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Robust fuzzy sliding mode control for tracking the robot manipulator in joint space and in presence of uncertainties

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SUMMARY
This paper proposes a simple fuzzy sliding mode control to achieve the best trajectory tracking for the robot manipulator. In the core of the proposed method, by applying the feedback linearization technique, the known dynamics of the robot's manipulator is removed; then, in order to overcome the remaining uncertainties, a classic sliding mode control is designed. Afterward, by applying the TS fuzzy model, the classic sliding mode controller is converted to fuzzy sliding mode controller with very simple rule base. The mathematical analysis shows that the robot manipulator with the new proposed control in tracking the robot manipulator in presence of uncertainties has the globally asymptotic stability. Finally, to show the performance of the proposed method, the controller is simulated on a robot manipulator with two degrees of freedom as case study of the research. Simulation results demonstrate the superiority of the proposed control scheme in presence of the structured and unstructured uncertainties.

KEYWORDS: Robot manipulator; Sliding mode control; Fuzzy; Uncertainties.

1. Introduction

Robotic manipulators are not only exorbitantly coupled, abundantly nonlinear, and time-varying systems but also have to travail from uncertainties in their dynamics, such as nonlinear friction, payload variation, and external disturbance. These properties exacerbate the system performance and stability, and they are formidable to find a punctual dynamical model for the model-based control system design.[1–3]

In the past, there have been a lot of control methods, which have been presented to control robotic manipulators. These approaches usually assume that the manipulator is enduring a constant payload or has only its own mass. Some of these apply adaptive systems to determine the robot manipulator model and others use a concise model of the robot manipulator. In ref. [4], a Takagi–Sugeno (T-S) fuzzy model-based sliding mode controller has been introduced to robot manipulators' control. Moreover, rapid online closed-loop identification with generalized proportional integral control method in ref. [5], adaptive fuzzy controllers in refs. [6–8], and T-S adaptive fuzzy controllers in ref. [9] have been presented to control robot manipulators with constant payload.

Some mechanical systems and robotic manipulators with time-varying unknown payload have also been proposed to control by expiating the influences of varying payload. An adaptive controller has been proposed in ref. [10] to control the mechanical system where the mutating payload is determined by bounded time functions. In ref. [11], the controller expiated the effect of the varying payload by a sliding mode controller, which is applied to control robot manipulator. Eventually, in ref. [12], to acquire zero steady-state error, a double-layer sliding-mode control has been proposed

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to control robot manipulator, and a proportional–integral (PI)-based disturbance observer has been used to estimate the time-varying payload.

However, as it has been explained before, the dynamic equations of robot manipulator not only have the loading fluctuation but also have other uncertainties such as friction, disturbance, and unmodeled dynamics; if this part of uncertainties in controller design is not considered, then there would be no guarantee for stability of the closed-loop system.

In order to obliterate chattering of global sliding mode control, merging sliding mode control with neural networks control emerged to be a good idea, and many researchers have published various control schemes based on this concept.[13–17] A few momentous ideas seem to be prevailing. The first-mentioned research strives to apply neural networks as an observer in the determination of equivalent control[13] and in some cases of disturbances.[14] Moreover, for online identification of model errors, a sliding mode controller with a modified switching function that produces a low-chattering control is used in ref. [15] in parallel with artificial neural networks, which impose the controller performance. In ref. [16], a new approach, which blends sliding mode control and neural networks, is presented, the weights of which are delineated by a fuzzy supervisory controller. Fuzzy neural networks have both fuzzy control and neural control privileges,[17] and were exerted for multi-link robots. However, design of fuzzy neural networks' fuzzy rules is very difficult and no criterion is valid. Therefore, the computation volume of these methods is too large, and hence the practical implementation of these methods is very difficult.

The methods of robust control and adaptive control and also the combination of these two methods of control have been used recently for tracking of robot manipulator in research. In refs. [18–22], by using a robust nonlinear control and joint space (or task space information), some mechanisms are presented to overcome uncertainties in the dynamics of the robot manipulator. The combination of adaptive control and robust control causes a mechanism that has advantages of both methods to control the tracking of robot manipulator in presence of structured and unstructured uncertainties.[23,24] In refs. [25, 26], the sliding mode control method and combining of sliding mode control and also the neural network and classical control methods are used for tracking control of robot manipulator. In some of these researches, not only the structured and unstructured uncertainties in the robot manipulator dynamics are considered but the uncertainties in actuator dynamics are also considered in controller design.[26,27] The mathematical proving and simulations' results demonstrate very good performance of these methods. However, the existing complexities of these controllers' design engender unwillingness to their practical implementation in industry, because one of the factors that increases the use of proportional-integral-differential (PID) controllers in industrial manipulators tracking control is the existing simplicities in design of these kinds of controllers.

In this paper, fuzzy sliding mode controller for tracking control of the position of robot manipulator in the presence of existing structured and unstructured uncertainties is presented in a simple instruction. In contradiction to the classic sliding mode control, the proposed control does not contain the chattering phenomena. The design structure of this controller is such that it decreases control input amplitude sensitivity with respect to the variation coefficients of the controller. Hence, by changing the controller coefficient, actuators become saturated slower in comparison to the classic methods.

The paper is structured as follows. Section 2 presents the robot manipulator dynamic equations and existing features in this dynamics. In Section 3, the feedback linearization method is used for eliminating the known dynamics of the robot manipulator. Section 4 contains the designing steps of the classic sliding mode for controlling the tracking of robot manipulator. Due to convert of classic sliding mode control to fuzzy sliding mode control, the TS fuzzy model is presented in Section 5. Section 6 is designated to the details of the fuzzy sliding mode control design. In Section 7, the advantages of the proposed control are presented. Section 8 contains dynamic equations and the details of the two-degree-of-freedom robot manipulator, which is used as the paper's case study. Afterward, in Section 9, the performance of the proposed control is presented by using of simulation in three steps, and finally in Section 10, the results of this paper are provided.

2. Problem Description

The dynamics of a rigid robot with n rotating links can be described by the following second-order nonlinear differential equation,[18–20]

$$D(q)\ddot{q} + N(q,\dot{q})\dot{q} + G(q) + \tau_d = \tau(t), \tag{1}$$

where $q, \dot{q}$, and $\ddot{q}$ are angular position vectors, angular speed vectors, and angular acceleration vectors of rotating joints respectively, $D(q) \in R^{n\times n}$ is the symmetric positive definite inertia matrix of the robot, $N(q,\dot{q}) \in R^n$ is the vector containing coriolis and centrifugal forces, $G(q) \in R^n$ is the gravitational torque, $\tau_d \in R^n$ is the sum of system model errors' torque and external disturbances' torque, and $\tau(t) \in R^n$ is the vector of applied joint torques that are actually the control inputs.

