J. Plasma Physics (2013), vol. 79, part 5, pp. 789–795. © Cambridge University Press 2013
doi:10.1017/S0022377813000457

Effect of non-extensivity during the collision between inward and outward ion acoustic solitary waves in cylindrical and spherical geometry

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(Received 7 February 2013; revised 25 March 2013; accepted 26 March 2013; first published online 14 May 2013)

Abstract. The head-on collision between two cylindrical/spherical ion acoustic solitary waves (IASWs) in un-magnetized plasmas comprising inertial ions and q-non-extensive electrons and positrons is investigated using the extended version of the Poincaré–Lighthill–Kuo perturbation method. How the interactions are taking place in cylindrical and spherical geometry are studied, and the collision is shown at different times. The non-planar geometry can modify analytical phase shifts following the head-on collision are derived. The effects of q-non-extensive electrons and positrons on the phase shift are studied. It is shown that the properties of the interaction of IASWs in cylindrical and spherical geometry are very different.

1. Introduction

Electrons are often accelerated to energies of tens of million electron volts (MeV) by the electric field induced during disruptive instability in tokamaks (Wesson et al. 1989). The resulting beam of runway electrons can carry up to about half of the original plasma current. At these high energies, electron–positron pairs can be created in collisions between the runaway electrons and the background plasma ions and electrons. Helander and Ward (2003) estimated the number of such pairs and discussed the fate of the positrons created in this way. The experiments (Tsytovich and Wharton 1978; Surko et al. 1989; Greaves et al. 1994; Greaves and Surko 1995) have established the possibility of creating a non-relativistic electron–positron plasma in the laboratory. There are at least two schemes in which the non-relativistic electron–positron plasma can be produced in the laboratory. In one scheme, a relativistic electron beam impinges on a high Z-target, where positrons are produced copiously. The relativistic pair plasma is then trapped in a magnetic mirror and is expected to cool rapidly by radiation (Trivelpiece 1972). In another scheme, positrons are accumulated from a radioactive source. The production of pure positron plasmas (Trivelpiece 1972; Surko and Murphy 1990; Greaves et al. 1994) now makes it possible to consider performing laboratory experiments on electron–positron plasmas. A natural extension of this research is to learn how to accumulate and store sufficient numbers of positrons so that they behave as a collective, multi-body system. Surko et al. (1989) have developed a method to accumulate and store positrons in an electrostatic trap using a tungsten moderator and inelastic collisions with nitrogen gas. The resulting positron gas fulfills the requirements on density n and temperature T for it to act collectively as a classical, single-component positron plasma. The electron–positron plasmas are occurring in many astrophysical environments such as the early universe neutron stars (Rees 1983), active galactic nuclei (Miller and Witta 1987), or pulsar magnetosphere (Michel 1982) and in solar atmosphere (Hansen and Emshie 1988) together with small numbers of ions. Indeed, electron–positron plasmas represent the larger class of equal-mass plasmas, a class of plasmas that may offer plasma physical properties quite different from those of conventional ion–electron plasmas. Clearly, the properties of wave motions in an electron–positron–ion (e-p-i) plasma should be different from those in two-component electron–positron plasmas. A great deal of attention has been paid to study e-p-i plasmas during the last three decades (Popel et al. 1995; Chatterjee and Ghosh 2011; Eslami et al. 2011).

An important source of information about the nature and characteristics of ion acoustic solitary wave (IASW) structures which propagate in a plasma is the behavior which is shown under interaction. For this very reason the collision of two nonlinear waves is a subject of great interest. It is well known that when two solitary structures propagate in a one-dimensional medium, they can undergo two different sorts of interactions. One occurs when they move in the same direction with different velocities. This process can be investigated in the frame of the inverse scattering method (Gardner et al. 1967) and shows that after the overtaking the two waves emerge without changes in shape but with a phase shift. The other type of interaction concerns two solitary waves which propagate in opposite directions and undergo a quasi-elastic head-on collision (Zabusky and Kruskal 1965; Su and Miura 1980) which modifies the trajectories of motion. The variation of phase shift and the trajectories of the two solitary waves after collision were studied by a remarkable number of researchers (Xue 2004a, b; Chatterjee et al. 2010; Ghosh et al. 2011, 2012a,b,c; Kundu et al. 2012) by using the extended version of Poincaré–Lighthill–Kuo (PLK) method.

