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Sectoral inflation under fragmentation of information

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Abstract

We examine the role of fragmentation of information in explaining the dynamics of sectoral inflation. Using the quarterly survey of firms' prices and costs in Japan, we first document two empirical facts: the sensitivity of sectoral inflation to changes in sectoral costs monotonically decreases with the dispersion of changes in (i) current costs and (ii) those in the past. A direct application of the dispersed information model can reconcile the fact (i) but fails to reconcile the fact (ii). We then extend the standard imperfect information model to construct a dynamic general equilibrium model that features fragmentation of information, wherein a finite number of groups of firms exist and firms in the same group share common idiosyncratic noises in their signals. Using this model, we find that the degree of fragmentation of information plays a crucial role in explaining these empirical facts.

Keywords: inflation dynamics; imperfect information; semi-public signals

JEL classification: E31; D80; L11

1. Introduction

Inflation dynamics has been of central issue in macroeconomics, and among others, the literature on imperfect information models has significantly contributed to the debate on the properties of inflation dynamics.[1] One key finding in the literature is that the sensitivity of aggregate inflation to the changes in aggregate economic condition is decreasing with the variance of the noises in private signals about the aggregate economic condition, explaining empirical observations on inflation dynamics, such as gradual and delayed responses to monetary policy (Woodford (2003); Nimark (2008); Angeletos and La'O (2009); Hellwig and Venkateswaran (2009)). This is because if firms use their idiosyncratic variables as private signals to infer developments in aggregate variables, higher heterogeneity in their signals and thus beliefs complicate coordination and decreases the sensitivity of their prices to aggregate variables.

While imperfect information models effectively replicate the properties of aggregate inflation dynamics, their fit to disaggregated inflation dynamics at the industry level (hereafter referred to as sectoral inflation), which is regarded as a fundamental building block for understanding aggregate inflation, has not yet been studied extensively.[2] There are a few exceptions. First, Mackowiak et al. (2009) examine the speed of the response of sectoral prices to aggregate and sector-specific shocks. They find that sectoral prices respond to sector-specific shocks immediately, whereas their response to aggregate shocks is gradual. They further demonstrate that standard sticky price models can only match this finding under extreme assumptions, while rational inattention models can match it without relying on such assumptions. Second, Kato and Okuda (2017) and Kato et al. (2021) find that sectoral inflation persistence decreases with market concentration and demonstrated that this fact can be reconciled by dispersed information models. However, to the authors' knowledge, none of them directly examine the relationship between changes in aggregate (sectoral) conditions and firms' pricing decisions.

Against this backdrop, we investigate whether dispersed information models can explain the observed patterns in sectoral inflation. We utilize data from the Tankan survey, which collects firms' answers to the questions regarding changes in their input costs and pricing in Japan (Hereafter, changes in input costs and inflation are proxied by these answers, respectively). Using these proxies, we find supporting evidence for dispersed information models, while also yielding a puzzling result. As the distribution of the firms' answers to the questions on changes in their (i) current and (ii) past input costs of the firms become more dispersed, the sectoral inflation becomes less sensitive to the changes in the sectoral costs, where the former is proxied by the sectoral average of the firms' answers to the questions on their price changes and the latter is proxied by the sectoral average of firms' answers to the questions on the changes of their costs.

From the viewpoint of consistency, standard dispersed information models can replicate the fact (i). By featuring the dispersion of idiosyncratic noises in the signals for the sectoral costs, the fact (i) is straightforward because an increase in the variance of idiosyncratic noise strengthens strategic uncertainty regarding the price setting, and dampen the sensitivity of prices of the firms.

However, we demonstrate that the changes in the variance of idiosyncratic noises fail to reconcile the second fact (ii). The dispersed information models suggest that the higher dispersion of firms' costs in the past does not decrease the sensitivity of sectoral inflation to changes in sectoral costs. When firms observe all past variables, the dispersion of firms' costs in the past does not affect current sectoral prices. In addition, when firms do not observe some of the past variables and learn them from observable competitors' prices (Amador and Weill (2010) and Amador and Weill (2012)) as noisy signals, the higher dispersion of firms' costs and prices in the past increases the sensitivity of sectoral inflation to changes in sectoral costs by making prior beliefs softer.

To reconcile these empirical observations, we expand upon standard dispersed information models of sectoral pricing by incorporating the concept of information fragmentation introduced by Morris and Shin (2007). In this study, the fragmentation of the information is defined as the situation in which only a subset of the firms within each industry receive the common signals.[3] In this economy, firms determine their prices amid strategic complementarity across firms. The firms' costs consist of sectoral components (hereafter, sectoral costs), which are persistent and shared among all firms within each industry, as well as transitory and firm-specific components (hereafter, idiosyncratic costs). Unlike standard dispersed information models, the idiosyncratic costs are shared by a subset of firms in the sector, while the distribution of the idiosyncratic costs are known to all firms. Regarding information structures, firms observe all firms' past costs (prices) and their own current costs. However, they do not observe their competitors' current costs (prices) or the components of past and current costs. In this environment, firms can accurately infer only some proportion of their competitors' costs, while they update their prior beliefs on the other competitors' costs by using their own costs and past prices of the other firms as noisy signals. Hence, we regard the number of idiosyncratic costs as the degree of fragmentation of information in the economy.

In this setup, we show that an increase in the degree of fragmentation of information reduces the sensitivity of the prices to changes in their own costs (i) in the current and (ii) previous periods. Regarding the former, under higher degree of fragmentation in the current period, firms share costs with smaller number of competitors. As a result, their' inference on their competitors' costs becomes noisier, leading to a smaller update of their beliefs about their competitors' costs. Moreover, under strategic complementarity, firms hesitate to adjust their prices based on their updated beliefs, which are more dispersed due to the fragmentation. With respect to the latter, under higher degree of fragmentation in the previous period, firms are able to learn about sectoral costs in the previous period more precisely. This is because, when the idiosyncratic costs in the previous period become more fragmented, it becomes easier for firms to infer sectoral costs in the previous period by aggregating competitors' costs, as a larger part of idiosyncratic components cancels out each other. As firms' beliefs about the sectoral costs in the previous period are used to form their prior beliefs about their competitors' current costs, this leads to more precise prior beliefs and thus a smaller update of beliefs when they receive noisy signals. These mechanisms serve to reconcile the aforementioned empirical observations (i) and (ii).[4]

To the best of our knowledge, our model represents the first attempt to generalize the assumption of noise structures in dispersed information models by incorporating the concept of information fragmentation. The model is more generalized than standard ones in the literature, as the case in which the number of idiosyncratic costs is one corresponds to public signals, and the case where the number of idiosyncratic costs is infinite corresponds to private signals discussed in standard dispersed information models.

The remainder of this study is organized as follows. Section 2 documents two empirical facts about sectoral prices, observed in the quarterly survey of firms in Japan. Section 3 outlines the general equilibrium model, while Section 4 presents the theoretical predictions regarding the two empirical facts in the context of information fragmentation. Section 5 concludes the study.

2. Empirical facts

Using quarterly survey data of Japanese firms, this section documents two empirical facts: As the distribution of firms' (i) current and (ii) past answers about "changes in input prices" within each sector becomes more dispersed across firms, their "changes in output prices" becomes less responsive to the "changes in input prices" at sector level.

2.1 Dataset

We utilize sectoral series from the Short-term Economic Survey of Enterprises in Japan (Tankan survey). The survey has been conducted by the Bank of Japan on a quarterly basis since the 1970s and contains around 10,000 sample firms chosen to represent country-wide firm size and industry distribution. In the survey, firms are asked about changes in output and input prices, among other variables.[5]

While the data is qualitative in the sense that firms are asked to choose one of the three numbers representing their judgment (1) rise, (2) unchanged, and (3) fall, the data enables us to observe the relationship between the sectoral averages of fluctuations in firms' costs (changes in *sectoral costs*, proxied by the sectoral average of the judgements about changes in input prices) and their underlying pricing decisions (*sectoral inflation*, proxied by the sectoral average of the judgements about changes in output prices).[6] Besides, the survey provides information about the within-sector distribution of firms' judgments regarding changes in input costs (distribution of firms' *own costs*).

In this section, for brevity, we denote the answer of firm $i \in I$ in sector s about $J \in \{\pi, \Delta Cost\}$ in period t by $J_t(i; s)$ where π and $\Delta Cost$ represent individual firms' inflation and changes in their cost, respectively. In the survey, numbers are assigned to each qualitative answer as follows: "Rise" ($J_t(i; s) = 1$), "Unchanged" ($J_t(i; s) = 0$), and "Fall" ($J_t(i; s) = -1$).[7] Then, sectoral diffusion index (DI) ($J_t(s)$) is calculated as $J_t(s) = 100\frac{1}{I}\sum_{i=1}^{I} J_t(i; s)$. The sample period of our dataset includes 1974/2Q-2023/2Q and 25 sectors are included in the dataset.[8] The dataset contains the sectoral DIs and the share of each answer ($J_t(i; s) \in \{-1, 0, 1\}$) in each sector.[9] Supplementary Appendix A shows the summary statistics of our dataset.

2.2 Proxies of sectoral inflation, changes in sectoral costs, and fragmentation of information

2.2.1 Sectoral inflation

We first define a proxy for sectoral inflation. Again, the proxy for the inflation of the goods sold by firm i in sector s is denoted by $\pi_t(i;s)$. The proxy for the sectoral inflation in sector s is then defined as the average of each firm's inflation, i.e.,

$$\pi_t(s) \equiv 100\frac{1}{I}\sum_{i=1}^{I}\pi_t(i;s).$$

2.2.2 Changes in sectoral costs

Next, we define a proxy of the changes in sectoral costs. The change in a firm's cost in sector s is denoted by $\Delta Cost_t(i;s)$, Then, the change in sectoral costs is defined as follows:

$$\Delta Cost_t(s) \equiv 100\frac{1}{I}\sum_{i=1}^{I}\Delta Cost_t(i;s).$$

2.2.3 Degree of dispersion

We define three indicators to measure the degree of dispersion regarding the changes in sectoral costs. While the first measure is our baseline measure, we also examine with the second and third measures for robustness.

