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Beat wave excitation of electron plasma wave by coaxial cosh-Gaussian laser beams in collisional plasma

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Abstract

This paper presents a scheme for beat wave excitation of an electron plasma wave (EPW) by cross-focusing of two intense cosh-Gaussian (ChG) laser beams in an under dense collisional plasma. The plasma wave is generated on account of beating of two ChG laser beams of frequencies ω_1 and ω_2. Starting from Maxwell's equations, coupled differential equations governing the evolution of spot size of laser beams with distance of propagation have been derived by using Moment theory approach in Wentzel–Kramers–Brillouin approximation. The collisional nonlinearity depends not only on the intensity of first laser beam, but also on that of second laser beam. Therefore, dynamics of first laser beam affects that of other and hence cross-focusing of the two laser beams takes place. Numerical simulations have been carried out to investigate the effect of laser as well as plasma parameters on cross-focusing of laser beams and further its effect on power of excited EPW. It has been found that decentered parameters of the two laser beams have significant effect on power of EPW.

Keywords: Beat wave; cosh-Gaussian; cross-focusing; electron plasma wave; collisional plasma

1. INTRODUCTION

Ever since the invention of laser (Maiman, 1960), the area of laser–plasma interactions due to its vivid applications has been a topic of immense interest to physicists. There have been ongoing conscious efforts to consistently improve upon the understanding of the subject by carrying out comprehensive studies encompassing theoretical as well as experimental aspects. All these works have collectively contributed to the rapid development of potential applications, including fast ignition schemes for inertial confinement fusion (Tabak *et al.*, 1994; Deutsch *et al.*, 1996), laser-driven plasma-based particle accelerators (Tajima & Dawson, 1979; Geddes *et al.*, 2004), X-ray lasers (Eder *et al.*, 1994; Lemoff *et al.*, 1995), laser–plasma channeling (Singh & Walia, 2010), etc. Efficient coupling of laser energy with plasmas is an important prerequisite for the feasibility of all these applications. In the absence of an optical guiding mechanism, the interaction length is limited approximately to a Rayleigh length due to diffraction divergence of the laser beam. Therefore, diffraction broadening is one of the fundamental phenomena that negate the efficient coupling of laser energy with plasmas. In conventional optics, diffraction of laser beam can be averted by using optical fibers or by relying on the phenomenon of self-focusing. When an intense laser beam with non-uniform spatial intensity distribution along its wave front propagates through a collisional plasma, it produces non-uniform ohmic heating of plasma electrons. Due to spatial intensity distribution of the laser beam, the electrons will experience temperature change according to their radial position. The temperature change will translate into modification of dielectric properties of the plasma in such a way that the plasma starts behaving like a convex lens leading to self-contraction of transverse dimensions of the laser beam.

Several nonlinear effects such as excitation of electron plasma wave (EPW), filamentation of laser beam, stimulated Raman scattering, stimulated Brillouin scattering, etc. come to existence, during the propagation of intense laser beams through plasmas. Some of these phenomena lead to anomalous electron and ion heating, others to depleting and redirecting the incident laser flux. Therefore, theoretical as well as experimental investigations of some of these nonlinear phenomena are of paramount importance in inertial confinement fusion and other applications involving laser–plasma interactions.

The propagation of laser beams through plasmas can excite natural modes of vibration of plasmas, that is, EPW or ion-

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acoustic waves. The plasma waves can be excited either by beating two co-propagating laser beams, differing in frequencies by the plasma frequency, or by a single short pulse of duration equal to the plasma period. Recently, there has been a renewed interest in the nonlinear excitation of plasma waves in view of their various applications such as plasma-heating mechanism for laboratory fusion devices (Bruce *et al*., 1972), acceleration of charged particles to ultrahigh energies (Tajima & Dawson, 1979), generation of terahertz radiations, etc.

A lot of theoretical and experimental work has been reported related to the beat wave excitation of EPWs in the past. Rosenbluth and Liu (1972) investigated the growth and saturation of large amplitude plasma wave in cold homogeneous collisionless plasma due to the beating of two laser beams with frequencies much above the plasma frequency by taking into account the modulation of Lorentz force. Darrow *et al*. (1986, 1987), have reported a model for beat wave excitation of EPWs in rippled density plasma. Leemans *et al*. (1992) have examined the nonlinear dynamics of the laser-driven plasma beat wave in the presence of a strong short wavelength density ripple using the relativistic Lagrangian oscillator model. Sharma and Chauhan (2008) investigated the effect of cross-focusing of two coaxial laser beams on beat wave excitation in relativistic plasmas.

The vast majority of earlier works on excitation of EPW outlined above have been carried out under the assumption of uniform laser beam or laser beams having Gaussian distribution of intensity along their wave fronts. In contrast to this picture, a new class of laser beams known as ChG laser beams is gaining much interest among researchers because they possess high power and low divergence in comparison with Gaussian beams. A review of literature reveals the fact that no earlier theoretical investigation on beat wave excitation of EPW has been carried out for ChG laser beams. The aim of this paper is to delineate for the first time the effect of cross-focusing of two coaxial ChG laser beams on beat wave excitation of EPW in an underdense plasma having collisional nonlinearity.

This paper is organized as follows: In Section 2, nonlinear dielectric function of the plasma has been obtained. In Section 3, coupled differential equations governing the cross-focusing of the laser beams have been derived. In Section 4, expression for normalized power of EPW has been derived. Section 5 gives detailed conclusions of the results obtained.

2. DIELECTRIC FUNCTION OF PLASMA

Consider the propagation of two coaxial, linearly polarized laser beams having electric field vectors

$$\mathbf{E_j}(r,z,t) = A_j(r,z)\, e^{-\iota(\omega_j t - k_j z)}\mathbf{e_x} \tag{1}$$

through an under dense collisional plasma having dielectric function

$$\epsilon_j = 1 - \frac{\omega_p^2}{\omega_j^2} \tag{2}$$

where, $j = 1,\ 2$ and ω_j and k_j, respectively are angular frequencies and vacuum wave numbers of the laser beams, $A_j(r,z)$ are the slowly varying complex amplitudes of the laser beams and

$$\omega_p^2 = \frac{4\pi e^2}{m} n_e \tag{3}$$

is the plasma frequency in the presence of laser beams. The initial intensity distribution of the laser beams along their wavefronts is assumed to be ChG and is given by Konar *et al*. (2007); Gill *et al*. (2011); Patil *et al*. (2012)

$$A_j A_j^\star|_{z=0} = E_{j0}^2\, e^{-(r^2/r_j^2)} \cosh^2\left(\frac{b_j}{r_j} r\right) \tag{4}$$

where r_j are the radii of the laser beams at the plane of incidence, that is, at $z = 0$, E_{j0} are the axial amplitudes of the electric fields of the laser beams, b_j/r_j are the parameters associated with cosh function and are known as cosh factors. Such ChG laser beams can be produced in the laboratory by superposition of two decentered Gaussian laser beams having same spot size and that are in phase with each other (Lu *et al*., 1999). Hence, the parameters b_j are also known as decentered parameters.