Following are the properties for a multi-link robot described by Eq. (1) in general.[18–20]

Property 1. $D(q)$ is a positive symmetric definite matrix, and its inverse matrix $D^{-1}(q)$ is valid. $D(q)$ is uniformly bounded by $\underline{D} \leq D(q) \leq \overline{D}$, where $\underline{D}$ and $\overline{D}$ are positive constants that depend on the mass properties of robot manipulators.

Property 2. $\dot{D}(q) - 2N(q,\dot{q})$ is a skew-symmetric matrix, i.e., it meets the following equation:

$$x^T(\dot{D}(q) - 2N(q,\dot{q}))x = 0. \tag{2}$$

Assumption. We have the following assumption aboutτ_d:

$$\|\tau_d\| \leq T, \tag{3}$$

where T is a positive constant.

3. Use of Feedback Linearization to Eliminate Known Part of Dynamics

The object of trajectory tracking control for multi-link robots is that the actual angular position q can track the expected angular position $q_d \in R^n$ as exactly as possible. To achieve this goal, the control input can be represented as follows:

$$\tau(t) = \hat{D}(q)V(t) + \hat{N}(q,\dot{q})\dot{q} + \hat{G}(q), \tag{4}$$

where $\hat{D}(q)$, $\hat{N}(q,\dot{q})$,and $\hat{G}(q)$ are known parts of $D(q)$, $N(q,\dot{q})$, and $G(q)$ respectively, and $V(t)$ is a new control vector. Equation (4) is substituted into Eq. (1) and is rearranged as follows:

$$D(q)\ddot{q} = \hat{D}(q)V(t) + \hat{N}(q,\dot{q})\dot{q} - N(q,\dot{q})\dot{q} + \hat{G}(q) - G(q) - \tau_d. \tag{5}$$

By defining $\hat{N}(q,\dot{q}) - N(q,\dot{q}) = \Delta N(q,\dot{q})$ and $\hat{G}(q) - G(q) = \Delta G(q)$, Eq. (5) is simplified as follows:

$$D(q)\ddot{q} = \hat{D}(q)V(t) + \Delta N(q,\dot{q})\dot{q} + \Delta G(q) - \tau_d. \tag{6}$$

Equation (6) is rearranged as follows:

$$\ddot{q} = D^{-1}(q)\hat{D}(q)V(t) + D^{-1}(q)(\Delta N(q,\dot{q})\dot{q} + \Delta G(q) - \tau_d). \tag{7}$$

$V(t)$ is added and subtracted into Eq. (7) and this equation is simplified as

$$\ddot{q} = V(t) + (D^{-1}(q)\hat{D}(q) - I)V(t) + D^{-1}(q)(\Delta N(q,\dot{q})\dot{q} + \Delta G(q), -\tau_d). \tag{8}$$

By defining $\eta = (D^{-1}(q)\hat{D}(q) - I)V(t) + D^{-1}(q)(\Delta N(q,\dot{q})\dot{q} + \Delta G(q) - \tau_d)$, we have

$$\ddot{q} = V(t) + \eta. \tag{9}$$

Remark 1. According to above defining, η includes all the existing uncertainties, that is, if the system does not have structured and unstructured uncertainties then $\eta = 0$.

In Eq. (9), input control $V(t)$ is selected as

$$V(t) = \ddot{q}_d - W(t), \tag{10}$$

where $\ddot{q}_d$ is the desired acceleration in the joint space, and $W(t)$ is the new control law. Equation (10) is substituted into Eq. (9) to obtain

$$\ddot{q} = \ddot{q}_d - W(t) + \eta. \tag{11}$$

By defining $q - q_d = e(t)$, Eq. (11) is simplified as follows:

$$\ddot{e}(t) = -W(t) + \eta. \tag{12}$$

Remark 2. $e(t)$ is tracking error in the joint space and $\dot{e}(t)$ and $\ddot{e}(t)$ are its first and second derivatives with respect to time respectively.

By defining $e(t) = X_1(t)$ and $\dot{e}(t) = X_2(t)$, the state space model of Eq. (12) is of the following form:

$$\begin{cases} \dot{X}_1(t) = X_2(t), \\ \dot{X}_2(t) = -W(t) + \eta. \end{cases} \tag{13}$$

4. Classic Sliding Mode Control Design for Tracking of Robot Manipulator

For designing of sliding mode control, we must define sliding surface. Sliding surface is selected as follows:

$$S(t) = CX_1(t) + X_2(t), \tag{14}$$

where C is the vector with constant coefficients. In this section, the control action $W(t)$ is designed in such a way that the output is capable of tracking the desired path. Moreover, the tracking error and all its derivatives will tend to zero. In the sliding mode control design $W(t)$ consists of two parts: $W_{eq}(t)$, equivalent control, and $W_S(t)$, switching control[25]:

$$W(t) = W_{eq}(t) + W_S(t). \tag{15}$$

In the sliding phase, where $S(t) = 0$ and $\dot{S}(t) = 0$, the equivalent term $W_{eq}(t)$ is designed to keep the system on the sliding surface. In the approaching phase, where $S(t) \neq 0$, the switching term $W_S(t)$ is designed to satisfy the reaching condition, $S(t)\dot{S}(t) < 0$.

For designing the part $W_{eq}(t)$, the derivative of Eq. (14) is supposed to be equal to zero:

$$\dot{S}(t) = C\dot{X}_1(t) + \dot{X}_2(t) = 0. \tag{16}$$

Equation (13) is substituted in Eq. (16):

$$C\dot{X}_1(t) - W(t) + \eta = 0. \tag{17}$$

In the design of $W_{eq}(t)$, it is assumed that the sliding surface is zero. So the task of $W_{eq}(t)$ is preventing the sliding surface from changes. According to this assumption, $W_S(t)$ in this part of the design may be considered as zero. By considering the above points and substituting Eq. (15) in (17),

$$C\dot{X}_1(t) - W_{eq}(t) + \eta = 0. \tag{18}$$

Finally, $W_{eq}(t)$ is derived from the above equation:

$$W_{eq}(t) = C\dot{X}_1(t) + \eta. \tag{19}$$

Concerning Eq. (19), we can conclude that

$$\|W_{eq}(t)\| \leq C\dot{X}_1(t) + \|\eta\|, \tag{20}$$

where the $\|\circ\|$ symbol is the norm. According to Eq. (20), $W_{eq}(t)$ can be set as follows:

$$W_{eq}(t) = C\dot{X}_1(t) + \|\eta\| \,. \tag{21}$$

Now the $W_S(t)$ is designed in a way that the sliding surface tends to zero. So the following Lyapunov candidate function is introduced:

$$V(S(t)) = \frac{1}{2}S^T S. \tag{22}$$

The derivative of the Lyapunov candidate function with respect to time is

$$\dot{V}(S(t)) = \dot{S}^T S. \tag{23}$$

From Eqs. (14) and (23), we conclude that

$$\dot{V}(S(t)) = (C\dot{X}_1(t) + \dot{X}_2(t)^T S. \tag{24}$$

From Eqs. (13), (15), and (24), it results that

$$\dot{V}(S(t)) = (C\dot{X}_1(t) - (W_{eq}(t) + W_S(t)) + \eta)^T S. \tag{25}$$

We substitute Eq. (21) in Eq. (25) to get

$$\dot{V}(S(t)) = (C\dot{X}_1(t))^T S - (C\dot{X}_1(t))^T S - \|\eta\| \, S - W_S^T(t)S + \eta^T S. \tag{26}$$

It is resulted from Eq. (26) that to have the inequality $\dot{V}(S(t)) < 0$ satisfied, the following condition has to be met:

$$W_S(t) = \begin{cases} \rho & if\, S(t) > 0 \\ -\rho & if\, S(t) < 0 \end{cases}, \tag{27}$$

where ρ is a constant positive factor. Concerning Eqs. (15), (21), and (27), we have

$$W(t) = \begin{cases} W^+(t) = W_{eq}(t) + \rho if\, S(t) > 0 \\ W^-(t) = W_{eq}(t) - \rho if\, S(t) < 0 \end{cases}. \tag{28}$$

5. T-S Fuzzy Model

The T-S fuzzy logic system is given in the following form of IF–THEN rules:

$$R^i : IF\; x_1(t)\; is\; A_{1i}\; and\; ....\; and\; x_q(t)\; is\; A_{qi}\; THEN\; u_i(t) = f_i(X(t), t), \; i = 1, \ldots, r, \tag{29}$$

where R^i represents the ith fuzzy inference rule, x_j and A_{ij} ($i = 1, \ldots, r$ and $j = 1, \ldots, q$) are the premise variables and fuzzy sets respectively, and r is the number of fuzzy IF–THEN rules.

Following the fuzzy inference method of the T-S fuzzy system, the control input $U(t)$ of the overall system can be obtained in the weighted average form along the trajectories $X(t)$:

$$U(t) = \frac{\sum_{i=1}^{r} w_i(X(t)) f_i(X(t), t)}{\sum_{i=1}^{r} w_i(X(t))}, \tag{30}$$

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where $T_d = [^{T_{d_1}}_{T_{d2}} \ ^{\sin(t)}]$, and $X(t) \in R^q$, the weight functions are defined as

$$w_i(X(t)) = \prod_{j=1}^{q} A_{ij}(x_j(t)), \tag{31}$$

where $A_{ij}(x_j(t))$ is the grade of membership of $x_j(t)$ in the fuzzy set A_{ij}. The weight functions $w_i(X(t))$ are non-negative and measurable, and usually satisfy

$$\sum_{i=1}^{r} w_i(X(t)) > 0, \quad \text{for all } t > 0. \tag{32}$$

6. Fuzzy Sliding Mode Control Design for Tracking of Robot Manipulator

The proposed T-S fuzzy model-based sliding mode control is based on the intuitive feedback control strategy. Thus, the fuzzy inference rule base is established as

$$\begin{aligned} &R^1 : IF\ S(t)\ is\ positive\ THEN\ W(t) = W^1(t) = W^+(t),\\ &R^2 : IF\ S(t)\ is\ negative\ THEN\ W(t) = W^2(t) = W^-(t). \end{aligned} \tag{33}$$

Finally, the system control $W(t)$ can be obtained through the center of gravity defuzzification method,

$$W(t) = \frac{\sum_{i=1}^{2} w_i(S(t))W^i(t)}{\sum_{i=1}^{2} w_i(S(t))}, \tag{34}$$

where $w_i(S(t))$ is the same as the one defined in Eq. (31). Therefore, the designed control input in its general form is as follows:

$$\begin{cases} \tau(t) = \hat{D}(q)V(t) + \hat{N}(q,\dot{q})\dot{q} + \hat{G}(q) \\ V(t) = \ddot{q}_d - W(t) \\ W(t) = \frac{\sum_{i=1}^{2} w_i(S(t))W^i(t)}{\sum_{i=1}^{2} w_i(S(t))} \\ W^1(t) = W_{eq}(t) + \rho \quad ifS(t) > 0 \\ W^2(t) = W_{eq}(t) - \rho \quad ifS(t) < 0 \\ W_{eq}(t) = C\dot{X}_1(t) + \|\eta\| \end{cases}. \tag{35}$$

Remark 3. To calculate $\|\eta\|$, Property 1, Property 2, and the following equation can be used:

$$\eta = (D^{-1}(q)\hat{D}(q) - I)V(t) + D^{-1}(q)(\Delta N(q,\dot{q})\dot{q} + \Delta G(q) - \tau_d). \tag{36}$$

Design guidance of the proposed control

According to the above discussion, the procedure for designing a fuzzy sliding mode controller for robot manipulator is described as follows:

1. Specify $\hat{D}(q)$, $\hat{N}(q,\dot{q})$, and $\hat{G}(q)$ as known part of $D(q)$, $N(q,\dot{q})$, and $G(q)$ respectively. Then specify upper and lower bounds of these.
2. Specify the desired paths in the joint space.
3. Define the tracking error variable, $e_i(t)$s.
4. Specify $S(t)$ sliding surface by selecting C as a vector with positive constant coefficients.

5. Specify $W_{eq}(t)$ by using Eq. (21).
6. Design the T-S fuzzy model control laws $W^1(t)$ and $W^2(t)$, and the related fuzzy membership functions.
7. Build fuzzy inference rule base.
8. Defuzzify the fuzzy variables through the center of gravity method to get the crisp control law $W(t)$.
9. Specify input control of robot manipulator by using Eq. (4).