First, we define the following indicator based on the distribution of the answers by firms in each sector (s).

$$d_{s,t} \equiv \frac{1}{I}\sum_{i=1}^{I}\left(\Delta Cost_t(i;s) - \frac{\Delta Cost_t(s)}{100}\right)^2$$

This indicator represents the degree of dispersion in changes in firms' own costs within each industry (s) in period t. Because firms' own costs are noisy signals about sectoral costs for them, this indicator is expected to capture the degree of fragmentation of information on changes in sectoral costs across firms in the sector.[10] This measure is intuitive and straightforward, but there could be systematic correlation between the change of the costs ($\Delta Cost_t(s)$) and the dispersion ($d_{s,t}$). Because firms' answers are qualitative and unable to fully capture the magnitude of the changes in their costs, their answers tend to be more concentrated when sectoral costs more substantially change.[11]

Second, we define an alternative measure of dispersion by conditioning with the magnitude of the change in sectoral cost. This is given by:

$$\hat{d}_{s,t} \equiv d_{s,t} - \bar{d}(\Delta Cost_t(s)),$$

which can be calculated by the following algorithm:

- Step 1. We split the samples (i.e., all industries in each period) into ten groups based on the percentile rank of their $\Delta Cost_t(s)$. For example, the sample (sector s in period t) is assigned to group 1 if $\Delta Cost_t(s)$ is smaller than the 10th percentile, to group 2 if $\Delta Cost_t(s)$ is between the 10th and 20th percentiles, and so on.
- Step 2. We then calculate the median dispersion of changes in input prices in each group ($\bar{d}(\Delta Cost_t(s))$) for all groups.
- Step 3. Finally, we calculate relative dispersion ($\hat{d}_{s,t}$) by subtracting the median dispersion from the dispersion as defined earlier ($d_{s,t}$).

The advantage of this indicator is that it controls the potential systematic relationship between $\Delta Cost_t(s)$ and $d_{s,t}$ by focusing on the relative dispersion within similar values of $\Delta Cost_t(s)$.[12] In fact, while correlation between $\Delta Cost_t(s)$ and $d_{s,t}$ across all periods and industries is moderately positive (0.24), the correlation between $\Delta Cost_t(s)$ and $\hat{d}_{s,t}$ is close to zero (−0.09), implying that this measure successfully avoids the potential influence of systematic relationship between $\Delta Cost_t(s)$ and $d_{s,t}$.

Finally, for further robustness, we develop the dummy variable for measuring relative dispersion as follows:

$$D_{s,t} \equiv 1 \text{ if } d_{s,t} \geq \bar{d}(\Delta Cost_t(s)), \text{ otherwise } 0$$

where the dummy variable is calculated as follows:

Step 1. Following the calculation of relative dispersion, we split all sample (i.e., all industries in each period) into ten groups based on the percentile rank of their $\Delta Cost_t(s)$.

Step 2. We then calculate the median dispersion of changes in input prices in each group ($\bar{d}(\Delta Cost_t(s))$) for all groups.

Step 3. Finally, if the dispersion of changes in input prices of a sample ($d_{s,t}$) is higher than the median in the same group ($\bar{d}(\Delta Cost_t(s))$), we assign one to its dummy variable ($D_{s,t} = 1$), and zero otherwise ($D_{s,t} = 0$).

Similar to the previous measure, this measure succeeds in avoiding the potential influence of systematic relationship between $\Delta Cost_t(s)$ and $d_{s,t}$ as the correlation between $\Delta Cost_t(s)$ and $\hat{d}_{s,t}$ is nearly zero (−0.02).

2.3 Regression analysis

Using the proxies defined in the previous section, we examine whether the degree of fragemenation N_t on sectoral costs significantly affects the sensitivity of sectoral inflation to changes in sectoral costs.[13]

2.3.1 Dispersion in changes in firms' own costs

We examine how the current and past dispersion in changes in firms' own costs ($d_{s,t}$, $d_{s,t-1}$) affects the response of sectoral inflation ($\pi_t(s)$) to changes in sectoral costs ($\Delta Cost_t(s)$). Specifically, we estimate the following fixed- and period-effects model by ordinary least squares (OLS).

$$\pi_t(s) = \beta^A(s) + \beta_t^A + \left(\beta_1^A(s) + \beta_2^A d_{s,t} + \beta_3^A d_{s,t-1}\right) \Delta Cost_t(s) + \beta_4^A(s)\Delta Demand_{t-1}(s) + \epsilon_t^A(s), \tag{1}$$

where $\beta^A(s)$ represents sector-level fixed-effects, β_t^A indicates time-effects, and $\epsilon_t^A(s)$ is an error term. $\Delta Demand_{t-1}(s)$ controls the lagged changes in demand surveyed in Tankan survey,[14] where the lag is taken to mitigate endogeneity. Our interest in this regression is on the signs and significance of $\hat{\beta}_2^A$ and $\hat{\beta}_3^A$. Specifically, if $\hat{\beta}_2^A$ ($\hat{\beta}_3^A$) is negative, it means that the response of sectoral inflation to changes in sectoral costs decreases in accordance with the current (past) dispersion of firms' own costs on sectoral costs.

As shown in Table 1, the estimates of coefficients ($\hat{\beta}_2^A$ and $\hat{\beta}_3^A$) are all negative and statistically significant at least at five percent significance level.[15] These indicate that an increase in fragmentation of information on sectoral costs, either in the current and previous periods, lowers the sensitivity of sectoral inflation to sectoral costs.

Table 1. Regression results: ordinary least squares with dispersion

Dependent variable: Changes in output price (1974/2Q-2023/2Q)						
	Ordinary Least Squares					
	(1)	(2)	(3)	(4)	(5)	(6)
Changes in input price	−0.86***		−0.60***	−0.59***		−0.44**
× Dispersion of changes in input price	−(0.2)		(0.19)	(0.18)		(0.20)
Changes in input price		−0.83***	−0.57***		−0.50***	−0.32***
× Lagged dispersion of changes in input price		(0.19)	(0.17)		(0.08)	(0.08)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	Yes	Yes	Yes
Number of observations	3,766	3,766	3,766	3,766	3,766	3,766
Adjusted R-squared	0.82	0.82	0.82	0.88	0.88	0.88

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

Table 2. Regression results: ordinary least squares with relative dispersion

Dependent variable: Changes in output price (1974/2Q-2023/2Q)						
	Ordinary Least Squares					
	(1)	(2)	(3)	(4)	(5)	(6)
Changes in input price	−0.78***		−0.48**	−0.55***		−0.38*
× Dispersion of changes in input price	(0.20)		(0.21)	(0.19)		(0.21)
Changes in input price		−0.86***	−0.66***		−0.53***	−0.39***
× Lagged dispersion of changes in input price		(0.18)	(0.19)		(0.07)	(0.09)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	Yes	Yes	Yes
Number of observations	3,766	3,766	3,766	3,766	3,766	3,766
Adjusted R-squared	0.82	0.82	0.82	0.88	0.88	0.88

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

For robustness, we estimate the same equation by replacing the dispersion of firms' own costs $(d_{s,t}, d_{s,t-1})$ with relative dispersion $(\hat{d}_{s,t}, \hat{d}_{s,t-1})$.

$$\pi_t(s) = \beta^B(s) + \beta_t^B + \left(\beta_1^B(s) + \beta_2^B \hat{d}_{s,t} + \beta_3^B \hat{d}_{s,t-1}\right) \Delta Cost_t(s) + \beta_4^B(s) \Delta Demand_{t-1}(s) + \epsilon_t^B(s), \tag{2}$$

where $\beta^B(s)$ represents sector-level fixed-effects, β_t^B indicates time-effects, and $\epsilon_t^B(s)$ is an error term. As before, if the signs of $\hat{\beta}_2^B$ $(\hat{\beta}_3^B)$ is negative, it indicates that the response of sectoral inflation to changes in sectoral costs decreases in accordance with the current (past) dispersion of firms' own costs on sectoral costs. Table 2 shows the estimation results and the estimated $\hat{\beta}_2^B$ and $\hat{\beta}_3^B$ are negative and statistically significant at least at ten percent significance level, confirming the results in Table 1.

Finally, we estimate the same equation by replacing the dispersion of firms' own costs $(d_{s,t}, d_{s,t-1})$ by dummy variable $(D_{s,t}, D_{s,t-1})$. Specifically, we estimate the following fixed- and period-effects model by OLS.

Table 3. Regression results: ordinary least squares with dummy

Dependent variable: Changes in output price (1974/2Q-2023/2Q)						
	Ordinary Least Squares					
	(1)	(2)	(3)	(4)	(5)	(6)
Changes in input price	-0.08^{***}		-0.07^{***}	-0.05^{**}		-0.04^{**}
$\times$ Dispersion of changes in input price	(0.02)		(0.02)	(0.02)		(0.02)
Changes in input price		-0.08^{***}	-0.06^{***}		-0.04^{**}	-0.03^{**}
$\times$ Lagged dispersion of changes in input price		(0.02)	(0.01)		(0.01)	(0.01)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	Yes	Yes	Yes
Number of observations	3,766	3,766	3,766	3,766	3,766	3,766
Adjusted R-squared	0.82	0.82	0.82	0.88	0.88	0.88

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

$$\pi_t(s) = \beta^C(s) + \beta_t^C + \left(\beta_1^C(s) + \beta_2^C D_{s,t} + \beta_3^C D_{s,t-1}\right) \Delta Cost_t(s) + \beta_4^C(s) \Delta Demand_{t-1}(s) + \epsilon_t^C(s), \tag{3}$$

where $\beta^C(s)$ represents sector-level fixed-effects, β_t^C indicates time-effects, and $\epsilon_t^C(s)$ is an error term. As previously mentioned, if the signs of $\hat{\beta}_2^C$ ($\hat{\beta}_3^C$) are negative, it suggests that the sectoral inflation response to changes in sectoral costs diminishes in line with the current (past) relative dispersion of firms' own costs. Table 3 presents the estimation outcomes, and the estimated $\hat{\beta}_2^C$ and $\hat{\beta}_3^C$ are negative and statistically significant at least at five percent significance level, corroborating the findings in Table 1.

