Transient self-focusing of an intense laser pulse in magnetized plasmas under non-paraxial approximation

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Abstract

Non-paraxial approximation based study of transient self-focusing of an intense short-pulse laser in plasma has been investigated by considering the effect of a transverse magnetic field. The laser with non-uniform distribution of intensity exerts a ponderomotive force on electrons and sets in an ambi-polar diffusion of the plasma. The ambient magnetic field, however, strongly inhibits the process, when the electron Larmor radius is comparable to or shorter than the laser spot size. As the plasma density is depleted, the laser beam becomes more self-focused. This study addresses a significant enhancement in the laser self-focusing rate by including the correction terms due the off-axis approximation.

Keywords: Laser-Plasma interactions; Non-linearity; Self-focusing

1. INTRODUCTION

The interaction of an intense short-pulse laser with plasmas is a major area due to its relevance to laser-driven fusion, laser-driven particle accelerator, X-ray laser, and other related areas (Modena *et al.*, 1995; Esarey *et al.*, 1996; Shukla, 1999; 1986; Suk *et al.*, 2001; Umstadter, 2001; Bingham *et al.*, 2004; Fuerbach *et al.*, 2005; Gibbon, 2005; Hoffmann *et al.*, 2007; Gupta *et al.*, 2007; Sandhu *et al.*, 2005; Hora, 2007; Flippo *et al.*, 2007; Chen *et al.*, 2008). In these applications, it is required to propagate the laser pulse up to several Rayleigh lengths while preserving an efficient interaction with the plasma. In the last decades, significant progress in this direction has been made and continues to be a subject of great fascination among researchers (Chen & Sudan, 1993; Chessa *et al.*, 1998; Ren *et al.*, 2001; Leemans *et al.*, 1998). It occurs when the refractive index of the plasma is an increasing function of intensity. This dependence may be through local electron density depression, caused by ponderomotive force and subsequent plasma diffusion. Durfee and Milchberg (1993) experimentally observed plasma-channeling of a short-pulse laser to long distances. These studies, conducted with long pulses, have observed the phenomena of self-focusing of laser beams in plasma under a wide variety of conditions. It is caused by the intensity dependent refractive index that may arise due to a relativistic mass effect, ponderomotive force induced plasma diffusion or non-uniform Ohmic heating and subsequent plasma diffusion. The relativistic mass non-linearity arises instantly, however, it requires very high laser intensity such as that observed when the electron quiver-velocity is comparable to the velocity of light in vacuum. The two non-linearity's require a relaxation time to settle down, as they involve physical displacement of the plasma from the region of high-intensity to outside the domain of the beam. Ponderomotive non-relativistic self-focusing at higher laser intensities becomes dominant (Sodha *et al.*, 1976; Hauser *et al.*, 1988).

During particle-in-cell simulations of laser produced plasmas, researchers have observed the strong flows of relativistic electrons axially co-moving with the laser pulse and showing the existence of strong direct current magnetic fields that can influence non-linear phenomena (Pukhov & Meyer-ter-Vehn, 1996; Gahn *et al.*, 1999). Borghesi *et al.* (1998) have also observed multi-mega-gauss azimuthal magnetic-field generation with picosecond and sub-picosecond laser pulses. For direct electron acceleration, Tsakiris *et al.* (2000) used the laser created plasma channel with the radial electric field and the self-generated magnetic field. The magnetic field is very important in electron acceleration by a high power laser (Gupta & Suk, 2006; Liu *et al.*, 2006).

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It enhances the electron energy and collimates the accelerated electron beam. Wagner *et al.* (2004) proposed an experimental scheme to accelerate the electrons in the presence of a static magnetic field parallel to the laser's magnetic field. The magnetic field changes the dispersion relation of the laser beam and modifies the plasma electron density. It is very important in the context of laser self-focusing.

