If n is odd, the subset of values of p between 0 and 1 for which our statement is true has total length S_n given by

$$S_n = \sum_{i=0}^{\frac{1}{2}(n-1)} \left(\frac{i+\frac{1}{2}}{n} - \frac{i}{n+1} \right) + \sum_{j=\frac{1}{2}(n+1)}^{n} \left(\frac{j+1}{n+1} - \frac{j-\frac{1}{2}}{n} \right)$$

which evaluates immediately to give

$$S_n = \sum_{i=0}^{\frac{1}{2}(n-1)} \left(\frac{i+\frac{1}{2}}{n} - \frac{i}{n+1} \right) + \sum_{i=0}^{\frac{1}{2}(n-1)} \left(\frac{i+\frac{n}{2}+\frac{3}{2}}{n+1} - \frac{i+\frac{n}{2}}{n} \right)$$

$$= \sum_{i=0}^{\frac{1}{2}(n-1)} \left(\frac{n+3}{2(n+1)} - \frac{n-1}{2n} \right) = \frac{3n+1}{4n}.$$

If n is even, a similar analysis shows that $S_n = \dfrac{3n+4}{4(n+1)}$, once we realise that, in this case, all the values in the 'missing interval' $\dfrac{\frac{1}{2}n}{n+1} < p < \dfrac{\frac{1}{2}n+1}{n+1}$ must also be included in the sum. From both cases, we see that $S_n \to \frac{3}{4}$ as $n \to \infty$, as claimed.

NICK LORD
Tonbridge School, Kent TN9 1JP

A comment on Schnell and Mendoza

Schnell and Mendoza [1] derive the interesting formula $\int f^{-1}(y)\,dy = xf(x) - \int f(x)\,dx$, where $y = f(x)$. It can also be obtained from the standard A-level 'trick' of treating $\int g(x)\,dx$ as $\int 1.g(x)\,dx$. Using this method

$$\int f^{-1}(y)\,dy = \int 1.f^{-1}(y)\,dy = yf^{-1}(y) - \int y\left[f^{-1}(y)\right]'dy.$$

We note that since $f'(x) = \dfrac{dy}{dx}$ and $f^{-1}(y) = x$, $\left[f^{-1}(y)\right]' = \dfrac{dx}{dy} = \dfrac{1}{f'(x)}$, giving $\int f^{-1}(y)dy = xf(x) - \int \dfrac{y}{f'(x)}dy = xf(x) - \int f(x)dx$, since $\dfrac{dy}{f'(x)} = dx$.

Reference

1. S. Schnell and C. Mendoza, A formula for integrating inverse functions, *Math. Gaz.*, **84** (March 2000) pp. 103-104.

COLIN FOSTER
King Henry VIII School, Warwick Road, Coventry CV3 6AQ
e-mail: *c@foster77.co.uk*

https://doi.org/10.1017/S0025557200003284 Published online by Cambridge University Press