Most of these are confined to unbounded planar geometry, which may not be a realistic situation in laboratory devices and space. Recently, theoretical studies (Mamun and Shukla 2001, 2002, 2011; Xue 2003; Mamun and Mannan 2011) have shown that the properties of solitary waves and shock in bounded non-planar cylindrical/spherical geometry are very different from those in unbounded planar geometry and therefore a great deal of attention is paid to understand the nonlinear phenomena like solitons, shocks, etc. (Hershkowitz and Romesser 1974; Maxon and Viecelli 1974; Sahu and Roychudhury 2004) in the non-planar geometry.

Li (2010) has recently studied the interaction of IASWs in a non-planar unmagnetized quantum plasma consisting of electrons, positrons, and ions by employing the quantum hydrodynamic model and the Korteweg–de-Vries (KdV) description. He also derived phase shifts for compressive and rarefactive IASW collisions.

2. Non-extensive distribution in the governing equation

We consider the nonlinear propagation of finite amplitude non-planar (cylindrical and spherical) ion acoustic waves in a three-component collisionless, unmagnetized plasma composed of q-non-extensive distributed electrons and positrons and inertial ions. Thus, at equilibrium, we have $n_{i0} + n_{p0} = n_{e0}$, where n_{i0}, n_{e0}, and n_{p0} are, respectively, ion, electron, positron unperturbed number density. The dynamics of ion acoustic waves (IAWs) in such e-p-i plasma can be described by the following set of normalized equations:

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial r} = -\frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \phi}{\partial r}\right) = n_e - n_p - n_i, \tag{3}$$

where $\nu = 0$ in case of 1D planar geometry and $\nu = 1, 2$ in case of non-planar cylindrical (spherical) geometries. We normalize ion number density (n_i) and ion fluid velocity (u_i) by the ion equilibrium density n_{i0} and the ion-acoustic speed $C_i = (\frac{K_B T_e}{m_i})^{\frac{1}{2}}$, respectively, where m_i is the ion mass, K_B being the Boltzmann constant, and T_e is the electron temperature. The electrostatic potential ϕ is normalized to $\frac{K_B T_i}{e}$, where e is the magnitude of the electron charge. The space variable is normalized to the ion Debye radius, $\lambda_D = (\frac{K_B T_i}{4\pi n_{i0} e^2})^{\frac{1}{2}}$, and the time variable is normalized to the ion plasma frequency, $\omega_{pi} = (\frac{4\pi n_{i0} e^2}{m_i})^{\frac{1}{2}}$.

The Boltzmann distributed electrons and positrons are taken to be valid for the macroscopic ergodic equilibrium state. But Maxwell, i.e. Boltzmann distribution may be inadequate to describe the long-range interactions in unmagnetized collision less plasma where the non-equilibrium stationary state exists. Space plasma observations clearly indicate the presence of ion and electron populations that are far away from their thermodynamic equilibrium (Shukla et al. 1986; Ghosh and Bharuthram 2008). A new statistical approach (Renyi 1955), namely non-extensive statistics or Tsallis statistics based on the derivation of Boltzmann–Gibbs–Shannon (BGS) extropic measure (Tsallis 1988), is proposed to study the cases where Maxwell distribution is considered inappropriate. This was first acknowledged by Renyi (1955) and afterwards proposed by Tsallis (1988), where the entropic index q characterized the degree of non-extensivity of the considered system. The parameter q that undergoes the generalized entropy of Tsallis is linked to the underlying dynamics of the system and measures the amount of its non-extensivity. In statistical mechanics and thermodynamics, systems characterized by the property of non-extensivity are systems for which the entropy of the whole is different from the sum of the entropies of respective parts. In other words, the generalized entropy of the whole is greater than the sum of the entropies of the parts if $q < 1$ (superextensivity) whereas the generalized entropy of the system is smaller than the sum of the entropies of the parts if $q > 1$ (subextensivity). In accordance with the evidences (Lima et al. 2000; Leubner 2008; Tribeche and Merriche 2011; Ghosh 2012, 2012a; Yasmin et al. 2012) the q-entropy may provide a convenient frame for the analysis of many astrophysical scenarios, such as stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy cluster. It may be noted that for $q < -1$, the q-distribution is unnormalizable. In the extensive limiting case $(q \to 1)$, the q-distribution reduces to the well-known Maxwell–Boltzmann velocity distribution.