7. Advantages of the Proposed Control

Some points considered in the proposed control design have a prominent role in its practical implementation. These points are as follows:

1. In this control method, the bound of uncertainties can be decreased as much as possible because of using the feedback linearization. Then the fuzzy sliding mode controller is designed through applying the T-S fuzzy model and an inference engine composed of a very brief rule base (only two rules).
2. Lowering the bound of uncertainties engenders decreasing of control input amplitude. Hence, in robot manipulator control with the proposed method, it is possible to use actuators with lower power. Therefore, the economic cost of robot manipulator manufacturing would be reduced.
3. This control method is free of undesirable chattering phenomenon. Moreover, it can handle structured and unstructured uncertainties, whereas adaptive methods are weak in coping with unstructured uncertainties.[18–22]
4. Another benefit of the designed controller is its light burden of computations, which is an important figure in practical implementation and online control cases. In the control of industrial complex systems, the computational burden of the control action is very significant, as the heavy computational burden of the control action is not only expensive but can also cause instability of the closed-loop system.[18–27]
5. For reducing the tracking errors in most of the controllers, which are designed to track the robot manipulator, the controllers' coefficients have to be increased. On the other hand, increasing the controllers' coefficients engender the increasing of the control input amplitude and saturation of robot manipulator's actuators. By considering Eq. (34), it would be concluded that in the proposed method the sensitivity of the control input amplitude is decreased with respect to controller coefficient's variation. Hence, the concern about the saturation of the actuators would be fulfilled partially.

8. Case Study of Two-Link Elbow Robot Manipulator

In order to verify the performance of proposed control scheme, as an illustration, we will apply the above-presented controller to a two-link elbow robot manipulator as shown in Fig. 1. The dynamics of the two-link elbow robot manipulator can be described in the following differential equations[1]:

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \ddot{q} + N(q,\dot{q})\dot{q} + G(q) + T_d = \tau(t) \tag{37}$$

$$D_{11} = m_1 l_{c_1}^2 + m_2\left(l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2}\cos(q_2)\right) + I_1 + I_2, \tag{38}$$

$$D_{12} = D_{21} = m_2\left(l_{c_2}^2 + l_1 l_{c_2}\cos(q_2)\right) + I_2, \tag{39}$$

$$D_{22} = m_2 l_{c_2}^2 + I_2, \tag{40}$$

$$N(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_{c_2}\dot{q}_2\sin(q_2) & -m_2 l_1 l_{c_2}(\dot{q}_1+\dot{q}_2)\sin(q_2) \\ m_2 l_1 l_{c_2}\dot{q}_1\sin(q_2) & 0 \end{bmatrix}, \tag{41}$$

$$G(q) = \begin{bmatrix} (m_1 l_{c_1} + m_2 l_1) g\cos(q_1) + m_2 l_{c_2} g\cos(q_1+q_2) \\ m_2 l_{c_2} g\cos(q_1+q_2) \end{bmatrix}, \tag{42}$$

$$\tau_d = \begin{bmatrix} T_{d_1} \\ T_{d_2} \end{bmatrix}, \tag{43}$$

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Table I. Parameters of two-link elbow robot.

$l_1 = 1$	$l_2 = 1$	$l_{c_1} = 0.5$
$l_{c_2} = 0.5$	$m_1 = 15$	$m_2 = 6$
$I_1 = 5$	$I_2 = 2$	
	$T_{d_1} = T_{d_2} = 10$	$g = 9.8$

Table II. Desired path and initial condition.

$q_{d_1} = 0.3\sin(3t)$	$q_{d_2} = 0.2\sin(3t)$
$q_1(0) = 0.05$	$q_2(0) = 0$

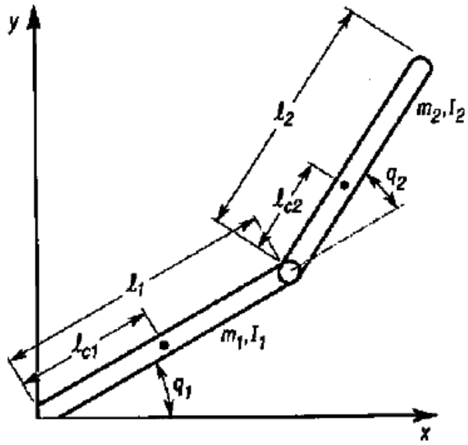

Fig. 1. Two-link elbow robot manipulator.

where q_i for $i = 1,\ 2$ denotes the joint angle, l_i is the link length, m_i is the link mass, I_i is the link's moment of inertia given in the center of mass, l_{c_i} is the distance between the center of mass of link and the ith joint, τ_{d_i} is the disturbance and unmodeled dynamics, and $\tau(t)$ is the torque input.

The link's parameters are estimated with a gain of 0.9 from real values given in Table I. We set the controller with $C = [1010]$ and$\rho = 5$. Desired paths in joint space and initial condition are expressed in Table II.

9. Simulation Results

In this section, in order to demonstrate the proposed control performance, the simulations are provided in three steps. This section tries to present clearly the advantages of the proposed control step-by-step.

Simulation 1. In this step of simulation, in addition to the proposed control, one classic sliding mode controller for robot manipulator tracking control is designed to show the performance of these two controllers, which would show the advantages of the proposed fuzzy sliding mode control. After applying the classic sliding mode control toward the robot manipulator, it is concluded that the joint tracking errors are converged to zero in less than 1 s according to Fig. 2. Therefore, the sliding mode controller in robot manipulator tracking control and in presence of the structured and unstructured uncertainties would have a desirable performance. However, by considering Figs. 3 and 4, intensive chattering appeared in robot manipulator control input. The existing of these undesirable phenomena can stimulate the dynamic modes of the robot manipulator.