To sum up, the estimation results in Tables 1, 2, and 3 indicate that the response of sectoral inflation to changes in sectoral costs is decreasing in the degree of fragmentation of (i) current and (ii) past information.[16] In what follows, we reconcile these findings with a theoretical model which generalizes the standard dispersed information model by introducing the concept of fragmentation of information, defined as the situation in which changes in firms' costs are shared only with a subset of their competitors. We then show that changes in the dispersion of firm-specific costs, the key parameter in standard dispersed information model, can replicate only (i), while changes in the degree of fragmentation of information can account for both (i) and (ii).

3. The model

To explore the underlying mechanism behind the empirical facts, we extend the standard dispersed information models (Woodford (2003); Nimark (2008); Angeletos and La'O (2009); Hellwig and Venkateswaran (2009)) by considering more generalized information structures.

Specifically, we construct a dynamic general equilibrium model of firms' pricing where firms are engaged in within-sector competition, observe their own costs, and flexibly set prices. The key feature is that the firms' costs which works as noisy signals on the sectoral costs, is shared by $1/N_t$ proportion of firms within the sector. We suppose N_t is greater than one. The proportion decreases with a higher degree of fragmentation of information (N_t). To examine the implications of the changes in the degrees of fragmentation of information not only at present, but also in the past for firms' price setting, we assume that firms cannot observe sectoral costs and infer the costs based on their own costs and competitors' prices set in the past.

3.1 Setting

The setting of our model is largely based on Mackowiak et al. (2009). The economy is populated by a representative household and a continuum of monopolistic competitive firms $i \in [0,1]$ in a continuum of sectors that produce differentiated goods indexed by $s \in [0,1]$ to satisfy household demand. Each representative household consumes whole income, and there is no saving in equilibrium. Time is discrete and indexed by $t \in \{1,2,3,\ldots\}$.

3.1.1 Households

Preferences of the representative household are described as a function of consumption, C_t, and labor, L_t, by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t - L_t\right),$$

where $\beta \in (0,1)$ is the discount rate. The household's consumption is described by the CES consumption aggregator:

$$C_t = \left[\int_0^1 (C_t(s))^{\frac{\tilde{\eta}-1}{\tilde{\eta}}}\, ds\right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}},$$

$$C_t(s) = \left[\int_0^1 (C_t(i;s))^{\frac{\eta-1}{\eta}}\, di\right]^{\frac{\eta}{\eta-1}},$$

where $\tilde{\eta} > 0$ is the elasticity of the substitution across sectors, $\eta > 0$ is the elasticity of the substitution across goods in the same sector, and $C_t(i;s)$ is the consumption of each good in sector s.

3.1.2 Firms

Preferences of the representative household imply the following demand for firms i in sector s:

$$C_t(i;s) = \left(\frac{P_t(i;s)}{P_t(s)}\right)^{-\eta} \left(\frac{P_t(s)}{P_t}\right)^{-\tilde{\eta}} C_t,$$

where $P_t \equiv \left[\int_0^1 P_t^{1-\tilde{\eta}}(s) ds\right]^{\frac{1}{1-\tilde{\eta}}}$ and $P_t(i;s) \equiv \left[\int_0^1 P_t^{1-\eta}(i;s) di\right]^{\frac{1}{1-\eta}}$ are the aggregate price index and the sectoral price index, respectively.[17]

Each firm produces a single goods using the following production technology:

$$Y_t(i;s) = A_t(i;s) L_t^{\epsilon}(i;s),$$

where $A_t(i;s)$ is a firm-specific productivity and $\epsilon \in (0,1)$ is the degree of diminishing returns.

3.1.3 Market clearing

Market clearing implies $Y_t(i;s) = C_t(i;s)$ for each firm, and $Y_t = C_t$ in the economy. Aggregate nominal demand, Q_t, is given by the following cash-in-advance constraint:

$$Q_t = P_t C_t.$$

In the subsequent analysis, we use lower-case variables to indicate logarithms of the corresponding upper-case variables (i.e., $p_t \equiv \log P_t$).

3.1.4 Optimal price setting

We first derive the optimal price-setting rule with flexible prices, assuming perfect information about the current nominal variables. We then describe the change in the environment under imperfect information. During each period t, firm i in sector s sets the optimal price as follows:

$$p_t(i;s) = \mu + mc_t(i;s) \tag{4}$$

where $\mu \equiv \eta/(\eta-1) > 0$ is the markup and $mc_t(i;s)$ is the firm i's nominal marginal cost. The nominal marginal cost is defined as nominal wage w_t net of the marginal product of labor:

$$mc_t(i;s) = w_t + (1-\epsilon)\, l_t(i;s) - a_t(i;s) - \log(\epsilon).$$

Using the production technology defined above, we express labor input as: $l_t(i;s) = [y_t(i;s) - a_t(i;s)]/\epsilon$. Then the nominal marginal cost is rewritten as:

$$mc_t(i;s) = w_t + \frac{1-\epsilon}{\epsilon} y_t(i;s) - \frac{1}{\epsilon} a(i;s) - \log(\epsilon).$$

The optimal labor supply condition for the representative household is:

$$w_t - p_t = c_t,$$

and the consumer demand is:

$$c_t(i;s) = -\eta\left(p_t(i;s) - p_t(s)\right) - \tilde{\eta}\left(p_t(s) - p_t\right) + c_t. \tag{5}$$

We derive the optimal price-setting rule for firm i in sector s by using Equations (4), (5), the market clearing conditions ($y_t(i;s) = c_t(i;s)$ for all i and s and $y_t = c_t$), and the cash-in-advance constraint, $y_t = q_t - p_t$, as follows:[18]

$$p_t(i;s) = r_1 p_t(s) + r_2 p_t + (1 - r_1 - r_2)\, x_t(i;s) + \xi, \tag{6}$$

where

$$\begin{aligned}
x_t(i;s) &= q_t - a_t(i;s),\\
\xi &= \frac{\epsilon}{\epsilon + \eta(1-\epsilon)}(\mu - \log(\epsilon)),\\
r_1 &= \frac{(\eta - \tilde{\eta})(1-\epsilon)}{\epsilon + \eta(1-\epsilon)},\\
r_2 &= \frac{(\tilde{\eta} - 1)(1-\epsilon)}{\epsilon + \eta(1-\epsilon)},
\end{aligned}$$

and $p_t(s) = \int_0^1 p_t(i;s)di$. Equation (6) shows that the optimal pricing rule for firm i in sector s is a weighted average of sectoral prices ($p_t(s)$), aggregate prices (p_t) and (the component of) their cost ($x_t(i;s)$). The weight between them is determined by the parameters (r_1, r_2), which reflect the degree of strategic complementarity among firms in the same sector and across sectors, respectively. In what follows, for the sake of simplicity, without loss of generalizability, we normalize ξ as zero, and assume $\eta > \tilde{\eta} = 1$ which yields $r_2 = 0$. The optimal price for firm i in sector s is then simplified as:

$$p_t(i;s) = r p_t(s) + (1-r)\, x_t(i;s), \tag{7}$$

where

$$r = \frac{(\eta - 1)(1-\epsilon)}{\epsilon + \eta(1-\epsilon)}.$$

Equation (7) shows that the optimal pricing rule for firm i is a weighted average of sectoral prices ($p_t(s)$) and its own costs ($x_t(i;s)$). The weights for sectoral prices and firms' own costs are determined by the parameter r, which reflects the degree of strategic complementarity between firms within sectors.

3.1.5 Cost structures

In what follows, we focus on price dynamics in a sector and take the notation s away for the sake of simplicity. We assume that $a_t(i)(=a_t(i;s))$ comprises sectoral component (a_t) and firm-specific component $(a_t(i)-a_t)$, which is shared by $1/N_t$ proportion of competing firms in the sector. We then denote firms' prices which receive a shock of type $n \in \{1, 2, \dots, N_t-1, N_t\}$ by $p_t(i;n)$. Firms' costs that receive the shock of type n at time t is denoted as $x_t(n)$, and is decomposed as $x_t(n) = \widetilde{mc}_t + v_t(n)$ where $\widetilde{mc}_t \equiv q_t - a_t$ represents sectoral costs, and $v_t(n) \equiv a_t - a_t(n)$ represents firm-specific costs where $v_t(n) \sim \mathcal{N}(0, \tau_t^2)$. The parameter τ_t^2 indicates the variance of firm-specific costs at time t, which is randomly and exogenously given and common knowledge for households and firms.[19]

We assume that sectoral costs follow the random-walk process:

$$\widetilde{mc}_t = \widetilde{mc}_{t-1} + \epsilon_t, \tag{8}$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$.

Note that there are a variety of drivers affecting N_t, but one possible driver is the supply chain relationship. For example, the firms that share the same supplier may receive common shocks from the common supplier, and changes in the supply chain relationship could result in time-varying N_t.[20] To avoid further complexity, the model considers the shock structure as given.