To take into account the non-extensivity of electrons, we use the following q-distribution function:

$$f_e(v) = C_q \left\{1 + (q-1)\left[\frac{m_e v^2}{2T_e} - \frac{e\phi}{T_e}\right]\right\}^{\frac{1}{q-1}},$$

where ϕ stands for the electrostatic potential, and the remaining variables/parameters have their usual meaning. It may be noted that $f_e(v)$ is the particular distribution that maximizes the Tsallis entropy and therefore conforms to the laws of thermodynamics. The normalization constant C_q is given by

$$C_q = n_{e0}\frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)}\sqrt{\frac{m_e(1-q)}{2\pi T_e}} \quad \text{for } -1 < q < 1,$$

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$$C_q = n_{e0}\left(\frac{1+q}{2}\right)\frac{\Gamma\left(\frac{1}{q-1}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}\sqrt{\frac{m_e(q-1)}{2\pi T_e}} \quad \text{for } q>1,$$

where the parameter q stands for the strength of non-extensivity. It may be useful to note that when $q<-1$, the q-distribution is unnormalizable. It should be noted that for $q>1$, the q-distribution function exhibits a thermal cutoff on the maximum value allowed for the velocity of the particles, which is given by

$$v_{\max} = \sqrt{\frac{2T_e}{m_e}\left(\frac{e\phi}{T_e}+\frac{1}{q-1}\right)};$$

we get

$$n_e(\phi) = \int_{-\infty}^{\infty} f_e(v)dv, \quad \text{for } -1<q<1,$$

$$n_e(\phi) = \int_{-v\max}^{+v_{\max}} f_e(v)dv, \quad \text{for } q>1.$$

We also assume that the electrons follow q-non-extensive distribution. Integrating the q-distribution over all velocity space, one obtains the following non-extensive electron number density:

$$n_e = [1+(q-1)\phi]^{\frac{1+q}{2(q-1)}}.$$

As electrons and positrons have the same mass but opposite charge, it is expected that they will be described by a similar distribution. Similarly, as derived from above, the positrons follow the distribution $n_p = [1-(q-1)\phi\sigma]^{\frac{1+q}{2(q-1)}}$.

3. Derivation of KdV equations and phase shifts

Suppose that two solitary waves in non-planar geometry, R and L, have been excited in the system. The solitary wave R (L) is traveling outward (inward) from (to) the initial point of the coordinate system. The initial position (at time $t=0$) of the solitary wave R (L) is at $r=r_R$ ($r=r_L$), $r_L \gg r_R$. Since $0<r<\infty$, this consideration is different from Cartesian solitons, where $-\infty<x<\infty$ in one dimension. For the head-on collision between two Cartesian solitons, one can consider that one soliton is at $x\approx-\infty$ and the other one is at $x\approx\infty$ at the initial time $t=-\infty$ (see Huang and Velarde 1996; Xue 2004a; Li 2010). After some time they interact following a collision and then depart each other. In order to investigate the head-on collision between two solitary waves in non-planar geometry, we extend the PLK method (Shukla et al. 1986) to the non-planar geometry. We anticipate that the collision will result in phase shifts in their post-collision trajectories. Thus, we introduce the following transformation:

$$\xi = \varepsilon(r-ct-r_A)+\varepsilon^2 A_1(\eta,L)+\varepsilon^3 A_2(\eta,\xi,L)+\cdots, \quad (4)$$

$$\eta = \varepsilon(r+ct-r_B)+\varepsilon^2 B_1(\xi,L)+\varepsilon^3 B_2(\eta,\xi,L)+\cdots, \quad (5)$$

$$L=\varepsilon^3 r, \quad (6)$$

where ξ and η denote the trajectories of two solitons traveling toward each other, and c is the unknown phase velocity of IASWs. The variables $A_1(\eta,L)$ and $B_1(\xi,L)$ are also to be determined.

Introducing the asymptotic expansion,

$$n_i = 1+\varepsilon^2 n_1+\varepsilon^3 n_2+\varepsilon^4 n_3+\cdots, \quad (7)$$

$$u_i = u_0+\varepsilon^2 u_1+\varepsilon^3 u_2+\varepsilon^4 u_3+\cdots, \quad (8)$$

$$\phi = \varepsilon^2\phi_1+\varepsilon^3\phi_2+\varepsilon^4\phi_3+\cdots, \quad (9)$$

where u_0 is the drift fluid velocity. Substituting (4)–(9) into (1)–(3) and equating the quantities with equal power of ε, we obtain coupled equations in different orders of ε. To the leading order, we have

$$-c\frac{\partial n_1}{\partial\xi}+c\frac{\partial n_1}{\partial\eta}+\left(\frac{\partial u_1}{\partial\xi}+\frac{\partial u_1}{\partial\eta}\right)=0, \quad (10)$$

$$-c\frac{\partial u_1}{\partial\xi}+c\frac{\partial u_1}{\partial\eta}+\left(\frac{\partial\phi_1}{\partial\xi}+\frac{\partial\phi_1}{\partial\eta}\right)=0, \quad (11)$$