Under paraxial approximations, several analyses relevant to this study have been performed (Chen *et al.,* 1993; Chess *et al.,* 1998; Ren *et al.,* 2001; Sun *et al.,* 1987; Gupta *et al.,* 2007; Esarey *et al.,* 1997). In these analyses, the main assumption was the consideration of paraxial approximations. When the scale of transverse variation is much larger than the scale length in the propagation direction, the variation of the transverse intensity distribution is negligible (transverse dynamic is neglected) and may be ignored under the paraxial approximation theory. If the self-focusing is a dominant process, this approximation would not be valid. To calculate the total energy of the laser pulse, one needs to consider the transverse distribution of intensity since this can influence the laser-plasma interactions. Johnson and Chu (1974) conducted experiments to find out the electron density evaluation and pulse self-focusing in laser-produced plasmas through hydrodynamics expansion of the radial plasma profile. Liu and Tripathi (2001) have studied the steady state self-focusing of a laser beam in plasma in the presence of an azimuthal magnetic field. They find that due to uneven focusing in two transverse directions, laser spot acquires an elliptic cross-section. Recently, Gupta *et al.* (2007) proposed the introduction of a localized plasma density ramp for the relativistic self-focusing of an intense laser. Moreover, a theory of steady static self-focusing in plasma, perpendicular to the direction of electromagnetic wave propagation, immersed in a uniform magnetic field, has been developed by Gupta *et al.* (2007). The non-linearity arises through the ponderomotive force and subsequent redistribution of plasma along the magnetic field.

In order to understand these non-linear phenomena, we propose a study of self-focusing of a laser beam under the non-paraxial approximations, where features a magnetic field to guide the laser beam. The results of the present study demonstrate that it is possible to under the actual non-linear focusing of the laser beam if the non-paraxial approximation is considered with the guiding magnetic field. In this research, we report a theoretical model of transient self-focusing of a short laser pulse, with Gaussian radial intensity distribution, in magnetized plasma, under non-paraxial ray approximation. Non-linearity arises due to the ponderomotive force and subsequent ambi-polar diffusion of the plasma. The magnetic field also affects the subsequent plasma diffusion, consequently the laser self-focusing. In Section 2, we deduce the equations governing non-linear high-order electron velocities (or radial electron flow) and subsequent evolution of plasma density via ambi-polar diffusion. The effect of this has been calculated by using effective plasma permittivity. In Section 3, we solve the wave equation to obtain an equation for beam width parameter. This equation is solved numerically to study the variation of the beam width parameter (which is the function of laser spot size) as a function of axial distance at different times. The results are discussed in Section 4.

2. NON-LINEAR EFFECT

Consider plasma of uniform equilibrium electron density n_0 immersed in a static magnetic field $B_s \hat{y}$. An intense short-pulse laser, propagates through the plasma along z-direction, $\vec{E} = \hat{y} A(x, z, t) \exp[-i(\omega t - kz)]$, where $A^2 = A_{00}^2 g(t) \exp(-x^2/r_0^2)$ at $z = 0$, $g(t)$ characterizes the temporal shape of the pulse, step function $g(t) = 0$ for $t < 0$, and $g(t) = 1$ for $t > 0$. For $z > 0$, we may write, $|A|^2 = (A_{00}^2/f^2) g(t - z/\upsilon_g) \exp(-x^2/r_0^2 f^2)$, where $k = (\omega/c)(1 - \omega_p^2/\omega^2)^{1/2}$, $\upsilon_g = c(1 - \omega_p^2/\omega^2)^{1/2}$, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, r_0 is transverse half width of the laser, $f(z)$ is the beam width parameter, and $-e$ and m are the charge and mass of an electron. Here we have assumed that $\omega \gg \omega_c$ where ω_c is the electron cyclotron frequency. Further, we assume that the length of the plasma channel $L < r_0 \upsilon_g/c_s$, where c_s is the sound speed so that the plasma channel evolves almost simultaneously in the entire axial length of the plasma. As the channel evolves, the laser self-focuses with the distance of propagation. However, we address the issue of self-focusing later. The oscillatory velocity of the electrons induced by the laser can be given by $\vec{\upsilon} = e\vec{E}/mi\omega$. The laser also exerts a ponderomotive force on the electrons, $\vec{F}_p = e\nabla \phi_p || \hat{x}$, where $\phi_p = -(m/2e)\vec{\upsilon}.\vec{\upsilon}* = -(e/m\omega^2)\vec{E}.\vec{E}^*$. The quasi-static motion of electrons and ions is governed by the equations of motion

$$\frac{\partial \vec{\upsilon}}{\partial t} = -\frac{e}{m}\vec{E}_s - \frac{e}{mc}(\vec{\upsilon} \times \vec{B}_s) + \frac{e}{m}\nabla \phi_p, \qquad (1)$$

