

Letter to the Editor

Amplitude modulation of whistlers by modified ion-cyclotron perturbations in plasmas

P. K. SHUKLA, L. STENFLO and B. ELIASSON

Department of Physics, Umeå University, SE-90187 Umeå, Sweden
(bengt@tp4.ruhr-uni-bochum.de)

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Abstract. The nonlinear interaction between a large-amplitude whistler pump and the modified ion-cyclotron perturbations is studied. A nonlinear dispersion relation for the modulation/filamentation interaction is derived and solved numerically to investigate the instability properties. We discuss the relevance of the present study with regard to recent laboratory experiments where the modulation/filamentation of a whistler pump by the modified ion-cyclotron waves has been observed.

Whistlers are of fundamental interest in space and laboratory plasmas. In the near-Earth environment, whistlers are generated by various sources such as lightning in the atmosphere, particle streams in the cusp and magnetospheric turbulence and shocks, while in laboratory experiments, whistlers can be produced by antennas in the plasma. Non-thermal whistlers are usually guided along the geomagnetic field lines where they travel from one hemisphere to another. Two-dimensional whistlers in sheared magnetic fields can mediate fast magnetic field reconnection [1], which may play an important role for the electron energization [2, 3]. Instruments on board the CLUSTER spacecraft have been observing broadband intense electromagnetic waves in the whistler frequency range, correlated with density cavities near the plasmopause as well as at the magnetopause and in the terrestrial foreshock [4]. Observations from the Freja satellite [5] also exhibit the formation of envelope whistler solitary waves accompanied by plasma density cavities. The linear and nonlinear properties of whistler waves have been demonstrated in various experiments. The ducting of whistler waves in a density trough was experimentally demonstrated by Stenzel [6] and was later interpreted as the action of the antenna near-zone field which heats the electrons [7]. A review of whistler-related phenomena in space and laboratory plasmas is contained in [8]. Recent laboratory experiments with a non-stationary magnetic field [9, 10] show an axial (along the magnetic field) self-focusing of whistlers and the formation of isolated wavepackets. Experiments also reveal the trapping of whistlers in density depleted ducts [11] and in magnetic ducts consisting of an enhanced magnetic field in a plasma with homogeneous density [12]. It is likely that nonlinear phenomena associated with electron whistlers may also emerge in high-energy laser-plasma experiments where extremely large

(giga-Gauss) magnetic fields have been observed [13]. Accounting for the spatio-temporal-dependent whistler ponderomotive force [14, 15], investigations of the modulation and filamentation of finite-amplitude whistlers interacting with magnetosonic waves [16–19] and dispersive Alfvén waves [20] have been carried out. The linear self-focusing of frequency modulated whistlers [21] and the nonlinear formation of density modulated whistlers [22] were investigated theoretically and numerically by the present authors. Recently, Sutherland et al. [23] reported experimental evidence of a four-wave decay interaction involving a whistler (helicon) pump and modified electrostatic ion-cyclotron waves (MEICWs). This is a new channel of transformation of energy of the high-frequency whistler wave into low-frequency modified ion-cyclotron waves.

We present here a model which describes the four-wave interaction scenario relevant to the experimental observations [23]. A large-amplitude whistler pump interacting with MEICWs will generate upper and lower whistler sidebands that form an envelope of waves. The ponderomotive force of the latter in turn reinforces the MEICWs in a plasma with an external magnetic field $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z -axis and B_0 is the strength of the magnetic field. Due to the nonlinear interactions between the MEICWs and a right-hand circularly polarized whistler pump, there appears a modulated whistler with an electric field $\mathbf{E}_\perp = E(\tau, z, \mathbf{r}_\perp)(\hat{\mathbf{x}} + i\hat{\mathbf{y}})\exp(-i\omega t + ikz) + \text{complex conjugate}$ (where $\hat{\mathbf{x}}$ ($\hat{\mathbf{y}}$) is the unit vector along the x - (y -)axis, the wave frequency ω and the wavenumber k are related by $\omega = k^2 c^2 \omega_{ce} / \omega_{pe}^2 \ll \omega_{ce}$, c is the speed of light in vacuum, ω_{ce} is the electron gyrofrequency and ω_{pe} is the electron plasma frequency), which evolves according to [17]