$$n_1 = \frac{(1+p\sigma)(q+1)}{2(1-p)}\phi_1. \quad (12)$$

Solving the above we get

$$\phi_1 = \phi_\xi(\xi,L)+\phi_\eta(\eta,L), \quad (13)$$

$$n_1 = \frac{(1+p\sigma)(q+1)}{2(1-p)}[\phi_\xi(\xi,L)+\phi_\eta(\eta,L)], \quad (14)$$

$$u_1 = \frac{1}{c}\phi_\xi(\xi,L)-\frac{1}{c}\phi_\eta(\eta,L), \quad (15)$$

and with the solvability condition (i.e. the condition to obtain a uniquely defined n_1, u_1 from (14)–(15) when ϕ_1 is given by (13)), the phase velocity $c=\sqrt{2\frac{(1-p)}{(1+p\sigma)(q+1)}}+u_0$ and $\sqrt{\frac{2(1-p)}{(1+p\sigma)(q+1)}}-u_0$ is also obtained. The unknown functions ϕ_ξ and ϕ_η will be determined from the next orders. Relations (13)–(15) imply that, at the leading order, we have two waves, one of which $\phi_\xi(\xi,\tau)$ is traveling to the outward, and the other one $\phi_\eta(\eta,\tau)$ is traveling to the inward.

Furthermore, the next higher order leads to

$$\begin{aligned}2c\frac{\partial^2 u_3}{\partial\xi\partial\eta} &= \frac{\partial}{\partial\xi}\left[\frac{\partial\phi_\xi}{\partial L}+A\frac{\partial}{\partial\xi}\left(\phi_\xi\frac{\partial\phi_\xi}{\partial\xi}\right)+B\frac{\partial^3\phi_\xi}{\partial\xi^3}+\frac{v}{2L}\phi_\xi\right]\\ &-\frac{\partial}{\partial\eta}\left[\frac{\partial\phi_\eta}{\partial L}+A\frac{\partial}{\partial\eta}\left(\phi_\eta\frac{\partial\phi_\eta}{\partial\eta}\right)+B\frac{\partial^3\phi_\eta}{\partial\eta^3}+\frac{v}{2L}\phi_\eta\right]\\ &+\left(2\frac{\partial A_1}{\partial\eta}-D\phi_\eta\right)\frac{\partial^2\phi_\xi}{\partial\xi^2}-\left(2\frac{\partial B_1}{\partial\xi}-D\phi_\xi\right)\frac{\partial^2\phi_\eta}{\partial\eta^2}.\end{aligned} \quad (16)$$

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Integrating the above equation with respect to variables ξ and η yields

$$2\alpha c u_3 = \int\left(\frac{\partial\phi_\xi}{\partial L} + A\frac{\partial}{\partial\xi}\left(\phi_\xi\frac{\partial\phi_\xi}{\partial\xi}\right) + B\frac{\partial^3\phi_\xi}{\partial\xi^3} + \frac{\nu}{2L}\phi_\xi\right) d\eta$$
$$-\int\left(\frac{\partial\phi_\eta}{\partial L} + A\frac{\partial}{\partial\eta}\left(\phi_\eta\frac{\partial\phi_\eta}{\partial\eta}\right) + B\frac{\partial^3\phi_\eta}{\partial\eta^3} + \frac{\nu}{2L}\phi_\eta\right) d\xi$$
$$+\int\int\left(2\frac{\partial A_1}{\partial\eta} - D\phi_\eta\right)\frac{\partial^2\phi_\xi}{\partial\xi^2}\, d\xi d\eta$$
$$-\int\int\left(2\frac{\partial B_1}{\partial\xi} - D\phi_\xi\right)\frac{\partial^2\phi_\eta}{\partial\eta^2}\, d\xi d\eta, \tag{17}$$

where $A = \frac{3(1+p\sigma)(q+1)}{4(1-p)} - \frac{(3-q)(1-p\sigma^2)}{4(1+p\sigma)}$, $B = \frac{(1-p)}{(1+p\sigma)(q+1)}$, and $D = \frac{(1+p\sigma)(q+1)}{4(1-p)} + \frac{(3-q)(1-p\sigma^2)}{4(1+p\sigma)}$.

The first term on the right-hand side of (17) will be proportional to η because the integrand function is independent of η. The second term on the right-hand side of (17) will be proportional to ξ because the integrand function is independent of ξ. Thus, the two terms of (17) are all secular terms, which must be eliminated to avoid spurious resonances.

