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COIN ASSAYING AND COMMODITY MONEY

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We build a model of search and matching in which agents trade using coins that are imperfectly recognizable, but have access to a coin inspection technology—known as coin assaying—that reveals the intrinsic content of coins for a fee. We consider two sources of imperfect information: counterfeit coins and clipping. With counterfeits, coin assaying reduces the extent of inefficiencies associated with imperfect recognizability of coins (namely lower traded quantities and lower trading frequencies). Yet coin assaying does not necessarily increase welfare, because it unmasks counterfeits that then trade at a discount, reducing total output. With clipping, we show that agents clip for two reasons: in the hope of passing an inferior coin for a superior one, and to reduce the purchasing power of coins that are too valuable. Although coin assaying could remove the first type of clipping, it had no effect on the second.

Keywords: Commodity Money, Asymmetric Information, Coin Assaying, Clipping

1. INTRODUCTION

What are agents' incentives to reveal information? What is the effect of information revelation on welfare? In this paper we study these questions within the context of the commodity money system.

In the commodity money system money was made of precious metal coins (usually gold or silver) whose exchange value was influenced by their intrinsic content. Assessing such intrinsic content (weight and fineness) was not straightforward, however. Clipping and wear, the variety of coins in circulation, and the imperfect technology of coinage all got in the way of a rapid evaluation of coins' value. As noted by Munro (2012), imperfections in the minting technology were

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indeed key in why no coin struck in one session of minting had exactly the same size, shape, and weight as the other coins in that lot. In particular, good coins (i.e., coins of high intrinsic content) could be hard to tell apart from bad coins (coins of lower intrinsice content) without in-depth investigation. As shown in Velde et al. (1999), henceforth VWW, such variation in quality created an adverse selection problem whereby some mutually beneficial trades either did not occur (good coins were hoarded when not recognized) or occurred at a discounted price (good coins traded at a discount when not recognized).

In addition to hoarding and discounting, agents typically responded in two additional ways to this information problem. They could alter good coins by removing some of their intrinsic content and then try to pass them on as unaltered good coins. This operation was known as *clipping*. Or they could certify the intrinsic content of their coins via an expert, an operation known as *coin assaying* [Watherston (1847)]. Both clipping and assaying are amply documented by economic historians [e.g., Sargent and Velde (2002)]. The main contribution of this paper is to study the impact of clipping and coin assaying on trade, output, and welfare within the framework developed by VWW. Despite their importance, to our knowledge there does not exist any study of their impact.

Although our study is framed within the historical context of commodity money, the questions it raises apply to the broader context of asset trading under imperfect information. There are good and bad assets that are hard to distinguish, and opportunities to produce bad assets (as with clipping) or certify the value as good assets (as with coin assaying). Given that those assets are monetary and therefore improve welfare by expanding the set of feasible trades, what happens to welfare when these two opportunities are thrown in? Similar questions are asked in the context of the recent financial crisis [Gorton and Ordoñez (2012)]. It turns out sometimes more information does not necessarily improve welfare [Andolfatto and Martin (2012); Andolfatto et al. (2014)]. As will be clear shortly, a similar result emerges in our economy.

When the proportion of high- and low-quality coins is exogenous, we show that transparency in the quality of coins increases welfare by suppressing the adverse selection problem impeding the number of trades. The coin assaying technology increases the extensive margin (i.e., the number of trades) because coin sellers obtain better terms of trade, thereby increasing their willingness to sell. This intuition, however, does not carry through when the adverse selection problem translates into smaller quantities traded rather than less trade. The reason is simply that the decision by high-quality coin holders to certify their coins unmasks holders of low-quality coins who were trading it at a premium. The screening technology substitutes a lottery (a low quantity with probability p or a high quantity with probability $1 - p$) for an average quantity with probability 1. Risk aversion ensures that agents prefer the sure payment to the lottery, i.e., imperfect information to the certification of the high-quality coin.

We then study whether greater transparency increases welfare when sellers are allowed to choose the quality of their coins. We do so by allowing agents to clip their good coins, consume the difference, and trade with the resulting bad coins. In an environment where coin assaying was available, one would expect clipping to survive because of the high cost of operating the certification technology. But evidence suggests the opposite: despite availability and affordability of coin assaying, clipping was pervasive until the mechanization of minting (toward the middle of the seventeenth century), which allowed the production of coins with regular edges. With this process, a design was impressed around or on the edge of the coin, making clipping more difficult by making it apparent if the coin had been altered.[1] In this section, we show that the trade-off behind the decision to clip is richer than simply comparing costs and benefits of the assaying technology. We show in particular that there were two forces behind clipping. First, because coins were hard to tell apart, agents clipped their coins in the hope of passing off an inferior coin for a good one and pocketing the proceeds. But there also existed a second motive behind clipping: agents clip coins that are too heavy, i.e., whose purchasing power is too high [Cipolla (1956)]. By reducing their intrinsic content, and hence their purchasing power, this increases the gains from trade by raising the marginal utility and reducing the marginal cost of the consumed output. Although coin assaying can remove clipping caused by imperfect asset recognizability, it cannot remove clipping motivated by too-heavy coins. This finding may explain why clipping was pervasive during the commodity money system despite the well-documented availability and affordability of coin assaying (more on this in the text).

From a modeling point of view, we fit our story within the search and matching theory of money, in the vein of Trejos and Wright (1995) and VWW (1999). These papers have several advantages for the problem at hand. First, this class of models is explicit about the frictions that allow some assets to be used as media of exchange [Williamson and Wright (2010)]. Second, they naturally nest decentralized trading, a feature that fits the commodity money system well. Third, the liquid asset is indivisible in our model. Although this assumption can be relaxed, building on Shi (1997) and Lagos and Wright (2005), we view indivisibility as descriptive of the economy we want to model. The history of the commodity money, and to some extent of the financial system, is also about the technological difficulties of achieving a system with portable, divisible. and recognizable assets that would make them as liquid as fiat money. Indivisibility, in the form of the lack of small change. was indeed a major impediment to trade in the commodity money system and has been abundantly documented [Glassman and Redish (1988), Munro (1988), Redish (2000), Sargent and Velde (2002)]. Although we focus on the answer to the information problem, we also want to maintain the other defining characteristics of the commodity system, especially indivisibility.[2] We will nonetheless discuss the relaxation of our key assumptions along the way and show how the results are affected.

Our paper is closest to the research that investigates the role of money when agents have private information about the goods or assets they trade. In those economies money works as a substitute for information acquisition on goods quality [Brunner and Meltzer (1971); King and Plosser (1986)] and alleviates the moral hazard problem by reducing the incentive to produce "lemons" [Williamson and Wright (1994), Trejos (1999), Berentsen and Rocheteau (2004)]. Here we ask: What if money itself can be a lemon? Imperfect recognizability of money is also central to models of counterfeiting in the fiat money system, in particular Green and Weber (1996), Kullti (1996), Williamson (2002), Nosal and Wallace (2007), and Quercioli and Smith (2015). To our knowledge, however, this paper is the first to study the impact of both clipping and coin assaying. Though framed in the context of the commodity monety system, our work also relates to the literature on the role played by information in market transparency. See for instance Morris and Shin (2002) and Gorton and Odonez (2012), where more information does not necessarily lead to higher welfare [Hirshleifer (1971)].

2. THE ENVIRONMENT

The background economy is essentially VWW. The economy is populated by a [0, 1] continuum of infinitely lived agents indexed by k and there are $I \geqslant 3$ types of nonstorable goods. A type $k \in I$ agent consumes good k and produces good $k+1$, ruling out barter trade. Agents meet bilaterally according to an anonymous random-matching Poisson process with arrival rate α. They discount the future at a rate $\frac{1}{1+r} > 0$.

To trade, agents use precious metal coins, and each agent can hold at most one coin. Coins are of two types, light (L) and heavy (H). We will first study an economy where money stocks are exogenous (the proportion of light and heavy coins is fixed). In this environment light coins will be called counterfeit coins and correspond to full weight low fineness coins. Later we endogenize the composition of the money supply by letting agents choose whether to keep their heavy coin, or turn it into a light coin by clipping it. Light coins will then correspond to clipped coins (low weight full fineness).

We let M_i be the measure of agents holding a coin of type $i = \{\mathrm{L}, \mathrm{H}\}$ with $M = M_{\mathrm{H}} + M_{\mathrm{L}}$, so that $1 - M$ represents the fraction of sellers. Because commodity money always has an alternative usage (as a consumption good, for example), each coin yields to its owner a flow of utility proportional to its intrinsic content: γ_{L} for a light coin and $\gamma_{\mathrm{H}} > \gamma_{\mathrm{L}}$ for a heavy coin. Note that if agents decide to hoard heavy coins, for instance, those coins are still part of the money supply because they can be used in trade any time.

We denote as $\beta = \frac{\alpha}{I}(1 - M)$ the probability per unit of time of a single-coincidence-of-wants meeting, i.e., a buyer meeting a seller who produces his consumption good. In such a meeting, it is assumed that terms of trade are formed via bargaining, where (for simplicity) the buyer has all the bargaining power. That is, when a buyer chooses to make an offer, his offer leaves the seller indifferent between accepting and refusing. If the buyer decides to trade, agents swap their inventories so that the buyer becomes a seller and vice versa. Consuming q units of their consumption good yields agents $u(q)$ with $u(0) = 0$, $u'(q) > 0$, and

$u''(q) < 0$. Producing q units of their production good costs agents $c(q) = q$. Further, there is a unique $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$.

The information problem on coins is captured as follows: Although buyers always know the type of their coin (i.e., buyers can evaluate the quality of their coin at no cost), sellers cannot always tell the true intrinsic content of the coin that the buyer offers to pay with. Specifically, we assume that when presented with a coin, a seller learns its true quality via a common knowledge signal that is informative with probability $\theta \in (0, 1)$ and uninformative with probability $1 - \theta$. When the signal is uninformative, a seller cannot tell what type of coin the buyer is offering. An interpretation is that buyers have full knowledge of their coin because they have more time to inspect it.

Based on historical records [e.g., Spufford (1986), Bompaire (1987), Gandall and Sussman (1997), Bompaire (2007)], we allow buyers to rent a coin-assaying device for a fee δ. This technology achieves two operations: the weighing of the coin and the testing of its fineness. Weight is determined using precise scales, and fineness is evaluated by rubbing the coin on a special stone (the touchstone) and comparing the color of the trace left with that of needles of known fineness [Gandall and Sussman (1997)]. A more precise assay involves melting down a sample of coins to weigh the quantity of pure metal. For obvious reasons, this last test was limited to large payments involving many coins.[3] Coin assaying was often intermediated via agents specialized in monetary affairs such as moneychangers or goldsmiths [Spufford (1986); Bompaire (2007)]. There is ample evidence of their activity as coin assayers in medieval Europe [see, e.g., De Roover (1948); Favreau (1964); Bonnet (1973); Chevalier (1973); De La Roncière (1973); Bompaire (1987)] but also in ancient Greece and in the Roman Empire [Lothian (2003)], in the Byzantine empire [Kaplanis (2003)], in the Islamic world [Udovitch (1975)], and in China [Von Glahn (1996)]. Coin assaying was thus a central feature of the commodity money system.

