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INTEREST RATE RULES AND EQUILIBRIUM STABILITY UNDER DEEP HABITS

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This paper studies the determinacy of equilibrium in a new Keynesian model with deep habits under different interest rate rules. The main finding is that an interest rate rule satisfying the Taylor principle is no longer a sufficient condition to guarantee determinacy. Including interest rate smoothing and a response to output deviations from steady state significantly enlarges the regions of determinacy. However, under all the simple interest rate rules considered, determinacy is not guaranteed for a very high degree of deep habits. Deep habits give rise to countercyclical markups, which is in line with empirical evidence and makes them an appealing feature in the study of demand shocks. The countercyclicality of markups also leads to multiple equilibria because of self-fulfilling expectations for a high degree of deep habit formation.

Keywords: Taylor Principle, Interest Rate Rules, Sticky Prices, Deep Habits

1. INTRODUCTION

An interest rate rule where the nominal interest rate adjusts more than one for one in response to inflation is said to satisfy the Taylor principle. This is shown by Woodford (2001) to be a necessary and sufficient requirement to guarantee a locally unique rational expectations equilibrium in a standard new Keynesian model. However, a number of papers have pointed out the limitation of the Taylor principle in avoiding indeterminacy and aggregate fluctuations due to self-fulfilling fluctuations, when departing from standard modeling assumptions. These include among others Benhabib et al. (2001) and Carlstrom and Fuerst (2001), who consider different modeling choices for money, Gali et al. (2004), who consider a model with rule-of-thumb consumers, and Sveen and Weinke (2005), who model firm-specific capital. The conditions for determinacy of a unique equilibrium are

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thus model-dependent, and so the robustness of simple interest rate rules to model specification is a concern.

In this paper, I show how introducing deep habits into a model affects the performance of simple interest rate rules, and assess whether the Taylor principle is a sufficient condition for determinacy. I analyze a standard new Keynesian model economy, and in this framework allow households to exhibit deep habits, which is essentially external habit formation (or keeping up with the Joneses) on a good-by-good basis. Habit formation is a desirable feature in macroeconomic models because it helps account for the hump-shaped and persistent response of consumption to various shocks in the economy. Studying a model with deep habits, as introduced in Ravn et al. (2006), is of special interest because it is a more generalized version of habit formation, as agents form habits over consumption of individual goods that form the composite consumption good.

Deep habit formation give rise to the same consumption Euler equation, but unlike the more widely used habit formation at the level of a single aggregate good, they have additional consequences for the supply side of the economy. They render the firm's pricing problem dynamic, even in the absence of nominal rigidities, and give rise to time-varying markups of price over marginal cost. The implied countercyclical markups are consistent with the findings of the empirical literature [e.g., Rotemberg and Woodford (1999)], and additionally act as a transmission mechanism for the observed effects of demand shocks. For instance, deep habits with their implied countercyclical movements of markups have been shown to successfully explain the rise in wages and consumption in response to a government spending shock, an empirical observation that most standard models fail to predict [see Ravn et al. (2007) or Zubairy (2009)].

The main findings of this paper can be summarized as follows. In a model with deep habits, if the monetary authority follows a rule where the nominal interest rate responds strictly to current inflation, then the Taylor principle is too weak a condition to yield stability. I also show that including interest rate smoothing and a response to output deviations from steady state in the monetary policy rule significantly enlarges the regions of determinacy. However, under all the simple interest rate rules considered here, when the nominal interest rate responds to contemporaneous variables, determinacy is not always guaranteed for very high degrees of deep habits.

The economic intuition behind deep habits giving rise to sunspot equilibria is as follows: Suppose agents in the economy expect higher demand. Because of deep habit formation in the model, a unit sold in the current period increases expected sales in the next period. This leads to firms lowering markups in order to hook a larger customer base to carry to the next period. Lowering markups cause firms to increase labor demand which in turn leads to a higher wage. Agents substitute from leisure to consumption as a result and consumption demand goes up. For a high degree of deep habit formation, these positive effects on overall demand are large enough so that expectations are self-fulfilling.

The remainder of the paper is as follows: Section 2 describes a simple model with deep habits, and how the introduction of deep habits affects the Phillips curve. Section 3 describes the equilibrium properties of the model and analyzes the conditions required for the determinacy of a local unique equilibrium under various simple interest rate rules. Section 4 examines the robustness of these results under different parameter values, extension of the model to include capital and government, and habit formation at the level of a single aggregate good. And finally, Section 5 concludes.

2. CANONICAL NEW KEYNESIAN MODEL WITH DEEP HABITS

I am considering a model economy that features optimizing households and a continuum of profit-maximizing firms producing intermediate goods. This is a canonical new Keynesian model and the only departure is the presence of deep habit formation, or habit formation at the level of intermediate goods, for private consumption goods, as first introduced in Ravn et al. (2006).

2.1. Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0, 1]$. Each household $j \in [0, 1]$ derives utility from consumption x_t and disutility from labor supply h_t and seeks to maximize lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(x_t^c, h_t\right). \tag{1}$$

The introduction of deep habits means that the agents do not form habits at the level of the aggregate consumption basket, given here by x_t^c, but at the level of individualized goods. This is then habit formation for a narrower category of goods. Thus, the variable x_t^c is a composite of habit-adjusted consumption of a continuum of differentiated goods indexed by $i \in [0, 1]$,

$$x_t^c = \left[\int_0^1 (c_{it} - b^c c_{it-1})^{1-\frac{1}{\eta}} di\right]^{1/(1-\frac{1}{\eta})}. \tag{2}$$

In principle, households could exhibit different degrees of habit formation across the different individualized goods, but for tractability, I assume the degree to be the same across the differentiated goods. The parameter $b^c \in [0, 1)$ measures the degree of external habit formation, and when b^c is zero, the households do not exhibit deep habit formation. For any given level of consumption of x_t^c, purchases of each individual variety of goods $i \in [0, 1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t. The optimal level of demand, c_{it} for $i \in [0, 1]$, is then given by

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t^c + b^c c_{it-1}, \tag{3}$$

where P_t is a nominal price index defined as $P_t \equiv (\int_0^1 P_{it}^{1-\eta} di)^{\frac{1}{1-\eta}}$.