Hence, we have

$$\frac{\partial\phi_\xi}{\partial L} + A\phi_A\frac{\partial\phi_\xi}{\partial\xi} + B\frac{\partial^3\phi_\xi}{\partial\xi^3} + \frac{\nu}{2L}\phi_\xi = 0, \tag{18}$$

$$\frac{\partial\phi_\eta}{\partial L} + A\phi_\eta\frac{\partial\phi_\eta}{\partial\eta} + B\frac{\partial^3\phi_\eta}{\partial\eta^3} + \frac{\nu}{2L}\phi_\eta = 0. \tag{19}$$

The third and fourth terms in (17) are not secular terms in this order, they could be secular in the next order. Hence, we have

$$2\frac{\partial A_1}{\partial\eta} = D\phi_\eta, \tag{20}$$

$$2\frac{\partial B_1}{\partial\xi} = D\phi_\xi. \tag{21}$$

Equations (18) and (19) are the two side-traveling wave KdV equations in the reference frames of ξ and η respectively. One of their special solutions are of (18) and (19),

$$\phi_\xi = U_A\Theta_A^{2\nu/3} sech^2\left[\Gamma_A^{1/2}\Theta_A^{\nu/3}\left(\xi - \frac{1}{3}AU_A\Theta_A^{2\nu/3}L\right)\right], \tag{22}$$

$$\phi_\eta = U_B\Theta_B^{2\nu/3} sech^2\left[\Gamma_B^{1/2}\Theta_B^{\nu/3}\left(\eta - \frac{1}{3}AU_B\Theta_B^{2\nu/3}L\right)\right], \tag{23}$$

where U_A and U_B are two constants which denote the amplitude of two IASWs R and L respectively. Here we have used the definitions $\Theta_A = L_A/L$, $\Theta_B = L_B/L$, $\Gamma_A = AU_A/12B$, $\Gamma_B = AU_B/12B$ with $L_A = \epsilon^3 r_R$ and $L_B = \epsilon^3 r_L$. It is noted that the amplitude decreases drastically as L(r) increases in cylindrical and spherical geometries. In (18) and (19), A and B are the coefficients of nonlinearity and dispersion respectively. If we set $\nu = 0$, (18) and (19) will become the planar KdV equations. $\nu = 1$ and 2 corresponds to the cylindrical and spherical KdV equations respectively.

Inserting (22) and (23) into (20) and (21), we can obtain the leading phase functions A_1 and B_1 due to the collision of IASWs as follows:

$$A_1 = \frac{D}{2}U_B\Gamma_B^{-1/2}\Theta_B^{\nu/3}$$
$$\times\left[\tanh\left\{\Gamma_B^{1/2}\Theta_B^{\nu/3}\left(\eta\,|_t - \frac{1}{3}AU_B\Theta_B^{2\nu/3}L\right)\right\}\right.$$
$$\left. - \tanh\left\{\Gamma_B^{1/2}\Theta_B^{\nu/3}\left(\eta\,|_{t=0} - \frac{1}{3}AU_B\Theta_B^{2\nu/3}L\right)\right\}\right], \tag{24}$$

$$B_1 = \frac{D}{2}U_A\Gamma_A^{-1/2}\Theta_A^{\nu/3}$$
$$\times\left[\tanh\left\{\Gamma_A^{1/2}\Theta_A^{\nu/3}\left(\xi\,|_t - \frac{1}{3}AU_A\Theta_A^{2\nu/3}L\right)\right\}\right.$$
$$\left. - \tanh\left\{\Gamma_A^{1/2}\Theta_A^{\nu/3}\left(\xi\,|_{t=0} - \frac{1}{3}AU_A\Theta_A^{2\nu/3}L\right)\right\}\right], \tag{25}$$

where $\xi\,|_{t=0} = \varepsilon(r_L - r_R)$, $\eta\,|_{t=0} = \varepsilon(r_R - r_L)$, $\xi\,|_t = \varepsilon[(r_L - r_R) - 2ct]$, and $\eta\,|_t = \varepsilon[(r_R - r_L) + 2ct]$. By setting $d_0 = r_L - r_R$ as the initial separation between two solitary waves and assuming that $d_0 \gg 0$, we have

$$A_1 = \frac{D}{2}U_B\Gamma_B^{-1/2}\Theta_B^{\nu/3}$$
$$\left[\tanh\left\{\Gamma_B^{1/2}\Theta_B^{\nu/3}\left(\eta\,|_t - \frac{1}{3}AU_B\Theta_B^{2\nu/3}L\right)\right\} + 1\right]; \tag{26}$$

$$B_1 = \frac{D}{2}U_A\Gamma_A^{-1/2}\Theta_A^{\nu/3}$$
$$\left[\tanh\left\{\Gamma_A^{1/2}\Theta_A^{\nu/3}\left(\xi\,|_t - \frac{1}{3}AU_A\Theta_A^{2\nu/3}L\right)\right\} - 1\right]. \tag{27}$$