3.1.6 Information structures

We introduce imperfect information for firms as follows. Regarding the current variables, firms observe their own cost $(x_t(n))$ but they cannot disentangle sectoral component $(\widetilde{mc}_t)$ and firm-specific component $(v_t(n))$ from the observed cost $(x_t(n))$. Hence, $x_t(n)$ serves as noisy signals for $\widetilde{mc}_t$ where $v_t(n)$ is the noise whose variance is τ_t^2. N_t represents the degree of fragmentation of information and this generalizes the information structures of standard dispersed information models. Namely, if $N_t = 1$, the signals correspond to the public signals and if $N_t \to \infty$, the signals correspond to private signals.

Firms observe their own past costs and their competitors' prices while they cannot observe past sectoral costs. Moreover, as shown later, firms can form endogenous signal $(s_{t-1}(n))$ on $\widetilde{mc}_{t-1}$ after all firms set their prices in the previous period. Then the type n firms' information set in period t when they set their prices is then defined as:

$$\mathcal{I}_t(n) \equiv \left\{ x_t(n), \widehat{\mathcal{I}}_t(n) \right\},$$

where the firms' information set before observing the signal, $x_t(n)$, is given as:

$$\widehat{\mathcal{I}}_t(n) \equiv \left\{ \mathcal{I}_{t-1}(n), \left\{ p_{t-1}(j;n) \right\}_{j\in[0,1]}, s_{t-1}(n) \right\}.$$

Under these settings, firms' log-linearized best response functions about their pricing decisions (Equation (7)) under this imperfect information are given by:

$$\begin{aligned} p_t(i;n) &= r\mathbb{E}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\mathbb{E}\left[\widetilde{mc}_t + v_t(n)|\mathcal{I}_t(n)\right] \\ &= r\mathbb{E}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)x_t(n) \end{aligned} \tag{9}$$

where $p_t = \int_{i\in[0,1]} p_t(i;n)di$ represents the sectoral price. Note that the sectoral price, p_t, can be expressed as follows:

$$p_t = r\overline{\mathbb{E}}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right] = \frac{1}{N_t}\sum_{n=1}^{N_t} p_t(n), \tag{10}$$

where $\overline{\mathbb{E}}[\cdot] \equiv \int_{i\in[0,1]} \mathbb{E}[\cdot]di$ represents an operator of the average of firms' expectations and $p_t(n)$ represents the common price for the firms receiving a common shock of type n. Hence, unlike standard imperfect information models where firms receive private signals and cannot know any of their competitors' prices, in this model, firms can know a proportion of $1/N_t$ of their competitors' prices. In the following, the dynamics of sectoral prices are to be derived.

3.2 Equilibrium

This section derives the dynamics of equilibrium prices. In so doing, we first calculate the equilibrium prices by taking firms' prior beliefs about the current marginal costs as given. We then calculate the dynamics of firms' prior beliefs considering their learning.

3.2.1 Prices with exogenous prior beliefs

Denote the mean and the imprecision of the type n firms' prior beliefs about sectoral costs by $\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$ and $\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$, respectively. As shown in the following, firms' prior beliefs are common across all firms in the sector. Then, by taking this prior beliefs as given, we can calculate the equilibrium sectoral price as follows:

Lemma 1. *Given $\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$ and $\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$, the firm i's price and average price in the equilibrium are given by:*

$$p_t(i;n) = \alpha_t x_t(n) + (1-\alpha_t)\, \mathbb{E}\left[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)\right], \tag{11}$$

$$p_t = \alpha_t \left(\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right) + (1-\alpha_t)\, \mathbb{E}\left[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)\right], \tag{12}$$

where

$$\alpha_t \equiv \frac{1-r}{1-r\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)} \tag{13}$$

and

$$\widehat{\lambda}_t \equiv \frac{\mathbb{V}\left[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)\right]}{\mathbb{V}\left[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)\right] + \tau_t^2}. \tag{14}$$

Proof. *See Appendix A.1.* □

Equation (11) in Lemma (1) indicates that each firm's equilibrium price ($p_t(i;n)$) is a linear combination of its own cost ($x_t(n)$) and its prior beliefs about the sectoral cost ($\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$). The latter is included because the firm infers competitors' prices based on its beliefs about sectoral price. Similarly, Equation (12) indicates that the sectoral price is affected by prior beliefs while it also depends on the sectoral costs ($\widetilde{mc}_t$) and the averages of the firm-specific costs $\left(\frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right)$. Note that because N_t is finite, aggregation process does not eliminate the idiosyncratic variables.

Moreover, these equations show an important observation: the weights for the variables in period t $\left(x_t(n),\ \widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right)$ are monotonically decreasing in N_t as the following inequality holds:[21]

$$\frac{\partial \alpha_t}{\partial N_t} = -\frac{\left(1-\widehat{\lambda}_t\right)(1-r)\,r}{\left[r\left(1-\widehat{\lambda}_t\right) - N_t\left(1-r\widehat{\lambda}_t\right)\right]^2} < 0.$$

The intuition behind this relationship is as follows: If N_t is small, then each firm shares the same costs and the information with many of its competitors. In this situation, the firm does not hesitate to incorporate the signal ($x_t(n)$) into its price because it expects that many others will set the same prices. By contrast, if N_t is large, then each firm shares the same costs and the information with only a few of its competitors. In this situation, the firm does hesitate to set its price in accordance with the signal ($x_t(n)$) as it is afraid that the price may be very different from the others. In summary, the primary mechanism is that firms experiencing common shocks in their costs can effectively coordinate their prices with each other, and the degree of commonality in these shocks varies depending on N_t.

3.2.2. Endogenizing prior beliefs

Next, we endogenize prior belifs $\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$ and $\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$. Specifically, we calculate how firms generate endogenous signal on the sectoral costs based on competitors' prices in the previous period, and then how they update thier beliefs based on the endogenous signal. To this end, we model the process of firms' learning form their competitors' prices as follows. Suppose a firm's cost is type n. Then, by aggregating Equation (11) across firms whose cost type is not n, endogenous signal of $\widetilde{mc}_{t-1}$ is formed in period t as follows. The information included in the aggregated price $\left(\frac{1}{N_{t-1}-1}\sum_{n'=1,n'\neq n}^{N_{t-1}} p_{t-1}(n')\right)$ is given as:

$$\frac{1}{N_{t-1}-1}\sum_{n'=1,n'\neq n}^{N_{t-1}} p_{t-1}(n') = \alpha_{t-1}\left(\widetilde{mc}_{t-1} + \frac{1}{N_{t-1}-1}\sum_{n'=1,n'\neq n}^{N_{t-1}} v_{t-1}(n')\right) + (1-\alpha_{t-1})\,\mathbb{E}\left[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)\right].$$

Note that in this model, the prior beliefs become identical across firms, which is to be shown later. Hence, the following signal ($s_{t-1}(n)$), consisting of observed variables, is a noisy signal on $\widetilde{mc}_{t-1}$:

$$\begin{aligned} s_{t-1}(n) &\equiv \frac{\frac{1}{N_{t-1}-1}}{\alpha_{t-1}}\sum_{n'=1,n'\neq n}^{N_{t-1}} p_{t-1}(n') - \frac{1-\alpha_{t-1}}{\alpha_{t-1}}\mathbb{E}\left[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)\right] \\ &= \widetilde{mc}_{t-1} + \frac{1}{N_{t-1}-1}\sum_{n'=1,n'\neq n}^{N_{t-1}} v_{t-1}(n') \sim \mathcal{N}\left(\widetilde{mc}_{t-1}, \frac{1}{N_{t-1}-1}\tau_{t-1}^2\right). \end{aligned}$$

This signal is composed of the true past sectoral cost ($\widetilde{mc}_{t-1}$) and noises ($v_{t-1}(n)$) due to finite types of firm-specific costs within the industry. Combining this signal, and its own cost $x_{t-1}(n)$, each firm updates its beliefs about sectoral prices as below.

Lemma 2. *$\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$ and $\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$ are identical across firms and expressed as follows:*

$$\mathbb{E}\left[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)\right] = \sum_{s=1}^{\infty}\prod_{u=1}^{s}\frac{1-\lambda_{t-u}}{1-\lambda_{t-s}}\lambda_{t-s}\left(\widetilde{mc}_{t-s} + \frac{1}{N_{t-s}}\sum_{n=1}^{N_{t-s}} v_{t-s}(n)\right), \tag{15}$$

$$\mathbb{V}\left[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)\right] = \frac{\mathbb{V}\left[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)\right]\frac{1}{N_{t-1}}\tau_{t-1}^2}{\mathbb{V}\left[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)\right] + \frac{1}{N_{t-1}}\tau_{t-1}^2} + \sigma_t^2, \tag{16}$$

where

$$\lambda_{t-s} = \frac{\mathbb{V}\left[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)\right]}{\mathbb{V}\left[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)\right] + \frac{1}{N_{t-s}}\tau_{t-s}^2}, \tag{17}$$

and

$$\mathbb{V}\left[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)\right]=\frac{\mathbb{V}\left[\widetilde{mc}_{t-1-s}|\widehat{\mathcal{I}}_{t-1-s}(n)\right]\frac{1}{N_{t-1-s}}\tau^2_{t-1-s}}{\mathbb{V}\left[\widetilde{mc}_{t-1-s}|\widehat{\mathcal{I}}_{t-1-s}(n)\right]+\frac{1}{N_{t-1-s}}\tau^2_{t-1-s}}+\sigma^2_{t-s}, \tag{18}$$

for $s\geq 1$.