For $z > 0$, energy conserving ansatz for the intensity distribution of the ChG laser beams propagating along z-axis are given by

$$A_j A_j^\star = \frac{E_{j0}^2}{f_j^2} e^{-r^2/r_j^2 f_j^2} \cosh^2\left(\frac{b_j}{r_j f_j} r\right) \tag{5}$$

where $r_j f_j$ are the instantaneous radii of the laser beams. Hence, the parameters f_j are termed as dimensionless beam width parameters that are measure of both axial intensity and spot size of the laser beams. For $b_j = 0$, the ChG distribution gradually gets converted into usual Gaussian distribution that is,

$$\lim_{b_j \to 0} A_j A_j^\star = \frac{E_{j0}^2}{f_j^2} e^{-r^2/r_j^2 f_j^2}$$

The nonuniform intensity distribution along the wavefronts of the laser beams produce nonuniform heating of plasma electrons as a result of which redistribution of electrons takes place. The resultant distribution of electrons is given by Sodha *et al*., (1976)

$$n_e = n_0 \left(\frac{2T_0}{T_0 + T}\right)^{1-(s/2)} \tag{6}$$

where n_0 is the equilibrium electron density of plasma, T_0 is the equilibrium plasma temperature and T is the temperature of plasma in the presence of laser beams which is related to laser electric fields by

$$\frac{T}{T_0} = 1 + \sum_j \beta_j E_j E_j^\star \tag{7}$$

where $\beta_j = (e^2 M \,/\, 6K_0 T_0 m^2 \omega_j^2)$ are the coefficients of collisional nonlinearity; *e, m*, respectively are the electronic charge and mass, M is the mass of ions, and K_0 is the Boltzmann constant. Using Eqs (3)–(7) in Eq. (2) we get

$$\epsilon_j = 1 - \frac{\omega_{p0}^2}{\omega_j^2}\left\{1 + \frac{1}{2}\frac{\beta_j E_{j0}^2}{f_j^2} e^{-r^2/r_j^2 f_j^2} \cosh^2\left(\frac{b_j}{r_j f_j} r\right)\right\}^{(s/2)-1} \tag{8}$$

where $\omega_{p0}^2 = (4\pi e^2 \,/\, m) n_0$ is the plasma frequency in the absence of the laser beams. The parameter s describes the nature of collisions and can be defined through the dependence of collision frequency ν on electron's random velocity $\mathbf{v}$ and temperature T as $\boldsymbol{\nu} \propto (v^2 \,/\, T)^s$. For velocity-dependent collisions $s = 0$, for collisions between electrons and diatomic molecules $s = 2$ and for electron–ion collisions $s = -3$. Taking

$$\epsilon = \epsilon_{0j} + \phi_j(A_1 A_1^\star, A_2 A_2^\star) \tag{9}$$

we get,

$$\epsilon_{0j} = 1 - \frac{\omega_{p0}^2}{\omega_j^2} \tag{10}$$

and

$$\phi_j(A_1 A_1^\star, A_2 A_2^\star) = \frac{\omega_{p0}^2}{\omega_j^2}\left[1 - \left\{1 + \frac{1}{2}\sum_j \frac{\beta_j E_{j0}^2}{f_j^2} e^{-(r^2/r_j^2 f_j^2)} \cosh^2\left(\frac{b_j}{r_j f_j} r\right)\right\}^{(s/2)-1}\right] \tag{11}$$

where, ϵ_{0j} and ϕ_j, respectively, are the linear and nonlinear parts of the dielectric function of the plasma.

3. CROSS-FOCUSING OF LASER BEAMS

Starting from Ampere's and Faraday's laws for an isotropic, non-conducting, and non-absorbing medium ($\mathbf{J} = 0$, $\rho = 0$, $\mu = 1$), we get

$$\nabla \times \mathbf{B_j} = \frac{1}{c}\frac{\partial \mathbf{D_j}}{\partial t} \tag{12}$$

$$\nabla \times \mathbf{E_j} = -\frac{1}{c}\frac{\partial \mathbf{B_j}}{\partial t} \tag{13}$$

where $\mathbf{E}_j$ and $\mathbf{B}_j$ are electric and magnetic fields associated with the laser beams and $\mathbf{D_j} = \epsilon_j \mathbf{E_j}$ are the electric displacement vectors. Eliminating $\mathbf{B}_j$ from Eqs (12) and (13) it can be shown that electric field vectors $\mathbf{E}_j$ of the laser beams satisfy the wave equation

$$\nabla^2 \mathbf{E_j} + \frac{\omega_j^2}{c^2}\epsilon_j \mathbf{E_j} = 0 \tag{14}$$

In deriving Eq. (14) the polarization term $\nabla(\nabla.\mathbf{E_j})$ has been neglected under the assumption that either the transverse dielectric variations are weak or the plasma is significantly under dense.

Using Eq. (1) in Eq. (14) we get

$$\iota \frac{dA_j}{dz} = \frac{1}{2k_j}\nabla_\perp^2 A_j + \frac{k_j}{2\epsilon_{0j}}\phi_j(A_1 A_1^\star, A_2 A_2^\star) A_j \tag{15}$$

where

$$\nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

is the Laplacian along the transverse direction. Equation (15) is the well-known nonlinear Schrödinger wave equation that describes propagation characteristics of electromagnetic beams in nonlinear media under the assumption that wave-amplitude scale length along the z-axis is much larger as compared with characteristic scale in the transverse direction.

Due to its symmetrical properties, the wave Eq. (15) possesses a number of conserved quantities. In view of self-focusing of the laser beams, the two most important invariants are

$$I_{0j} = \int_0^{2\pi}\int_0^\infty A_j A_j^\star \, r\, dr\, d\theta \tag{16}$$

$$H_j = \int_0^{2\pi}\int_0^\infty \frac{1}{2k_j^2}(|\nabla_\perp A_j|^2 - F_j) r\, dr\, d\theta \tag{17}$$

where

$$F_j = \frac{1}{2\epsilon_{0j}}\int_0^{A_j A_j^\star} \phi_j(A_1 A_1^\star, A_2 A_2^\star) d(A_j A_j^\star) \tag{18}$$

The first invariant I_{0j} is merely a statement of the conservation of energy of the laser beams and second invariant H_j relates the wavefront curvature of the laser beams to the plasma nonlinearity. In moment theory approach the average value of a physical quantity M_j is defined as

$$\langle M_j^2 \rangle = \frac{1}{I_{0j}}\int_0^{2\pi}\int_0^\infty A_j A_j^\star M_j r\, dr\, d\theta \tag{19}$$

The quantity of particular importance from the standpoint of self-focusing is mean square radius of the laser beams,

$$\langle R_j^2 \rangle = \frac{1}{I_{0j}}\int_0^{2\pi}\int_0^\infty r^2 A_j A_j^\star \, r\, dr\, d\theta \tag{20}$$

Following the procedure of Lam *et al*. (1977), we get the following quasi-optic equation governing the evolution of mean-square radius of laser beams with distance of propagation

$$\frac{d^2}{dz^2}\langle a_j^2\rangle = 4\frac{H_j}{I_{0j}} - \frac{4}{I_{0j}}\int_0^{2\pi}\int_0^{\infty}\left(2F_j - \frac{1}{2\epsilon_{0j}}A_j A_j^{\star}\phi_j\right) r\,dr\,d\theta \quad (21)$$

Using Eqs (5), (11), (16)–(19), (20) in (21), we get the following coupled differential equations governing the cross-focusing of the laser beams.