Therefore, up to $O(\varepsilon^2)$, we can estimate the phase shifts in the head-on collision process of the two cylindrical/spherical solitary waves for weak head-on interaction. According to (4)–(6) and (26) and (27), we obtain the corresponding phase shifts ΔP_0 and ΔQ_0 if the initial separation between the two solitons is large enough, i.e. $r_L \gg r_R$, and the observation time is much larger than the collision time, i.e. $t \gg t_c = (r_L - r_R)/2$, as

$$\Delta P_0 = -2\varepsilon^2\frac{D}{2}\left(\frac{12BU_B}{A}\right)^{1/2}\left(\frac{r_L}{r}\right)^{\nu/3}, \tag{28}$$

$$\Delta Q_0 = 2\varepsilon^2\frac{D}{2}\left(\frac{12BU_A}{A}\right)^{1/2}\left(\frac{r_L}{r}\right)^{\nu/3}. \tag{29}$$

4. Results and discussions

In this paper we have investigated the collision phenomenon between two cylindrical and spherical IASWs in a plasma consisting of inertial ions, q-non-extensive distributed electrons, and positrons by using the extended version of PLK method. It is well known that soliton

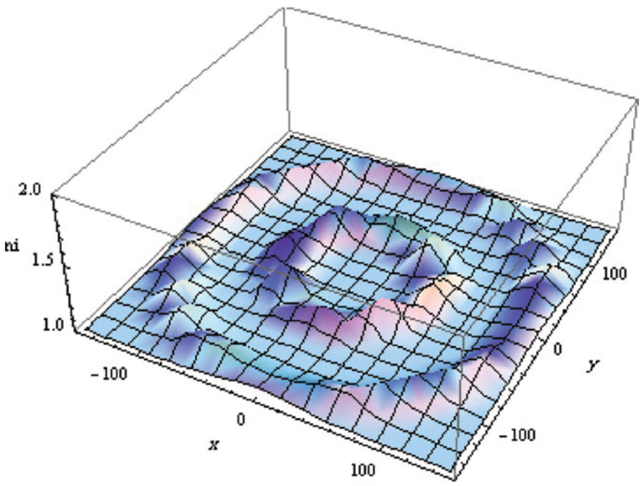

Figure 1. (Colour online) Graphs of cylindrical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $q = 1.5$, $r_R = 57.5$, $r_L = 137.5$, $t = 0$.

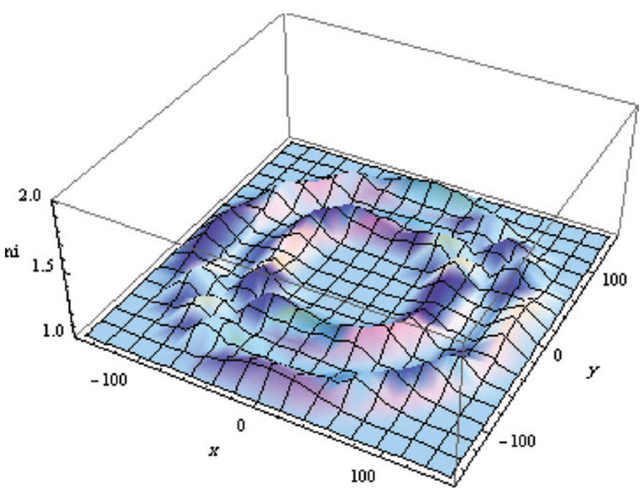

Figure 2. (Colour online) Graphs of cylindrical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $q = 1.5$, $r_R = 57.5$, $r_L = 137.5$, $t = 30$.

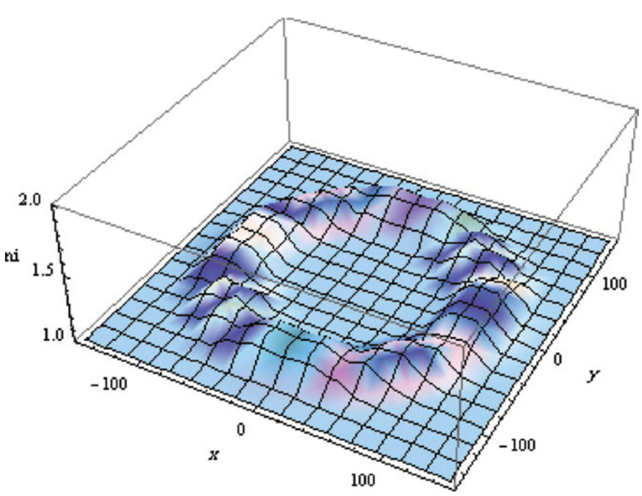

Figure 3. (Colour online) Graphs of cylindrical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $q = 1.5$, $r_R = 57.5$, $r_L = 137.5$, $t = 60$.