Proof. *See Appendix A.2.* □

Equations (15) and (16) in Lemma 2 indicate that firms' prior beliefs about sectoral costs are fully endogenized variables in this model, and the beliefs rely not only on the past sectoral costs, but also on the firm-specific costs due to imperfect information. Furthermore, both the mean and imprecision of the prior beliefs depend on the degree of fragmentation of information in the past ($N_{t-1}, N_{t-2}, ...$), which is an important channel through which past information structures affect the firms' current beliefs.

3.2.3 Prices with learning

Combining the equilibrium prices (11) and (12) with Equations (15) and (16), equilibrium prices are derived as follows:

Proposition 1. *The firm i's price and sectoral price in the equilibrium are given by:*

$$p_t(i;n)=\alpha_t x_t(n)+(1-\alpha_t)\left[\sum_{s=1}^{\infty}\prod_{u=1}^{s}\frac{1-\lambda_{t-u}}{1-\lambda_{t-s}}\lambda_{t-s}\left(\widetilde{mc}_{t-s}+\frac{1}{N_{t-s}}\sum_{n=1}^{N_{t-s}}v_{t-s}(n)\right)\right] \tag{19}$$

$$\begin{aligned}p_t=\alpha_t&\left(\widetilde{mc}_t+\frac{1}{N_t}\sum_{n=1}^{N_{t-1}}v_t(n)\right)\\&+(1-\alpha_t)\left[\sum_{s=1}^{\infty}\prod_{u=1}^{s}\frac{1-\lambda_{t-u}}{1-\lambda_{t-s}}\lambda_{t-s}\left(\widetilde{mc}_{t-s}+\frac{1}{N_{t-s}}\sum_{n=1}^{N_{t-s}}v_{t-s}(n)\right)\right]\end{aligned} \tag{20}$$

where α_t, $\widehat{\lambda}_t$, λ_{t-s}, *and* $\mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]$ *for* $s\geq 0$ *are, respectively, given by Equations (13), (14), (17), and (18).*

Proof. *Combining Lemma 1 and 2 yields Equations (19) and (20).* □

Proposition 1 shows the full-blown equilibrium prices, which depend on the current and an infinite number of past variables. The weight for the current variables in Equations (19) and (20) are the same and it depends not only on the degree of fragmentation in the current period (N_t), but also the degrees of fragmentation in the previous periods ($N_{t-1}, N_{t-2}, \ldots$) via $\widehat{\lambda}_t$. Similarly, the weight also depends not only on the variance of idiosyncratic costs in the current period (τ_t^2), but also on those in past periods ($\tau^2_{t-1}, \tau^2_{t-2}, \ldots$). The next section examines the theoretical relationship between the sensitivity of sectoral prices to the sectoral average of the firms costs and the degree of fragmentation of information, which we examined empirically in Section 2. Although Section 2 examines the sensitivity of sectoral inflation to changes in sectoral average costs, for the sake of simplicity, we first show the property of the equilibrium prices and then connect it to that of sectoral inflation later.

3.3 Property of the equilibrium prices

This section unveils the property of the equilibrium sectoral prices derived in Proposition 1. Specifically, our focus is on how the sensitivity of the sectoral price (p_t) to sectoral average of the firms' costs $\left(\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right)$ depends on the parameters of information structures (N_t, N_{t-1}, N_{t-2}. . .., and τ_t^2, τ_{t-1}^2, τ_{t-2}^2, ...). In this respect, α_t in Equation (13) corresponds with the sensitivity as below:

$$\alpha_t \equiv \frac{1-r}{1-r\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)} = \frac{\partial p_t}{\partial\left(\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right)}.$$

Then, the following Proposition holds.

Proposition 2. *(i) α_t is decreasing in N_t, N_{t-1}, N_{t-2}. . .. (ii) α_t is decreasing in τ_t^2, but increasing in τ_{t-1}^2, τ_{t-2}^2, ...*

Proof. *See Appendix A.3* □

Proposition 2-(i) first indicates that as the firms' costs in the current period become more fragmented across firms ($N_t \uparrow$), the sectoral prices (p_t) become less sensitive to current sectoral costs $\left(\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right)$. The intuition is as follows. If the information is less fragmented, firms know more about what others know about sectoral costs. In such a case, strategic uncertainty among firms is low and firms do not hesitate to adjust their prices to changes in their own costs. In contrast, if the information is more fragmented, firms do not know what others know about sectoral costs, and the strategic uncertainty among firms becomes high. Thus, the firms are reluctant to adjust their prices. In addition, Proposition 2-(i) also indicates that as firms' own costs in the past become more fragmented ($N_{t-s} \uparrow$ for $s \geq 1$), sectoral prices become less sensitive to current sectoral costs as well. If firms' costs in the past become more fragmented, the endogenous signals ($\{s_{t-u}(n)\}_{u=1}^{\infty}$) become more precise as the idiosyncratic noise (i.e., firm-specific costs) cancels each other out. As a result, firms' prior beliefs about sectoral costs become more precise and harder, making the update of the beliefs and price adjustment slower.

Next, Proposition 2-(ii) indicates that as the variance of firm-specific costs in the current period becomes larger ($\tau_t^2 \uparrow$), the sectoral prices become less sensitive to current sectoral costs. If the variance is large, it is difficult for firms to infer sectoral costs from their own costs. Hence, firms update their beliefs more gradually, which results in slower price adjustment. Proposition 2-(ii) also indicates that the sensitivity becomes higher as the variance of firm-specific costs in the past periods becomes larger ($\tau_{t-s}^2 \uparrow$ for $s \geq 1$). If the variance of firm-specific costs is high in the previous period, the endogenous signals are less precise as the variance of idiosyncratic noise is large. This results in less precise and softer prior beliefs about the sectoral costs, making the update of the beliefs and price adjustment faster.

These results highlight a critical difference between two key parameters. While the degree of fragmentation in the current and past periods has qualitatively the same effects on sensitivity — higher fragmentation leading to lower sensitivity, the impact of the variance of the noise on sensitivity differs between the current and past periods. The next section examines which parameter change can consistently account for the empirical findings in Section 2. Before that we show the case of standard imperfect information models as follows.

3.3.1 Standard imperfect information environments

In standard imperfect information environments, N_{t-s} is infinite in all periods ($s \geq 0$). In this extreme case, Proposition 1 is transformed as follows.

Corollary 1. *Suppose $N_{t-s} \to \infty$ for all $s \geq 0$. Then, the firm i's price and sectoral price in the equilibrium are given by:*

$$p_t(i;n) = \left(\frac{1-r}{1-r\widehat{\lambda}_t}\right) x_t(n) + \left(1 - \frac{1-r}{1-r\widehat{\lambda}_t}\right) \widetilde{mc}_{t-1}, \tag{21}$$

$$p_t = \left(\frac{1-r}{1-r\widehat{\lambda}_t}\right) \widetilde{mc}_t + \left(1 - \frac{1-r}{1-r\widehat{\lambda}_t}\right) \widetilde{mc}_{t-1} \tag{22}$$

where

$$\widehat{\lambda}_t \equiv \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2}, \tag{23}$$

and $\alpha_t \equiv \frac{1-r}{1-r\hat{\lambda}_t}$ is decreasing in τ_t^2 while invariant to $\tau_{t-1}^2, \tau_{t-2}^2, \ldots$.

Proof. *See Appendix A.4.* □

Corollary 1 indicates that under standard imperfect information models, prices depend on the variables only in the current and previous periods, and do not depend on the variables for more than two periods ago. τ_t^2 reduces the sensitivity of sectoral prices to sectoral costs. More importantly, unlike Proposition 2, τ_{t-s}^2 for $s \geq 1$ has no relationship with the sensitivity. When $N_{t-s} \to \infty$, the endogenous signals become perfectly accurate as the idiosyncratic noises are fully canceled out. Hence, the variance of the past firm-specific costs does not affect the efficiency of the learning. Compared with Proposition 2, the takeaway from this result is that α_t never decreases in τ_{t-s}^2 for $s \geq 1$.

4. Mapping from the model to empirical analysis

This section shows the mapping from the model in Section 3 to the empirical results in Section 2. Specifically, it examines whether the changes in two types of parameters of information structures, that is, the degree of fragmentation of information and the variance of firm-specific costs can consistently account for our empirical findings or not.

4.1 Mapping of the variables

We first define which variables in our model correspond to *changes in sectoral costs* ($\Delta Cost_t(s)$), *sectoral inflation* ($\pi_t(s)$), and *distribution of firms' costs* ($d_{s,t}$) in the empirical analysis in Section 2. The change in sectoral costs is written as,

$$\Delta Cost_t(s) \equiv \left(\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right) - \left(\widetilde{mc}_{t-1} + \frac{1}{N_{t-1}}\sum_{n=1}^{N_{t-1}} v_{t-1}(n)\right). \tag{24}$$

Moreover, from Equations (20) and (24), sectoral inflation can be written as follows:

$$\begin{aligned} \pi_t(s) &\equiv p_t - p_{t-1} \\ &= \alpha_t \Delta Cost_t(s) + (1-\alpha_t)\left[\sum_{s=1}^{\infty}\prod_{u=1}^{s}\frac{1-\lambda_{t-u}}{1-\lambda_{t-s}}\lambda_{t-s}\Delta Cost_{t-s}(s)\right], \end{aligned} \tag{25}$$

where α_t is defined in Equation (13). Hence, the sensitivity of sectoral inflation to changes in sectoral costs, estimated by the regressions in Section 2.3 corresponds to α_t.[22]

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Finally, *dispersions of firms' costs* $\left(d_{s,t}, d_{s,t-1}\right)$ are proxied in our model as follows:[23]

$$d_{s,t} \equiv \int_0^1 (\Delta Cost_t(i;s) - \Delta Cost_t(s))^2\, di = \left(\frac{N_t - 1}{N_t}\right) \tau_t^2 + \left(\frac{N_{t-1} - 1}{N_{t-1}}\right) \tau_{t-1}^2, \tag{26}$$

$$d_{s,t-1} \equiv \int_0^1 (\Delta Cost_{t-1}(i;s) - \Delta Cost_{t-1}(s))^2\, di = \left(\frac{N_{t-1} - 1}{N_{t-1}}\right) \tau_{t-1}^2 + \left(\frac{N_{t-2} - 1}{N_{t-2}}\right) \tau_{t-2}^2. \tag{27}$$

4.2 Mapping of the regression results

Based on these mappings of the variables, the following Proposition shows theoretical predictions on the relationship between dispersion of changes in firms' costs and sensitivity of sectoral inflation to sectoral costs, driven by the fragmentation of information.