By applying fuzzy sliding mode control toward the robot manipulator, it is concluded according to Fig. 5 that the robot manipulator tracking errors are converged to zero. Hence, the proposed control does its duties properly in the presence of all uncertainties. By comparing Figs. 2 and 5, it is concluded that the tracking errors due to applying fuzzy sliding mode control are more than tracking errors due to applying the classic sliding mode control. However, by considering Figs. 6 and 7, it is concluded that the control inputs due to applying fuzzy sliding mode control are free of chattering. Although, in

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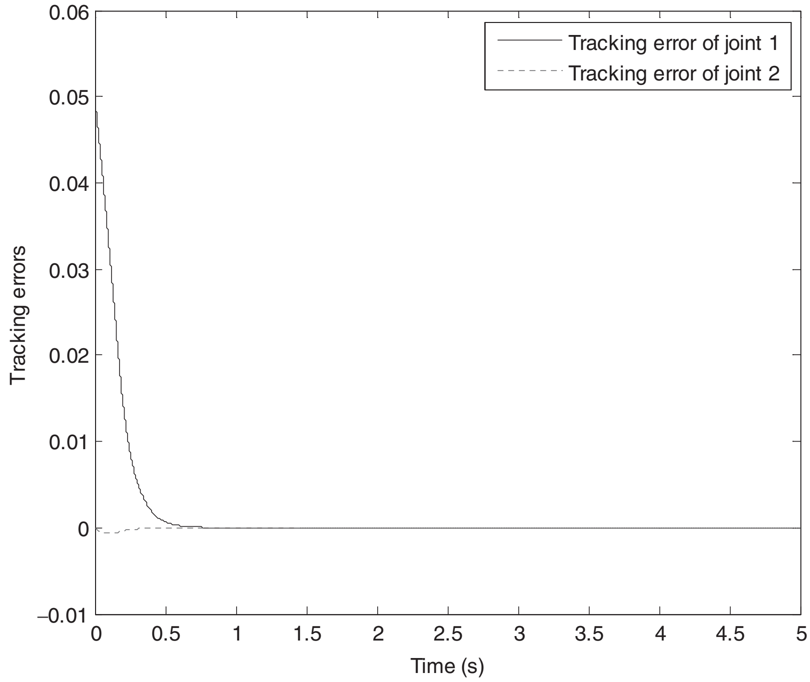

Fig. 2. Tracking errors of the robot manipulator by applying the classic sliding mode control.

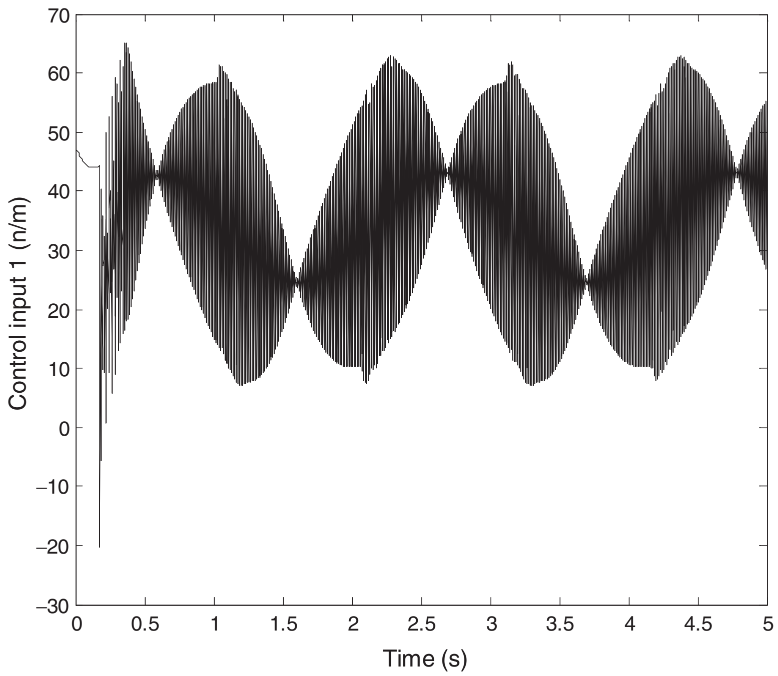

Fig. 3. Control input 1 of the robot manipulator due to applying the classic sliding mode control.

classic sliding mode control method, various mechanisms such as defining one boundary layer around the zero sliding surface and increasing the degree of sliding surface dynamics have been presented, each of these mechanisms in practical implementation level contains some problems.[28] Therefore, it can be concluded that although robot manipulator tracking control has low value of error, unlike classic sliding mode control, the proposed control is capable of practical implementation in industrial robot manipulator tracking control.

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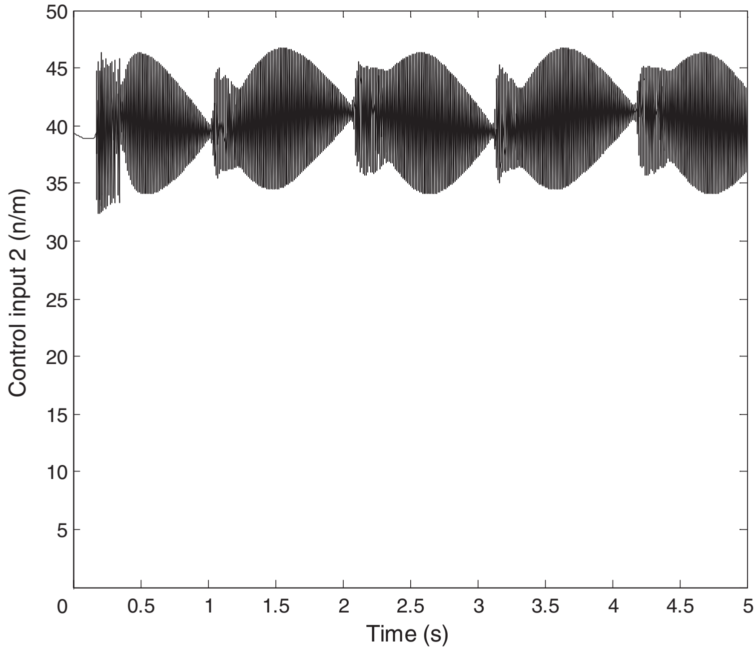

Fig. 4. Control input 2 of the robot manipulator due to applying the classic sliding mode control.

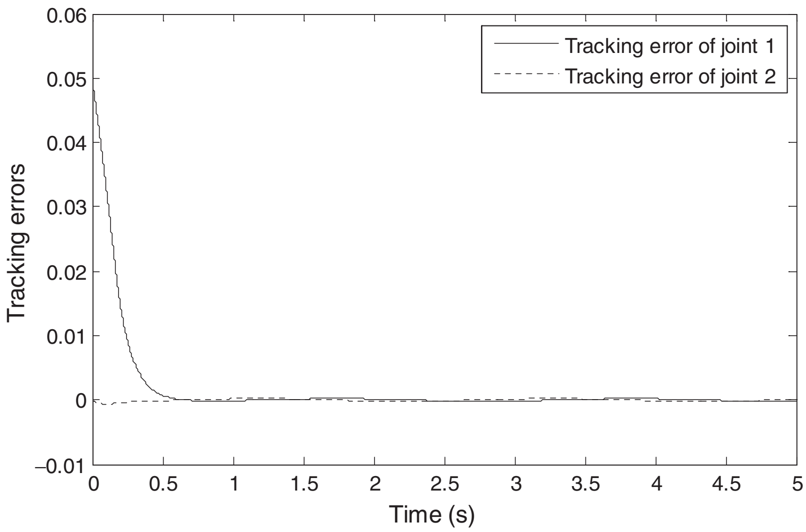

Fig. 5. Robot manipulator tracking errors due to applying the fuzzy sliding mode control.