$$m_i \frac{\partial \vec{\upsilon}_i}{\partial t} = e\vec{E}_s, \qquad (2)$$

where magnetic force on the ions has been neglected owing to their large mass. Non-linearity arises due to the ponderomotive force and subsequent ambi-polar diffusion of the plasma. Taking the z-components of Eqs. (1) and (2) in the ambi-polar diffusion process ($\upsilon_{iz} \approx \upsilon_z$), we get $\partial \upsilon_z/\partial t = -\omega_c \upsilon_x$. Now, using the continuity equation, $\partial n/\partial t + (\partial/\partial x)n\upsilon_x) = 0$, we can get the perturbed plasma density $\Delta n = (n_0/\omega_c)\partial \upsilon_z/\partial x$, where we have used $n = n_0 + \Delta n$ and the expression of υ_x obtained in above calculation. Now, we expand the z-component of the velocity in the non-paraxial region as $\upsilon_z = \upsilon_1 x + \upsilon_3 x^3 + \upsilon_5 x^5$, where even power of x turn out to be zero, hence it has been dropped. Using this expansion of υ_z and expanding ϕ_p in power of x, we get, on equating coefficients of equal

powers of x on both sides,

$$\frac{\partial^2 v_1}{\partial t^2} = -\frac{m}{m_i}\omega_c^2 v_1 - \frac{e}{m_i}\phi_{p0}\omega_c\left(\frac{-2}{r_0^2 f^2}\right), \tag{3}$$

$$\frac{\partial^2 v_3}{\partial t^2} = -\frac{m}{m_i}\omega_c^2 v_3 - \frac{e}{m_i}\phi_{p0}\omega_c\left(\frac{2}{r_0^4 f^5}\right), \tag{4}$$

$$\frac{\partial^2 v_5}{\partial t^2} = -\frac{m}{m_i}\omega_c^2 v_5 - \frac{e}{m_i}\phi_{p0}\omega_c\left(-\frac{2}{r_0^6 f^7}\right), \tag{5}$$

where $\phi_{p0} = eA_{00}^2/m\omega^2$ and $\omega_c = eB_s/mc$. Using the expansion of v_z in the expression of Δn, we obtain $\Delta n = (\Delta n)_0 + (\Delta n)_2 x^2 + (\Delta n)_4 x^4$, where $(\Delta n)_0 = (n_0/\omega_c)v_1$, $(\Delta n)_2 = (3n_0/\omega_c)v_3$, and $(\Delta n)_4 = (5n_0/\omega_c)v_5$. Using $n = n_0 + \Delta n$ in the expression of non-linear dielectric constant of plasma, the modified dielectric constant of the plasma can be written as $\varepsilon = \varepsilon_0 + \varepsilon_2 x^2 + \varepsilon_4 x^4$, where $\varepsilon_0 = 1 - (\omega_p^2/\omega^2)$, $\varepsilon_2 = (3\omega_p^2 v_3/\omega^2 \omega_c)$, and $\varepsilon_4 = (5\omega_p^2 v_5/\omega^2\omega_c)$. It is noted that the quantity ε_4 arises due to the non-paraxial approximation. In paraxial region, where the higher order terms can be neglected, the quantity ε_4 turns out to be zero.

3. SELF-FOCUSING

The wave equation governing by the propagation of the laser is

$$\nabla^2 \vec{E} = \frac{4\pi}{c^2}\frac{\partial \vec{J}}{\partial t} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \tag{6}$$

where $\vec{J} = -ne\vec{v}$ is the current density. Using the expression of laser electric field in Eq. (6), in the quasi-static approximation, one obtains, $\nabla_\perp^2 A + 2ik(\partial A/\partial z') + \varepsilon_2(\omega^2/c^2)x^2 A + \varepsilon_4(\omega^2/c^2)x^4 A = 0$, where $z' = z - v_g t$, We introduce an Eikonal, A as $A = A_0(x, z')e^{iS(x,z')}$, where $A_0(x, z')$ and $S(x, z')$ are the real function of z, and z'. By substitute the value of A and separating the real and imaginary part of in above expression, the following set of equations is obtained.