Proposition 3. *If the degrees of fragmentation of information* $(N_t, N_{t-1}, N_{t-2}, \ldots)$ *vary, the sensitivity* α_t *decreases with* $d_{s,t}$ *and* $d_{s,t-1}$*.*

Proof. *See Appendix A.5.* □

Proposition 3 formally shows that the degrees of fragmentation of information $(N_t, N_{t-1}, N_{t-2}, \ldots)$ can generate the empircally consistent relationship between α_t and $\left(d_{s,t}, d_{s,t-1}\right)$ — the sensitivity α_t is decreasing in the dispersion $d_{s,t}$ and d_{s,t_1}. Intuition for this result follows that of Proposition 2: if the degree of fragmentation of information in the current period (N_t) is higher, then firms react less to the noisy signals due to the higher strategic uncertainty. Moreover, if the degree of fragmentation of information in the past periods $(N_{t-1}, N_{t-2}, \ldots)$ is higher, firms find it easier to learn past sectoral costs from competitors' past prices, leading to slower belief updating and thus slower price adjustment in the current period. At the same time, the increase in the degree of fragmentation of information leads to higher dispersion of firms' costs, making firms' costs more dispersed.

By constrast, as shown in the next Proposition, the changes in the variances of firm-specific costs $\left(\tau_t^2, \tau_{t-1}^2, \tau_{t-2}^2, \ldots\right)$ cannot consistently explain the empirical observation above.

Proposition 4. *If the variances of firm-specific costs* $\left(\tau_t^2, \tau_{t-1}^2, \tau_{t-2}^2, \ldots\right)$ *vary, the sensitivity* α_t *could decrease with* $d_{s,t}$*, but* α_t *increases with* $d_{s,t-1}$*.*

Proof. *See Appendix A.6.* □

Proposition 4 indicates that the variances of firm-specific costs $\left(\tau_t^2, \tau_{t-1}^2, \tau_{t-2}^2, \ldots\right)$ can generate the empircally consistent relationship between α_t and $d_{s,t}$ documented in Section 2 — the sensitivity is decreasing in the dispersion of the current period. However, the proposition also indicates that they cannot generate the empirically consistent relationship between α_t and $d_{s,t-1}$ as the sensitivity in the model (data) is increasing (decreasing) in the dispersion in the past period. Intuition for these results follows that of Proposition 2. Namely, when the variance of the noisiness in the signal of firm-specific costs in the current period (τ_t^2) is higher, firms react less to the noisy signals, keeping their prices less affected. When the variance of the noisiness of the signal in the past period (τ_{t-s}^2 for $s \geq 1$) is higher, firms find it more difficult to learn past sectoral costs from competitors' past prices,[24] leading to stronger belief updating and faster price adjustment. Simultaneously, the increase in the variance leads to higher dispersion of firms' costs, making firms' costs more dispersed.

To sum up, the empirically observed relationship between α_t and $d_{s,t}$, and the relationship between α_t and $d_{s,t-1}$ shown in Section 2 can be explained consistently with the change of degrees of fragmentation of information $(N_t, N_{t-1}, N_{t-2}, \ldots)$, newly introduced parameters of the information structure. However, the two relationships cannot be reconciled consistently with the change of variance of the noisy signals of the firm-specific costs $\left(\tau_t^2, \tau_{t-1}^2, \tau_{t-2}^2, \ldots\right)$, key parameters in standard imperfect information models.

5. Concluding remarks

We examined how the fragmentation of information contributes to the dynamics of sectoral inflation. Utilizing data from quarterly surveys of prices and costs from firms in Japan, we reported two empirical observations: the sensitivity of sectoral inflation to changes in sectoral costs consistently decreases as the dispersion of changes in both (i) current costs and (ii) past costs. Additionally, we found that while standard imperfect information models can explain the first (i), it falls short in explaining the second (ii). To address this gap, we expanded upon the standard dispersed information model to develop a dynamic general equilibrium framework that incorporates information fragmentation. In this extended model, we introduced a finite number of idiosyncratic noises, each shared by a proportion of firms in the market. Through our analysis, we found that the increase in the number of idiosyncratic costs, representing the fragmentation of information, plays a critical role in elucidating these empirical observations. This finding highlights the importance of the new parameter of information structures, that is, the degree of fragmentation of information.

Our research can be extended in multiple directions. One possible extension is to endogenize cost and information structures by explicitly modeling the supply chain network. Another extension is to estimate our dispersed information model with data (Melosi (2014) and Melosi (2017)). These extensions could provide additional insights potentially useful for understanding the role of information frictions in inflation dynamics.

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Supplementary material. To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100524000610

Notes

1 Regarding the empirical analyses on inflation dynamics, Atkeson and Ohanian (2001), Roberts (2006), Mavroeidis et al. (2014), Bullard (2018), Gagnon and Collins (2019), and Hazell et al. (2022) have studied the changing behavior of the Phillips curve in the U.S. Mourougane and Ibaragi (2004), Veirman (2007), Nishizaki et al. (2014), Kaihatsu and Nakajima (2018), Kaihatsu et al. (2023), and Kishaba and Okuda (2023) have investigated the Phillips curve in Japan. Regarding the related literature on imperfect information models, see Lucas (1972), Lucas (1973), Mankiw and Reis (2002), Woodford (2003), Adam (2007), Fukunaga (2007), Nimark (2008), Angeletos and La'O (2009), Hellwig and Venkateswaran (2009), Mackowiak and Wiederholt (2009), Dupor et al. (2010), Aoki et al. (2019), Okuda et al. (2019), Okuda et al. (2021), and Afrouzi (2023).
2 Existing literature reports that the dynamics of sectoral inflation is starkly dispersed across industries and indicates that the investigation of the determinants of sectoral inflation dynamics is essential for examining firms' price-setting behaviors. For instance, see Bils and Klenow (2004), Nakamura and Steinsson (2008), Klenow and Kryvtsov (2008), and Boivin et al. (2009).
3 These signals are referred to as semi-public signals by Morris and Shin (2007) because they lie between public and private signals.
4 Note that an increase in the degree of fragmentation of information and an increase in the variance of idiosyncratic noises affect firms' learning differently. Namely, the former increases the precision of learning because it makes the noises more idiosyncratic while keeping the variance of each noise unchanged. The latter decreases the precision of learning because it makes the variance of each noise higher while keeping the heterogeneity of the noises unchanged.

5 Specific questionnaires are as follows: *(a) change in input prices (judgment of changes in the yen-based purchasing prices of main raw materials, processing fees for subcontractors, and/or prices of main purchasing merchandise paid by the responding enterprise),* and *(b) change in output prices (judgment of changes in the yen-based selling prices of major products and services provided by the responding enterprise).*
6 Andrade et al., (2022) utilize a qualitative dataset to examine the empirical validity of Lucas (1972) island model-type information frictions on the firm side.
7 The set of potential answers is $J_t(i;s) \in \{-1, 0, 1\}$.
8 The sectors in our sample are specifically Textiles, Lumber and Wood products, Pulp and Paper, Chemicals, Petroleum and Coal products, Ceramics, Stone and Clay, Iron and Steel, Nonferrous metals, Food and Beverages, Processed metals, General-purpose, Production and Business oriented machinery, Electrical machinery, Transportation machinery, Other manufacturing, Construction, Real estate, Wholesaling, Retailing, Transport and Postal activities, Information communication, Electric and Gas Utilities, Services for businesses, Services for individuals, Accommodations, Eating and Drinking services, Mining and Quarrying of stone and gravel.
9 The data is available from the Bank of Japan's website: https://www.boj.or.jp/en/statistics/outline/exp/tk/extk.htm/.
10 Supplementary Appendix B shows the developments in this measure in each industry.
11 For details, see Supplementary Appendix C.
12 Supplementary Appendix C shows the 10th percentiles of $\Delta Cost_t(s)$ and also indicates the median dispersion of changes in input price in each group.
13 Note that because both dependent variable ($\pi_t(s)$) and explanatory variable ($\Delta Cost_t(s)$) are qualitative measures in the sense that firms do not choose the magnitude/size of changes in their price and costs, the results do not indicate quantitative implications.
14 We control the demand conditions by using the sectoral averages of firms' judgments about *domestic supply and demand conditions for products and services in their industry*: $\Delta Demand_{t-1}(s) \equiv 100\frac{1}{I}\sum_{i=1}^{I}\Delta Demand_{t-1}(i;s)$ where the potential choices are "Excess demand" ($\Delta Demand_{t-1}(i;s) = 1$), "Almost balanced" ($\Delta Demand_{t-1}(i;s) = 0$) or "Excess supply" ($\Delta Demand_{t-1}(i;s) = -1$).
15 For brevity, this table does not report the estimates $\hat{\beta}_1^A(s)$ for each industry. However, the sum of the coefficients for $\Delta Cost_t(s)$, $\left(\hat{\beta}_1^A(s) + \hat{\beta}_2^A d_{s,t} + \hat{\beta}_3^A d_{s,t-1}\right)$, is positive for all industries with observed ranges of $d_{s,t}$ and $d_{s,t-1}$. The coefficient for $\Delta Demand_{t-1}(s)$, $\hat{\beta}_4^A(s)$, is also positive in almost all industries. The same applies to the other tables.
16 The results are robust even if we use Weighted Least Squares. For details, see Supplementary Appendix D.
17 See Supplementary Appendix E.1 for the derivation of the demand function for each firm *i* in sector *s*.
18 Supplementary Appendix E.2 shows the derivation of the price setting rule.
19 For the sake of simplicity, we impose this assumption. However, because decisions by households and firms are static in this model and τ_t^2 is assumed known, the relationship between firms' prices and τ_t^2 does not change, irrespective of the process of τ_t^2.
20 Albagli et al., (2022) argue that firms form their inflation expectations based on the price changes along with their supply chain.
21 Because α_t is continuous and differentiable with respect to N_t, we take derivative of α_t with respect to N_t as if N_t is a real number. Note that the monotonic relationship between α_t and continuous N_t holds for an integer N_t.
22 The lags of changes in costs in Equation (25) may imply omitted variable bias in the empirical regression equations. For robustness, Supplementary Appendix D shows that the estimation results remain overall unchanged when including lagged changes in sectoral costs over one to eight periods as control variables.
23 For derivation, see Supplementary Appendix E.3. Note that in the model there are an infinite number of firms in sector *s*, while the number of firm-specific shocks *n* could be finite.
24 In an extreme case in which the degree of fragmentation of information is infinite ($N_{t-s} \to \infty$ for all $s \geq 0$), the learning informs the firms perfect information about the sectoral costs and thus, the impact of the increase in the variance of firm-specific costs in the past on the sensitivity is zero.