Simulation 2. In this simulation step, the performance of the proposed control is considered in the presence of disturbance and time-variant un-model dynamics, $T_d = [\begin{smallmatrix} T_{d_1} & \sin(t) \\ T_{d2} & \end{smallmatrix}]$. The values of T_{d_1} and $T_{d_1 2}$ in simulation are presented in Table I. According to Fig. 8, it is shown that by applying the proposed control, robot manipulator tracking errors are converged toward zero in less than 1 s. By comparing Figs. 5 and 8, it is concluded that the tracking errors are more in presence of time-variant disturbances. In Fig. 9, the robot manipulator control inputs are presented. The figure shows that not only the control inputs in presence of the time-variant disturbance are free of any type of chattering but are also smooth, soft, and in permitted interval in aspect of amplitude.

Simulation 3. In this step of the simulation, for decreasing the tracking error due to time-variant disturbance, the coefficient ρ, which in the previous simulations was equal to 5, is changed to 10. Figure 10 shows that the changing of the coefficient worked very well and robot manipulator

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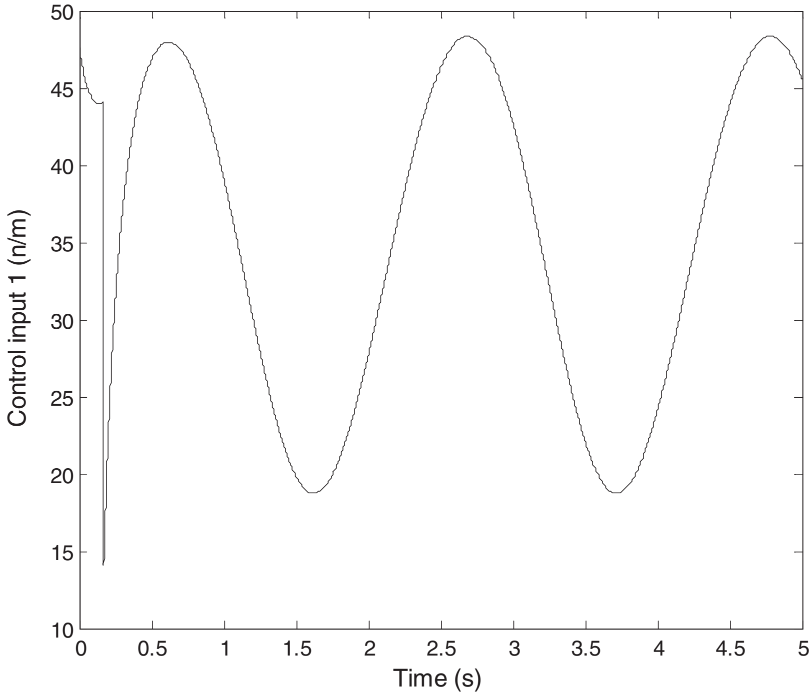

Fig. 6. Control input 1 of robot manipulator due to applying the fuzzy sliding mode control.

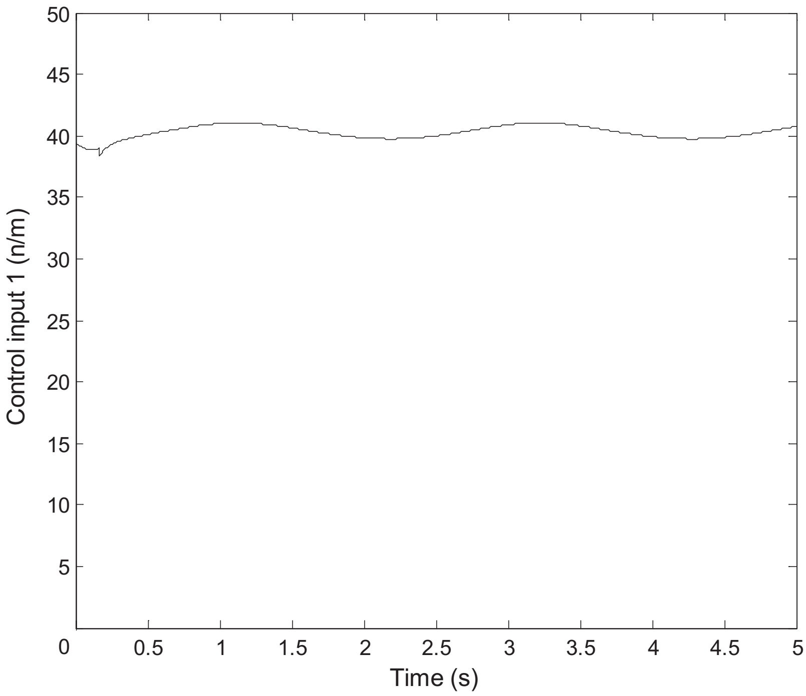

Fig. 7. Control input 2 of robot manipulator due to applying the fuzzy sliding mode control.

tracking errors are improved significantly. In many control methods, which have been presented for robot manipulator tracking control, the tracking errors can be reduced by increasing controller's coefficient. However, this issue, along with increasing of the control input amplitude, causes actuators' saturation; whereas by considering Fig. 11, it is concluded that the increasing of the proposed control input amplitude is not too sensitive with the increasing of coefficient ρ. Therefore, by applying this

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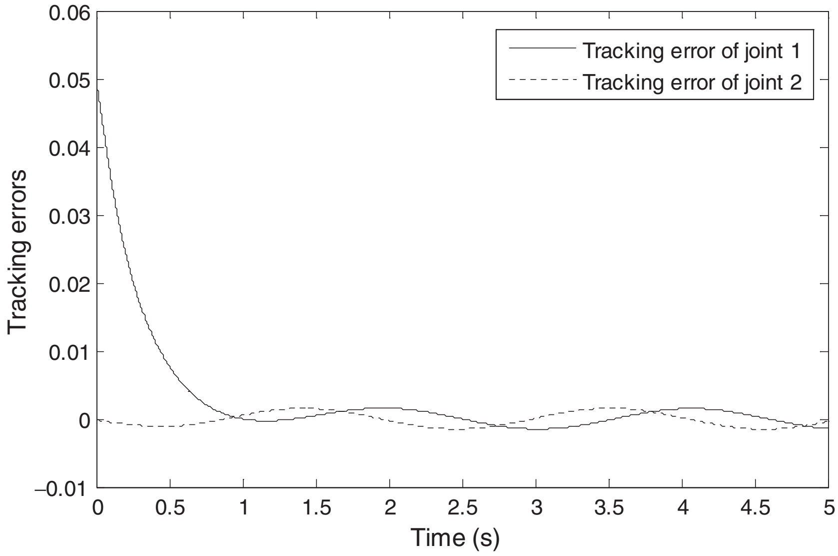

Fig. 8. Robot manipulator tracking errors due to applying the fuzzy sliding mode control in presence of time-variant disturbance.

