of intervisibility, this plot must be analysed in conjunction with the Figure 7b, which depicts the percentages from columns 4, 6 and 8 of Table 2.

To view graphically the effect of the intervisibility problem, Figure 7c was obtained with the two error growth linear equations: one fitting to the errors obtained on the periods of optimum intervisibility and another one fitting to the errors obtained when the user employed less satellites than the ones viewed at the station. The former is the equation already shown in Figure 7a and which gives the spatial decorrelation of the GPS errors. The latter equation estimates the errors on all the epochs when the intervisibility had an added effect on the error growth. Therefore, the difference between the two equations corresponds to the effect of the forced selection of satellites.

The problem for users is that they do not know when they are viewing the same satellites as the Reference Station or when they are seeing fewer satellites, and therefore they do not know which of the equations is the adequate for each epoch. If there were no obstructions to the sky on the four sites and all the mask angles were equal to the Reference Station one, both these equations should have the same intercept value, meaning that the intervisibility had no impact on a zero baseline receiver, because it would see always exactly the same satellites as the Reference Station. However, all receivers were placed on harbours, where it is difficult to put the antenna so as to avoid all sky-shading, particularly as a temporary mounting, and the Lisbon receiver had to be configured with a mask angle higher than that of the beacon. These are the reasons why the equation fitting to the errors obtained when one or more satellites were not simultaneously visible did not converge with the other equation for a distance of zero.

Nevertheless, Figure 7c allows us to quantify the effect of the lack of intervisibility on the results of this trial by calculating the difference between the two plotted linear equations, as $0{\cdot}414+0{\cdot}0014\ S$. The intercept value represents the average effect of the local obstructions. Assuming that the loss of satellites due to local shading was similar on all four sites (which is not necessarily true, but is an acceptable simplification which helps in this evaluation), it is possible to say that the corresponding loss of accuracy on each site amounted to 0·414 m, which is also the loss of accuracy at a zero baseline.

The inclination of the function: $0{\cdot}0014\ S$ is the quantification of the degradation of performance occurred in the epochs when the user is seeing fewer satellites than the station, compared with the ideal situation, in which users have a clear view of the sky. This is illustrated by Figure 7d, which depicts the error growth component caused by the lack of intervisibility, in ideal situations. The main conclusion from this plot is that on ideal sites, with no sky shading, the effect of the lack of intervisibility is very low, for the usual ranges of DGPS beacons. Having a clear view of the sky is common at sea, particularly off-shore, but in harbours the visibility of the sky is generally limited by mountains, buildings, cranes, etc. The corresponding degradation of performance depends on the sky shading and is therefore different from port to port. Therefore, users inside ports may expect a more pronounced error growth due to lack of intervisibility. Assuming that the four ports where this trial took place are representative of the average situation (in what concerns sky shading), then the effect of the lack of intervisibility corresponds to the shadowed area on Figure 7c.

Nevertheless, the corresponding loss of accuracy only occurs during the periods when the user is viewing fewer satellites than the Reference Station. This period is relatively small for shorter baselines and increases for longer baselines. This means that users close to the Reference Station will have an error which is mainly determined by the DGPS *noise floor* plus the spatial decorrelation of satellite ephemeris and ionospheric/tropospheric errors, while more distant users will suffer an additional degradation of accuracy, caused by the lack of intervisibility, which will cause a small error growth on increasingly longer periods of time.

8. CONCLUSION.

The differential technique eliminates most GPS errors, but the corrections can lose their validity with time (due to temporal decorrelation) and with the displacement between Reference Station and user (due to spatial decorrelation). The errors which decorrelate with increasing age of corrections are satellite clock errors, ephemeris errors, ionospheric errors and tropospheric errors, but all of these change very slowly and, hence, are strongly correlated over many minutes. Therefore, differential corrections are expected to remain valid for extended periods of time. To confirm this, a small trial was conducted, but the receiver employed had the limitation of reverting to stand-alone positioning after 4 minutes without corrections. The trial showed that after 4 minutes without corrections the positioning error grew only 0·5 to 1 m, but it can reasonably be accepted that even 10 to 15 minutes after being calculated the differential corrections remain valid, although most DGPS receivers revert to stand-alone positioning much before that.

With such slow temporal decorrelation, currently it is decorrelation with distance the most important factor determining DGPS accuracy.

The errors which decorrelate with distance are satellite ephemeris, ionospheric and tropospheric errors. Combining the decorrelation of each of these errors (estimated by Parkinson and Enge), the expected Pseudo-Range error can be predicted theoretically as less than 0·44 m, for every 100 km separation between Reference Station and user (95% estimate). Additionally, there is another effect which contributes to the error growth with increasing distance and that is whether the Reference Station and the user can see the same satellites.

IALA and the USCG consider for maritime DGPS networks a rate of accuracy degradation of approximately 1 m for each 150 km distance from the broadcast site, i.e. approximately 0·67 m for each 100 km separation. To evaluate the error growth with increasing distance to the Reference Station, a trial was devised in which seven DGPS receivers were installed along the Portuguese coastline, at approximately 50 nm intervals from the Reference Station. The PC which was recording data from one of the receivers (sited at *Nazaré*) failed but the other 6 receivers gathered enough data to estimate the error growth. According to the errors observed by the 6 receivers, the rate of accuracy degradation of the Portuguese DGPS network is 0·22 m for each 100 km distance from the Reference Station. This means the error growth is lower than the theoretical estimate published by IALA and the USCG (0·67 m per 100 km). Therefore, a reasonable approximation to estimate the achievable accuracy at a given point is to take the typical error near the Broadcast Station (on the order of 0·5 m to 1 m) and add an additional 0·4 m of error for each 100 nm of separation from the Broadcast Station. Furthermore, the spatial decorrelation of the individual GPS errors was isolated from the lack of intervisibility, allowing the conclusion that, for the usual ranges of DGPS stations, the latter effect is very small, particularly if the users set no mask angle and have no obstructions to the visibility of the sky (which is generally the case on open seas). In harbours, the effect of the lack of intervisibility is more significant, but nevertheless not as much as the spatial decorrelation of the individual errors.

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