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Appendix A. Proofs

A.1 Proof of Lemma 1

From Equations (9) and (10), firm i's price and sectoral price are, respectively given as:

$$p_t(i;n) = r\mathbb{E}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)x_t(n), \tag{A.1}$$

and

$$p_t = r\overline{\mathbb{E}}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right]. \tag{A.2}$$

We employ a "brute-force solution." By taking expectations about (A.2) over $\mathcal{I}_t(n)$, type n firms' expectations about the sectoral price are given as:

$$\mathbb{E}\left[p_t|\mathcal{I}_t(n)\right] = r\mathbb{E}\overline{\mathbb{E}}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\mathbb{E}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right].$$

Firms' average expectations about the sectoral price are then given by:

$$\overline{\mathbb{E}}\left[p_t|\mathcal{I}_t(n)\right] = \int_{i\in[0,1]} \mathbb{E}\left[p_t|\mathcal{I}_t(n)\right] di = r\overline{\mathbb{E}}^2\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\overline{\mathbb{E}}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right],$$

and firms' expectations about the other firms' average expectations about the sectoral price are:

$$\mathbb{E}\overline{\mathbb{E}}\left[p_t|\mathcal{I}_t(n)\right] = r\mathbb{E}\overline{\mathbb{E}}^2\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\mathbb{E}\overline{\mathbb{E}}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right].$$

By repeating these steps, we obtain the following equation:

$$\mathbb{E}\overline{\mathbb{E}}^j\left[p_t|\mathcal{I}_t(n)\right] = r\mathbb{E}\overline{\mathbb{E}}^{j+1}\left[p_t|\mathcal{I}_t(n)\right] + (1-r)\mathbb{E}\overline{\mathbb{E}}^j\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v(n)|\mathcal{I}_t(n)\right].$$

By recursively solving this equation, $\mathbb{E}[p_t|\mathcal{I}_t(n)]$ is given as:

$$\mathbb{E}\left[p_t|\mathcal{I}_t(n)\right] = (1-r)\sum_{j=0}^{\infty} r^j \mathbb{E}\overline{\mathbb{E}}^j\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right]. \tag{A.3}$$

Therefore, by plugging Equation (A.3) into Equation (A.1), we have:

$$p_t(i;n) = r(1-r)\sum_{j=0}^{\infty} r^j \mathbb{E}\overline{\mathbb{E}}^j\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right] + (1-r)x_t(n). \tag{A.4}$$

Next, we calculate $\mathbb{E}\overline{\mathbb{E}}^j\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right]$. Through the filtering process about sectoral costs, $\widetilde{mc}_t$, firms form expectations about $\mathbb{E}\left[\widetilde{mc}_t|\mathcal{I}_t(n)\right]$ as:

$$\mathbb{E}\left[\widetilde{mc}_t|\mathcal{I}_t(n)\right] = \widehat{\lambda}_t x_t(n) + (1-\widehat{\lambda}_t)\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$$

where $\widehat{\lambda}_t \equiv \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]/(\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)] + \tau_t^2)$. Note that, to be shown later, the prior beliefs, $\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$, are common across firms. Then, we have the following relationship.

$$\begin{aligned}\mathbb{E}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right] &= \frac{N_t-1}{N_t}\mathbb{E}\left[\widetilde{mc}_t|\mathcal{I}_t(n)\right] + \frac{1}{N_t}x_t(n)\\ &= \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)x_t(n) + \left(\frac{N_t-1}{N_t}(1-\widehat{\lambda}_t)\right)\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)].\end{aligned} \tag{A.5}$$

By taking averages about Equation (A.5) across firms, we have:

$$\begin{aligned}\overline{\mathbb{E}}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right] &= \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right]\\ &\quad + \left(\frac{N_t-1}{N_t}(1-\widehat{\lambda}_t)\right)\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)].\end{aligned} \tag{A.6}$$

Using Equation (A.6), $\mathbb{E}\overline{\mathbb{E}}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right]$ is given by:

$$\begin{aligned}&\mathbb{E}\overline{\mathbb{E}}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right]\\ &= \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)\mathbb{E}\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right] + \left(\frac{N_t-1}{N_t}(1-\widehat{\lambda}_t)\right)\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]\\ &= \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)^2 x_t(n) + \left[1 - \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)^2\right]\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)].\end{aligned}$$

By repeating the same steps, we have:

$$\begin{aligned}\mathbb{E}\overline{\mathbb{E}}^j\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right] &= \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)^{j+1} x_t(n)\\ &\quad + \left[1 - \left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)^{j+1}\right]\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)].\end{aligned} \tag{A.7}$$

By plugging (A.7) into equation (A.4), we obtain Equations (11) and (12) as follows: First, $p_t(i;n)$ is given as:

$$\begin{aligned}
&p_t(i;n)\\
&= r(1-r)\sum_{j=0}^{\infty} r^j \mathbb{E}\overline{\mathbb{E}}^j\left[\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)|\mathcal{I}_t(n)\right] + (1-r)x_t(n)\\
&= r(1-r)\sum_{j=0}^{\infty} r^j\left[\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)^{j+1} x_t(n) + \left[1-\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)^{j+1}\right]\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]\right]\\
&\quad +(1-r)x_t(n)\\
&= \left[\frac{1-r}{1-r\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)}\right]x_t(n) + \left[1-\frac{1-r}{1-r\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)}\right]\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]
\end{aligned}$$

and by taking averages across firms, p_t is derived as:

$$\begin{aligned}
p_t = &\left[\frac{1-r}{1-r\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)}\right]\left(\widetilde{mc}_t + \frac{1}{N_t}\sum_{n=1}^{N_t} v_t(n)\right)\\
&+\left[1-\frac{1-r}{1-r\left(\frac{N_t-1}{N_t}\widehat{\lambda}_t + \frac{1}{N_t}\right)}\right]\overline{\mathbb{E}}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)].
\end{aligned}$$

□

A.2 Proof of Lemma 2

We show how firms update their beliefs about sectoral costs based on the signal above. First, $\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]$ is updated to $\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_t(n)]$ as follows:

$$\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_t(n)] = \lambda_{t-1}\left(\frac{N_{t-1}-1}{N_{t-1}}s_{t-1}(n) + \frac{1}{N_{t-1}}x_{t-1}(n)\right) + (1-\lambda_{t-1})\,\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)],$$

where λ_{t-1} represents the weight for the combined signal $\frac{N_{t-1}-1}{N_{t-1}}s_{t-1}(n) + \frac{1}{N_{t-1}}x_{t-1}(n)$ and given as:

$$\lambda_{t-1} \equiv \frac{\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]}{\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)] + \frac{1}{N_{t-1}}\tau_{t-1}^2}.$$

When the firm update $\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]$ to $\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_t(n)]$, it attempts to utilize all the information in period $t-1$. Because there are signals with n different types of independent noises whose imprecision is identical, it is optimal to form the combined signal as follows:

$$\begin{aligned}
&\frac{N_{t-1}-1}{N_{t-1}}s_{t-1}(n) + \frac{1}{N_{t-1}}x_{t-1}(n)\\
&= \frac{N_{t-1}-1}{N_{t-1}}\left(\widetilde{mc}_{t-1} + \frac{1}{N_{t-1}-1}\sum_{n'=1,n'\neq n}^{N_{t-1}} v_{t-1}(n')\right) + \frac{1}{N_{t-1}}\left(\widetilde{mc}_{t-1} + v_{t-1}(n)\right)\\
&= \widetilde{mc}_{t-1} + \frac{1}{N_{t-1}}\sum_{n=1}^{N_{t-1}} v_{t-1}(n) \sim \mathcal{N}\left(\widetilde{mc}_{t-1}, \frac{1}{N_{t-1}}\tau_{t-1}^2\right).
\end{aligned}$$

Because all firms use this identical combined signal, their prior beliefs are identical as well.