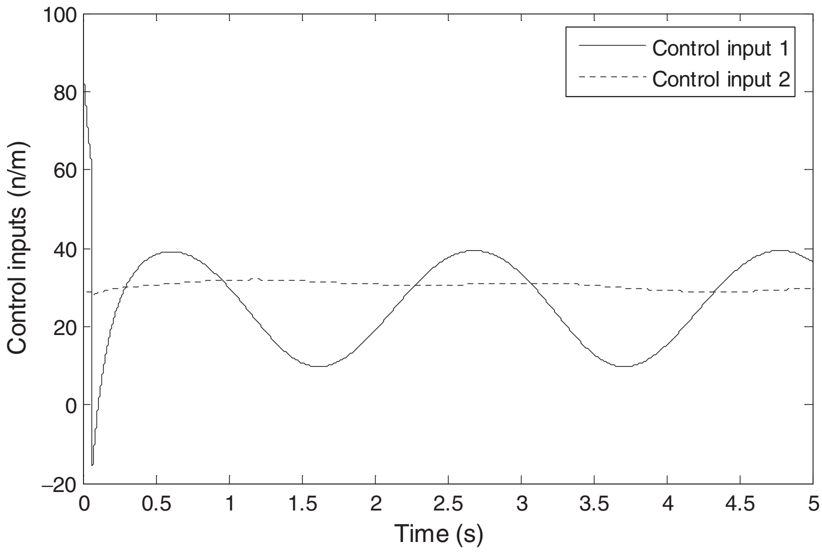

Fig. 9. Robot manipulator control inputs due to applying fuzzy sliding mode control in presence of time-variant disturbance.

change, not only the tracking errors are reduced but the system also is not encountered by increasing the control input amplitude and actuators' saturation.

10. Conclusions

In this paper, by combining the feedback linearization and T-S fuzzy model methods, the fuzzy sliding mode controller for robot manipulator control in presence of structured and unstructured uncertainties was presented. The new method not only has a simple design method but it also does not have the problems of the classic sliding mode control. In the proposed method, the feedback linearization along with elimination of known dynamics reduced the remaining bounds of uncertainties. Furthermore, the T-S fuzzy model method not only compensated the information shortages of designers about structured and unstructured uncertainties but it also reduced the control input mathematical calculation volume and the control input amplitude sensitivity in respect of increasing of controller's coefficient. The mathematical analyses show that the closed-loop system with proposed control in presence of

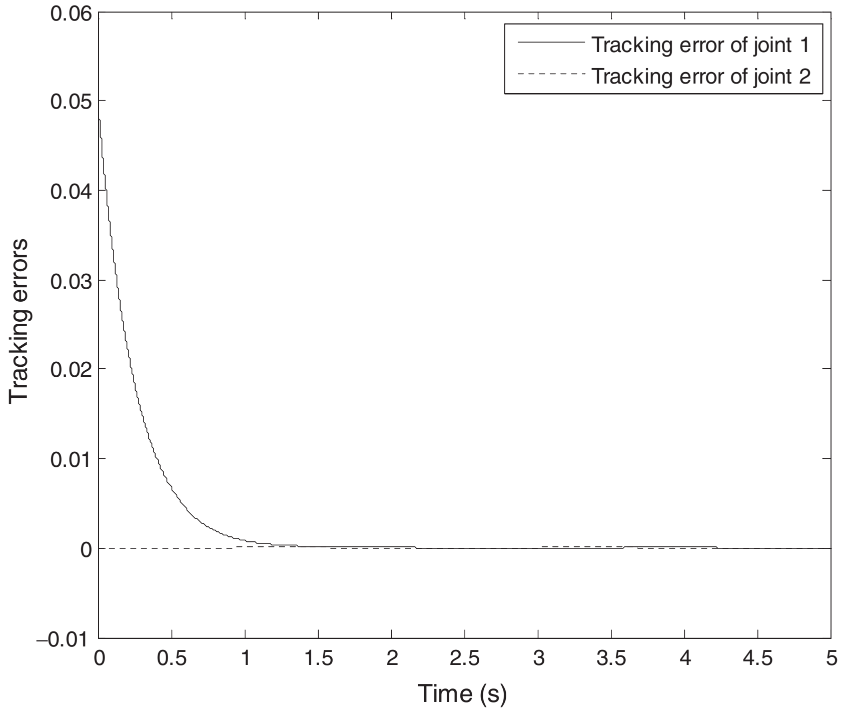

Fig. 10. Reduction of robot tracking errors in presence of the time-variant disturbance with increasing of coefficient ρ.

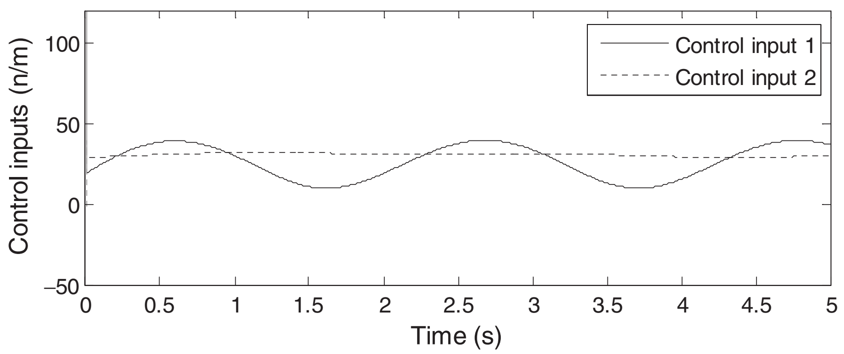

Fig. 11. Robot manipulator control inputs in presence of the time-variant disturbance and increasing of coefficient ρ.

structured and unstructured uncertainties has the globally asymptotic stability. For the observation of performance of the proposed control, some simulations in three steps on robot manipulator with two degrees of freedom were implemented. The simulation design steps and their results assert that the proposed method is strong to overcome the existing uncertainties.