$$\frac{\partial A_0^2}{\partial z'} + \left(\frac{\partial S}{\partial x}\right)\frac{\partial A_0^2}{\partial x} + \left(\frac{\partial^2 S}{\partial x^2} + \frac{1}{x}\frac{\partial S}{\partial x}\right)A_0^2 = 0, \tag{7}$$

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial x}\right)^2 - \frac{\varepsilon_2}{\varepsilon_0}x^2 - \frac{\varepsilon_4}{\varepsilon_0}x^4 = \frac{1}{kA_0}\left(\frac{\partial^2 A_0}{\partial x^2} + \frac{1}{x}\frac{\partial A_0}{\partial x}\right). \tag{8}$$

In non-paraxial approximation (Akhmanov *et al.*, 1968) we express the A_0^2 as given earlier and the Eikonal S up to the second power of x^2. Equating the coefficients of various powers of x on both sides, we obtain the equation governing the beam width parameter as

$$\frac{\partial^2 f}{\partial z'^2} = \frac{1}{R_d^2 f^3} - \frac{\varepsilon_2}{\varepsilon_0}f - \frac{r_0^2 \varepsilon_4}{\varepsilon_0}f, \tag{9}$$

where $R_d = kr_0^2$ is the diffraction length. The first term on the right side is due to diffraction divergence. The second is due to non-linear refraction in paraxial region, and the last one shows self-focusing beyond the axial region. Now, we introduce the dimensionless variables, $\xi = z'/R_d$, $\tau = t\omega$, $V_3 = (v_3 r_0^2/\omega)$, and $V_5 = (v_5 r_0^4/\omega)$, then Eqs. (4), (5), and (9) can be written as

$$\frac{\partial^2 V_3}{\partial \tau^2} = -\frac{m}{m_i}\left(\frac{\omega_c^2}{\omega^2}\right)V_3 - \frac{m}{m_i}\frac{\omega_c}{\omega}\left(\frac{e\phi_{p0}}{m\omega^2 r_0^2}\right)\left(\frac{2}{f^5}\right), \tag{10}$$

$$\frac{\partial^2 V_5}{\partial \tau^2} = -\frac{m}{m_i}\left(\frac{\omega_c^2}{\omega^2}\right)V_5 - \frac{m}{m_i}\frac{\omega_c}{\omega}\left(\frac{e\phi_{p0}}{m\omega^2 r_0^2}\right)\left(-\frac{2}{f^7}\right), \tag{11}$$

$$\frac{\partial^2 f}{\partial \xi^2} = \frac{1}{f^3} - \frac{R_d^2}{r_0^2 \varepsilon_0}\frac{\omega}{\omega_c}\left(\frac{\omega_p^2}{\omega^2}\right)(3V_3 f) - \frac{R_d^2}{r_0^2 \varepsilon_0}\frac{\omega}{\omega_c}\left(\frac{\omega_p^2}{\omega^2}\right)(5V_5 f). \tag{12}$$

Eq. (11) and the last term on the right-hand side in Eq. (12) do not appear in the case of the paraxial approximation study; these equations arise due to the off-axis approximations. The results change significantly by using the latest approximation. Using the boundary conditions $f(\xi = 0, \tau) = 1$, $(\partial f/\partial \xi) = 0$ at $\xi = 0$ for all times (corresponding to an initially plane wave front) and $V_3(\xi, 0) = 0$, $(\partial V_3/\partial \tau) = 1$, $V_5(\xi, 0) = 0$, $(\partial V_5/\partial \tau) = 1$ at $\tau = 0$, we solved Eqs. (10)–(12) by using the Runga-Kutta technique. For this calculation, we use the typical parameters, laser wavelength $\lambda_0 = 1.053~\mu m$ ($\omega = 2 \times 10^{15}~rad/$ sec.), and $n_0 = 0.2n_{cr} = 2.2 \times 10^{20}/\lambda_0^2(\mu).cm^{-3}$ with a magnetic field of about $B_s = 12 \times 10^6~Gauss$. The laser intensity is $I_0 = 5 \times 10^{14}~W/cm^2$ with the laser spot size of $r_0 = 10$ μm. For an incident laser power exceeding a particular value known as the threshold power a self-focusing of the laser beam should occur. This threshold power can be estimated by balancing the diffraction and non-linear effects. For MW power of laser, the threshold power $(P_{cr}(W) = 1.15 \times 10^4 T_e$, where T_e is the plasma electron temperature) depends on the temperature. However, for high-intensity lasers, the critical power $[P_{cr}(W) = 2.16 \times 10^{30}/n_0(cm^{-3})]$ depends mainly on the density of the plasma electron. In our calculation, the critical power can be calculated by equating both terms on the right-hand side of Eq. (12). From our theory, for the given experimental parameters, the threshold power of self-focusing turns out to be about $P_{cr}(W) = 1.05 \times 10^{10}~W$.