Thanks to coin assaying, a buyer is able to certify the quality (weight and fineness) of his coin in front of the seller and produce a certificate of quality, which unambiguously reveals the quality of the coin, light or heavy. It should be noted that sellers have no incentive to rent the technology, because of the buyer-takes-all assumption. As will be clear shortly, buyers holding coins of lower quality have no incentive to rent the technology either, because they actually benefit from the information problem by trading their coins above their full information value in some circumstances. Therefore, only buyers holding good coins may want to pay for the technology at equilibrium.

3. COUNTERFEITS AND COIN ASSAYING

We start with an economy in which money stocks are given: there is a fraction M_{L} of buyers holding counterfeit coins and there is a fraction M_{H} of agents holding genuine coins. Counterfeiting was widespread in the commodity money system [Ashley (1888, p. 172)]. Munro (2000), for instance, provides a detailed account of "the war of the gold nobles" between England and Flanders in the late fourteenth century: On October 1, 1388, the Flemish count, Duke Philip the Bold of Burgundy, began striking counterfeit imitations of the English gold nobles struck in London's Tower Mint and the Calais Stable mint (Calais was a recently conquered enclave on the French side of the English Channel). Although they were identical in weight and alloy to its British counterpart in the first issues, Flemish authorities soon started minting nobles of lower fineness, making it difficult for agents to tell genuine nobles from fake ones. Similar examples can be found in Bompaire (2007) and Jambu (2007).

The sequence of events is as follows: At the beginning of the period, each buyer decides whether to rent the coin-testing technology or not. He then searches for a seller. If he has the technology and finds a seller producing his consumption good, he uses the technology to show the quality of his coin and then makes an offer. If the parties agree to trade, the seller produces the agreed-on quantity and they swap inventories. The technology is returned at the end of the period by the former buyer.[4,5]

To conduct the study we proceed as follows. First, we characterize the equilibria in which agents choose not to use the coin assaying technology. Then we characterize the equilibria where agents choose to use the coin assaying technology. We will see that both equilibria coexist for some parameter values (i.e., there is multiplicity), and that there also exists a mixed-strategy equilibrium at which agents randomize over the use of coin assaying.

Let us denote as λ_{ij} the probability (endogenously determined) that a buyer with a coin of type $i \in \{\mathrm{L}, \mathrm{H}\}$ wants to trade with a seller of type $j \in \{\mathrm{K}, \mathrm{U}\}$, where K means that the quality of the coin is known to the seller (the signal is informative) and U means that the quality of the coin is unknown (the signal is uninformative). The (steady-state) Bellman equation for a buyer with a light coin is

$$V_{\mathrm{L}} = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_{\mathrm{L}} + \beta\theta \max\limits_{\lambda_{\mathrm{LK}}} \left[\lambda_{\mathrm{LK}}\left[u(q_{\mathrm{L}}) + V_0\right] + (1-\lambda_{\mathrm{LK}})\, V_{\mathrm{L}}\right] \\ +\beta\,(1-\theta) \max\limits_{\lambda_{\mathrm{LU}}} \left[\lambda_{\mathrm{LU}}\left[u(\bar{q}) + V_0\right] + (1-\lambda_{\mathrm{LU}})\, V_{\mathrm{L}}\right] \\ +\,(1-\beta)\, V_{\mathrm{L}} \end{array}\right\}. \qquad (1)$$

Multiplying by $(1+r)$ and rearranging yields the flow version of the Bellman equation,

$$r V_{\mathrm{L}} = \gamma_{\mathrm{L}} + \beta\theta \max_{\lambda_{\mathrm{LK}}} \lambda_{\mathrm{LK}}\left[u(q_{\mathrm{L}}) + V_0 - V_{\mathrm{L}}\right] + \beta\,(1-\theta) \max_{\lambda_{\mathrm{LU}}} \lambda_{\mathrm{LU}}\left[u(\bar{q}) + V_0 - V_{\mathrm{L}}\right]. \qquad (2)$$

Equation (2) gives the flow return to a buyer holding a light coin, made up of three components. The first part gives the periodic return on holding the coin, γ_{L}. The second part corresponds to the probability that he meets a producer and there is a single coincidence of wants, β, multiplied by the probability that the seller recognizes the light coin, θ, times the net gain from trading the light coin against q_{L}, which is equal to consuming q_{L} and switching from buyer with a light coin to producer, that is, $u(q_L) + V_0 - V_L$, times the probability that he decides to trade with him, λ_{LK}. The last part has a similar interpretation. The difference is that because the coin is not recognized, it is not traded for q_L but for an average quantity $\bar{q}$ defined in equation (8) in the following.

Similarly, the flow Bellman equation for a buyer holding a heavy coin is given by

$$rV_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} [u(q_H) + V_0 - V_H] + \beta (1-\theta) \max_{\lambda_{HU}} \lambda_{HU} [u(\bar{q}) + V_0 - V_H]. \quad (3)$$

From the take-it-or-leave-it bargaining protocol, the informed seller is indifferent between not trading or producing q_i for the buyer and becoming a buyer with a coin of type i. Therefore, the offers made by buyers satisfy

$$V_0 = -q_i + V_i \text{ for } i \in \{L, H\}. \quad (4)$$

Similarly, the uninformed seller is indifferent between not producing and producing and trading $\bar{q}$ against the unknown coin, so that

$$V_0 = -\bar{q} + \pi V_H + (1-\pi) V_L, \quad (5)$$

where π is the probability that the buyer has a heavy coin given that he wants to trade,

$$\pi = \frac{\lambda_{HU} M_H}{\lambda_{HU} M_H + \lambda_{LU} M_L}.$$

Because sellers never get any utility from trade, we have $V_0 = 0$ and then $V_H = q_H$ and $V_L = q_L$. Once we insert these values into (2) and (3), we obtain

$$rq_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK} [u(q_L) - q_L] + \beta (1-\theta) \max_{\lambda_{LU}} \lambda_{LU} [u(\bar{q}) - q_L], \quad (6)$$

$$rq_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} [u(q_H) - q_H] + \beta (1-\theta) \max_{\lambda_{HU}} \lambda_{HU} [u(\bar{q}) - q_H], \quad (7)$$

with

$$\bar{q} = \pi q_H + (1-\pi) q_L. \quad (8)$$

Finally, the λ_{ij} satisfy the following incentive conditions: for $i \in \{L, H\}$,

$$\lambda_{iK} = \begin{cases} 1 \text{ if } u(q_i) - q_i \geq 0 \\ 0 \quad \text{otherwise,} \end{cases} \quad (9)$$

$$\lambda_{iU} = \begin{cases} 1 \text{ if } u(\bar{q}) - q_i \geq 0 \\ 0 \quad \text{otherwise.} \end{cases} \quad (10)$$

As shown by VWW (1999), there exist three types of pure-strategy monetary equilibria in this commodity money economy where coin assaying is not available: (i) both coins circulate *by weight* (heavy coins are traded only when recognized, i.e., $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = 1$ and $\lambda_{HU} = 0$); (ii) both coins circulate *by tale* (unrecognized coins trade at the same price; hence there is a premium on light coins and a discount on heavy ones, i.e., $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = \lambda_{HU} = 1$); and (iii) only light coins circulate (i.e., singlecurrency equilibrium with $\lambda_{LK} = \lambda_{LU} = 1$ and $\lambda_{HK} = \lambda_{HU} = 0$).

To show how the coin-testing technology impacts circulation and welfare, we will start by considering the first two of these equilibria[6] and offer an agent the opportunity to deviate from his strategy and rent the assaying technology. This will add a nondeviating condition for each equilibrium. For instance, a by-weight equilibrium will now be an equilibrium at which light coins always circulate, heavy coins circulate only when recognized, *and* no buyer holding a heavy coin deviates by renting the technology to certify his coin. Note that all this happens at time 0, at which agents have a one-time chance to deviate from the equilibrium we characterize. Possible mixed-strategy equilibria where agents randomize between hoarding the heavy coin, trading it at a discount, or certifying it will also be characterized.

3.1. By-Weight Equilibrium

With circulation by weight, a heavy coin trades only if it is recognized by the seller. It follows that light coins always circulate whether recognized or not (and at the same price q_L) because unrecognized circulating coins can only be light coins. Therefore $\lambda_{LK} = \lambda_{LU} = 1$. From (9), the light coin circulates in informed meetings if $\lambda_{LK} = 1$ equivalent to $u(q_L) \geq q_L$, and the heavy coin circulates in informed meetings if $\lambda_{HK} = 1$ equivalent to $u(q_H) \geq q_H$. These two conditions are met if $r > \gamma_H$.[7] Finally, heavy coins do not circulate when not recognized if $\lambda_{HU} = 0$, which from (10) is equivalent to $u(\bar{q}) = u(q_L) \leq q_H$, because $\bar{q} = q_L$ when unrecognized heavy coins are hoarded.

Assume first that coin assaying is not available. Inserting the preceding values for λ and q into (6) and (7), a by-weight equilibrium is a list (q_L, q_H) given by[8]

$$rq_L = \gamma_L + \beta \left[u(q_L) - q_L\right], \tag{11}$$

$$rq_H = \gamma_H + \beta\theta \left[u(q_H) - q_H\right] \tag{12}$$

that satisfies

$$r \geq \gamma_H, \tag{13}$$

$$q_H \geq u(q_L). \tag{14}$$

Set to equality, equation (14) together with (11)and (12) defines the by-weight frontier (BWF), as in VWW, which is the set of points in (r, θ) space such that the pair $q_L = q_L(r, \theta)$ and $q_H = q_H(r, \theta)$ that solves (11) and (12) satisfies (14) with equality. A by-weight equilibrium exists for all points in the parameter space (r, θ) to the right of $r = \gamma_H$ and to the left of the BWF (see Figure 1).[9]

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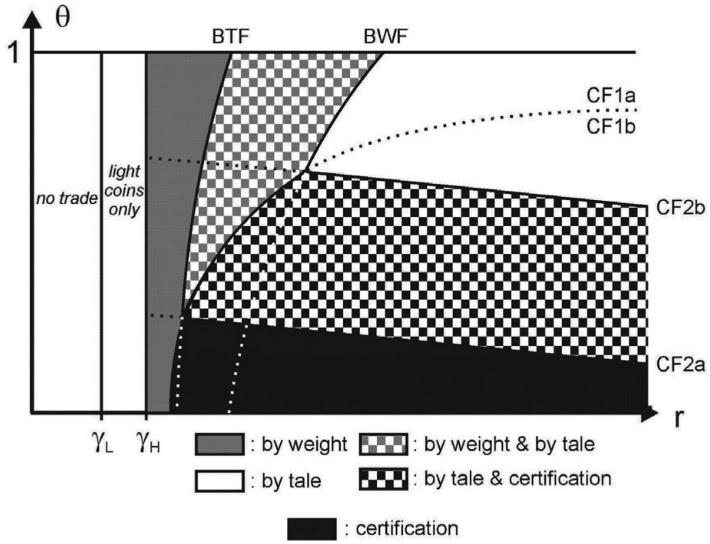

FIGURE 1. Equilibrium region in the (r, θ) space.