References

1. M. Spong, S. Hutchinson and M. Vidyasagar, *Robot Modeling and Control* (Wiley, New York, 2006).
2. M. H. Khooban, A. Alfi and D. Nazari Maryam Abadi, "Teaching-learning-based optimal interval type-2 fuzzy PID controller design: A nonholonomic wheeled mobile robots," *Robotica*. Available on CJO2013. doi:10.1017/S0263574713000283.
3. M. H. Khooban, A. Alfi, N. Nazari Maryam Abadi, "Control of a class of nonlinear uncertain chaotic systems via an optimal type-2 fuzzy PID controller," *IET. Sci. Meas. Technol.* **7**, 1–9 (2013).
4. Y.-W. Liang, S.-D. Xu, D.-C. Liaw and C.-C. Chen, "A study of TS model-based SMC scheme with application to robot control," *IEEE Trans. Ind. Electron.* **55**(11), 3964–3971 (2008).

https://doi.org/10.1017/S0263574713000702 Published online by Cambridge University Press

5. J. Becedas, J. R. Trapero, V. Feliu and H. S. Ramírez, "Adaptive controller for single-link flexible manipulators based on algebraic identification and generalized proportional integral control," *IEEE Trans. Syst. Man Cybern. B.* **39**(3), 735–751 (2009).
6. S. S. Neo and M. J. Er, "Adaptive fuzzy controllers of a robot manipulator," *Int. J. Syst. Sci.* **27**(6), 519–532 (1996).
7. Y. Jin, "Decentralized adaptive fuzzy control of robot manipulators," *IEEE Trans. Syst. Man Cybern. B.* **28**(1), 47–57 (1998).
8. R.-J. Wai and Z.-W. Yang, "Adaptive fuzzy neural network control design via a TS fuzzy model for a robot manipulator including actuator dynamics," *IEEE Trans. Syst. Man Cybern.B.* **38**(5), 1326–1346 (2008).
9. C.-H. Hyun, C.-W. Park and S. Kim, "TS fuzzy model based indirect adaptive fuzzy observer and controller design," *Inf. Sci.* **180**(11), 2314–2327 (2010).
10. P. R. Pagilla, B. Yu and K. L. Pau, "Adaptive control of time varying mechanical systems: Analysis and experiments," *IEEE/ASME. Trans. Mechatronics* **5**(4), 410–418 (2000).
11. A. Rojko and K. Jezernik, "Sliding-mode motion controller with adaptive fuzzy disturbance estimation," *IEEE. Trans. Ind. Electron.* **51**(5), 963–971 (2004).
12. F. Cupertino, D. Naso, E. Mininno and B. Turchiano, "Sliding-mode control with double boundary layer for robust compensation of payload mass and friction in linear motors," *IEEE. Trans. Ind. Appl.* **45**(5), 1688–1696 (2009).
13. H. Morioka, K. Wada, A. Sabanovic and K. Jezernik, "Neural network based chattering free sliding mode control," **In**: *Proceedings of the 34th SICE Annual Conference*. International Session Papers, IEEE (1995) pp. 1303–1308.
14. K. Jezernik, M. Rodič and R. Šafarič and B. Curk, "Neural network sliding mode robot control," *J. Robtica* **15**(1), 23–30 (1997).
15. H. Hu and P. Y. Woo, "Fuzzy supervisory sliding mode and neural network control for manipulators," *IEEE. Trans. Ind. Electron.* **53**(3), 929–940 (2006).
16. C. T. Li and Y. H. Tan, "Neural sliding mode control for systems with hysteresis" **In**: *Proceedings of IEEE International Symposium on Intelligent Control*, Limassol, Cyprus (2005) pp. 467–472.
17. Y. Yildiz, A. Sabanovic and K.Abidi, "Sliding mode neuro controller for uncertain systems," *IEEE. Trans. Ind. Electron.* **54**(3), 1676–1684 (2007).
18. M. M. Fateh and M. R. Soltanpour, "Robust task-space control of robot manipulators under imperfect transformation of control space," *Int. J. Innovative Comput. Inf. Control.* **5**(12), 3949–3960 (2009).
19. M. R. Soltanpour and S. E. Shafiei, "Robust backstepping control of robot manipulator in task space with uncertainties in kinematics and dynamics," *J. Electron. Electr. Eng. Autom. Robot.* **96**(8), 75–80 (2009).
20. M. R. Soltanpour and M. Siahi, "Robust control of robot manipulator in task space," *Int. J. Appl. Comput. Math.* **8**(2), 227–238 (2009).
21. M. R. Soltanpour, M. M. Fateh and A. R. Ahmadifard, "Nonlinear tracking control on a robot manipulator in the task space with uncertain dynamics," *J. Appl. Sci. Asian Network Sci. Inf.* **8**(23), 4397–4403 (2008).
22. M. R. Soltanpour and S. E. Shafiei, "Design and stability analysis of a robust impedance control system for a robot manipulator," *Stud. Inf. Control.* **17**(1), 34–40 (2010).
23. M. R. Soltanpour and S. E. Shafiei, "Robust adaptive control of manipulators in the task space by dynamical partitioning approach," *J. Electron. Electr. Eng. Autom. Robot.* **101**(5), 73–78 (2010).
24. M. R. Soltanpour and M. M. Fateh, "Adaptive robust control of robot manipulators in the task space under uncertainties," *Aust. J. Basic Appl. Sci.* **1**(3), 308–322 (2009).
25. M. R. Soltanpour and M. M. Fateh, "Sliding mode robust control of robot manipulators in the task space by suupport of feedback linearization and backstepping control," *World Appl. Sci. J.* **6**(1), 70–76 (2009).
26. S. E. Shafiei and M. R. Soltanpour, "Neural network sliding-model-pid controller design for electrically driven robot manipulators," *Int. J. Innovative Comput. Inf. Control* **5**(12), 3949–3960 (2011).
27. M. R. Soltanpour, J. Khalilpour and M. Soltani, "Robust nonlinear control of robot manipulator with uncertainties in kinematics, dynamics and actuator models," *Int. J. Innovative Comput. Inf. Control* **8**(8), 5487–5498 (2012).
28. M. H. Khooban and M. R. Soltanpour, "Swarm optimization tuned fuzzy sliding mode control design for a class of nonlinear systems in presence of uncertainties," *J. Intell. Fuzzy Syst.* **24**(2), 383–394 (2013).