Note that consumption of each variety is decreasing in its relative price, P_{it}/P_t, and increasing in the level of habit-adjusted consumption, x_t^c. At the same time, the demand function has a second price-inelastic component, given by $b^c c_{it-1}^C$. When there is an increase in aggregate demand, the price-elastic part gets a higher weight, which implies that price elasticity is procyclical, and because markup is given by the inverse of the price elasticity, it is countercyclical. Additionally, firms are forward-looking and internalize that the demand function has a backward-looking term. When they expect high future demand, they have an additional incentive to lower their markups in order to appeal to a broader customer base and carry it over to the following period.

Households have access to a risk-free nominal bond, B_t^j, that pays a gross nominal interest rate R_t in period $t+1$. They also pay lump-sum taxes in the amount T_t per period, and receive dividends from ownership of firms, ϕ_t. The household's period-by-period budget constraint is given by

$$P_t x_t^j + \omega_t + B_t^j + T_t = R_{t-1} B_{t-1}^j + W_t h_t^j + \phi_t^j, \tag{4}$$

where $\omega_t = b^c \int_0^1 P_{it} c_{it-1} di$ and W_t is the nominal wage rate. The household chooses sequences for x_t^j, h_t^j, and B_t^j so as to maximize the utility function (1) subject to (4), and a no-Ponzi game constraint.

The first-order conditions from the optimizing household's problem are

$$\frac{-U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)} = \frac{W_t}{P_t}, \tag{5}$$

$$U_x\left(x_t^j, h_t^j\right) = \beta R_t E_t U_x\left(x_{t+1}^j, h_{t+1}^j\right) \frac{P_t}{P_{t+1}}. \tag{6}$$

The first equation equates the marginal rate of substitution between consumption and leisure to the real wage, and the second equation is the standard Euler equation.

2.2. Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0, 1]$ produces output using labor services h_{it} as factor input, with a production technology given by $F(h_{it})$. The firm is assumed to satisfy demand at the posted price. Formally, $F(h_{it}) \geq c_{it}$.

The objective of the firm is to choose contingent plans for P_{it} and h_{it} in order to maximize the present discounted value of dividend payments, given by $E_o \sum_{t=0}^{\infty} q_t P_t \phi_t^i$, where q_t is a pricing kernel determining the period-zero value of utility from one unit of a composite good in period t,[1] and

$$\phi_t^i = P_{it} c_{it} - W_t h_{it} - P_t \frac{\alpha}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2. \tag{7}$$

Note that sluggish price adjustment is introduced, following Rotemberg (1982), by assuming that the firms incur a quadratic price adjustment cost for the goods they produce, and the parameter α is the degree of price stickiness. This modeling choice of price stickiness produces aggregate dynamics qualitatively similar to those in the pricing mechanism based on Calvo (1983).[2]

The firm's problem is to maximize profits and solves the problem

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} q_t \left\{ P_{it} c_{it} - W_t h_{it} - P_t \frac{\alpha}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 \right.$$
$$\left. + P_t \mathrm{mc}_{it} \left[F(h_{it}) - c_{it} \right] + P_t \nu_{it} \left[\left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t + b c_{it-1} - c_{it} \right] \right\}.$$

The first-order conditions w.r.t h_{it}, c_{it}, and P_{it} are

$$W_t = P_t mc_{it} F_h(h_{it}), \tag{8}$$

$$\nu_{it} = \frac{P_{it}}{P_t} - mc_{it} + b E_t \nu_{it+1} \frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t}, \tag{9}$$

$$c_{it} - \nu_{it} \eta \left(\frac{P_{it}}{P_t} \right)^{-\eta-1} x_t - \alpha \left(\frac{P_{it}}{P_{it-1}} - 1 \right) \frac{P_t}{P_{it-1}}$$
$$+ \alpha E_t \frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t} \left(\frac{P_{it+1}}{P_{it}} - 1 \right) \frac{P_{it+1}}{P_{it}^2} \frac{P_t^2}{P_t} = 0. \tag{10}$$

Equation (8) implies that the markup of price over marginal cost is a wedge between the marginal product of labor and the real wage.[3] Equation (9) states that the value of selling an additional unit of a good, ν_{it}, is the sum of the short-run profit of the sale and the future expected profits associated with it. Because of deep habits, a unit sold in the current period increases sales by an additional b units in the next period. Equation (10) equates the costs and benefits of a unit increase in relative price P_{it}/P_t. The first term is an increase in revenue, followed by the cost in the form of a decline in demand that the price change induces, and finally the loss from the price adjustment cost.

2.3. Monetary Policy Rule

The log-linearized monetary policy rule is assumed to have the form

$$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1 - \alpha_R) \left(\alpha_\pi \hat{\pi}_t + \alpha_Y \hat{y}_t \right), \tag{11}$$

where $\alpha_R \geq 0$, $\alpha_\pi \geq 0$, and $\alpha_Y \geq 0$ and $\hat{R}_t$, $\hat{\pi}_t$, and $\hat{y}_t$ represent nominal interest rate, inflation, and output log deviations from the respective steady states.