$$\frac{d^2 f_1}{d\xi^2} + \frac{1}{f_1}\left(\frac{df_1}{d\xi}\right)^2 = \left(\frac{1+e^{-b_1^2}(1-b_1^2)}{2(1+b_1^2)}\right) \frac{1}{f_1^3} + \left(\frac{s}{2}-1\right)\left(\frac{\omega_{p0}r_1}{c}\right)^2\left(\frac{e^{-b_1^2}}{1+b_1^2}\right) J_1(f_1, f_2) \quad (22)$$

$$\frac{d^2 f_2}{d\xi^2} + \frac{1}{f_2}\left(\frac{df_2}{d\xi}\right)^2 = \left(\frac{r_1}{r_2}\right)^4\left(\frac{\omega_1}{\omega_2}\right)^2\left(\frac{\epsilon_{01}}{\epsilon_{02}}\right)\left[\left(\frac{1+e^{-b_2^2}(1-b_2^2)}{2(1+b_2^2)}\right)\frac{1}{f_2^3} + \left(\frac{s}{2}-1\right)\left(\frac{\omega_{p0}r_1}{c}\right)^2\left(\frac{e^{-b_2^2}}{1+b_2^2}\right) J_2(f_1, f_2)\right] \quad (23)$$

where $\xi = z / k_1 r_1^2$ is the dimensionless distance of propagation and,

$$J_1(f_1, f_2) = \left\{\frac{\beta_1 E_{10}^2}{f_1^3}(T_1 - b_1 T_2) + \frac{\beta_2 E_{20}^2}{f_1^3}\left(\frac{r_1^2}{r_2^2}\frac{f_1^4}{f_2^4}T_3 - b_2\frac{r_1}{r_2}\frac{f_1^3}{f_2^3}T_4\right)\right\}$$

$$J_2(f_1, f_2) = \left\{\frac{\beta_1 E_{10}^2}{f_2^3}(T_3 - b_1 T_5) + \frac{\beta_2 E_{20}^2}{f_2^3}\left(\frac{r_1^2}{r_2^2}\frac{f_1^4}{f_2^4}T_6 - b_2\frac{r_1}{r_2}\frac{f_1^3}{f_2^3}T_7\right)\right\}$$

$$T_1 = \int_0^{\infty} x^3 e^{-2x^2}\cosh^4(b_1 x) G(x)dx$$

$$T_2 = \int_0^{\infty} x^2 e^{-2x^2}\cosh^3(b_1 x)\sinh(b_1 x) G(x)dx$$

$$T_3 = \int_0^{\infty} x^3 e^{-x^2} e^{-(r_1 f_1/r_2 f_2)^2 x^2} \cosh^2(b_1 x)\cosh^2\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right) G(x)dx$$

$$T_4 = \int_0^{\infty} x^2 e^{-x^2} e^{-(r_1 f_1/r_2 f_2)^2 x^2}\cosh^2(b_1 x) \cosh\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right)\sinh\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right) G(x)dx$$

$$T_5 = \int_0^{\infty} x^2 e^{-x^2} e^{-(r_1 f_1/r_2 f_2)^2 x^2}\cosh(b_1 x)\sinh(b_1 x) \cosh^2\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right)\sinh\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right) G(x)dx$$

$$T_6 = \int_0^{\infty} x^3 e^{-2(r_1 f_1/r_2 f_2)^2 x^2}\cosh^4\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right) G(x)dx$$

$$T_7 = \int_0^{\infty} x^3 e^{-2(r_1 f_1/r_2 f_2)^2 x^2}\cosh^3\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right) \sinh\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right) G(x)dx$$

$$G(x) = \left\{1 + \frac{1}{2}\frac{\beta_1 E_{10}^2}{f_1^2}e^{-x^2}\cosh^2(b_1 x) + \frac{1}{2}\frac{\beta_2 E_{20}^2}{f_2^2}e^{-(r_1 f_1/r_2 f_2)^2 x^2}\cosh^2\left(b_2\frac{r_1 f_1}{r_2 f_2}x\right)\right\}^{(s/2)-2}$$

$$x = \frac{r}{r_1 f_1}$$

For initially plane wavefronts Eqs (22) and (23) are subjected to boundary conditions $f_j = 1$ and $df_j / d\xi = 0$ at $\xi = 0$.

Equations (22) and (23) are the coupled nonlinear differential equations governing the relativistic cross-focusing of two coaxial ChG laser beams in an under dense plasma. Analytical solutions to these equations are not possible. We therefore seek numerical computational techniques to investigate the beam dynamics. Before that, it is worth noting to understand the physical mechanisms that originate various terms on the right-hand sides (RHS) of Eqs (22) and (23). The first terms on RHS of Eqs (22) and (23) are responsible for diffraction divergence of the laser beams and have their origin in the Laplacian $\nabla_{\perp}^2$, appearing in nonlinear wave Eq. (15). The second terms on RHS of these equations arise under the combined effect of relativistic mass nonlinearity and nonlinear coupling between the two laser beams. These terms are responsible for nonlinear refraction of the laser beams. It is the relative competition between the two terms that determine the focusing/defocusing of the laser beams in plasma.

Equations (22) and (23) have been solved for the following set of laser–plasma parameters

$$\omega_1 = 1.78\times 10^{15}\ \text{rad/s};\quad \omega_2 = 1.98\times 10^{15}\ \text{rad/s}$$
$$r_1 = 15\ \mu\text{m};\quad r_2 = 16.67\ \mu\text{m}$$
$$T_0 = 10^6\ \text{K}$$

to analyze the effect of decentered parameters and plasma density on cross focusing of the laser beams and beat wave excitation of EPW.

Figures 1 and 2 illustrate the effect of decentered parameter b_1 on focusing/defocusing of first laser beam. The plots in Fig. 1 depict that as the value of decentered parameter of first laser beam increases for $0 \le b_1 < 1$, there is increase in the extent of its self-focusing. This is due to the fact that for $0 \le b_1 < 1$ the intensity profile of ChG laser beam