Assume now that coin assaying is available. Suppose that every buyer plays the by-weight equilibrium, and one buyer contemplates deviating and certifying his coin. If he does not deviate, he receives $rV_{\mathrm{H}} = rq_{\mathrm{H}}$, given by (12). If he does deviate, he pays δ to rent the technology, certifies the coin in front of the seller, and makes a take-it-or-leave-it (deviant) offer $\tilde{q}_C$ to the seller such that

$$-\tilde{q}_C + V_{\mathrm{H}} = V_0. \tag{15}$$

With this offer, the seller is indifferent between producing a quantity $\tilde{q}_C$ and becoming a holder of an uncertified heavy coin, V_{H}, or staying as a producer, V_0. Note that the deviant buyer believes that future buyers will not deviate and then not use the assaying technology; hence the V_{H} in equation (15).[10] Because $V_0 = 0$ from the take-it-or-leave-it protocol, equation (15) implies that

$$\tilde{q}_C = V_{\mathrm{H}} = q_{\mathrm{H}}. \tag{16}$$

That is, the deviant buyer certifying his coin actually asks the seller for exactly the same quantity as if he were not deviating, $\tilde{q}_C = q_{\mathrm{H}}$.

Denoting as $\tilde{\lambda}_C$ the deviant buyer's strategy for whether to trade the certified heavy coin or not, the Bellman equation for the deviator who certifies is

$$\tilde{V}_C = \frac{1}{1+r}\left\{-\delta + \gamma_{\mathrm{H}} + \beta \max_{\tilde{\lambda}_C}\left[\tilde{\lambda}_C\left[u(\tilde{q}_C) + V_0\right] + \left(1 - \tilde{\lambda}_C\right) V_{\mathrm{H}}\right] + (1-\beta)\, V_{\mathrm{H}}\right\}. \tag{17}$$

Note that with probability $\beta(1 - \tilde{\lambda}_{\mathrm{H}}) + (1 - \beta)$ he does not trade, returns the technology, and moves back to holding an uncertified heavy coin. Using $\tilde{q}_C = q_{\mathrm{H}}$

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and $V_0 = 0$, the flow version (assuming he wants to trade the certified heavy coin) is

$$r\tilde{V}_C = \gamma_{\mathrm{H}} - \delta + \beta\left[u(q_{\mathrm{H}}) - q_{\mathrm{H}}\right] + V_{\mathrm{H}} - \tilde{V}_C. \tag{18}$$

Equation (18) says that the net gain from deviating and renting the technology is equal to the periodic return on the heavy coin minus the rent for the technology, plus the net gains from trading the heavy coin at its full information value in all single-coincidence-of-wants meetings, plus the net gain from swapping from deviator back to holding an uncertified heavy coin that trades by weight.

In the end there is no incentive to deviate if the payoff to trading the heavy coin by weight is larger than the payoff to deviating and shopping with a certified heavy coin, that is,

$$V_{\mathrm{H}} > \tilde{V}_C, \tag{19}$$

which, using (12) and (18), yields

$$\delta > \beta\,(1-\theta)\left[u\,(q_{\mathrm{H}}) - q_{\mathrm{H}}\right] + V_{\mathrm{H}} - \tilde{V}_C. \tag{20}$$

Inequality (20) says that a buyer will *not* deviate by certifying his heavy coin if the cost of expertise is higher than the benefit.

To characterize the frontier between circulation by weight and certification (labeled CF*1a*), we insert the indifference condition between deviating or not, i.e. $V_{\mathrm{H}} = \tilde{V}_C$, into (20) and set it to equality. This gives

$$\delta = \beta\,(1-\theta)\left[u\,(q_{\mathrm{H}}) - q_{\mathrm{H}}\right], \tag{21}$$

Although buyers holding genuine coins can have them certified now, a by-weight equilibrium still exists to the right of $r = \gamma_{\mathrm{H}}$ and to the left of BWF and CF*1a*. See Figure 1. We postpone comments to Section 4.3.

3.2. By-Tale Equilibrium

With circulation by tale, both light and heavy coins trade at the same price $\bar{q}$ when not recognized, so that $\lambda_{\mathrm{LK}} = \lambda_{\mathrm{LU}} = \lambda_{\mathrm{HK}} = \lambda_{\mathrm{HU}} = 1$ (cf. VWW). As an example of circulation by tale, Grierson (1988) notes that in Egypt in the late Middle Ages the circulation of many counterfeits of the Venetian ducat triggered an undervaluation of the real coin. From (9), the two coins circulate in informed meetings if $u(q_{\mathrm{L}}) \geq q_{\mathrm{L}}$ and $u(q_{\mathrm{H}}) \geq q_{\mathrm{H}}$, which again requires $r > \gamma_{\mathrm{H}}$. Finally, heavy coins circulate at a discount when not recognized if $\lambda_{\mathrm{HU}} = 1$, which from (10) is equivalent to $u(\bar{q}) \geq q_{\mathrm{H}}$.

Assume first that coin assaying is not available. Inserting those values into (6) and (7), a by-tale equilibrium is a list $(q_{\mathrm{L}}, q_{\mathrm{H}})$ given by

$$rq_{\mathrm{L}} = \gamma_{\mathrm{L}} + \beta\theta\left[u(q_{\mathrm{L}}) - q_{\mathrm{L}}\right] + \beta\,(1-\theta)\left[u\,(\bar{q}) - q_{\mathrm{L}}\right], \tag{22}$$

$$rq_{\mathrm{H}} = \gamma_{\mathrm{H}} + \beta\theta\left[u(q_{\mathrm{H}}) - q_{\mathrm{H}}\right] + \beta\,(1-\theta)\left[u\,(\bar{q}) - q_{\mathrm{H}}\right], \tag{23}$$

satisfying the two conditions

$$r \geq \gamma_{\mathrm{H}}, \tag{24}$$

$$u(\bar{q}) \geq q_{\mathrm{H}}. \tag{25}$$

Set to equality, equation (25) together with (22) and (23) defines the by-tale frontier (BTF). A by-tale equilibrium exists for all points in the parameter space (r, θ) to the right of the BTF (see Figure 1).

Assume now that coin assaying is available. Suppose that every buyer plays the by-tale equilibrium and one contemplates deviating and renting the technology. If he does not deviate, he receives $rV_{\mathrm{H}} = rq_{\mathrm{H}}$, given by (23). If he deviates, he pays δ to rent the equipment and makes a deviating offer $\tilde{q}_C$ to the seller that also satisfies (15) so that $\tilde{q}_C = V_{\mathrm{H}} = q_{\mathrm{H}}$. The continuation payoff to the deviating buyer with a heavy coin $\tilde{V}_C$ is again given by (18). Therefore a buyer does not deviate if $V_{\mathrm{H}} > \tilde{V}_C$, which, using (18) and (23), transforms into

$$\delta > \beta\,(1-\theta)\,[u\,(q_{\mathrm{H}}) - u(\bar{q})] + V_{\mathrm{H}} - \tilde{V}_C. \tag{26}$$

Per equation (26), the heavy coin will not be certified if the cost of certification is greater than the increase in gains from trade due to the full recognizability of heavy coins $\beta\,(1-\theta)\,[u(q_{\mathrm{H}}) - u(\bar{q})]$, plus the net gain from shifting from deviator back to playing by tale, that is, $V_{\mathrm{H}} - \tilde{V}_C$. Inserting the indifference condition $V_{\mathrm{H}} = \tilde{V}_C$ into (26) and setting it to equality yields

$$\delta = \beta\,(1-\theta)\,[u\,(q_{\mathrm{H}}) - u(\bar{q})]\,. \tag{27}$$

We label this frontier CF$2a$. In Figure 1, to the right of BTF and CF$2a$, a circulation-by-tale equilibrium exists, even though coin assaying is available.

3.3. Assaying Equilibrium

We now conduct a mirror exercise, i.e., characterize the region where agents certify their heavy coin and have no incentive to deviate and trade it uncertified. Because the deviator can trade an uncertified heavy coin by weight or by tale (as seen in the preceding), we will have to consider both deviations.

Let q_{C} be the quantity traded against a certified heavy coin and let V_{C} be the Bellman equation for the buyer with a certified heavy coin. Because all unrecognized coins can only be light coins in a full-certification equilibrium, light coins circulate at full information value ($\lambda_{\mathrm{LK}} = \lambda_{\mathrm{LU}} = 1$) with associated payoff

$$rq_{\mathrm{L}} = \gamma_{\mathrm{L}} + \beta\,[u(q_{\mathrm{L}}) - q_{\mathrm{L}}]\,. \tag{28}$$

For a holder of a certified heavy coin, we have

$$rq_{\mathrm{C}} = \gamma_{\mathrm{H}} - \delta + \beta\,[u(q_{\mathrm{C}}) - q_{\mathrm{C}}]\,. \tag{29}$$

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Equations (28) and (29) describe a full-information economy with a periodic return on light coins equal to γ_L and a periodic return on heavy coins equal to $\gamma_H - \delta$. From (9) the circulation of light coins requires $r \geq \gamma_L$ whereas that of certified heavy coins requires $r > \gamma_H - \delta$, although we will see that it is dominated by another constraint.