2.4. Phillips Curve under Deep Habits

The detailed derivations of the steady state and log-linearized equations are shown in the Appendix. In this simple model with deep habits, and under the assumption that the interest rate only responds to deviations of inflation from the steady state (i.e., $\alpha_R = \alpha_Y = 0$), the log-linearized Phillips curve in terms of marginal cost is given as

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{h}{\alpha}[\eta(1-b) - (1-b\beta)]\hat{mc}_t$$
$$+ b\frac{h}{\alpha}\left[\beta\left(E_t \hat{y}_{t+1} - \frac{1}{1-b}\hat{y}_t\right) - \frac{b\beta+1}{(1-b)}(\hat{y}_t - \hat{y}_{t-1}) - \beta E_t \hat{\nu}_{t+1}\right]. \quad \textbf{(12)}$$

Note that in the case of no deep habits, i.e., $b = 0$, this simplifies to the standard new Keynesian Phillips curve. The presence of deep habits modifies the Phillips curve in several different ways. First, the presence of deep habits affects the impact of marginal cost on inflation. Deep habits also introduce a backward-looking term into the Phillips curve through the impact of the habit stock on the current period's demand, which would indicate that the inflation dynamics displays more inertia. The presence of deep habits introduces additional forward-looking terms in the form of $E_t \hat{\nu}_{t+1}$ and $E_t \hat{y}_{t+1}$. Notably, for a given path of inflation, an increase in the expected future value of sales, $E_t \hat{\nu}_{t+1} > 0$, reduces the markup, which is $-\hat{mc}_t$. In addition, an increase in current demand, $\hat{y}_t > 0$, also reduces the markup.

3. EQUILIBRIUM DYNAMICS

3.1. Equilibrium Stability in the Model

As shown in the Appendix, the system of linearized equilibrium conditions can be reduced to four linear difference equations, namely the Phillips curve, the dynamic equation describing the final goods markup, an equation connecting past and current consumption, and the Euler equation. The system of equation is given as follows:

$$\begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{\nu}_{t+1} \\ \hat{c}_t \\ \hat{c}_{t+1} \end{pmatrix} = C \begin{pmatrix} \hat{\pi}_t \\ \hat{\nu}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix}.$$

The coefficients of the matrix C are functions of the steady state values and parameters of the model. This system can now be used to infer if the equilibrium is determinate by comparing the number of roots of the matrix C outside the unit circle relative to the number of nonpredetermined variables. In this model, there is one predetermined variable, c_{t-1}, and three nonpredetermined state variables, c_t, π_t, and ν_t . I do not analytically derive necessary conditions for the existence of local equilibria because the analytical eigenvalues of the 4×4 nonsparse matrix C are too messy.

TABLE 1. Calibrated parameters

β	η	h	α
0.9902	6	0.5	17.5

Instead, I present numerical results. The model is calibrated to quarterly frequency. The discount factor β, is set at $1.03^{-1/4}$, which implies a steady-state annualized real interest rate of 3%. Goods elasticity of substitution η is set at 6, which implies a steady state markup of 20% percent in the absence of deep habits, consistent with average markup values discussed in Rotemberg and Woodford (1992). Also, the steady state labor h is set at 0.5. In the baseline calibration, the price stickiness parameter, α, following Schmitt-Grohé and Uribe (2004), who model price stickiness with a Rotemberg price adjustment cost, is set to be 17.5.[4]

I characterize regions in the parameter space for which the equilibrium is determinate by computing the number of explosive eigenvalues of C for combinations of the monetary policy parameter α_π and the deep habit parameter b. A determinate equilibrium requires three explosive eigenvalues of the system. Figure 1 shows the determinacy region as I vary the deep habit parameter along with the inflation

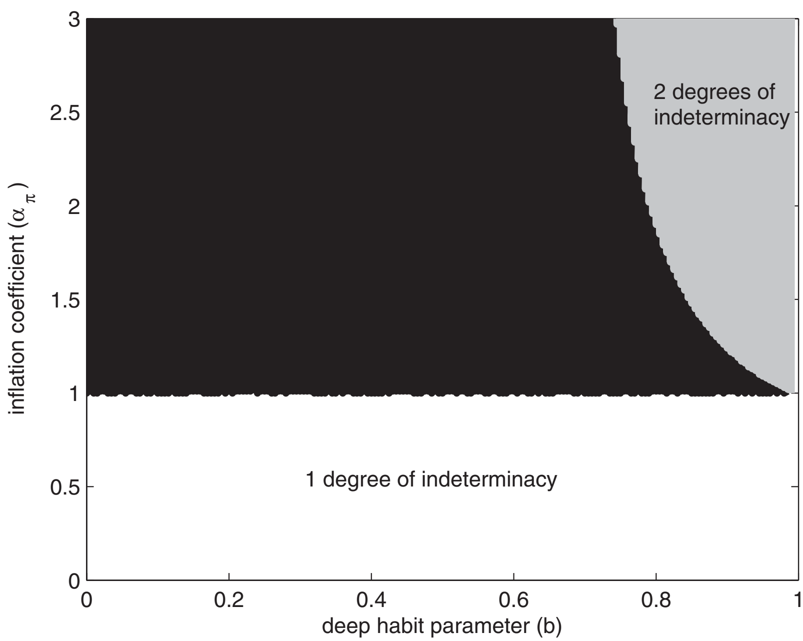

FIGURE 1. Region of determinacy under baseline calibration and a monetary rule that responds only to inflation with $\alpha_R = \alpha_Y = 0$. The equilibrium is determinate in the black region, and the white and gray regions corresponds to regions of indeterminacy.

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coefficient under a monetary rule responding only to inflation. Note that $\alpha_\pi > 1$ is a necessary condition for determinacy, and for $\alpha_\pi < 1$, there are only two eigenvalues outside the unit circle, and so the economy exhibits one degree of indeterminacy. For high values of the deep habit formation parameter, $\alpha_\pi > 1$ is not a sufficient condition to guarantee a stable equilibrium and Figure 1 shows there is only one eigenvalue outside the unit circle in the right quadrant. To sum up, this figure shows that the Taylor principle is no longer a sufficient condition to ensure the existence of a local unique equilibrium in the case of a high degree of deep habit formation.