$$i(\partial_\tau + V_g \partial_z)E + \frac{S_z}{2} \partial_z^2 E + \frac{S_\perp}{2} \nabla_\perp^2 E + \omega \left(N - \frac{2V_z}{V_g} \right) E = 0, \quad (1)$$

which should replace [23, equation (2)]. Here, τ is the slow time scale, $|\delta E / \delta \tau| \ll |\omega E|$. We stress that [23] does not describe the physics of modulated whistler waves accurately. Here $V_g = 2\omega/k$ is the whistler group velocity, $S_z = V_g/k = 2S_\perp$ is the coefficient of group dispersion, $N = n_{e1}/n_0 \ll 1$, n_{e1} and V_z are the electron density and magnetic-field-aligned electron fluid velocity perturbations associated with the MEICWs and n_0 is the unperturbed electron number density. The electron continuity equation gives

$$\partial_\tau N \approx -\partial_z V_z. \quad (2)$$

Averaging the parallel component of the electron momentum equation over $2\pi/\omega$, we obtain [24]

$$m_e \partial_\tau V_z = \partial_z (e\phi - T_e N) + \frac{\omega_{pe}^2}{4\pi n_0 \omega \omega_{ce}} \left[\partial_z + \frac{2}{V_g} \partial_\tau \right] |E|^2, \quad (3)$$

where m_e is the electron mass, ϕ is the ambipolar potential associated with the MEICWs, T_e is the electron temperature and the third term in the right-hand side represents the whistler ponderomotive force [17]. The ions participating in the dynamics of the MEICWs are coupled to the electrons through the ambipolar potential. The expression for the ion number density perturbation n_{i1} is thus obtained by combining the linearized ion continuity and momentum equations. We have

$$(\partial_\tau^2 + \omega_{ci}^2) n_{i1} = \frac{cn_0 \omega_{ci}}{B_0} \nabla_\perp^2 \phi, \quad (4)$$

where ω_{ci} is the ion gyro-frequency. Our equations are closed by invoking $n_{i1} = n_{e1}$.