In Figure 1, we have depicted three-dimensional surface plot for the variation of the beam width parameter (f) with the normalized distance (ξ) at different values of time (τ) for $(R_d^2/r_0^2) = 50$, $(\omega_p^2/\omega^2) = 0.1$, and $(e\phi_{p0}/r_0^2\omega^2~m) = 0.05$, where the magnetic field has been considered zero in order to compare it with the next case. It is shown that the beam width parameter decreases with the propagation distance and reaches a minimum value, after which it increases with the propagation distance due to the saturation of non-linearity. It is also observed that the pulse diverges early in time. As time passes the non-linearity induced self-convergence takes over the diffraction-divergence and the pulse starts focusing gradually up to $2R_d$. At $\tau = 2000$ the
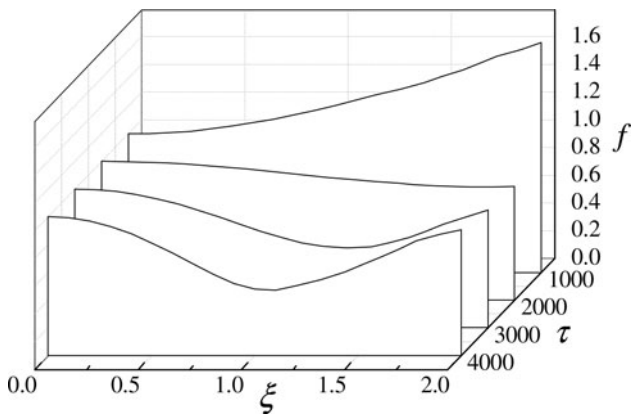
Fig. 1. Surface-diagram for beam width (f) with the propagation distance (ξ) and the time (τ) for $I_0 = 5 \times 10^{14}$, W/cm^2 (with $\lambda_0 = 1.053$ μm), (R_d^2/r_0^2) = 50, (ω_p^2/ω^2) = 0.1, and ($e\phi_{p0}/r_0^2 \, \omega^2 m$) = 0.05 (here the magnetic field has been considered zero).

pulse focuses over a length of $1.8R_d$. At $\tau \geq 3000$ the beam has dug a significant cavity in the plasma and it focuses over a length of $\sim 1.2R_d$.

In Figure 2, we have seen the effect of the magnetic field on laser self-focusing. The three-dimensional surface plot shows the beam width parameter with the propagation distance at different times for ω_c/ω= 0.105(corresponding to the magnetic field strength of B_s = 12 MG with the given laser wavelength of $\lambda_0 = 1.053$ μm). In the presence of a transverse magnetic field, the beam width parameter decreases rapidly and does not increase much. The plasma density soon minimizes on the axis corresponding to the radial direction due to the effect of a magnetic field. Hence, the laser receives the condition for self-focusing and becomes focused. From our simulations, it is also observed that the magnetic field not only strengthens, but also increases the rate of self-focusing of the laser pulse. From Figure 2, one may see that the beam width parameter acquires a minimum value in very short propagation distance compared to the former case.
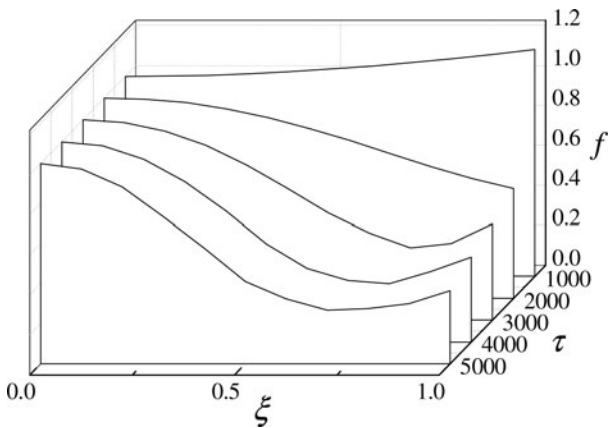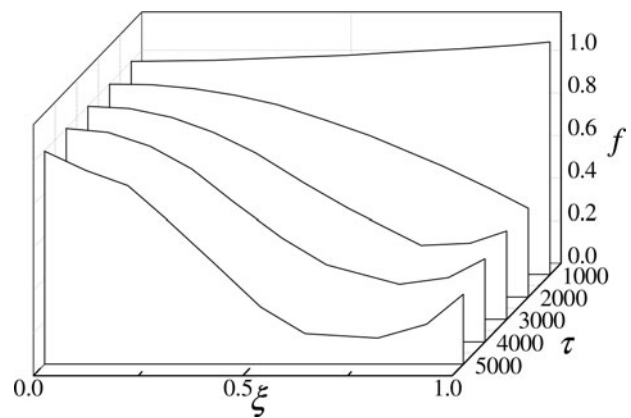
For example, the beam width parameter reaches the value of 0.3 in a distance of $\sim 0.6R_d$ at time $\tau = 5000$. Hence, it is concluded that as the magnetic field enhances the rate of self-focusing of a laser in plasma. Furthermore, under non-paraxial approximation, we need to consider the off-axis effects of the laser pulse on plasma dynamics. The calculation shows that due to the off-axis plasma behavior this approximation significantly affects self-focusing.