Assume now that a buyer contemplates deviating by not certifying his heavy coin. Let us denote as $\tilde{\lambda}_{HK}$ the probability for the deviant buyer of trading the uncertified heavy coin when it is recognized, and let $\tilde{q}_H$ be the corresponding quantity purchased. This quantity is determined by the take-it-or-leave-it offer $-\tilde{q}_H + V_C = V_0$. Because $V_0 = 0$, we have $\tilde{q}_H = q_C$; i.e., the deviant buyer offers to buy the same quantity as if the coin were certified. Similarly, let $\tilde{\lambda}_{HU}$ be the probability for the deviant buyer of trading the heavy coin when it is not recognized, in which case the coin is inferred to be light because all unrecognized coins have to be light at an equilibrium with full certification. The Bellman equation for the deviator is then

$$\tilde{V}_H = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_H + \beta\theta \max\limits_{\tilde{\lambda}_{HK}} \left[\tilde{\lambda}_{HK}\left[u(\tilde{q}_H) + V_0\right] + \left(1-\tilde{\lambda}_{HK}\right)V_C\right] \\ +\beta(1-\theta)\max\limits_{\tilde{\lambda}_{HU}}\left[\tilde{\lambda}_{HU}\left[u(q_L) + V_0\right] + \left(1-\tilde{\lambda}_{HU}\right)V_C\right] \\ +(1-\beta)V_C \end{array}\right\}. \quad \mathbf{(30)}$$

Using $\tilde{q}_H = q_C$ and $V_0 = 0$ this simplifies into

$$r\tilde{V}_H = \gamma_H + \beta\theta\max_{\tilde{\lambda}_{HK}}\tilde{\lambda}_{HK}\left[u(q_C) - q_C\right] + \beta(1-\theta)\max_{\tilde{\lambda}_{HU}}\tilde{\lambda}_{HU}\left[u(q_L) - q_C\right] + V_C - \tilde{V}_H. \quad \mathbf{(31)}$$

In the end, there is no incentive to deviate from certification and play either by-weight or by-tale if

$$V_C > \tilde{V}_H. \quad \mathbf{(32)}$$

Deviation to by-weight. Assume first that a buyer with a heavy coin deviates by trading the uncertified coin only when it is recognized, i.e,. by-weight ($\tilde{\lambda}_{HK} = 1$ and $\tilde{\lambda}_{HU} = 0$). When these values are inserted into (31), his payoff is

$$r\tilde{V}_H = \gamma_H + \beta\theta\left[u(q_C) - q_C\right] + V_C - \tilde{V}_H. \quad \mathbf{(33)}$$

Proceeding as in the preceding section, the indifference condition is given by $V_C = \tilde{V}_H$, so that the frontier between certification and circulation by weight, denoted as CF*1b*, is characterized by

$$\delta = \beta(1-\theta)\left[u(q_C) - q_C\right] \quad \mathbf{(34)}$$

and represented in Figure 1. Note that it is identical to CF*1a*. An equilibrium with certification exists to the right of CF1$a \equiv$ CF1b and to the left of BWF.[11]

Deviation to by-tale. Assume now that the buyer deviates by trading the uncertified heavy coin at a discount when not recognized, that is, by tale ($\tilde{\lambda}_{HK} = \tilde{\lambda}_{HU} = 1$).

When these values are inserted into (31), his payoff is

$$r\tilde{V}_{\mathrm{H}} = \gamma_{\mathrm{H}} + \beta\theta\,[u(q_{\mathrm{C}}) - q_{\mathrm{C}}] + \beta\,(1-\theta)\,[u(q_{\mathrm{L}}) - q_{\mathrm{C}}] + V_{\mathrm{C}} - \tilde{V}_{\mathrm{H}}, \quad \textbf{(35)}$$

so that the frontier, denoted as CF2b, is given by

$$\delta = \beta\,(1-\theta)\,[u\,(q_{\mathrm{C}}) - u(q_{\mathrm{L}})]\,. \quad \textbf{(36)}$$

In contrast to CF1$a \equiv$ CF1b, CF2a and CF2b are different. An equilibrium with certification exists to the right of CF1$a \equiv$ CF1b and to the left of (or below) CF2b. It follows that circulation by tale and certification coexist as equilibria.[12]

Before we summarize our results, note from (21), (27), (34), and (36) that because $u(q) - q > 0$, if $\delta = 0$ (coin assaying is free, meaning the economy is now one of full information on coins), then all coins will be certified and will circulate at a price that reflects their intrinsic content. In contrast, if $\delta > \bar{\delta}$, agents do not assay their coins and the economy reverts to VWW. These results can be summarized in a proposition.

PROPOSITION 1. *If $\delta = 0$ all coins circulate at their full information value. For $0 < \delta < \bar{\delta}$ the possible equilibria, which exist in the regions shown in Figure 1, are as follows:*

- (i) *A no-trade equilibrium;*
- (ii) *A single-currency equilibrium in which only light coins circulate;*
- (iii) *A pure-strategy by-weight equilibrium in which light coins always circulate whereas heavy coins circulate only when recognized;*
- (iv) *A pure-strategy by-tale equilibrium in which both types of coins always circulate and light and heavy coins trade at the same price when not recognized;*
- (v) *A pure-strategy certification equilibrium in which light coins circulate and heavy coins are certified and circulate;*
- (vi) *A mixed-strategy certification equilibrium in which light coins circulate and some heavy coins are certified whereas the rest are not and trade at the same price as light coins when not recognized.*

First note that, as δ decreases, CF1$a \equiv$ CF1b shifts to the left and becomes steeper, whereas both CF2a and CF2b shift up and get closer to each other. Eventually, as noted in Proposition 1, as δ reaches 0 all heavy coins are certified and both coins circulate at their full information value.

Assuming now that $\delta > 0$, certification of heavy coins is an equilibrium when information on coins is low and the discount rate not too high, because this is where gains from trade are large enough to compensate for the cost of assaying.[13] Importantly, there can be multiple equilibria as the circulation-by-tale and certification regions overlap. To understand this result, note that in order to trade the coin by tale (deviating from certification), buyers understand that their unrecognized heavy coin will be treated as a light coin by sellers, because all other unrecognized coins are necessarily light ones in an equilibrium with certification. But when considering deviating from by-tale to certification, buyers understand that their unrecognized heavy coin will be treated as a weighted average of the two coins. Because unrecognized coins are valued more at a by-tale equilibrium than at a certification equilibrium, the two symmetric deviations do not yield the same payoff. As a result, the two equilibrium regions overlap. Finally, thanks to adding only coin assaying to their model, we are able to fully partition VWW's (1999) set of equilibria (compare Figure 1, p. 302, in their paper and Figure 1 in this paper).

Given estimates provided by historians on the low cost of coin assaying (between 0.3% and 1% of the transaction), this suggests that unless restrictions on coin assaying applied, the extent of the low-quantity (by tale) and low-frequency (by weight) inefficiencies must have been quite limited. For instance, if we use γ_H and γ_L as a proxy for the intrinsic content of the coins and set $\delta = 0.01 * \gamma_H$ then all by-weight equilibria disappear and only a narrow band of by-tale equilibria survives in the region where θ is close to 1.

Although similar to VWW, type (i) and type (ii) equilibria are also worth commenting on. As in VWW, if coins have sufficient intrinsic value (both γ_L/r and γ_L/r are high), they will not circulate. If only γ_L/r is too high, then only light coins will circulate. Such a single-currency equilibrium exists if the heavy coins are too intrinsically valuable to be used as money, whereas the light coins are not.[14] In a type (iii) by-weight equilibrium, an observer of the economy would distinguish two types of coins, each circulating at its own price, which reflects its intrinsic content. A by-weight equilibrium delivers the following version of Gresham's Law: In the absence of light coins, heavy coins would be used in all trades. But when light coins are present, heavy coins do not trade in meetings with uninformed sellers, who would rather wait than trade their heavy coin at a discount. By contrast, in a type (iv) by-tale equilibrium, buyers obtain a premium on heavy coins and a discount on light coins with informed sellers, but they trade either coin at the same price with uninformed sellers. It is the higher discount rate that makes buyers with heavy coins more impatient and therefore more willing to trade their unrecognized heavy coin at a discount rather than hoard it.

3.4. Welfare

PROPOSITION 2. *(i) When coin assaying triggers a shift from a by-weight equilibrium to certification, welfare increases. (ii) When coin assaying triggers a shift from the pure strategy by-tale equilibrium to a certification equilibrium, welfare decreases.*

The proof of Part (i) is straightforward. When coins trade by weight, unrecognized circulating coins can only be light coins; therefore light coins always circulate at their full information value. It follows that if holders of heavy coins opt for certification, it does not impact on buyers holding light coins, but it increases their own payoff. These observations imply that welfare is higher with certification than with circulation-by-weight.

Things are different for circulation by tale (a formal proof can be found in Appendix A.2). With circulation by tale, when her counterfeit coin is not recognized, a buyer can still purchase the average quantity $\bar{q}$. But when holders of genuine coins certify, all coins become fully recognizable, because uncertified coins can only be counterfeits now. This means that certification substitutes a lottery (q_L with probability M_L and q_C with probability M_H) for a sure payment $\bar{q}$ in former uninformed meetings. Concavity of the utility function implies that agents prefer the sure payment to the lottery, or $u[E(q)] > E[u(q)]$, i.e., circulation by tale to certification. It also implies that, assuming the economy settles on an equilibrium with certification that overlaps with a by-tale equilibrium, which we have shown to be possible, agents would collectively be better off dropping the coin inspection technology, but have no incentive individually to do so. To that extent, historical restrictions on the use of coin assaying may actually have increased welfare.[15]

Although some form of indivisibility is desirable in modelling the commodity money system, an alternative to fully divisible money would be to introduce lotteries on the delivery of the coin, in the vein of Berentsen et al. (2001). There is little evidence, however, that such a mechanism was used during the commodity money era, despite the lack of small change (we discuss some ingenious ways to circumvent the lack of small change, such as cutting coins in half or quarters, at the end of Section 5). But we should be aware that the use of lotteries is likely to eliminate the pooling type equilibria that we have if one appeals to the Cho–Kreps refinement. Randomization would indeed give the buyer holding a heavy coin another way (on top of coin assaying) to signal that his coin was of high intrinsic value.

4. CLIPPING

We now assume that there is just one type of coin in circulation, but that buyers can clip it. Clipping consisted in removing some of the metal of a coin by cutting or shaving the edges using tin snips or shears. We assume that clipping is costless and takes the form of the buyer permanently removing a fixed portion of the metal of which the coin is made (we discuss later what happens if we let buyers choose this portion). Consistent with historical evidence, we assume that the proceeds of clipping are sold to bullion dealers, who export the metal. Latimer (2001), for instance, reports that clipping in England in 1180–1220 aimed at exporting the extracted metal to Amsterdam. Quinn (1996) documents similar types of bullion exports in the later part of the seventeenth century, also in England.[16] An alternative would be to melt the proceeds of clipping into additional coins. One advantage of assuming that the proceeds are exported is that they do not alter the stock of money, and hence we have matching probabilities in $\beta = \frac{\alpha}{I}(1-M)$. Note that the severity of clipping could vary greatly. In seventeenth- and eighteenth-century France and Spain, for instance, clipping was mostly done by shaving a small percentage of the metal [Jambu (2007); Royo (2012)]. But in other instances coins were simply cut in half, as in medieval and seventeenth-century England [Redish (2000)].

To facilitate comparison with the preceding section, we call the unclipped coin heavy (H) and the clipped coin light (L). Algebraically, we assume that the price paid by bullion dealers exactly compensates for the difference in periodic rates of return between the clipped coin and the unclipped coin. That is, whereas unclipped coins yield γ_{H} per period, clipped coins yield $\gamma_{\mathrm{L}} < \gamma_{\mathrm{H}}$, so that if a buyer decides to clip his coin, he sells the proceed in exchange for a lump-sum (utility-equivalent) payment of $\frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r}$. He then tries to trade the clipped coin. As in the preceding section, buyers know whether their coins are clipped or not, but a seller is informed about the type of the coin offered in payment with probability θ. In this section we will characterize the equilibria with and without clipping, assuming no coin-assaying technology is available. In the next section, we will study the impact of coin assaying in this economy, in particular on clipping activities.