Finally, because $\widetilde{mc}_t$ follows the random-walk process (Equation (8)), we can write the belief updating process recursively as

$$\mathbb{E}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)] = \lambda_{t-1}\left(\widetilde{mc}_{t-1} + \frac{1}{N_{t-1}}\sum_{n=1}^{N_{t-1}} v_{t-1}(n)\right) + (1-\lambda_{t-1})\,\mathbb{E}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)],$$

where

$$\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)] = \frac{\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]\frac{1}{N_{t-1}}\tau_{t-1}^2}{\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)] + \frac{1}{N_{t-1}}\tau_{t-1}^2} + \sigma_t^2.\ (16) \tag{A.8}$$

By solving these equations backwardly, Equation (15) is derived. Also, because updating processes of equations for λ_{t-1} and $\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]$ hold for any periods, Equations (17) and (18) hold. □

A.3 Proof of Proposition 2

(i) In terms of N_t, the following inequality holds:

$$\frac{\partial\alpha_t}{\partial N_t} = -\frac{\left(1-\widehat{\lambda}_t\right)(1-r)\,r}{\left[r\left(1-\widehat{\lambda}_t\right) - N_t\left(1-r\widehat{\lambda}_t\right)\right]^2} < 0.$$

Regarding N_{t-1}, first:

$$\frac{\partial\alpha_t}{\partial\widehat{\lambda}_t} = \frac{r(1-r)\,N_t\,(N_t-1)}{\left(N_t(r\widehat{\lambda}_t-1) - r\widehat{\lambda}_t + r\right)^2} > 0$$

holds. Then, from Equation (14):

$$\frac{\partial\widehat{\lambda}_t}{\partial\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]} > 0$$

holds, and the following inequality also holds.

$$\frac{\partial\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial N_{t-1}} = -\frac{\mathbb{V}^2[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]\tau_{t-1}^2}{\left(\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]N_{t-1} + \tau_{t-1}^2\right)^2} < 0.$$

Therefore,

$$\frac{\partial\widehat{\lambda}_t}{\partial N_{t-1}} = \underbrace{\frac{\partial\widehat{\lambda}_t}{\partial\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}}_{>0}\underbrace{\frac{\partial\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial N_{t-1}}}_{<0} < 0$$

holds, and thus the following inequality holds:

$$\frac{\partial\alpha_t}{\partial N_{t-1}} = \underbrace{\frac{\partial\alpha_t}{\partial\widehat{\lambda}_t}}_{>0}\underbrace{\frac{\partial\widehat{\lambda}_t}{\partial N_{t-1}}}_{<0} < 0.$$

Furthermore, from Equations (16) and (18):

$$\frac{\partial\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial\mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]} = \underbrace{\frac{\partial\mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]}}_{>0}\underbrace{\frac{\partial\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]}{\partial\mathbb{V}[\widetilde{mc}_{t-2}|\widehat{\mathcal{I}}_{t-2}(n)]}}_{>0}\cdots\underbrace{\frac{\partial\mathbb{V}[\widetilde{mc}_{t-s+1}|\widehat{\mathcal{I}}_{t-s+1}(n)]}{\partial\mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}}_{>0} > 0$$

holds, and

$$\frac{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}{\partial N_{t-1-s}} = -\frac{\mathbb{V}^2[\widetilde{mc}_{t-1-s}|\widehat{\mathcal{I}}_{t-1-s}(n)]\tau^2_{t-1-s}}{\left(\mathbb{V}[\widetilde{mc}_{t-1-s}|\widehat{\mathcal{I}}_{t-1-s}(n)]N_{t-1-s} + \tau^2_{t-1-s}\right)^2} < 0.$$

hold. Therefore, for $s \geq 1$, we have the following inequality:

$$\frac{\partial \alpha_t}{\partial N_{t-1-s}} = \underbrace{\frac{\partial \alpha_t}{\partial \widehat{\lambda}_t}}_{>0} \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}}_{>0} \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}}_{>0} \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}{\partial N_{t-1-s}}}_{<0} < 0.$$

(ii) In terms of τ^2_t, $\frac{\partial \alpha_t}{\partial \hat{\lambda}_t} > 0$ holds as shown in (i) and $\frac{\partial \hat{\lambda}_t}{\partial \tau^2_t} < 0$ hold from Equation (17). Thereby the following in equality holds:

$$\frac{\partial \alpha}{\partial \tau^2_t} = \underbrace{\frac{\partial \alpha}{\partial \widehat{\lambda}_t}}_{>0} \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \tau^2_t}}_{<0} < 0.$$

Next, regarding τ^2_{t-1}, from Equation (14) and

$$\frac{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial \tau^2_{t-1}} = \frac{N_{t-1}\mathbb{V}^2[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]}{\left(\tau^2_{t-1} + N_{t-1}\mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]\right)^2} > 0,$$

the following inequality holds,

$$\frac{\partial \widehat{\lambda}_t}{\partial \tau^2_{t-1}} = \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}}_{>0} \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial \tau^2_{t-1}}}_{>0} > 0$$

and thus, unlike the case of τ^2_t, the following inequality holds:

$$\frac{\partial \alpha}{\partial \tau^2_{t-1}} = \underbrace{\frac{\partial \alpha}{\partial \widehat{\lambda}_t}}_{>0} \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \tau^2_{t-1}}}_{>0} > 0.$$

Moreover, regarding τ^2_{t-1-s}, Equations (16) and (18), respectively, lead to

$$\frac{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]} = \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial \mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]}}_{>0} \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_{t-1}|\widehat{\mathcal{I}}_{t-1}(n)]}{\partial \mathbb{V}[\widetilde{mc}_{t-2}|\widehat{\mathcal{I}}_{t-2}(n)]}}_{>0} \cdots \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_{t-s+1}|\widehat{\mathcal{I}}_{t-s+1}(n)]}{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}}_{>0} > 0$$

for $s \geq 1$ and

$$\frac{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}{\partial \tau^2_{t-1-s}} = \frac{N_{t-1-s}\mathbb{V}^2[\widetilde{mc}_{t-1-s}|\widehat{\mathcal{I}}_{t-1-s}(n)]}{\left(\tau^2_{t-1-s} + N_{t-1-s}\mathbb{V}[\widetilde{mc}_{t-1-s}|\widehat{\mathcal{I}}_{t-1-s}(n)]\right)^2} > 0.$$

Finally, for $s \geq 1$, we have the following inequality:

$$\frac{\partial \alpha_t}{\partial \tau^2_{t-1-s}} = \underbrace{\frac{\partial \alpha_t}{\partial \widehat{\lambda}_t}}_{>0} \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}}_{>0} \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_t|\widehat{\mathcal{I}}_t(n)]}{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}}_{>0} \underbrace{\frac{\partial \mathbb{V}[\widetilde{mc}_{t-s}|\widehat{\mathcal{I}}_{t-s}(n)]}{\partial \tau^2_{t-1-s}}}_{>0} > 0.$$

$\square$

A.4 Proof of Collorary 1

Taking $N_{t-s} \to \infty$ for $s \geq 0$ in all equations of Proposition 1 yields Equations (21), (22), and (23). Moreover, the following inequality and equality hold:

$$\frac{\partial \alpha_t}{\partial \tau_t^2} = \underbrace{\frac{\partial \alpha_t}{\partial \widehat{\lambda}_t}}_{>0} \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \tau_t^2}}_{<0} < 0, \; \frac{\partial \alpha_t}{\partial \tau_{t-s}^2} = \underbrace{\frac{\partial \alpha_t}{\partial \widehat{\lambda}_t}}_{>0} \underbrace{\frac{\partial \widehat{\lambda}_t}{\partial \tau_{t-s}^2}}_{=0} = 0 \text{ for } s \geq 1.$$

□

A.5 Proof of Proposition 3

The dispersion $d_{s,t}$ in Equation (26) is monotonically increasing in the degree of fragmentation N_t and N_{t-1}, while the sensitivity α_t is monotonically decreasing in N_t and N_{t-1}, according to Proposition 2. This proves that α_t is decreasing in $d_{s,t}$. Furthermore, $d_{s,t-1}$ in Equation (27) is monotonically increasing in N_{t-1} and N_{t-2}, while α_t is monotonically decreasing in N_{t-1} and N_{t-2}, according to Proposition 2. Thereby, α_t is decreasing in $d_{s,t-1}$. Note that changes in $N_{t-3}, N_{t-4}, \ldots$ do not affect $\left(d_{s,t}, d_{s,t-1}\right)$ while α_t is monotonically decreasing in them, meaning that changes in $N_{t-3}, N_{t-4}, \ldots$ do not alter the negative relationship between α_t and $\left(d_{s,t}, d_{s,t-1}\right)$, generated by (N_t, N_{t-1}, N_{t-2}). □

A.6 Proof of Proposition 4

The dispersion $d_{s,t}$ in Equation (26) is monotonically increasing in τ_t^2, and α_t is monotonically decreasing in τ_t^2, according to Proposition 2. Combining these two, α_t is decreasing in $d_{s,t}$. However, $d_{s,t}$ in Equation (26) is monotonically increasing in τ_{t-1}^2, and α_t is monotonically increasing in τ_{t-1}^2 according to Proposition 2. In that case, α_t could also be increasing in $d_{s,t}$. By contrast, $d_{s,t-1}$ in Equation (27) is monotonically increasing in τ_{t-1}^2 and τ_{t-2}^2, while α_t is increasing in τ_{t-1}^2 and in τ_{t-2}^2, according to Proposition 2, indicating that the sensitivity α_t increases with $d_{s,t-1}$. Note that changes in $\tau_{t-3}^2, \tau_{t-4}^2, \ldots$ do not affect $\left(d_{s,t}, d_{s,t-1}\right)$ while α_t is monotonically increasing in these, meaning that the changes in $\tau_{t-3}^2, \tau_{t-4}^2, ..$ do not alter the relationship between α_t and $\left(d_{s,t}, d_{s,t-1}\right)$, generated by $\left(\tau_t^2, \tau_{t-1}^2, \tau_{t-2}^2\right)$. □

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