Figure 3 shows the same for higher magnetic field strength $\omega_c/\omega = 0.21$ (corresponding to the magnetic field strength of $B_s = 24$ MG with the given laser wavelength of $\lambda_0 = 1.053$ μm). However, in the presence of a strong magnetic field, the plasma density soon reaches its minimum on the axis. Hence, the external magnetic field enhances the rate of plasma diffusion. Due to the ambi-polar motion, plasma diffusion sets in early, and the electron's velocity increases with the normalized radial distance. With time the density of plasma decreases more on the axis, and the corresponding electron velocity increases due to the effect of the ponderomotive force. Due to the effect of a strong magnetic field, the non-linear velocity increases which contributes to the non-linearity responsible for the self-focusing of the laser. By comparing the results of Figures 2 and 3, strong self-focusing can be observed. What results is an increased rate of the self-focusing of a laser if a magnetic field of sufficient strength is applied.

4. CONCLUSION

An intense Gaussian laser pulse propagating through plasma exerts a ponderomotive force on plasma electrons. During early stages of the process, the plasma density is maximized on the axis. With the passes of time, the plasma electrons experience a ponderomotive force in the radial direction due to the Gaussian radial distribution of the intensity of the laser, causing an ambi-polar diffusion of the plasma. This leads to decreases in the plasma density on the axis,

Fig. 2. Surface-diagram for beam width (f) with the propagation distance (ξ) and the time (τ) for $I_0 = 5 \times 10^{14}$ W/cm^2 (with $\lambda_0 = 1.053$ μm), (R_d^2/r_0^2) = 50, (ω_p^2/ω^2) = 0.1, ($e\phi_{p0}/r_0^2 \, \omega^2 \, m$) = 0.05, and ω_c/ω = 0.105 (B_s = 12 MG).

Fig. 3. Surface-diagram for beam width (f) with the propagation distance (ξ) and the time (τ) for $I_0 = 5 \times 10^{14}$ W/cm^2 (with $\lambda_0 = 1.053$ μm), (R_d^2/r_0^2) = 50, ($\omega p^2/\omega^2$) = 0.1, ($e\phi_{p0}/r_0^2 \, \omega^2 \, m$) = 0.05, and ω_c/ω = 0.21(B_s = 24 MG).

and the occurrence of a density-depleted channel on the axis. This density depleted channel is very useful for laser guiding in plasmas. As the channel evolves, self-focusing of the laser becomes stronger since low density acts as a duct to guide and self-focus the laser beam, which further intensifies the wave maximum. The higher value of the magnetic field shows a strong self-focusing of a laser in plasma. Due to the higher-order plasma particle velocity consideration in non-paraxial approximation, the off-axis plasma distribution is affected by the laser pulse. In this calculation, the higher-order non-linearity terms have been included under non-paraxial approximations. Consequently their effects become important for the self-focusing of a laser in magnetized plasma.

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