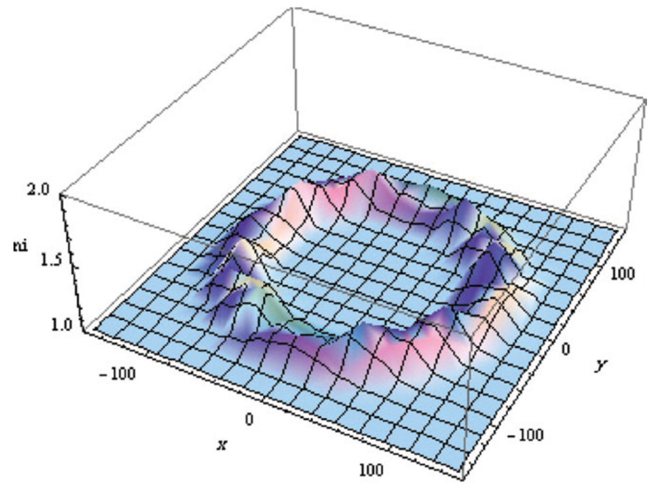

Figure 4. (Colour online) Graphs of cylindrical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $q = 1.5$, $r_R = 57.5$, $r_L = 137.5$, $t = 90$.

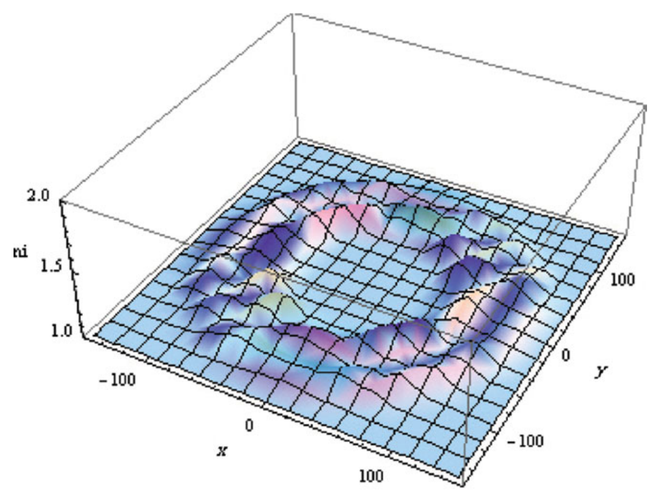

Figure 5. (Colour online) Graphs of cylindrical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $q = 1.5$, $r_R = 57.5$, $r_L = 137.5$, $t = 120$.

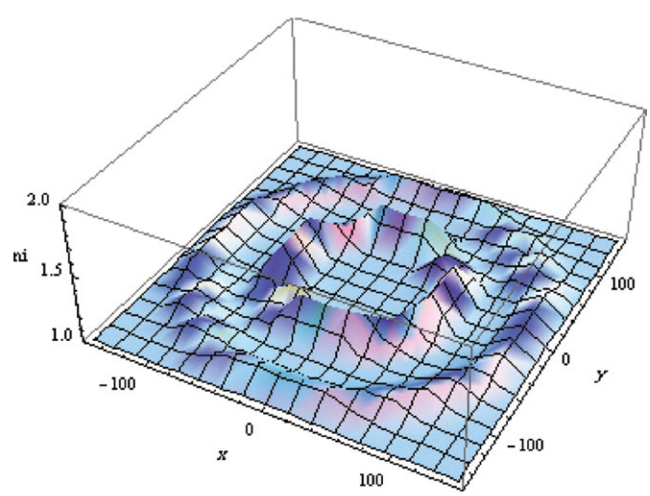

Figure 6. (Colour online) Graphs of cylindrical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $q = 1.5$, $r_R = 57.5$, $r_L = 137.5$, $t = 150$.