3.2. Intuition for Indeterminate Equilibria and Impulse Response Analysis

In the preceding section, I showed that for a very high degree of deep habits, a unique equilibrium converging to a steady state does not exist. To obtain further insight into this finding, I consider the same baseline model assuming that in the monetary policy rule the nominal interest rate responds only to current inflation.[5] Suppose that households anticipate an increase in aggregate demand, without any shocks to fundamentals to justify it. This increase in demand would be accompanied by an increase in hours worked, lower markups due to deep habits, and high inflation as the firms adjust prices to get to their desired markups. But an interest rate rule that has $\alpha_\pi > 1$ will generate a high real interest rate along the adjustment path and imply lower consumption and investment relative to the steady state. Thus it would not be possible to sustain a boom in demand, and so this is not consistent with rational expectations.

On the other hand, consider the case where the degree of deep habits is sufficiently high to allow multiple equilibria.[6] The impulse response functions for such an expansionary sunspot shock are shown in Figure 2. Here the model is calibrated so that the Taylor principle is satisfied, $\alpha_\pi = 1.5$, and the deep habit parameter, $b = 0.85$. Now even if the interest rate rule follows the Taylor principle, the higher degree of deep habits will drive the markups to be countercyclical to a greater extent. Note that the markup, say μ_t, is a wedge between the marginal product of labor and the real wage; i.e., $F_h(h_t) = \mu_t w_t$. This high deep-habit formation helps in driving the markup sufficiently far down so that for any given level of wage, the marginal product of labor falls, and so labor demand rises. This shift in the labor demand leads to a rise in real wages. The increase in wages causes the households to substitute away from leisure to consumption, and so the consumption of households rises. In other words, in this case the degree of deep habit formation leads to intratemporal substitution effects working in opposition to the intertemporal substitution effects.[7] This rise in consumption is an increase in realized demand, as anticipated by agents in the economy, thus leading to self-fulfilling expectations.

Schmitt-Grohe (1997) shows other models with variable countercyclical markups; in particular, the implicit collusion model of Rotemberg and Woodford (1992) and a variant of Gali (1994) with increasing returns to scale are also

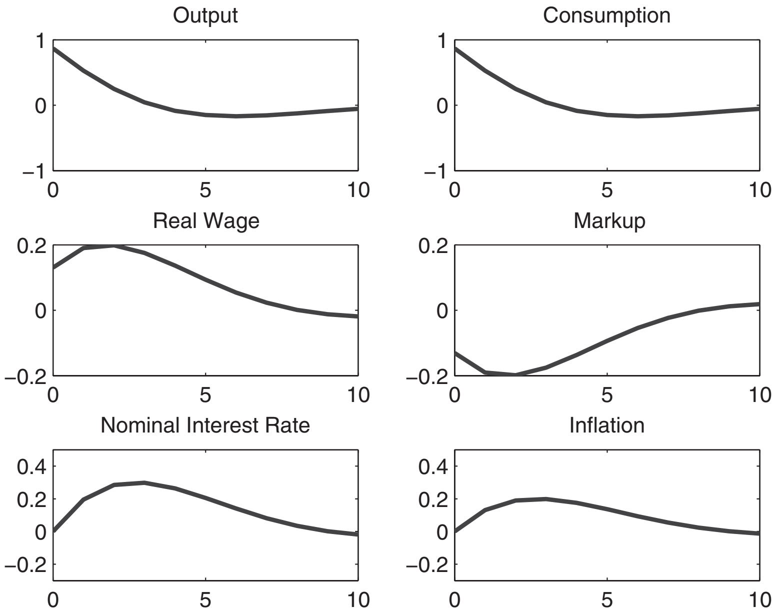

FIGURE 2. Response to a sunspot shock, where $b = 0.85$, $\alpha = 0$, and the monetary policy rule is given by $\hat{R}_t = 1.5\hat{\pi}_t$.

characterized by an indeterminate equilibrium for markup values in the upper range of empirical estimates.[8] The economic intuition in those cases is also similar.

3.3. Monetary Policy Rules and Indeterminacy

Figure 1, as discussed in Section 3.3, shows the determinacy region as I vary the deep habit parameter along with the inflation coefficient under a monetary rule responding only to inflation. It is apparent that for the case of no deep habit formation, $b = 0$, or low values of the deep habit parameter, a unique equilibrium is guaranteed for $\alpha_\pi > 1$. Woodford (2001) shows that in the case of a simple new Keynesian model, when there is a zero coefficient on the output gap in the monetary policy rule, namely $\alpha_Y = 0$, then $\alpha_\pi > 1$ satisfies the Taylor principle and guarantees the existence of a local unique equilibrium. Under deep habits, the Taylor principle is no longer a sufficient condition to ensure the existence of a local unique equilibrium in the case of a high degree of deep habit formation.

The next question that arises is if the region of determinacy can be enlarged by modifying the monetary policy rule. So far, only the case of $\alpha_\pi > 0$ has been considered, where $\alpha_R = \alpha_Y = 0$ in the monetary policy rule. Next, I formally analyze variation in these other policy rule coefficients.

In a standard new Keynesian model with interest rate smoothing, the Taylor principle implies that monetary policy should be active in the long run. So the

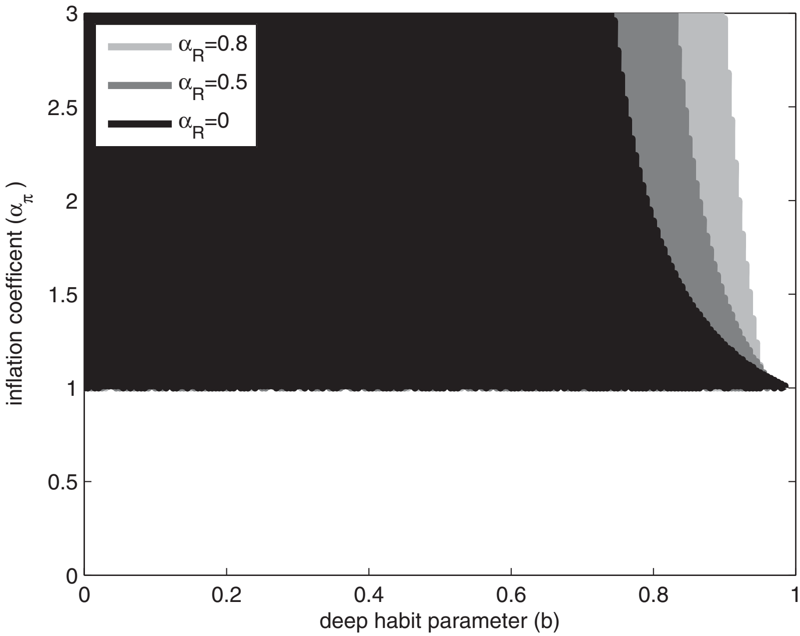

FIGURE 3. Regions of determinacy under varying degree of interest rate smoothing parameter, α_R, in the monetary policy rule and $\alpha_Y = 0$.