This section shares some elements with Section III on debasements in VWW (1999), where agents are offered a one-off chance by the mint to have their heavy coin swapped for a light coin plus some compensating payment. This should not be surprising, because a debasement was nothing but legalized clipping by the authorities. There are two differences, however. First, the clipper keeps the proceeds whereas in their paper the proceeds of a debasement are shared between the mint and the coin holder. Second, and more fundamentally, we propose a different partition of the set of parameters. In particular, we highlight the roles that both imperfect information and trading frictions play in clipping decisions. This prepares the ground for the next section, where we study the impact of coin assaying on clipping activities, which is our main goal.

4.1. No Coin Is Clipped

First, consider the equilibrium in which no coin is clipped, that is, $M = M_{\mathrm{H}}$. Because unrecognized coins can only be full-bodied (i.e., not clipped), the flow payoff to holding a full-bodied coin is

$$rV_{\mathrm{H}} = rq_{\mathrm{H}} = \gamma_{\mathrm{H}} + \beta\left[u(q_{\mathrm{H}}) - q_{\mathrm{H}}\right]. \qquad \textbf{(37)}$$

For such an equilibrium to exist, we need to check that no coin holder has an incentive to clip his coin, given that no one else clips.

Assume, then, that a buyer deviates and clips his coin. Let us denote as $\tilde{\lambda}_{\mathrm{LK}}$ the probability with which the deviant buyer trades the clipped coin if it is recognized, in which case he makes a take-it-or-leave-it offer $\tilde{q}_{\mathrm{L}}$ such that

$$-\tilde{q}_{\mathrm{L}} + \tilde{V}_{\mathrm{L}} = V_0, \qquad \textbf{(38)}$$

so that $\tilde{q}_{\mathrm{L}} = \tilde{V}_{\mathrm{L}}$. Note that in contrast to certification, clipping permanently alters the coin; hence $\tilde{V}_{\mathrm{L}}$ in (38) instead of V_{L}. Now let $\tilde{\lambda}_{\mathrm{LU}}$ denote the probability with which the deviant buyer trades the clipped coin if it is not recognized, in which case it trades as if it was not clipped, that is, q_{H}, because all unrecognized coins

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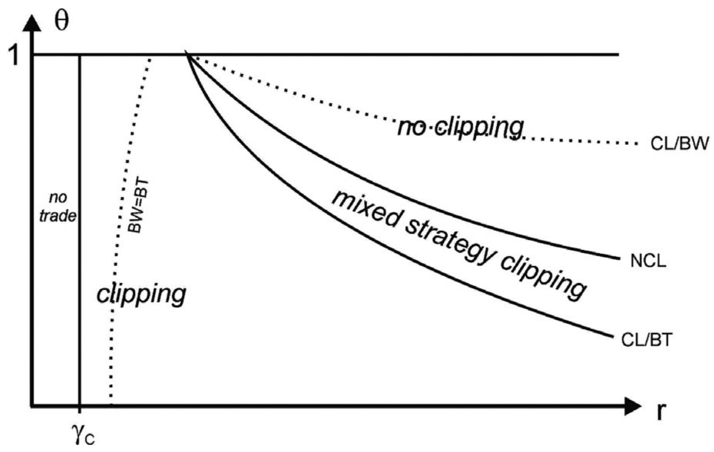

FIGURE 2. Equilibria with clipping.

are inferred to be full-bodied. The Bellman equation for a deviator is then

$$\tilde{V}_{\mathrm{L}} = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_{\mathrm{L}} + \beta\theta \max\limits_{\tilde{\lambda}_{\mathrm{LK}}}\left[\tilde{\lambda}_{\mathrm{LK}}\left\{u(\tilde{q}_{\mathrm{L}}) + V_0\right\} + \left(1-\tilde{\lambda}_{\mathrm{LK}}\right)\tilde{V}_{\mathrm{L}}\right] \\ +\beta\left(1-\theta\right)\max\limits_{\tilde{\lambda}_{\mathrm{LU}}}\left[\tilde{\lambda}_{\mathrm{LU}}\left\{u(q_{\mathrm{H}}) + V_0\right\} + \left(1-\tilde{\lambda}_{\mathrm{LU}}\right)\tilde{V}_{\mathrm{L}}\right] \\ +(1-\beta)\tilde{V}_{\mathrm{L}} \end{array}\right\}. \quad (39)$$

Multiplying both sides by $1+r$ and simplifying yields the flow payment to deviating by holding a clipped coin,

$$r\tilde{q}_{\mathrm{L}} = \gamma_{\mathrm{L}} + \beta\theta\left[u(\tilde{q}_{\mathrm{L}}) - \tilde{q}_{\mathrm{L}}\right] + \beta\left(1-\theta\right)\left[u(q_{\mathrm{H}}) - \tilde{q}_{\mathrm{L}}\right]. \quad (40)$$

In this equation it is assumed that the deviant trades the clipped coin when recognized, $u(\tilde{q}_{\mathrm{L}}) - \tilde{q}_{\mathrm{L}} > 0$, which implies that it is traded when not recognized, because $u(q_{\mathrm{H}}) - \tilde{q}_{\mathrm{L}} > u(\tilde{q}_{\mathrm{L}}) - \tilde{q}_{\mathrm{L}}$.

A buyer will not clip his coin if

$$V_{\mathrm{H}} > \frac{\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}}}{r} + \tilde{V}_{\mathrm{L}}, \quad (41)$$

which, using (37) and (40), yields

$$u(q_{\mathrm{H}}) - q_{\mathrm{H}} > \theta\left[u(\tilde{q}_{\mathrm{L}}) - \tilde{q}_{\mathrm{L}}\right] + (1-\theta)\left[u(q_{\mathrm{H}}) - \tilde{q}_{\mathrm{L}}\right]. \quad (42)$$

Figure 2 represents the area in the (r, θ) space where buyers do not clip their coins. Such an equilibrium exists above the *nonclipping frontier* (NCL), given by (42) set to equality, that is,

$$\theta\left[u(q_{\mathrm{H}}) - u(\tilde{q}_{\mathrm{L}})\right] = q_{\mathrm{H}} - \tilde{q}_{\mathrm{L}}. \quad (43)$$

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Equation (43) gives the set of points in (r, θ) space such that the pair $q_H = q_H(r, \theta)$ and $q_L = q_L(r, \theta)$ that solves (37) and (40) satisfies (42) with equality. We defery all explanations to Section 4.4, together with the welfare results.

4.2. All Coins Are Clipped

Now consider the case in which all coins are clipped, that is, $M = M_L$. Because unrecognized coins can only be clipped, the flow payoff to holding a clipped coin is

$$rV_L = rq_L = \gamma_L + \beta \left[u(q_L) - q_L\right]. \quad \mathbf{(44)}$$

For such an equilibrium to exist, we need to check that no coin holder keeps his coin unclipped, given that everyone else clips.

Assume that a buyer deviates, keeps his coin unclipped, and tries to trade it. Let us denote as $\tilde{\lambda}_{HK}$ the probability with which the deviant buyer trades the unclipped coin if it is recognized, in which case he makes a take-it-or-leave-it offer $\tilde{q}_H$ such that

$$-\tilde{q}_H + \frac{\gamma_H - \gamma_L}{r} + V_L = V_0, \quad \mathbf{(45)}$$

so that $\tilde{q}_H = \frac{\gamma_H - \gamma_L}{r} + q_L$. With this offer a seller is indifferent between rejecting the offer and staying as a producer with steady-state payoff V_0, or producing $\tilde{q}_H$, receiving the heavy coin, gaining $\frac{\gamma_H - \gamma_L}{r}$ by clipping it, and becoming a holder of a clipped coin with steady-state payoff V_L. Note here that by contrast to (38), the deviation has no permanent effect on the coin, as the deviant must believe that the recipient of the coin will clip it.

Let us denote as $\tilde{\lambda}_{HU}$ the probability with which the deviant buyer trades the unclipped coin if it is not recognized, in which case he has two options: he can pass it as a clipped coin, which is what sellers will infer the coin to be at this equilibrium, or he can hoard it and wait for a seller to recognize it and produce more goods accordingly. By analogy to the preceding section, we will describe the former as circulation by tale and the later as circulation by weight and will consider both as possible deviations.[17]

The Bellman equation for a deviator is then

$$\tilde{V}_H = \frac{1}{1+r} \left\{ \begin{array}{c} \gamma_H + \beta\theta \max\limits_{\tilde{\lambda}_{HK}} \left[\tilde{\lambda}_{HK} \left[u(\tilde{q}_H) + V_0\right] + \left(1 - \tilde{\lambda}_{HK}\right)\left(\frac{\gamma_H - \gamma_L}{r} + V_L\right)\right] \\ + \beta\left(1 - \theta\right) \max\limits_{\tilde{\lambda}_{HU}} \left[\tilde{\lambda}_{HU} \left[u(q_L) + V_0\right] + \left(1 - \tilde{\lambda}_{HU}\right)\left(\frac{\gamma_H - \gamma_L}{r} + V_L\right)\right] \\ + (1 - \beta)\left(\frac{\gamma_H - \gamma_L}{r} + V_L\right) \end{array} \right\}. \quad \mathbf{(46)}$$

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Multiplying both sides by $1+r$, using $\tilde{q}_{\mathrm{H}} = \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}}$, and simplifying yields the flow payment to deviating by holding an unclipped coin:

$$r\tilde{V}_{\mathrm{H}} = \gamma_{\mathrm{H}} + \beta\theta \max_{\tilde{\lambda}_{\mathrm{HK}}} \tilde{\lambda}_{\mathrm{HK}} [u(\tilde{q}_{\mathrm{H}}) - \tilde{q}_{\mathrm{H}}]$$
$$+\beta(1-\theta) \max_{\tilde{\lambda}_{\mathrm{HU}}} \tilde{\lambda}_{\mathrm{HU}} [u(q_{\mathrm{L}}) - \tilde{q}_{\mathrm{H}}] + \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} - \tilde{V}_{\mathrm{H}}. \quad \mathbf{(47)}$$

No deviation to by-weight: $\tilde{\lambda}_{\mathrm{HK}} = 1$ *and* $\tilde{\lambda}_{\mathrm{HU}} = 0$. First, for the unclipped coin to circulate by weight, we must have $u(q_{\mathrm{L}}) < \tilde{q}_{\mathrm{H}}$, which, using (45) and $V_{\mathrm{L}} = q_{\mathrm{L}}$, is equivalent to

$$u(q_{\mathrm{L}}) < \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + q_{\mathrm{L}}. \quad \mathbf{(48)}$$

When set to equality, this equation represents the *by-weight frontier* demarcating the combination of r and θ to the left of which any unclipped coin that is not recognized is hoarded. We label it BW in Figure 2.