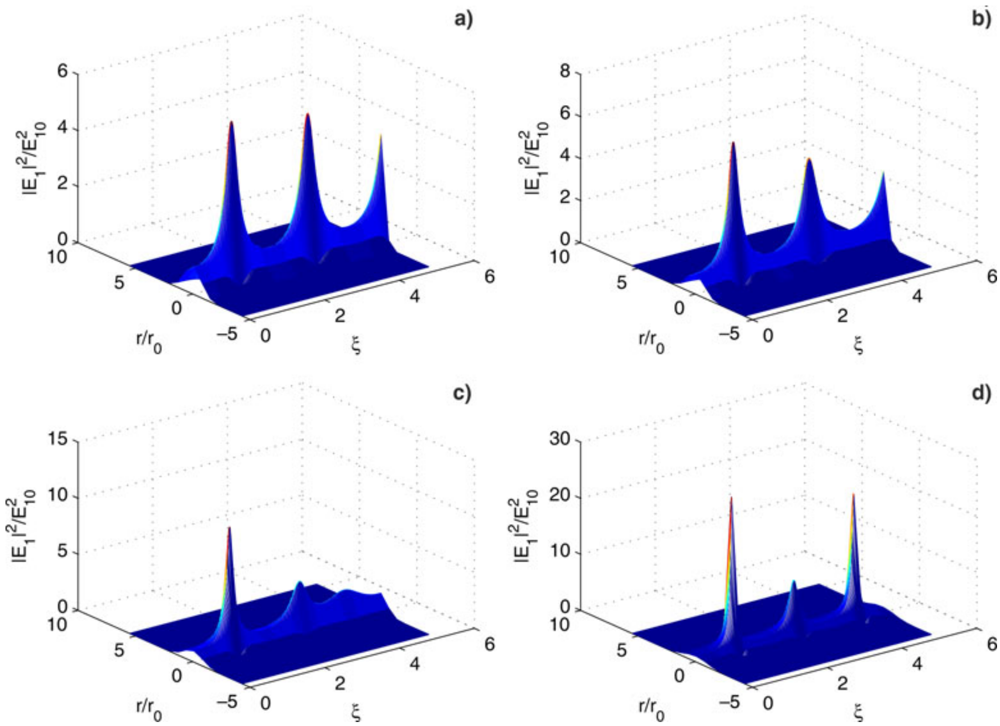

Fig. 1. Variation of normalized intensity of first beam with normalized distance of propagation ξ and radial distance r / r_1, keeping $(\omega_{p0}r_1 / c)^2 = 12$, $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.5$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of decentered parameter, (a) $b_1 = 0$, (b) $b_1 = 0.25$, (c) $b_1 = 0.50$, (d) $b_1 = 0.75$, respectively.

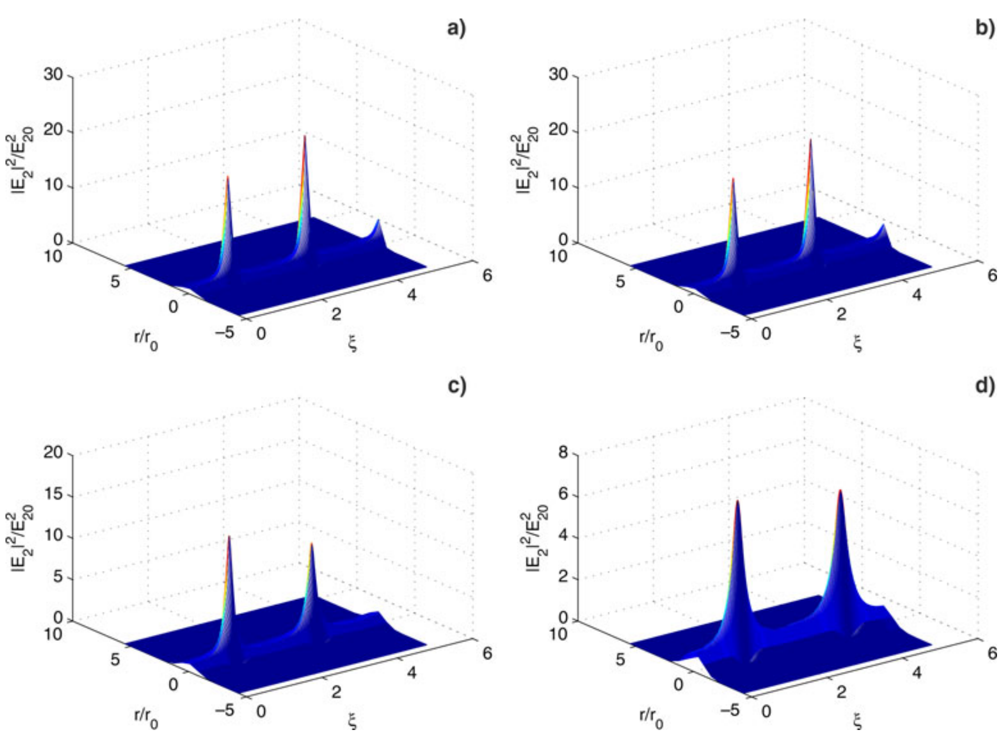

Fig. 2. Variation of normalized intensity of first beam with normalized distance of propagation ξ and radial distance r / r_1, keeping $(\omega_{p0}r_1 / c)^2 = 12$, $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of decentered parameter, (a) $b_1 = 1.0$, (b) $b_1 = 1.25$, (c) $b_1 = 1.50$, (d) $b_1 = 1.75$, respectively.

resembles to that of flat topped beam and extent of flatness increases with increase in the value of b_1. As a result of this nonlinear refraction of most of the transverse part of its wavefront opposes the diffraction divergence. Hence, increase in the value of b_1 from 0 to 1 leads to stronger focusing of the first laser beam. The plots in Fig. 2 depict that as the value of b_1 increases for $b_1 \geq 1$ there is decrease in extent of self-focusing of first laser beam. This is due to the fact that as the value of decentered parameter b_1 increases from 1 onwards up to 1.50 the diffractive term relatively dominates the refractive term but still maintaining the focusing character of the first laser beam. As the value of b_1 becomes more than 1.50 (or $b_1 > 1.50$) the diffractive term completely dominates over refractive term leading to complete defocusing of the first laser beam.

Figures 3 and 4 illustrate the effect of decentered parameter b_1 of first laser beam on focusing/defocusing of second laser beam. It is observed that with increase in the decentered parameter of first laser beam there is decrease in the extent of self-focusing of the second laser beam. This is due to the fact that with an increase in the value of b_1 the magnitude of refractive term in Eq. (23) decreases which leads to reduced focusing of second laser beam.

Figures 5 and 6 illustrate the effect of plasma density ($\omega_{p0}^2 r_1^2 / c^2$) on focusing/defocusing of laser beams. It is observed that with increase in plasma density there is increase in the extent of self-focusing of both the laser beams. This is due to the fact that with increase in plasma density the number of electrons contributing to the collisional nonlinearity also increases.

Figures 7 and 8 illustrate the effect of nature of collisions on focusing/defocusing of laser beams. It is observed that in plasmas dominant with collisions between electrons and diatomic molecules, no self-focusing of both the laser beams takes place. This is due to the fact that $s = 2$ corresponds to the situation where $n_e = n_0 = n_{0i}$ and hence no redistribution of electrons takes place. Thus, nonlinear terms in Eqs (22) and (23) are zero. Diffraction is dominant mechanism in this case.

4. BEAT WAVE EXCITATION

In the dynamics of excitation of EPW, it must be mentioned here that the contribution of ions is negligible because they only provide a positive background, that is, only plasma electrons are responsible for excitation of EPW. The background plasma density is modified via non-uniform ohmic heating of plasma. Therefore, amplitude of the EPW, which depends on the background electron density, gets strongly coupled to the laser beams. The generated plasma wave is governed by equation of continuity, equation of motion and Poisson's equation:

$$\frac{\partial N}{\partial t} + \nabla (N\mathbf{v}) = 0 \tag{24}$$

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - 3\frac{K_0 T_0}{N}\nabla N \tag{25}$$

