Fifth Meeting, March 11th, 1898.

Dr MORGAN, Vice-President, in the Chair.

An Analysis of all the Inconclusive Votes possible with 15 Electors and 3 Candidates.

By Professor STEGGALL.

A Suggestion for a Shortened Table of Five-Figure Logarithms.

By Professor STEGGALL.

Note on the Centre of Gravity of a Circular Arc.

By JOHN DOUGALL, M.A.

Mr Crawford's note on this subject, read at a recent meeting, reminds me of a method I gave to a class four or five years ago.

FIGURE 14.

Let AMB be an arc subtending an angle 2α at the centre O of a circle of radius a. The centre of gravity G_1 lies, from symmetry, on OM the line from O to the mid-point of the arc.

Let G_2 be the C.G. of an adjacent arc BNC of angle 2β.

If G be the C.G. of the whole arc AMBNC, the angle AOG is $\alpha+\beta$.

Thus $\angle G_1OG=\beta$ and $\angle G_2OG=\alpha$.
Also G_1GG_2 is a straight line.

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But $GG_1 : GG_2 =$ mass at G_2 : mass at G_1

$$= \beta : \alpha$$

and $$GG_1 : GG_2 = OG_1 \sin\beta : OG_2 \sin\alpha$$

$\therefore\ OG_1 \cdot \dfrac{\alpha}{\sin\alpha} = OG_2 \cdot \dfrac{\beta}{\sin\beta}$, and therefore each must be a constant. By taking the arc indefinitely small, we get the constant equal to a the radius, and therefore $OG_1 = \dfrac{a\sin\alpha}{\alpha}$.

It is curious to observe that the result may be deduced, though not quite so simply, from the mere consideration that G is in the line G_1G_2.

Thus $$\triangle G_1OG_2 = \triangle G_1OG + \triangle GOG_2$$

giving $$\frac{\sin(\alpha+\beta)}{OG} = \frac{\sin\alpha}{OG_1} + \frac{\sin\beta}{OG_2};$$

or, if we denote the function of α, $\dfrac{\sin\alpha}{OG_1}$, by $\phi(\alpha)$,

$$\phi(\alpha+\beta) = \phi(\alpha) + \phi(\beta)$$

and $\therefore\ \phi(\alpha) =$ a constant multiple of α, as before.

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