In addition, a buyer prefers to clip his coin rather than trade an unclipped coin by weight if

$$\frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} > \tilde{V}_{\mathrm{H}}. \quad \mathbf{(49)}$$

Using (44) and (47) and setting (49) to equality, the *clipping-versus-by-weight* equation for the frontier, labeled CL/BW, is given by

$$u(q_{\mathrm{L}}) - q_{\mathrm{L}} = \theta\,[u(\tilde{q}_{\mathrm{H}}) - \tilde{q}_{\mathrm{H}}], \quad \mathbf{(50)}$$

with $\tilde{q}_{\mathrm{H}} = \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + q_{\mathrm{L}}$. This frontier demarcates the combinations of r and θ below which all coins are clipped and no one has an incentive to keep his coin full-bodied and trade it by weight. In Figure 2 such an equilibrium exists to the right of $r = \gamma_{\mathrm{L}}$ and to the left of CL/BW and BW. Because BW stands on the left of CL/BW the latter is redundant.

No deviation to by-tale: $\tilde{\lambda}_{\mathrm{HK}} = \tilde{\lambda}_{\mathrm{HU}} = 1$. First, for the unclipped coin to circulate by tale we must have $u(q_{\mathrm{L}}) > \tilde{q}_{\mathrm{H}}$ or

$$u(q_{\mathrm{L}}) > \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + q_{\mathrm{L}}. \quad \mathbf{(51)}$$

When set to equality, the resulting equation characterizes the frontier that demarcates the combinations of r and θ to the right of which any unclipped coin that is not recognized trades at a discount. We label it BT in Figure 2 and note that $\mathrm{BT} \equiv \mathrm{BW}$.

In addition, a buyer prefers to clip his coin rather than trade an unclipped coin at a discount if

$$\frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} > \tilde{V}_{\mathrm{H}}. \quad \mathbf{(52)}$$

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Using (44) and (47) and setting (52) to equality, the *clipping-versus-by-tale* frontier, labeled CL/BT, is given by

$$u(q_L) - q_L = \theta\,[u(\tilde{q}_H) - \tilde{q}_H] + (1-\theta)\,[u(q_L) - \tilde{q}_H] \quad \textbf{(53)}$$

or

$$\theta\,[u(\tilde{q}_H) - u(q_L)] = \tilde{q}_H - q_L, \quad \textbf{(54)}$$

with $\tilde{q}_H = \frac{\gamma_H - \gamma_L}{r} + q_L$. This frontier demarcates the combinations of r and θ to the left of which all coins are clipped and no one has an incentive to keep his coin full-bodied and trade it at a discount. In Figure 2 such an equilibrium exists to the right of BT≡ BW and to the left of CL/BT.

4.3. Some Coins Are Clipped

In this equilibrium the payoff to clipping or sticking to one's full-bodied coin is the same, $\frac{\gamma_H - \gamma_L}{r} + V_L = V_H$. As a result buyers randomize so that the money stock is a mix of clipped and unclipped coins. It is easy to show that if such an equilibrium exists then unclipped coins must circulate by tale.[18]

Assume then that nonclipped coins circulate by tale. Given a pair (r, θ) in between CL/BT and NCL, an equilibrium is a triple (q_H, q_L, π) given by

$$rq_H = \gamma_H + \beta\theta\,[u(q_H) - q_H] + \beta\,(1-\theta)\,[u\,(\bar{q}) - q_H]\,, \quad \textbf{(55)}$$

$$rq_L = \gamma_L + \beta\theta\,[u(q_L) - q_L] + \beta\,(1-\theta)\,[u\,(\bar{q}) - q_L]\,, \quad \textbf{(56)}$$

$$\frac{\gamma_H - \gamma_L}{r} + q_L = q_H, \quad \textbf{(57)}$$

with $\bar{q} = \pi q_H + (1-\pi)q_L$. Using (55) and (56), (57) becomes

$$\theta\,[u(q_H) - u(q_L)] = q_H - q_L. \quad \textbf{(58)}$$

For instance, if $\pi = 1$ (no coin is clipped), then $\bar{q} = q_H$ in (55) and (56), and (58) transforms into equation (43) for NCL. And if $\pi = 0$ (all coins are clipped), then $\bar{q} = q_L$ and (58) transforms into equation (54) for CL/BT. That is, given $\theta < 1$, for any r between CL/BT and NCL, there exists a unique triple $[q_H(r, \theta), q_L(r, \theta), \pi(r, \theta)]$ that satisfies (55)–(57). Intuitively, as r increases away from CL/BT, agents clip less on average, which means that the fraction of unclipped coins π increases from 0 on CL/BT to reach 1 on NCL.

4.4. Welfare and Comments

Welfare is simply M times the buyer's payoff,

$$W_H = Mq_H, \quad \textbf{(59)}$$

$$W_L = M\left\{\frac{\gamma_H - \gamma_L}{r} + q_L\right\}, \quad \textbf{(60)}$$

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so that welfare is higher with clipping whenever agents chose to clip and vice versa.

When the discount rate is high, that is, when trading frictions are severe, quantities traded are small, and marginal utility of consuming goods is high, so that gains from trade are larger with full-bodied coins. As a result, agents prefer to keep their coin unclipped. If, in addition, information on coins is good (high θ), unclipped coins are frequently recognized, making it even more profitable to keep the coin full-bodied. But if the discount rate is low and/or information on coins is poor, then agents are better off clipping their coins. In the next section, we investigate how the availability of coin assaying impacts on clipping activities and welfare. We also detail two motives behind clipping.

4.5. The Optimal Amount of Clipping

Before doing so, let us briefly characterize the optimal amount of clipping if agents were allowed to choose the intrinsic content of their coins. First note that the value to holding a clipped coin, V_{L}, and the amount of good that can be purchased with a clipped coin, q_{L}, are both functions of its intrinsic content, γ_{L}, so that $V_{\mathrm{L}} = V_{\mathrm{L}}(\gamma_{\mathrm{L}})$ and $q_{\mathrm{L}} = q_{\mathrm{L}}(\gamma_{\mathrm{L}})$. The payoff to clipping, denoted V, is given by

$$V = \frac{\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}}(\gamma_{\mathrm{L}}).$$

Substituting for V_{L}, multiplying by r, and taking the derivative of the resulting expression with respect to γ_{L} yields

$$\frac{\partial q_{\mathrm{L}}}{\partial \gamma_{\mathrm{L}}} \left[\theta u'(q_{\mathrm{L}}) - 1\right] = 0,$$

so that

$$u'(q_{\mathrm{L}}) = \frac{1}{\theta}.$$

In particular, if agents are fully informed ($\theta = 1$), this simplifies to $u'(q_{\mathrm{L}}) = 1$, which simply says that the intrinsic content of the clipped coins must be such that gains from trade are maximized when it is traded (more on this in the next section). In contrast, if information on coins is very poor, coins will be heavily clipped.

5. CLIPPING AND COIN ASSAYING

In this section, we study the impact of a coin-assaying technology in an economy with clipping. The new sequence of events is as follows: At time 0, each buyer holding an unclipped coin decides whether to clip it, certify it, or leave it full-bodied. Then buyers and sellers search for each other. If a buyer decides to rent the technology and finds a seller producing his consumption good, he uses the technology to prove the quality of his coin to the seller and then makes an offer. If the

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parties agree to trade, they swap inventories so that the seller becomes a buyer and the buyer a seller. Finally, the former buyer (and new seller) returns the technology.

There can be three pure-strategy equilibria now (in addition to possible mixed-strategy equilibria): all coins are clipped, all coins are certified, and all coins are left full-bodied. A pure-strategy clipping equilibrium, for example, is one in which agents prefer to clip their coin over any of the two alternatives, i.e. not clipping the coin or certifying it. In order to characterize those pure-strategy equilibria, we will use what we have done in the preceding section and see how robust the clipping and no-clipping equilibria are to coin assaying. Then we will characterize equilibria at which all coins are certified.

Before we start, note that coins are not clipped in the upper right corner of Figure 2. Because all coins are full-bodied in this region, agents have no incentive to certify their unclipped coins. Therefore, if an equilibrium with certification exists, it must be in the region where coins are clipped and be such that agents prefer certification to clipping. At such an equilibrium, if an uncertified coin is spotted yet its intrinsic content cannot be identified, it is inferred to be a clipped coin.

5.1. All Coins Are Clipped

For an equilibrium with full clipping to exist when coin assaying is available, agents must prefer clipping to assaying their coin. The payoff to a nondeviant buyer is

$$rq_{\mathrm{L}} = \gamma_{\mathrm{L}} + \beta\,[u(q_{\mathrm{L}}) - q_{\mathrm{L}}]. \qquad \mathbf{(61)}$$

If he deviates, he pays δ to certify the coin and offers a quantity $\tilde{q}_{\mathrm{C}}$ such that

$$-\tilde{q}_{\mathrm{C}} + \frac{\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} = V_0, \qquad \mathbf{(62)}$$

which implies $\tilde{q}_{\mathrm{C}} = \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + q_{\mathrm{L}}$. The Bellman equation for such a deviator is given by

$$\tilde{V}_{\mathrm{C}} = \frac{1}{1+r}\left\{ \begin{array}{c} \gamma_{\mathrm{H}} - \delta + \beta \max\limits_{\tilde{\lambda}_{\mathrm{C}}} \left[\tilde{\lambda}_{\mathrm{C}}\,[u(\tilde{q}_{\mathrm{C}}) + V_0] + \left(1 - \tilde{\lambda}_{\mathrm{C}}\right)\left(\frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}}\right)\right] \\ +(1-\beta)\left(\frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}}\right) \end{array} \right\}. \qquad \mathbf{(63)}$$

This equation shows, in particular, that the buyer considers a one-time deviation and that the recipient of the certified coin would immediately return to the equilibrium by clipping it. Assuming the deviator trades the certified coin, his flow payoff is

$$r\tilde{V}_{\mathrm{C}} = \gamma_{\mathrm{H}} - \delta + \beta\,[u(\tilde{q}_{\mathrm{C}}) - \tilde{q}_{\mathrm{C}}] + \frac{\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} - \tilde{V}_{\mathrm{C}}. \qquad \mathbf{(64)}$$

Therefore, a buyer prefers clipping to certification if $\frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} > \tilde{V}_{\mathrm{C}}$ with corresponding frontier given by

$$\beta\,[u(q_{\mathrm{L}}) - q_{\mathrm{L}}] = -\delta + \beta\,[u(\tilde{q}_{\mathrm{C}}) - \tilde{q}_{\mathrm{C}}]. \qquad \mathbf{(65)}$$

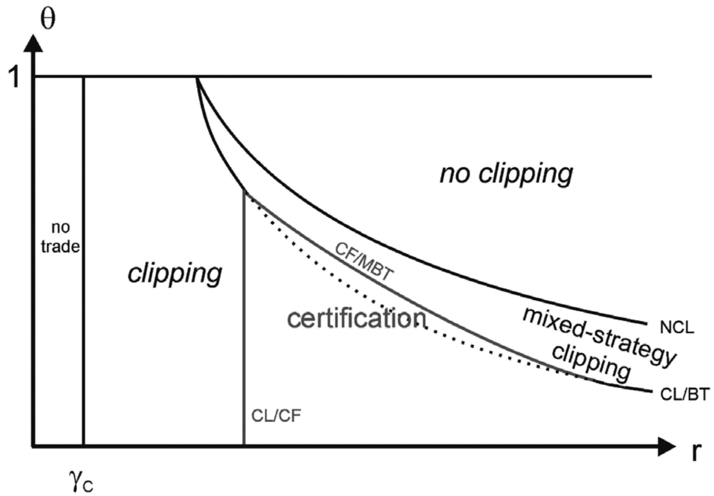

FIGURE 3. Equilibria with clipping and coin assaying.