$$\nabla.\mathbf{E} = -4\pi e N \tag{26}$$

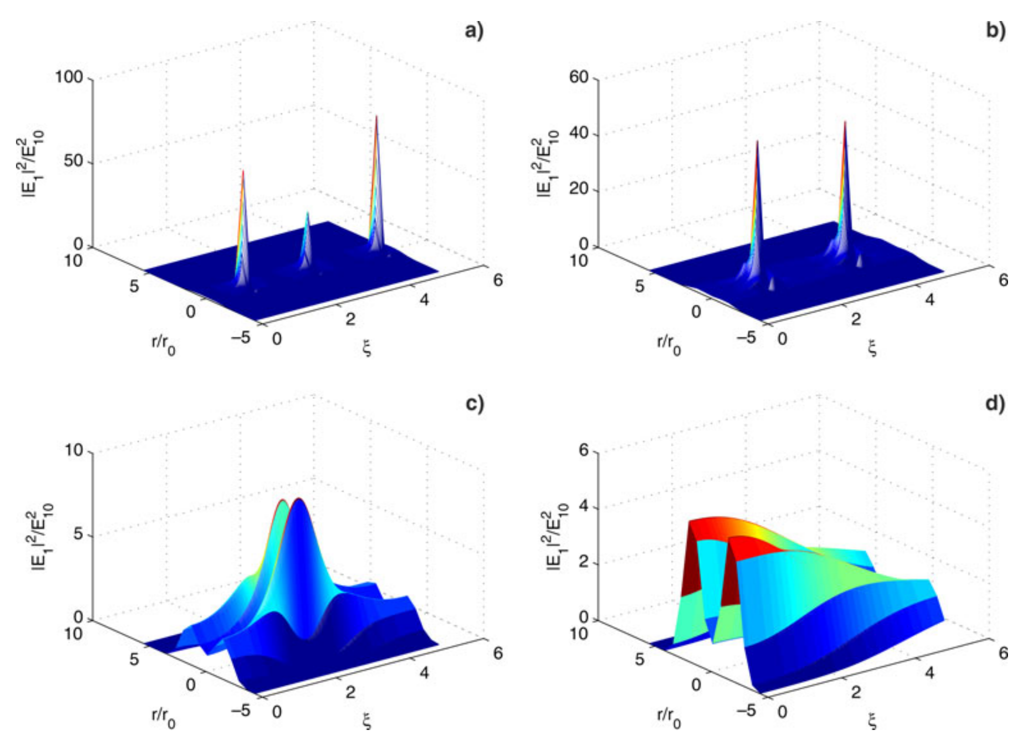

Fig. 3. Variation of normalized intensity of second beam with normalized distance of propagation ξ and radial distance r / r_2, keeping $(\omega_{p0} r_1 / c)^2 = 12$, $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of decentered parameter, (a) $b_1 = 0$, (b) $b_1 = 0.25$, (c) $b_1 = 0.50$, (d) $b_1 = 0.75$, respectively.

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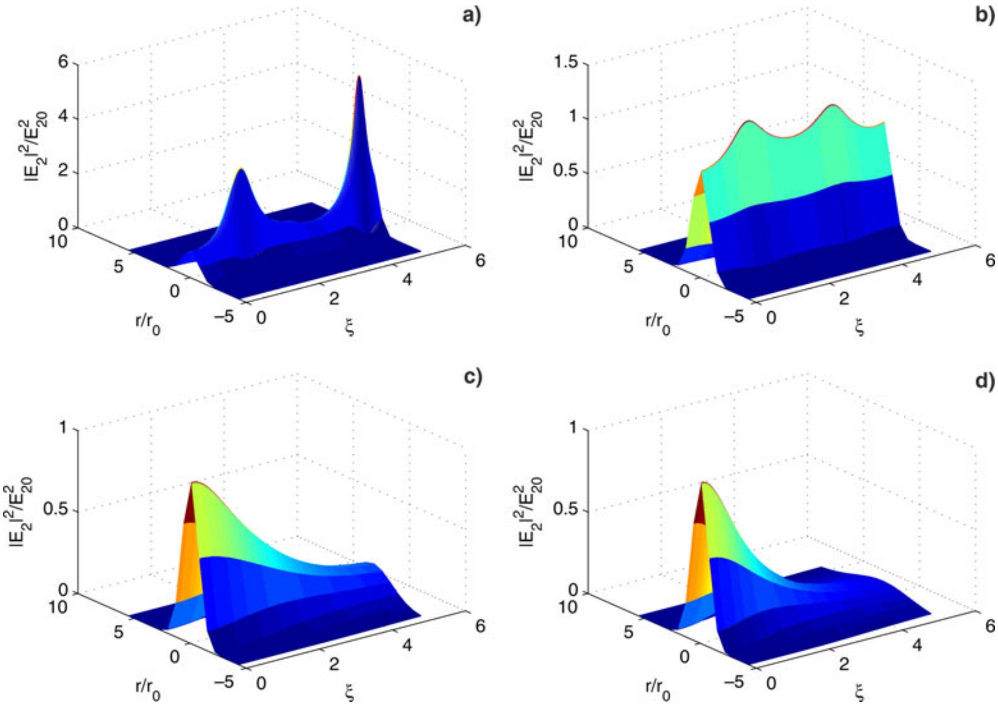

Fig. 4. Variation of normalized intensity of second beam with normalized distance of propagation ξ and radial distance $r\ /\ r_2$, keeping $(\omega_{p0}r_1\ /\ c)^2 = 12$, $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of decentered parameter, (a) $b_1 = 1.0$, (b) $b_1 = 1.25$, (c) $b_1 = 1.50$, (d) $b_1 = 1.75$, respectively.

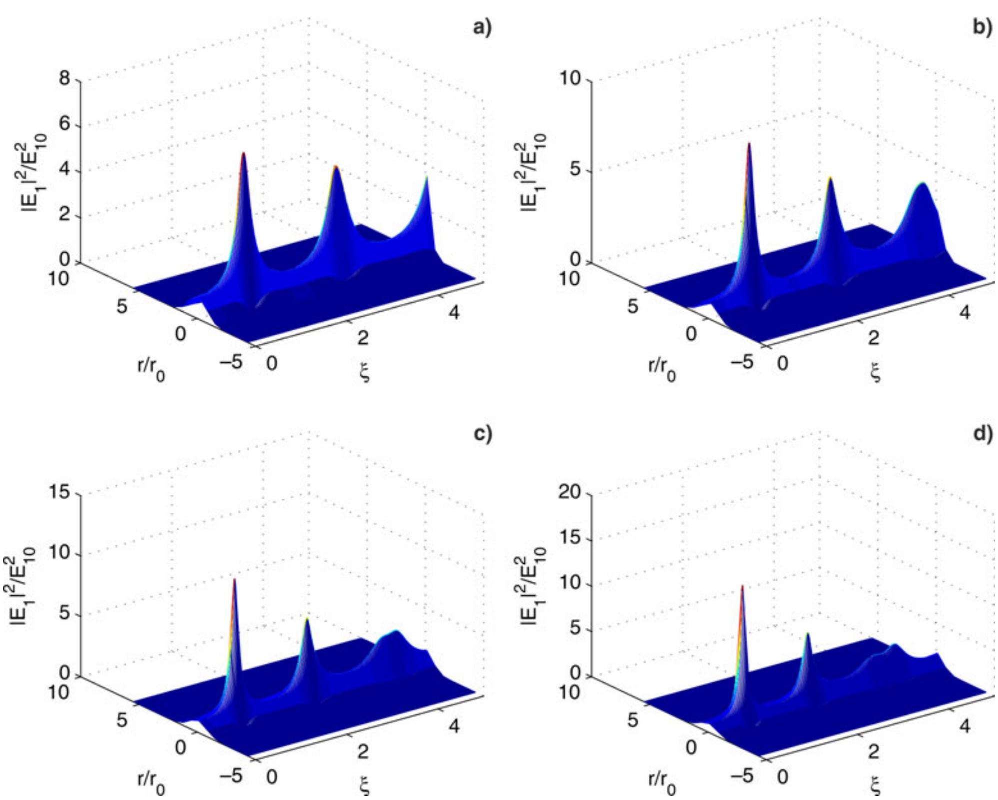

Fig. 5. Variation of normalized intensity of first beam with normalized distance of propagation ξ and radial distance $r\ /\ r_1$, keeping $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_1 = 0.25$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of normalized density, (a) $(\omega_{p0}r_1\ /\ c)^2 = 12$, (b) $(\omega_{p0}r_1\ /\ c)^2 = 14$, (c) $(\omega_{p0}r_1\ /\ c)^2 = 16$, (d) $(\omega_{p0}r_1\ /\ c)^2 = 18$, respectively.