particular value of α_R, the interest-rate-smoothing parameter, is irrelevant for determinacy, as long as $\alpha_\pi > 1$. I find, however, that with the introduction of deep habits, determinacy is not guaranteed for all $\alpha_R \in (0,1)$, even when $\alpha_\pi > 1$. In fact, the size of the region of indeterminacy shrinks gradually as α_R is increased, as shown in Figure 3. This suggests that inertial rules are more desirable as a way to attain macroeconomic stability. Next, the nominal interest rate is allowed to respond to deviations of output from steady state, and once again increasing α_Y widens the region of determinacy. As shown in Figure 4, there is a significant enlargement of the determinacy region between the case of no response to output deviations ($\alpha_Y = 0$) and the case of $\alpha_Y = 0.5$. For the case of $\alpha_Y = 1$, the region of determinate equilibria now also includes a very high degree of deep habit formation when there is a small response to inflation. So a response of nominal interest rate to economic activity is also a desirable feature for an interest rate rule to lead to determinacy.

The finding that combining active monetary policy with interest rate smoothing and responsiveness of nominal interest rate to economic activity improves the determinacy properties of the model is common across significantly different models.[9] Note, however, that allowing interest rate smoothing and/or response to economic activity still gives us indeterminacy for very high degrees of deep habits. The estimates for the deep habit parameter in the context of medium-scale

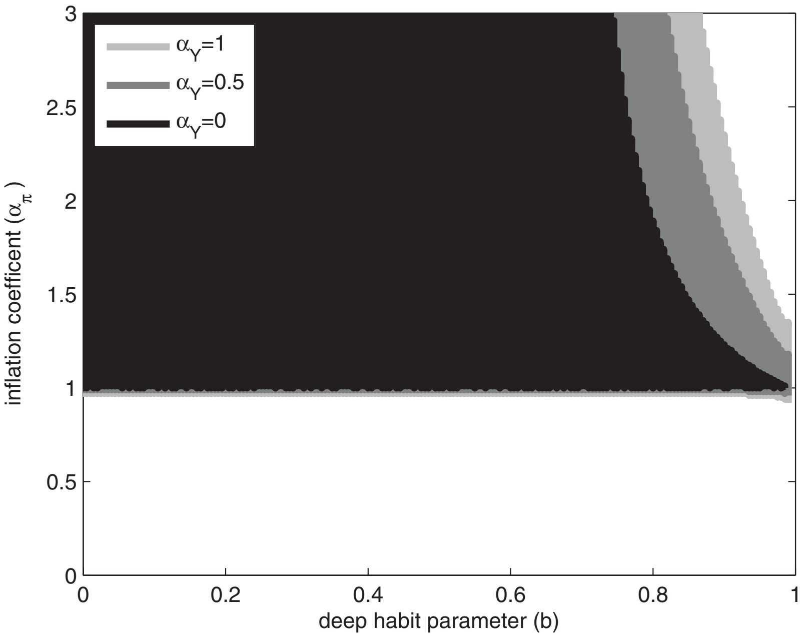

FIGURE 4. Regions of determinacy under varying degree of response to output gap, α_Y, in the monetary policy rule and $\alpha_R = 0$.

dynamic general equilibrium models as well as of simpler frameworks similar to the one considered here are usually between 0.6 and 0.9 [see Ravn et al. (2006) and Zubairy (2009)]. The equilibrium is determinate for these values of the deep habit parameter under some of the calibrations for α_R and α_Y considered here.

4. ROBUSTNESS ANALYSIS

4.1. Robustness to Parameter Values

This section considers the robustness of results to a different choice of parameter values for the price stickiness parameter, α. Figure 5 shows what the determinacy region looks like in the baseline model with the price stickiness parameter α increasing along the y-axis and the deep habit parameter b along the x-axis and under the assumption that $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$. Looking at the figure, it is clear that when there are no deep habits in the model, i.e., $b = 0$, a unique local equilibrium exists for all values of α. However, the degree of deep habit formation plays a crucial role and for high values of the deep habit parameter, the economy runs into a region of indeterminacy even though the nominal interest rate is adjusting more than one for one with inflation. Notice also that when there is no price stickiness (i.e., $\alpha = 0$) or for very low values of α, the model still has multiple equilibria for high degrees of habit formation, and the indeterminacy