Because agents compare two full-information options, the level of information θ plays no role in this decision; only the discount rate does. We label this *clipping-versus-certification* frontier CL/CF in Figure 3. To the left of that frontier, agents clip their coins despite the availability of certification.

5.2. All Coins Are Certified

Assume now that all coins are certified with associated payoff:

$$rq_{C} = \gamma_{H} - \delta + \beta\,[u(q_{C}) - q_{C}]\,. \tag{66}$$

A deviator who clips offers to buy a quantity $\tilde{q}_{L}$ such that

$$-\tilde{q}_{L} + \tilde{V}_{L} = V_0, \tag{67}$$

implying that $\tilde{V}_{L} = \tilde{q}_{L}$. Because unrecognized coins are inferred to be clipped, the Bellman equation for the deviator is

$$\tilde{V}_{L} = \frac{1}{1+r}\left\{\begin{array}{c}\gamma_{L} + \beta \max\limits_{\tilde{\lambda}_{L}}\left[\tilde{\lambda}_{L}\left\{u(\tilde{q}_{L}) + V_0\right\} + \left(1 - \tilde{\lambda}_{L}\right)\tilde{V}_{L}\right] \\ +(1-\beta)(\frac{\gamma_{H}-\gamma_{L}}{r} + \tilde{V}_{L})\end{array}\right\}, \tag{68}$$

so that

$$r\tilde{q}_{L} = \gamma_{L} + \beta\,[u(\tilde{q}_{L}) - \tilde{q}_{L}]\,. \tag{69}$$

It follows that the indifference condition between certification and clipping, $\frac{\gamma_{H}-\gamma_{L}}{r} + \tilde{V}_{L} = V_{C}$, yields the same cutoff r (or CL/CF) as in equation (65). To the right of that frontier agents certify their coins instead of clipping them. See Figure 3.

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Finally, let us look at what happens when only some coins are clipped, i.e., in the mixed-strategy clipping region of Figure 2. At such an equilibrium, buyers are indifferent between clipping or not, with associated payoffs $q_{\mathrm{H}} = \frac{\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}}{r} + q_{\mathrm{L}}$ given by (55) and (56). If one buyer deviates, he pays δ, certifies the coin, and makes the same offer as in (62) with payoff given by (63). A buyer would then prefer a clipped coin (or an unclipped coin, because he is indifferent between the two) to a certified one if

$$\frac{\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}}}{r} + V_{\mathrm{L}} = V_{\mathrm{H}} > \tilde{V}_{\mathrm{C}}, \tag{70}$$

which once set to equality yields the frontier equation

$$\delta = \beta \left[u\left(q_{\mathrm{C}}\right) - q_{\mathrm{C}}\right] - \beta \left\{\theta \left[u(q_{\mathrm{H}}) - q_{\mathrm{H}}\right] + (1-\theta)\left[u\left(\bar{q}\right) - q_{\mathrm{H}}\right]\right\}. \tag{71}$$

Here θ enters the equation for the frontier because of the mix of clipped and unclipped coins. We label this frontier CF/MBT in Figure 3. As can be seen, CF/MBT never touches NCL, indicating that there must be a minimum percentage of clipped coins (for a given δ) for agents to certify their unclipped coins rather than randomize between clipping or leaving the coins unaltered. On CF/MBT, agents are indifferent between certification, clipping, or trading the unclipped coin by tale.

When coin assaying is free ($\delta = 0$), the CL/CF frontier shifts to the left and stops where CL/BT and NCL intersect on $\theta = 1$, leaving space between γ_{L} and CL/CF where agents still clip despite free and full information on coins. There, clipping is not motivated by the hope of passing an inferior coin for a superior one. Clipping is due to the coin being too heavy, and clipping it makes it possible to bring its intrinsic content closer to the optimal intrinsic content from the buyer's point of view. There is nothing coin assaying can do against this type of clipping.[19]

Note that one could as well construct an example in which coins were too small with regard to the buyer's need. Clipping would then bring the intrinsic content of coins further away from the optimum, in which case agents would rather keep the coins full-bodied than reduce their size further. Even if we let buyers choose how much metal to remove from the coins, they would still not touch them.

One may wonder to what extent this type of clipping (reducing the purchasing power of the coins) is an artefact of the strong indivisibility assumption. If the menu of coins available to buyers makes it possible for them to carry their optimal purchasing power in the form of coins, then that sort of clipping will not exist. Strong evidence on the lack of small change and other curious practices suggest that this sort of clipping was present. For instance, it was once accepted practice to cut coins in half (or quarters) to produce two coins of half (or four coins of one-fourth) the value of the original coin. Ashley (1888) reports, for instance, that the first *round* halfpennies were first introduced in England only in 1220. This can easily be interpreted within our model as the second form of clipping, that motivated by too heavy coins in the face of an urgent need for change.

In the end, as can be seen by comparing Figures 2 and 3, the existence of assaying does impact the incentive to clip. In particular, when coins are difficult to tell apart (θ is small), assaying induces agents to certify their coins instead of clipping them. But again, assaying can do nothing against clipping motivated by too heavy coins.

5.3. Welfare

PROPOSITION 3. *(i) When coin assaying triggers a shift from clipping to certification, welfare increases; (ii) When coin assaying triggers a shift from a mixed-strategy clipping equilibrium to certification, welfare decreases.*

The proof for part (i) is straightforward: because welfare is M times the buyer's payoff (either all buyers hold clipped coins, or they all hold certified coins), if buyers chose certification over clipping, then welfare is higher with certification and vice versa. As for the proof of part (ii), it is similar to that of Proposition 2, the only difference being that the mix of light and heavy coins is now endogenous via clipping, whereas in Proposition 2 the mix was given. We therefore do not reproduce the proof (cf. Appendix A.2).

The intuition behind case (i) is as follows. Note first from Figure 3 that such full certification equilibrium requires the discount factor (r) to be higher than a threshold value (given by the intersection between the CL/CF frontier and the x-axis). In that region, as discussed in Section 5.2, clipping is motivated by informational frictions (θ is low so information on coins is poor), not by the coin being too "heavy." Clipping in that region is then agents' response to the information friction, which results in buyers passing the clipped coin for more than its value in uninformed meetings. In this environment, however, if assaying is not too expensive, there is a higher-payoff option: rather than clipping, it is to certify the coin and trade it at its full information value. But, of course, if information frictions are low (high θ), an even better option is neither to clip the coin nor to certify it. It is simply to trade it as is.

The intuition behind case (ii) is slightly more involved, but similar to that of Proposition 2. When the money supply is a mix of full-bodied and clipped coins (the "mixed strategy clipping" equilibrium), unrecognized coins trade by tale, so that clipped coins trade at a premium when not recognized, and full-bodied (unclipped) coins trade at a discount when not recognized. In this environment, the decision by holders of full-bodied coins to certify imposes a negative externality on holders of clipped coins. Although holders of full-bodied coins gain, thanks to certification (they would not certify their coins if it was not increasing their utility), this extra utility is more than canceled by the cost of certification and the loss incurred by holders of clipped coins; hence the fall in welfare.

6. CONCLUSION

Because money was made of precious metal, trade in the commodity money system suffered from clipping and the tedious task of evaluating the intrinsic content of the coins offered in payment. The goal of this paper was to evaluate the effect of a well-documented solution to the problem: coin assaying. Coin assaying, usually intermediated by moneychangers, was a central feature of the commodity money system. To our knowledge, however, no theoretical assessment of its impact on circulation, output, and welfare is available. The problem is somewhat similar in some of today's financial markets, where assets with monetary value are traded, raising the question of transparency and its effect on assets' liquidity and welfare.

Three main conclusions were reached. First, coin assaying did not necessarily increase welfare. Second, coin assaying could not get rid of all forms of clipping. Third, because coin assaying seems to have been relatively affordable, there must have been restrictions on its use, given the prevalence of information problems on coins.

Although some form of indivisibility should be part of a model of commodity money, it takes a convenient yet extreme form in our model: buyers can hold at most one coin. Our main results are unlikely to be challenged by introducing some divisibility, yet more work on the source of indivisibility and the lack of small change in the commodity money system is probably warranted.

NOTES

1. The invention of the steam-powered minting press in the first half of the nineteenth century further refined the minting process, allowing more homogenous coins with perfect edges to be produced, making it easier to learn quickly and easily the intrinsic value of coins [Sargent and Velde (2002)]. Counterfeiting coins also became more difficult as a result.

2. A similar approach is adopted in Wallace and Zhou (1998).

3. Although assaying determined fineness and scales determined weight, for the sake of simplicity we will use "coin assayin"' for both fineness and weight testing.

4. Renting the technology takes the form of a side payment of real output that can be consumed after the transaction by the coin assayer for an implied utility δ. The advantage of this approach is that it keeps the total number of coins constant.

5. The timing of assaying decisions by buyers is important. In our model, buyers decide whether to rent the technology before they know whether the seller has recognized the coin or not. Assuming otherwise would make the model more realistic, but less tractable, as there would always be a mix of certified and noncertified good coins in addition to bad coins. Moreover, because the assaying decision would no longer be based on the probability of an informative signal but on the realization of the signal itself, it would no longer be possible to partition the (r, θ) space.

6. Coin assaying is of no use in a single-currency equilibrium.

7. To see why $u(q_{\mathrm{H}}) \geq q_{\mathrm{H}}$ requires $r \geq \gamma_{\mathrm{H}}$, insert $\lambda_{\mathrm{H}} = 0$ into (7), which shows that the return to keeping home the coin is $rq_{\mathrm{H}} = \gamma_{\mathrm{H}}$ so that $q_{\mathrm{H}} = \gamma_{\mathrm{H}}/r$. There is then an incentive to deviate and trade the coin if $u(q_{\mathrm{H}}) - q_{\mathrm{H}} \geq 0$, which is equivalent to $q_{\mathrm{H}} \leq \hat{q}$, or $r\hat{q} \geq \gamma_{\mathrm{H}}$. Given that $u(q_{\mathrm{H}}) = q_{\mathrm{H}}^{\eta}$ and recalling that $c(q_{\mathrm{H}}) = q_{\mathrm{H}}$, we have $\hat{q} = 1$ so that $u(q_{\mathrm{H}}) - q_{\mathrm{H}} \geq 0$ requires $r \geq \gamma_{\mathrm{H}}$.