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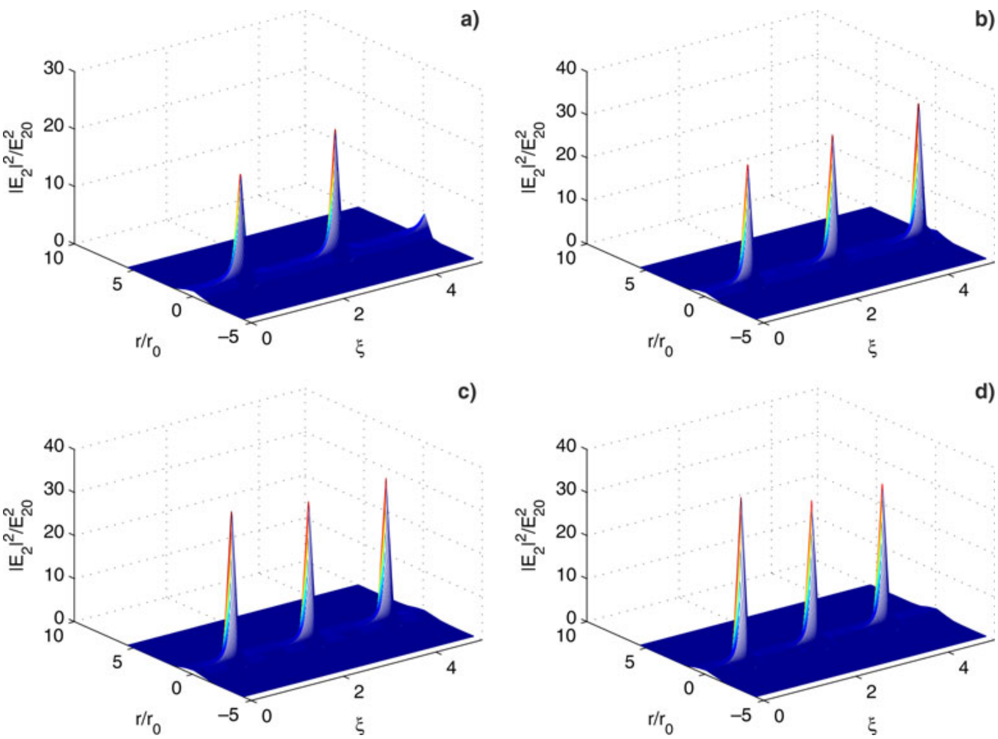

Fig. 6. Variation of normalized intensity of second beam with normalized distance of propagation ξ and radial distance $r\ /\ r_2$, keeping $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_1 = 0.25$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of normalized density, (a) $(\omega_{p0} r_1\ /\ c)^2 = 12$, (b) $(\omega_{p0} r_1\ /\ c)^2 = 14$, (c) $(\omega_{p0} r_1\ /\ c)^2 = 16$, (d) $(\omega_{p0} r_1\ /\ c)^2 = 18$, respectively.

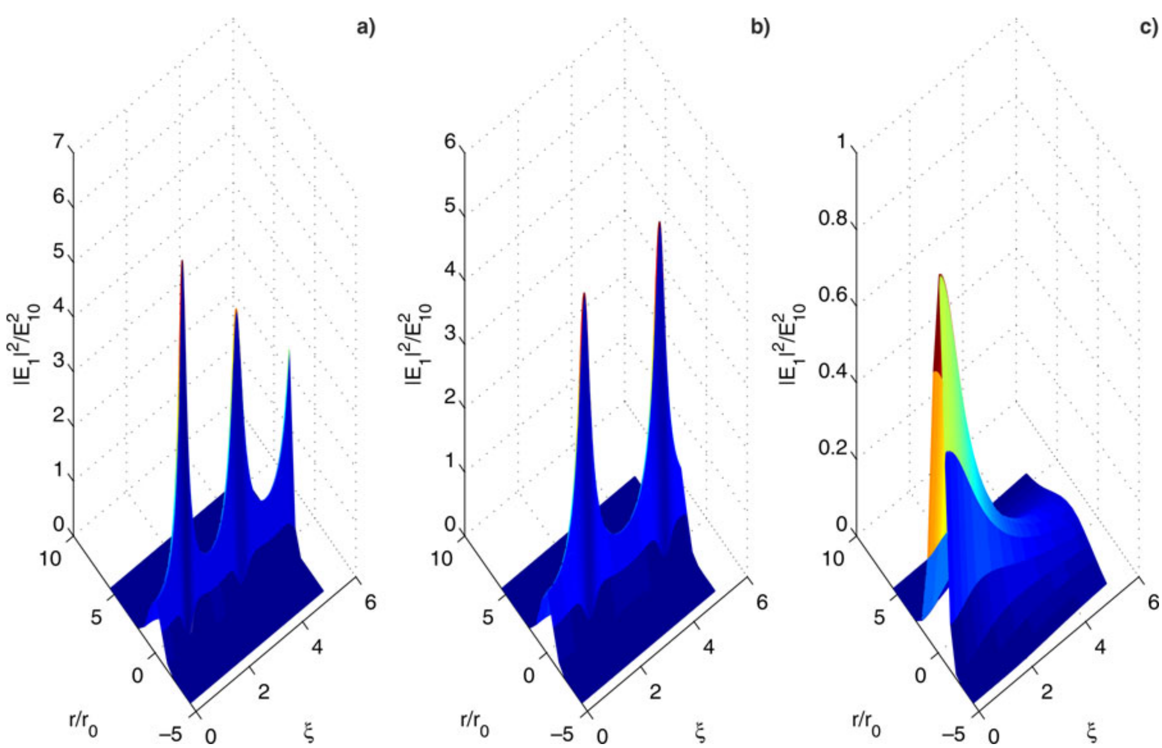

Fig. 7. Variation of normalized intensity of first beam with normalized distance of propagation ξ and radial distance $r\ /\ r_1$, keeping $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_1 = 0.25$, $b_2 = 0.25$, $(\omega_{p0} r_1\ /\ c)^2 = 12$ fixed and at different values of collisional parameter (a) $s = -3$,, (b) $s = 0$,, (c) $s = 2$, respectively.