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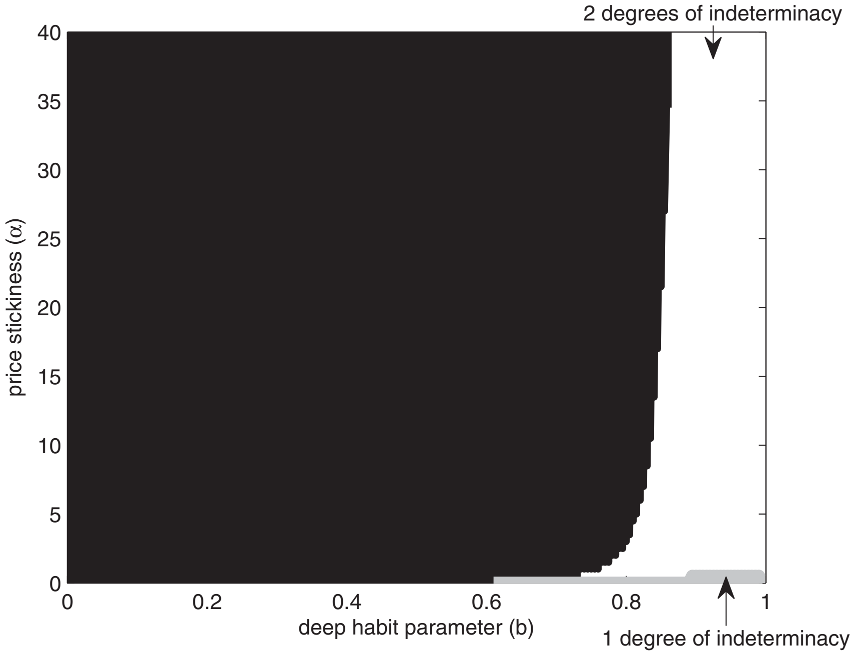

FIGURE 5. Robustness to degree of price stickiness. Region of determinacy for varying degree of price stickiness, α, under a monetary policy rule responding only to inflation with $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$. The equilibrium is determinate in the black region, and the white and gray regions corresponds to regions of indeterminacy.

is of degree one. For higher values of α and deep habit formation, the degree of indeterminacy is 2. To get some intuition behind this, consider equations (9) and (10) in the firm's problem, where it is clear that in addition to the presence of deep habits, price stickiness also affect markup dynamics. Price stickiness, measured by α, causes firms to smooth price increase over time in response to changes in marginal costs or aggregate demand. Thus both mechanisms amplify the effects of shocks to the economy, and for a high degree of price stickiness along with deep habit formation, lead to equilibrium indeterminacy.

4.2. Model of Deep Habits with Capital and Government Spending

In this section, I consider the effects of extending the model presented in Section 2 in several dimensions. I introduce capital accumulation, consider a more general formulation of deep habits, and introduce government spending into the model. This framework is close to the model considered in Ravn et al. (2007) and Zubairy (2009), where deep habits are considered as a transmission mechanism for demand shocks, namely government purchases shocks. The details of the model are given in the Online Appendix.

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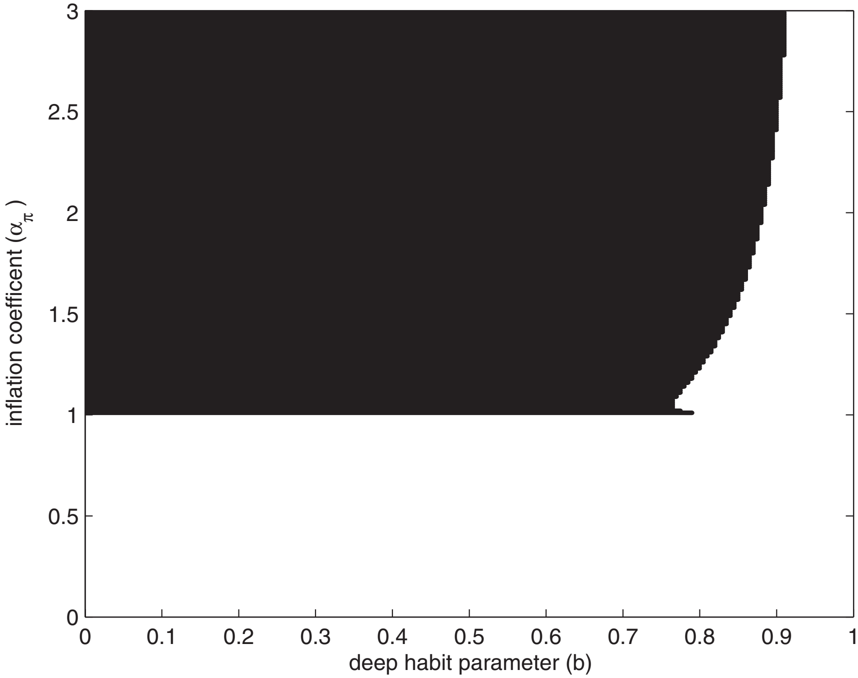

FIGURE 6. Robustness in extended model. Region of determinacy for the extended model with capital and government spending, and a monetary rule responding only to inflation with $\alpha_R = \alpha_Y = 0$.

Figure 6 shows the determinacy region for this extended model as I vary the deep habit parameter, along with the inflation coefficient, under a monetary rule responding only to inflation. It is apparent that for the case of no deep habit formation, $b = 0$, or low values of the deep habit parameter, a unique equilibrium is guaranteed for $\alpha_\pi > 1$, but this is not the case for a high degree of deep habit formation. Namely, $\alpha_\pi > 1$, although a necessary condition, is no longer sufficient to ensure determinate equilibrium.[10] Specifically, the response to inflation required to guarantee a determinate equilibrium is increasing in the degree of deep habit.

4.3. Habit Formation over a Single Aggregate Good

In this section we consider the more standard form of habit formation, where agents form habits over the composite consumption good, and each household maximizes its utility function,

$$U(c_t, h_t) = \frac{\left[(c_t - \theta \tilde{c}_{t-1})^{1-\nu} (1-h)^{\nu}\right]^{1-\sigma} - 1}{1-\sigma},$$