8. In general q_{L} and q_{H} are different across equilibria. In this paper, unless specified otherwise, q_{L} and q_{H} will implicitly refer to the q_{L} and q_{H} of the equilibrium we are considering.

9. Frontiers are derived numerically using the following functional forms and parameter values, none of which are critical to our results: $u(q) = \sqrt{q}$, $c(q) = q$, $\gamma_{\mathrm{H}} = 0.04$, $\gamma_{\mathrm{L}} = 0.02$, and $\delta = 0.005$. The algorithm is as follows. Step 1: Take one $\tilde{r} \geq \gamma_{\mathrm{H}}$ and find the equilibrium q_{H} and q_{L} given by (11)

and (12) for all $\theta \in (0, 1)$. Then pick the $\tilde{\theta}$ such that (14) holds with equality. Step 2: Repeat Step 1 for all $\tilde{r} \geq \gamma_{\mathrm{H}}$. All the couples $(\tilde{r}, \tilde{\theta})$ that satisfy (14) with equality constitute the frontier. All frontiers in the rest of the article are built using the same algorithm.

10. A deviation has then no permanent effect on the coin. As we will see later, the same cannot be said when a deviant buyer clips his coin, because the coin is then permanently damaged.

11. To see why CF*1a* ≡CF*1b*, let us denote as q_i^{bw}, q_i^{bt}, and q_i^c the quantities traded for a coin of type i in a by-weight, by-tale, and certification equilibrium, respectively. From (21), in CF*1a* we have $\beta\theta\left[u\left(q_{\mathrm{H}}^{bw}\right) - q_{\mathrm{H}}^{bw}\right] = \beta\left[u\left(q_{\mathrm{H}}^{bw}\right) - q_{\mathrm{H}}^{bw}\right] - \delta$. Inserting this into (12) gives $rq_{\mathrm{H}}^{bw} = \gamma_{\mathrm{H}} + \beta\left[u(q_{\mathrm{H}}^{bw}) - q_{\mathrm{H}}^{bw}\right] - \delta$, identical to (29). Then $q_{\mathrm{H}}^{bw} = q_{\mathrm{H}}^{c}$ in CF*1a* and we can substitute q_{H}^{bw} for q_{H}^{c} into (21), which yields (34), characterizing CF*1b*.

12. We show in Appendix A.1 that, in the region where circulation by tale and certification coexist, there also exists a unique mixed-strategy equilibrium in which some heavy coins are certified and some are not, called the mixed-strategy certification equilibrium.

13. The result that buyers certify more when both the cost of certification is lower and the probability of a meeting with an uninformed seller is higher is intuitive. Related conclusions can be found in Lester et al. (2011, forthcoming), but in a very different environment.

14. The no-trade and single-curreny equilibria are artefacts of the indivisibility assumption. Since Berentsen and Rocheteau (2002), it has been well known that the indivisibility of money generates inefficient terms of trade. We have a clear illustration, here as divisible coins or lotteries would allow both types of coins to circulate at low discount rates, at least in some trades.

15. See, e.g., Bigwood (1921) and Favreau (1964) for accounts of the restrictions applying to coin assaying in medieval Belgium and France, respectively.

16. See also Redish (2000) and Mayhew (2012).

17. Note that no such dilemma exists in a no-clipping equilibrium, because holders of clipped coins are always happy to pass on a clipped coin for a full-bodied one.

18. Assume that some coins are clipped and that nonclipped coins circulate by weight instead of by tale, i.e., are hoarded when not recognized. This implies that $\bar{q} = q_{\mathrm{L}}$ in (55) and (56), so that (55) transforms into $rq_{\mathrm{H}} = \gamma_{\mathrm{H}} + \beta\theta[u(q_{\mathrm{H}}) - q_{\mathrm{H}}] + \beta(1-\theta)[u(q_{\mathrm{L}}) - q_{\mathrm{H}}]$, (56) transforms into $rq_{\mathrm{L}} = \gamma_{\mathrm{L}} + \beta[u(q_{\mathrm{L}}) - q_{\mathrm{L}}]$, and (57) transforms into $q_{\mathrm{L}} = q_{\mathrm{H}}$, which is impossible.

19. This second motive is similar to endogenous money creation. Smaller coins may reduce inefficiencies on the intensive and extensive margins, which can lead to a positive effect when money is indivisible. Related papers on divisibility and money creation are Berentsen and Rocheteau (2002), Camera (2005), and Deviatov (2006).

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APPENDIX

A.1. EXISTENCE OF A MIXED-STRATEGY EQUILIBRIUM IN WHICH SOME HEAVY COINS ARE CERTIFIED AND SOME ARE NOT

Using superscript m for mixed strategy, if such an equilibrium exists, it is characterized by q_L^m, q_H^m, $\bar{q}^m$, q_C, and Σ, given by

$$rq_L^m = \gamma_L + \beta\theta\left[u(q_L^m) - q_L^m\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}^m\right) - q_L^m\right], \quad \textbf{(A.1)}$$

$$rq_H^m = \gamma_H + \beta\theta\left[u(q_H^m) - q_H^m\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}^m\right) - q_H^m\right], \quad \textbf{(A.2)}$$

$$rq_C^m = \gamma_H - \delta + \beta\left[u(q_C^m) - q_C^m\right], \quad \textbf{(A.3)}$$

$$\bar{q}^m = \frac{(1-\Sigma)\,M_H}{(1-\Sigma)\,M_H + M_L}q_H^m + \frac{M_L}{(1-\Sigma)\,M_H + M_L}q_L^m, \quad \textbf{(A.4)}$$

$$\delta = \beta\left[u\left(q_C^m\right) - q_C^m\right] - \beta\left\{\theta\left[u(q_H^m) - q_H^m\right] + (1-\theta)\left[u\left(\bar{q}^m\right) - q_H^m\right]\right\}, \quad \textbf{(A.5)}$$

where Σ is the proportion of heavy coins that are certified.

To see this, note that if $\Sigma = 0$, then $\bar{q}^m = \bar{q}^{bt}$, $q_H^m = q_H^{bt}$, and $q_L^m = q_L^{bt}$, where superscript bt stands for by-tale. Then, except in CF2a, we have $\delta > \beta[u(q_C^m) - q_C^m] - \beta\{\theta[u(q_H^{bt}) - q_H^{bt}] + (1-\theta)[u(\bar{q}^{bt}) - q_H^{bt}]\} = \beta(1-\theta)[u(q_H^{bt}) - u(\bar{q}^{bt})]$ and buyers are better off not certifying. If $\Sigma = 1$ then $\bar{q}^m = q_L^c$, given by (28), and $q_H^m = q_C$, given by (29), where superscript c stands for certified. Then, except in CF2b, we have $\delta < \beta[u(q_C^m) - q_C^m] - \beta\{\theta[u(q_H^c) - q_H^c] + (1-\theta)[u(q_L^c) - q_H^c]\} = \beta(1-\theta)[u(q_C^m) - u(\bar{q}^{bt})]$ and buyers are better off certifying. Hence there is a unique $\Sigma \in (0, 1)$ that satisfies (A.5), where q_L^m, q_H^m, q_C, and $\bar{q}^m$ are given by (A.1)–(A.4).

A.2. PROOF OF PROPOSITION 2, PART (II)

At each equilibrium, welfare is given by

$$\begin{aligned} rW_{bt} = {} & M_L\left\{\gamma_L + \beta\theta\left[u(q_L^{bt}) - q_L^{bt}\right] + \beta\left(1-\theta\right)\left[u(\bar{q}^{bt}) - q_L^{bt}\right]\right\} \\ & + M_H\left\{\gamma_H + \beta\theta\left[u(q_H^{bt}) - q_H^{bt}\right] + \beta\left(1-\theta\right)\left[u(\bar{q}^{bt}) - q_H^{bt}\right]\right\} \end{aligned} \quad \textbf{(A.6)}$$

and

$$rW_c = M_L\left\{\gamma_L + \beta\left[u(q_L^c) - q_L^c\right]\right\} + M_H\left\{\gamma_H - \delta + \beta\left[u(q_C) - q_C\right]\right\}, \quad \textbf{(A.7)}$$

where subscripts bt and c refer to by-tale and certification, respectively. Grouping all the terms in θ and $(1-\theta)$ in (A.6), we obtain

$$rW_{bt} = M_L\gamma_L + M_H\gamma_H + \theta\beta\left\{M_L\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} + (1-\theta)\,\beta\left\{(M_L + M_H)\,u(\bar{q}^{bt}) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\}. \quad \textbf{(A.8)}$$

From the concavity of u, we have

$$u(\bar{q}^{bt}) = u\left[\pi q_H^{bt} + (1-\pi)\,q_L^{bt}\right] > \pi u(q_H^{bt}) + (1-\pi)\,u\left(q_L^{bt}\right). \quad \textbf{(A.9)}$$

Using the definition of π, this inequality can be rewritten $(M_L + M_H)u(\bar{q}^{bt}) > M_L u(q_L^{bt}) + M_H u(q_H^{bt})$. It follows that for all θ we have $rW_{bt} > rW(\theta)$ with

$$rW(\theta) = M_L\gamma_L + M_H\gamma_H + \theta\left\{M_L\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\beta\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} + (1-\theta)\,\beta\left\{M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\}, \quad \textbf{(A.10)}$$

which simplifies to

$$rW(\theta) = M_L\left\{\gamma_L + \beta\left[u(q_L^{bt}) - q_L^{bt}\right]\right\} + M_H\left\{\gamma_H + \beta\left[u(q_H^{bt}) - q_H^{bt}\right]\right\}. \quad \textbf{(A.11)}$$

Note now that, when $\theta = 1$, from (22), (23), and (28) we have $q_L^{bt} = q_L^c$. Similarly, when $\theta = 1$, from (22), (23), and (29) we have $\gamma_H + \beta[u(q_H^{bt}) - q_H^{bt}] > \gamma_H - \delta + \beta[u(q_C) - q_C]$, so that $rW(\theta = 1) > rW_c$. Next, numerical simulation shows that $\partial rW_{bt}/\partial\theta < 0$ regardless of parameter values. No formal proof could be derived for this result, but it is due again to risk aversion and the fact that agents prefer an average payment for sure to a lottery [with payoffs satisfying an equation similar to (A.9) in the preceding]. Finally, because $rW_{bt}(\theta = 1) = rW(\theta = 1)$, we have $rW_{bt}(\theta = 0) > rW_{bt}(\theta = 1) = rW(\theta = 1) > rW_c$.

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