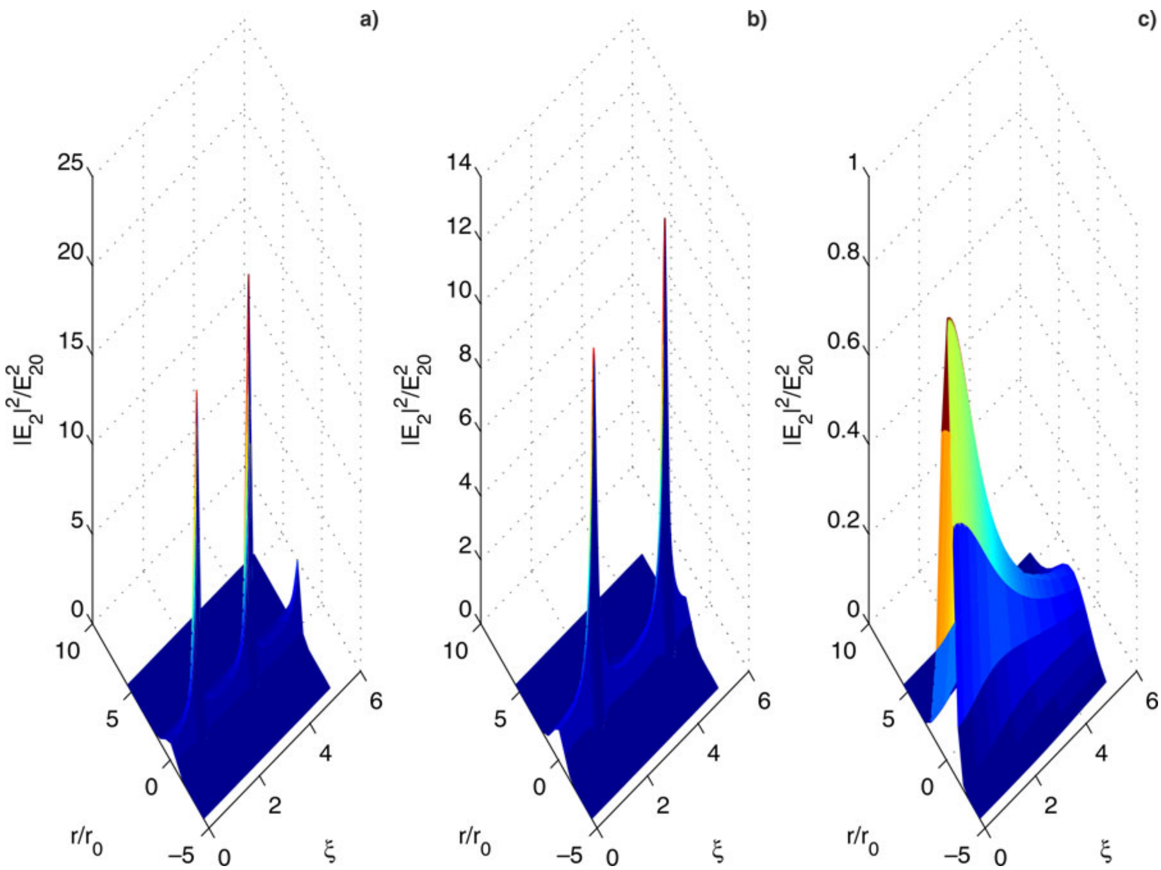

Fig. 8. Variation of normalized intensity of second beam with normalized distance of propagation ξ and radial distance r / r_2, keeping $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_1 = 0.25$, $b_2 = 0.25$, $(\omega_{p0} r_1 / c)^2 = 12$ fixed and at different values of collisional parameter (a) $s = -3$, (b) $s = 0$, (c) $s = 2$, respectively.

where N is the total electron density, $\mathbf{E}$ is the sum of electric fields of the laser beams and EPW

$$N = n_0 + n'$$

$$E = \sum_j \mathbf{E_j} + \mathbf{E}'$$

and

$$\mathbf{v} = \mathbf{v_e}$$

is the oscillatory velocity of electrons. Using linear perturbation theory, we get the following wave equation governing the dynamics of EPW

$$\frac{\partial^2 n'}{\partial t^2} - v_{\text{th}}^2 \nabla^2 n' + \omega_p^2 n' = \frac{e}{m} n_0 \nabla \sum_j E_j \tag{27}$$

Taking

$$n' = n_1 e^{\iota(\omega t - kz)}$$

where $k = \mathrm{k}_2 - \mathrm{k}_1$, we get the density perturbation associated with plasma wave

$$n_1 = \frac{e n_0}{m_0} \frac{1}{(\omega^2 - k^2 v_{\text{th}}^2 - \omega_p^2)} \sum_j \left(\frac{E_{j0}}{f_j} e^{-r^2/2r_j^2 f_j^2} F_j(f_j) \right) \tag{28}$$

where

$$F_j(f_j) = \frac{r}{r_j^2 f_j^2} \cosh\left(\frac{b_j}{r_j f_j} r\right) - \frac{b_j}{r_j f_j} \sinh\left(\frac{b_j}{r_j f_j} r\right)$$

Using Poisson's equation

$$\nabla \mathbf{E}' = -4\pi e n'$$

$$E' = E_{\text{e.p}} e^{\iota(\omega t - kz)}$$

we get

$$E_{\text{e.p}} = \frac{\iota}{k} \frac{\omega_{p0}^2}{(\omega^2 - k^2 v_{\text{th}}^2 - \omega_p^2)} \sum_j \left(\frac{E_{j0}}{f_j} e^{-r^2/2r_j^2 f_j^2} F_j(f_j) \right) \tag{29}$$

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Defining normalized power of EPW as

$$\eta = \frac{P_{\mathrm{e.p}}}{P_1}$$

where

$$P_{\mathrm{e.p}} = \frac{v_g}{8\pi}\int_0^\infty E_{\mathrm{e.p}} E_{\mathrm{e.p}}^\star 2\pi r dr$$

$$P_1 = \frac{c}{8\pi}\int_0^\infty A_1 A_1^\star 2\pi r dr$$

$$v_g = v_{\mathrm{th}}\left(1 - \frac{\omega_{p0}^2}{\omega^2}\right)^{1/2}$$

we get

$$\eta = 2\left(\frac{v_g}{c}\right) e^{-b_1^2} \frac{\omega_{p0}^2}{r_1^2 k^2 \omega^4}\int_0^\infty \frac{1}{D(\omega_1,\omega_2)} X(E_{10},E_{20})\, dx \qquad (30)$$

where

$$D(\omega_1,\omega_2) = \left\{1 - \frac{k^2 v_{\mathrm{th}}^2}{(\omega_2^2-\omega_1^2)} - \frac{\omega_{p0}^2}{(\omega_2-\omega_1)^2}\left(1+\sum_j \frac{1}{2}\frac{\beta_j E_{j0}^2}{f_j^2} e^{-(r_1 f_1/r_j f_j)^2 x^2}\cosh^2\left(b_j \frac{r_1 f_1}{r_j f_j} x\right)\right)^{(s/2)-1}\right\}^2$$

$$X(E_{10},E_{20}) = \left[\sum_j \frac{E_{j0}}{f_j} e^{-(1/2)(r_1 f_1/r_j f_j)^2 x^2}\left\{\left(\frac{r_1 f_1}{r_j f_j}\right)^2 x \cosh\left(b_j \frac{r_1 f_1}{r_j f_j} x\right) - b_j\left(\frac{r_1 f_1}{r_j f_j}\right)\sinh\left(b_j \frac{r_1 f_1}{r_j f_j} x\right)\right\}\right]^2$$

Equation (30) gives the normalized power of EPW generated as a result of beating of the two laser beams and has been solved numerically in conjunction with differential Eqs (22) and (23) for the same set of parameters as in Section 3.