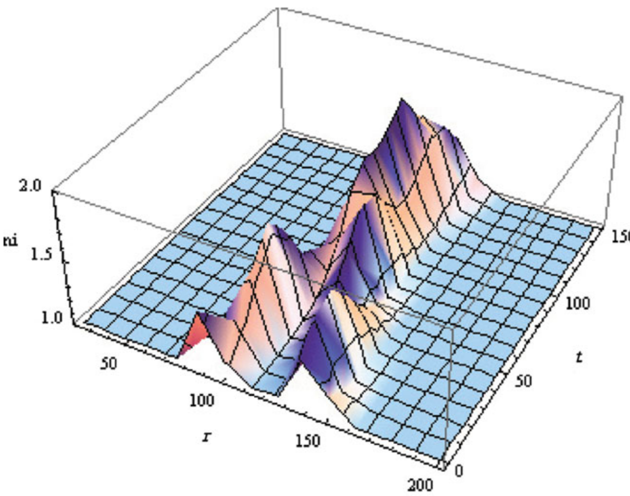

Figure 7. (Colour online) Graphs of spherical collision, $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $p = 0.01$, $\sigma = 0.01$, $r_R = 87.5$, $r_L = 137.5$, $q = 2.5$.

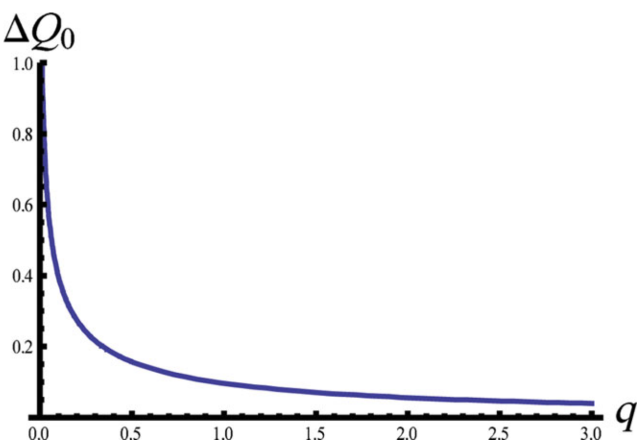

Figure 8. (Colour online) Graphs of phase shift ΔQ_0 vs. the non-extensive parameter q. $\varepsilon = 0.2$, $U_A = 1.5$, $U_B = 1$, $\sigma = 0.01$, $p = 0.01$, $r_R = 87.5$, $r_L = 137.5$, $r = 100$.

like solutions are formed due to the balance between nonlinearity and dispersion in a nonlinear dispersive media. Suppose that two solitary waves in non-planar geometry, R and L, have been excited in the system. The solitary wave R (L) is traveling outward (inward) from (to) the initial point of the coordinate system. The initial position (at time $t = 0$) of the solitary wave R (L) is at $r = r_R$ ($r = r_L$), $r_L \geqslant r_R$. Since $0 < r < \infty$, this consideration is different from Cartesian solitons where $-\infty < x < \infty$ in one dimension.

For cylindrical case, (22) and (23) are the outward (R) and inward (L) traveling wave KdV equations respectively. Now we want to show how the interaction takes place. At the initial position these two solitary waves, R and L, are at $r = r_R = 57.5$ and $r = r_L = 137.5$, respectively, at time $t = 0$. As time progresses, it is seen from Figs. 1–3 that the outward solitary waves R and inward solitary waves L come closer and closer and ultimately collide (Fig. 4). After colliding with each other they interchange their position and finally depart from each other so that the soliton R has the radius $r = r_L = 137.5$, and soliton L has the radius $r = r_R = 57.5$, which are depicted in Figs. 5 and 6 respectively. Similarly Fig. 7 shows the interaction of spherical ones.

Now we discuss the effect of physical variables on phase shifts. It is clear from (32) and (33) that the magnitude of the phase shifts due to the head-on collision is not only related to the physical parameters, i.e. ε, σ, β, U_A, and U_B but also modified by solitons initial positions r_R (r_L) and the geometry types ν. The latter does not exist in planar waves collision. For fixed physical parameters and the initial soliton position, the phase shifts are proportional to $r^{-\nu/3}$. That is, the collision-induced phase shifts in cylindrical/spherical geometry decrease with r according to $r^{-\nu/3}$. At the same physical parameters and soliton positions, the phase shifts in different geometries are different. When $\nu = 0$, the phase shifts given by (32) and (33) reduce to the planar case.

In all results, all physical parameters are dimensionless and $\varepsilon = 0.2$, $U_A = 1.5$, and $U_B = 1$ are used. The change of phase shifts (ΔQ_0) with respect to the change of q is plotted in Fig. 8. It has been seen from Fig. 8 that the phase shift decreases along with the increasing value of the non-extensive parameter q and finally tends to a constant magnitude. Thus, it can be stated that the non-extensive parameter q has a significant effect on phase shifts.

Acknowledgement

This scientific research is supported by SAP-DST Phase II. Reviewers' comments and suggestions are gratefully acknowledged by the authors, without which this paper would not have been in the present form.

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