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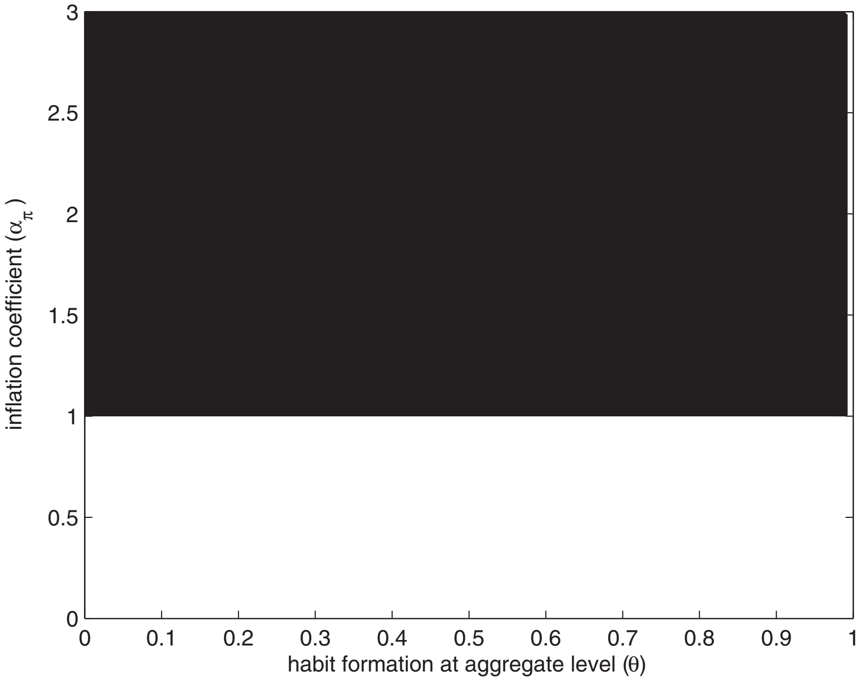

FIGURE 7. Robustness to habit formation over aggregate good. Region of determinacy in a model with nominal rigidities and superficial habits, and a monetary rule that responds only to inflation with $\alpha_R = \alpha_Y = 0$.

where $c_t = (\int_0^1 c_{it}^{1-\frac{1}{\eta}})^{1/(1-\frac{1}{\eta})}$. The parameter $\theta \in [0, 1)$ measures the degree of external habit formation, and $\tilde{c}_{t-1}$ is the average consumption last period. The demand function for good i for a household in this case is given by $c_{it} = (P_{it}/P_t)^{-\eta} c_t$.

This specification of external habit formation is the same as commonly found in the literature, in order to match the persistence in the consumption response to macroeconomic shocks. Figure 7 shows the region of determinacy for the model with superficial habits in the new Keynesian model. Note that the Taylor principle is a necessary and sufficient condition to guarantee determinacy in this framework. The indeterminacy region in Figure 1, for $\alpha_\pi > 1$, is therefore precisely due to how deep habits affect the firm's problem and give rise to countercyclical markups, and similar results do not hold for habit formation at the level of a single aggregate good, which affects only the demand side.

5. CONCLUSION

This paper shows how introducing deep habits into a model affects the performance of simple interest rate rules, where the nominal interest rate responds to inflation

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or to output or is subject to interest-rate smoothing. The results suggest that the Taylor principle is too weak a condition to guarantee stability. In this standard new Keynesian model with deep habits, including interest rate smoothing and a response to output deviations from the steady state in the interest rate rule significantly enlarge the regions of determinacy. But, under all these rules, the equilibrium is not uniquely determined for a very high degree of deep habits. The main intuition behind this finding is that at very high degrees of deep habits, the markups generated are large and countercyclical, with resulting effects on consumption and investment demand that counter otherwise stabilizing effects of changes in real interest rate due to the monetary policy rule.

This paper adds to the literature suggesting that the recommendations for monetary rules that render unique equilibria are model-dependent. It is thus important to be more careful and aware of these problems of indeterminacy when augmenting models with new features.

NOTES

1. This follows from equation (6) in the household's problem, $q_t P_t = \beta^t U_x(x_t, h_t)$.
2. The presence of deep habits alone makes the pricing problem dynamic, and so additionally accounting for dynamics due to Calvo-style pricing makes aggregation nontrivial.
3. The Lagrange multiplier on the constraint that output is determined by demand is in fact the marginal cost.
4. This value of the price stickiness parameter implies that firms change their price on the average every three quarters in a Calvo.Yun staggered-price-setting model, based on estimates of a linear new Keynesian Phillips curve by Sbordone (2002). Refer to the more detailed discussion in Schmitt-Grohé and Uribe (2004).
5. To understand specifically the role of deep habits, I consider the case where there are no adjustment costs for prices, so $\alpha = 0$, and in this case the degree of indeterminacy is 1. See Section 4.1 for further details about the interaction of price stickiness and deep habits.
6. Here I am considering the multiple equilibria characterized in the top right quadrant in Figure 1.
7. Note also that if in fact $\alpha > 0$ and there is price stickiness in the model, this further dampens the rise in inflation and the intertemporal substitution effects, and causes a further rise in demand.
8. In context of the model of deep habits, the higher degree of habit formation raises the steady state markup value and so, for instance, in the example shown previously, $b = 0.96$ corresponds to a steady state markup of 1.38 and is characterized by an indeterminate equilibrium. Schmitt-Grohe (1997) finds that the minimum markup leading to sunspot equilibria is around 1.7, which is significantly higher, but direct comparisons are difficult because of different calibration of other parameters.
9. Among others, Gali et al. (2004) and Sveen and Weinke (2005). Sveen and Weinke (2005) have output gap in the policy rule, which is the difference between output and its natural level (level of output absent any nominal rigidities).
10. The Online Appendix also explores the determinacy results for this extended model under different interest rate rules, and robustness to parameter values.