Figures 9 and 10 illustrate the effect of decentered parameter b_1 of first laser beam on power of generated plasma wave. It is observed that with an increase in value of b_1 there is decrease in power of generated plasma wave. This is due to the fact that power of plasma wave is decided by the focusing/defocusing behavior of the laser beam with higher intensity and with an increase in decentered parameter of first laser beam there is decrease in extent of self-focusing of second laser beam (i.e., the laser beam with higher initial intensity).

Figure 11 illustrates the effect of plasma density $(\omega_{p0}^2 r_1^2 / c^2)$ on power of plasma wave. It is observed that with increase in plasma density there is a substantial increase in power of plasma wave. This is due to the fact that increase in plasma density leads to increase in extent of self-focusing of both the laser beams.

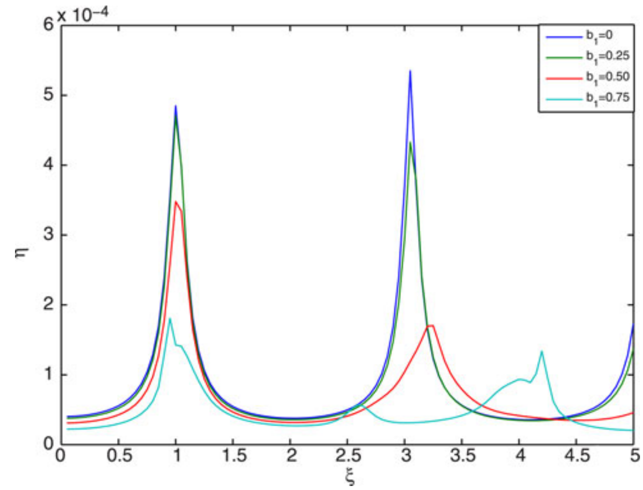

Fig. 9. Variation of normalized power η of beat wave with normalized distance of propagation ξ, keeping $(\omega_{p0} r_1 / c)^2 = 12$, $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of decentered parameter $b_1 = 0, 0.25, 0.50, 0.75$.

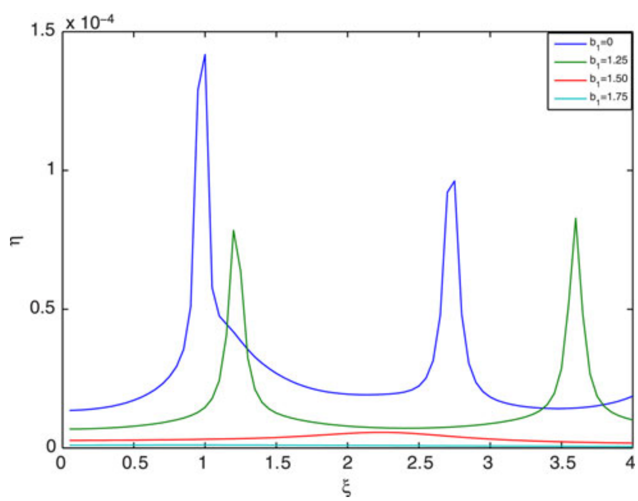

Fig. 10. Variation of normalized power η of beat wave with normalized distance of propagation ξ, keeping $(\omega_{p0} r_1 / c)^2 = 12$, $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of decentered parameter $b_1 = 1.0, 1.25, 1.50, 1.75$.

Figure 12 illustrates the effect of nature of collisions on power of plasma wave. It is observed that in case of collisions between electrons and diatomic molecules no plasma wave is generated. This is due to the fact that for $s = 2$, no self-focusing of the two laser beams takes place and hence there is no density gradient in the plasma.

5. CONCLUSIONS

In this paper authors have investigated the cross-focusing of two intense coaxial ChG laser beams in collisional plasma and subsequently its effect on beat wave excitation of EPW. Following important conclusions have been drawn from the present analysis:

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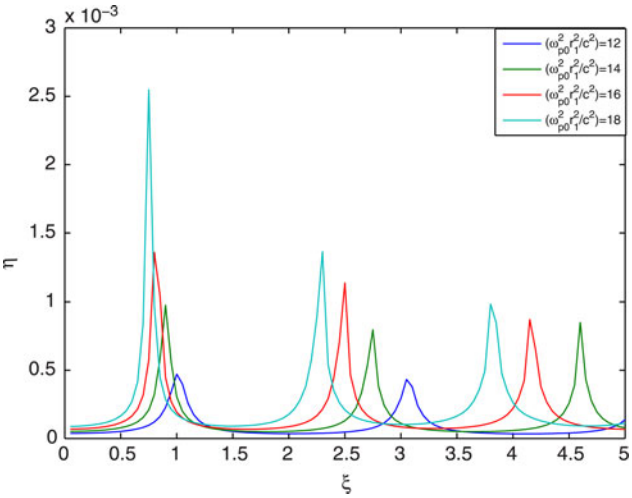

Fig. 11. Variation of normalized power η of beat wave with normalized distance of propagation ξ, keeping $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_1 = 0.25$, $b_2 = 0.25$, collisional parameter $s = -3$ fixed and at different values of normalized plasma density $(\omega_{p0} r_1 / c)^2 = 12, 14, 16, 18$.

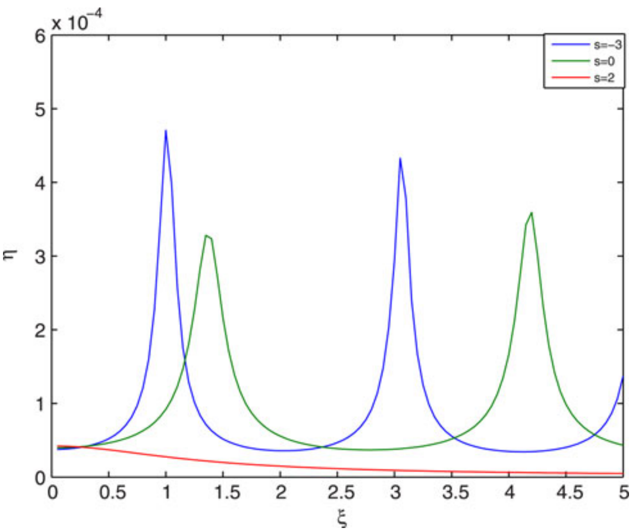

Fig. 12. Variation of normalized power η of beat wave with normalized distance of propagation ξ, keeping $\beta E_{10}^2 = 1.0$, $\beta E_{20}^2 = 1.50$, $b_1 = 0.25$, $b_2 = 0.25$, $(\omega_{p0} r_1 / c)^2 = 12$ fixed and at different values of collisional parameter $s = -3, 0, 2$.

- Increase in the value of decentered parameter of first laser beam for $0 \le b_1 < 1$ enhances the extent of self-focusing of first laser beam whereas, reduces that of second laser beam and vice versa.
- Increase in the value of decentered parameter of first laser beam for $b_1 \ge 1$ reduces the extent of self-focusing of both the laser beams.
- Maximum power of the generated plasma wave depends on the extent of self-focusing of the laser beam with higher intensity.
- With an increase of plasma density there is an increase in extent of self-focusing of both the laser beams as well as power of generated plasma wave.

The results of the present analysis may be of importance in various contexts of laser plasma physics. Besides its obvious relevance to inertial confinement fusion and beat wave accelerators, these results can also be helpful in other applications requiring laser beams with localized energy. The present analysis may serve as a guide for experimentalists working in the area of laser–plasma interactions.

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