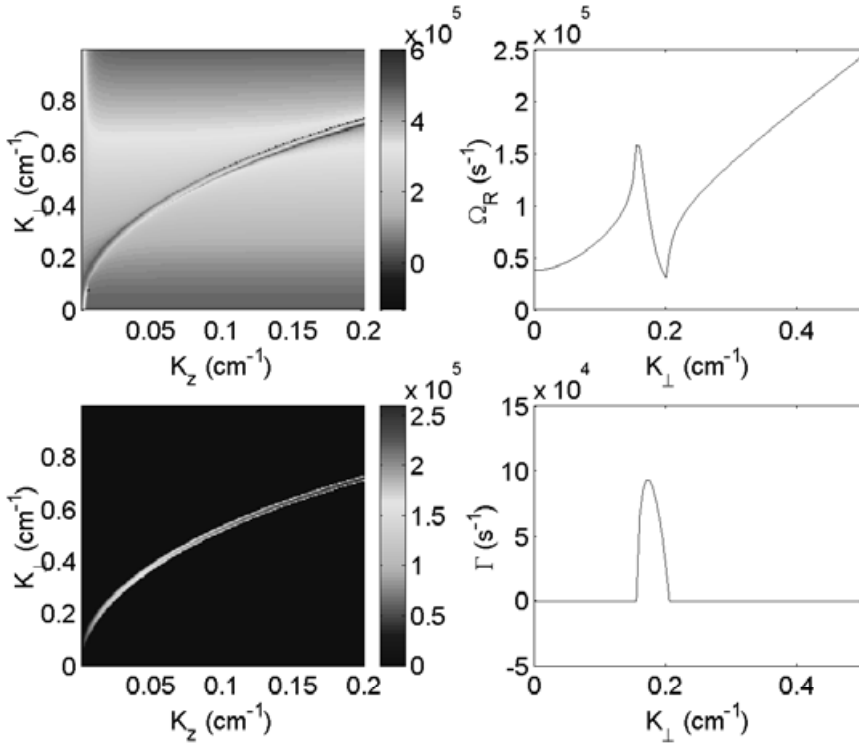


Figure 1. The real frequency Ω_R (upper left panel) and the growth rate Γ (lower left panel) as a function of K_z and K_\perp . The right upper and lower panels show the values of Ω_R and Γ at $K_z = 0.01 \text{ cm}^{-1}$ as a function of K_\perp . The parameters are: $B_0 = 158 \text{ G}$; $n_0 = 10^{13} \text{ cm}^{-3}$; $m_i/m_e = 73400$ (for argon); $E_0 = 170 \text{ V cm}^{-1}$; $C_s = 5 \times 10^5 \text{ cm s}^{-1}$; $\omega = 7.2 \times 2\pi \times 10^6 \text{ s}^{-1}$; and $\omega_{ci} = 3.8 \times 10^4 \text{ s}^{-1} = 6.0 \text{ kHz}$.

We now consider the MEICWs that are driven by the ponderomotive force of the helicons. We obtain from (2)–(4) the equation

$$(\partial_\tau^2 + \omega_{ci}^2 - C_s^2 \nabla_\perp^2) \partial_z^2 N + \frac{m_e}{m_i} \partial_\tau^2 \nabla_\perp^2 N = -\frac{\omega_{pe}^2}{4\pi n_0 m_i \omega \omega_{ce}} \left(\partial_z^2 + \frac{2}{V_g} \partial_{\tau z}^2 \right) \nabla_\perp^2 |E|^2, \quad (5)$$

where $C_s = (T_e/m_i)^{1/2}$ is the ion sound speed and m_i is the ion mass. We note that the right-hand side of (5) above is different from the right-hand side of [23, equation (3)], as the latter does not properly account for the whistler ponderomotive force [17]. The latter is an essential ingredient in the investigation of the modulational/filamentation instability [17, 24] of a constant-amplitude whistler pump with the electric field E_0 . The nonlinear dispersion relation corresponding to (1) and (5) can thus be derived following [24]. We obtain

$$\left[(\Omega - K_z V_g)^2 - \frac{1}{4} (S_z K_z^2 + S_\perp K_\perp^2)^2 \right] D = (S_z K_z^2 + S_\perp K_\perp^2) \left(1 - \frac{2\Omega}{K_z V_g} \right)^2 \frac{\omega_{pe}^2 K_\perp^2 |E_0|^2}{\omega_{ce} 4\pi n_0 m_i}, \quad (6)$$

where $D = (1 + m_e K_\perp^2 / m_i K_z^2) \Omega^2 - \omega_{ci}^2 - K_\perp^2 C_s^2$, and Ω and \mathbf{K} are the frequency and wavevector of the MEICWs, respectively. We solve (6) numerically by letting

$\Omega = \Omega_R + i\Gamma$ for the parameters in [23] and display the results for Ω_R and Γ in Fig. 1. We find an oscillatory instability, located approximately along the curve in (K_z, K_\perp) space where both D and $(\Omega - K_z V_g)^2 - (S_z K_z^2 + S_\perp K_\perp^2)^2/4$ are zero, as seen in the lower left panel of Fig. 1. From the upper panels of Fig. 1, we see that the frequencies of the unstable modes are of the same order as the ion-cyclotron frequency.

In conclusion, we have presented a model for the interaction between a large-amplitude whistler pump and the modified electrostatic ion-cyclotron perturbation. We have derived a nonlinear dispersion relation which takes into account the four-wave modulation/filamentation instability which generates the modified ion-cyclotron waves with frequencies comparable to the ion gyro-frequency. This has relevance for a recent laboratory experiment [23] where low-frequency waves with a frequency slightly lower than the ion-cyclotron frequency have been observed.

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