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APPENDIX: NEW KEYNESIAN MODEL WITH DEEP HABITS

A.1. EQUILIBRIUM CONDITIONS WITH FUNCTIONAL FORMS

Note that the functional form for utility function is $U(x_t, h_t) = (1-\phi)\log(x_t) + \phi\log(1-h_t)$, and we assume a linear production function, so that $F(h_t) = h_t$. All households are identical, so the consumption and labor supply across them is invariant. Additionally, because we consider a symmetric equilibrium, all firms charge the same price. Therefore, the following conditions characterize the equilibrium:

$$\frac{\phi x_t}{(1-\phi)(1-h_t)} = w_t, \tag{A.1}$$

$$\frac{1}{R_t}\frac{1}{x_t} = \beta E_t \frac{1}{x_{t+1}\pi_{t+1}}, \tag{A.2}$$

$$x_t = c_t - bc_{t-1}, \tag{A.3}$$

$$w_t = mc_t, \tag{A.4}$$

$$1 - mc_t - \nu_t + \beta b E_t \nu_{t+1} \frac{x_t}{x_{t+1}} = 0, \tag{A.5}$$

$$c_t - \nu_t \eta x_t - \alpha (\pi_t - 1) \pi_t + E_t \alpha\beta \frac{x_t}{x_{t+1}} (\pi_{t+1} - 1) \pi_{t+1} = 0, \tag{A.6}$$

and the resource constraint,

$$c_t + \frac{\alpha}{2} (\pi_t - 1)^2 = h_t. \tag{A.7}$$

A.2. STEADY STATE

We pin down $\pi = 1$ and $h = 0.5$, and the remaining steady state values are given as follows:

$$c = h, \qquad x = (1-b)c,$$

$$R = \frac{1}{\beta},$$

$$\nu = \frac{1}{\eta(1-b)},$$

$$mc = 1 + \nu(\beta b - 1), \qquad w = mc,$$

$$\phi = \frac{mc(1-h)}{h(1-b) + mc(1-h)}.$$

A.3. LOG-LINEARIZED EQUATIONS

Log-linearizing the equilibrium conditions around the steady state yields the following:

$$\hat{x}_t = \frac{1}{1-b}(\hat{c}_t - b\hat{c}_{t-1}),$$

$$\hat{w}_t = \hat{mc}_t,$$

$$\hat{h}_t = \hat{c}_t,$$

$$\hat{mc}_t = \left(\frac{1}{(1-b)} + \frac{h}{1-h}\right)\hat{c}_t - \frac{b}{1-b}\hat{c}_{t-1}.$$

Also, the monetary policy rule is given by the following, so we ignore interest rate smoothing and response to output deviations from steady state for now:

$$\hat{R}_t = \alpha_\pi \hat{\pi}_t.$$

We substitute these expressions into the log-linearized equations that follow:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{h}{\alpha}\hat{\nu}_t + \frac{h}{\alpha}\frac{b}{(1-b)}\hat{c}_{t-1} - \frac{h}{\alpha}\frac{b}{(1-b)}\hat{c}_t,$$

$$\frac{1}{1-b}(\hat{c}_{t+1} - (1+b)\hat{c}_t + b\hat{c}_{t-1}) = \alpha_\pi \hat{\pi}_t - E_t \hat{\pi}_{t+1},$$

$$
\begin{aligned}
E_t \hat{\nu}_{t+1} &= \frac{1}{b\beta}\hat{\nu}_t + \frac{1}{1-b}E_t\hat{c}_{t+1} \\
&\quad + \frac{\eta(1-b)+(b\beta-1)}{b\beta}\left(\left(\frac{1}{(1-b)}+\frac{h}{1-h}\right)\hat{c}_t - \frac{b}{1-b}\hat{c}_{t-1}\right) - \frac{1+b}{1-b}\hat{c}_t \\
&\quad + \frac{b}{1-b}\hat{c}_{t-1}.
\end{aligned}
$$

A.4. DETERMINACY OF EQUILIBRIUM

The equilibrium conditions can be written as follows, assuming perfect foresight:

$$
\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dfrac{-1}{1-b} \\ 0 & 0 & 1 & 0 \\ (1-b) & 0 & 0 & 1 \end{pmatrix}}_{A}
\begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{\nu}_{t+1} \\ \hat{c}_t \\ \hat{c}_{t+1} \end{pmatrix}
= \underbrace{\begin{pmatrix}
\dfrac{1}{\beta} & \dfrac{h}{\alpha\beta} & -\dfrac{h}{\alpha\beta}\dfrac{b}{1-b} & \dfrac{h}{\alpha\beta}\dfrac{b}{1-b} \\
0 & \dfrac{1}{\beta b} & \dfrac{b}{1-b}(1-X) & X\left(\dfrac{1-bh}{(1-b)(1-h)}\right) - \dfrac{1+b}{1-b} \\
0 & 0 & 0 & 1 \\
\alpha_\pi(1-b) & 0 & -b & (1+b)
\end{pmatrix}}_{B}
\begin{pmatrix} \hat{\pi}_t \\ \hat{\nu}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix},
$$

where $X = \frac{\eta(1-b)+(b\beta-1)}{b\beta}$. This can be further simplified, to write

$$
\begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{\nu}_{t+1} \\ \hat{c}_t \\ \hat{c}_{t+1} \end{pmatrix} = A^{-1}B \begin{pmatrix} \hat{\pi}_t \\ \hat{\nu}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix} = C \begin{pmatrix} \hat{\pi}_t \\ \hat{\nu}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix},
$$

where

$$
C = \begin{pmatrix}
\frac{1}{\beta} & \frac{h}{\beta\alpha} & -\frac{bh}{(1-b)\beta\alpha} & \frac{bh}{(1-b)\beta\alpha} \\
-\frac{1}{\beta}+\alpha_\pi & \frac{1}{b\beta}-\frac{h}{\beta\alpha} & \frac{bh}{\alpha\beta}+\frac{b-((1-b)\eta+b\beta-1)\beta}{1-b}-\frac{b}{1-b} & \frac{(1-bh)\beta((1-b)\eta+b\beta-1)}{b(1-b)(1-h)}-\frac{bh}{(1-b)\beta\alpha} \\
0 & 0 & 0 & 1 \\
(1-b)\alpha_\pi - \frac{(1-b)}{\beta} & \frac{-h(1-b)}{\beta\alpha} & \frac{bh}{\beta\alpha}-b & 1+b-\frac{bh}{\beta\alpha}
\end{pmatrix}.
$$

We now use this system to infer whether the equilibrium is determinate by comparing the number of roots of the matrix C outside the unit circle with the number of nonpredetermined variables, which in this case is three.