Proceedings of the Edinburgh Mathematical Society (2017) **60**, 877–879
DOI:10.1017/S0013091516000468

CORRIGENDUM AND ADDENDUM: ABSTRACT M- AND ABSTRACT L-SPACES OF POLYNOMIALS ON BANACH LATTICES

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(Received 30 October 2015)

Published in *Proceedings of the Edinburgh Mathematical Society* **58**(3) (2015), 617–629.

Abstract In this short note, we correct and reformulate Theorem 3.1 in the paper published in *Proceedings of the Edinburgh Mathematical Society* **58**(3) (2015), 617–629.

Keywords: homogeneous polynomials; positive tensor products; AM- and AL- spaces

2010 *Mathematics subject classification:* Primary 46G25; 46B28; 46B42

Recall that a Banach lattice is said to have a *Fatou norm* if $0 \leqslant x_\alpha \uparrow x$ implies $\|x_\alpha\| \uparrow \|x\|$. In [**2**, Theorem 3.1], if the domain spaces are not atomic, then the range space F should have a Fatou norm.* Actually, [**2**, Theorem 3.1] can be reformulated as follows.

Theorem 3.1.

Case 1. *Let F be a Dedekind complete AM-space. The following are equivalent.*

(1) *If $E, E_1, \dots, E_n$ are AL-spaces, then $\mathcal{L}^r(E_1, \dots, E_n; F)$ and $\mathcal{P}^r({}^nE; F)$ are AM-spaces.*

(2) *F has a Fatou norm.*

Case 2. *If F is an AM-space and $E, E_1, \dots, E_n$ are separable atomic AL-spaces, then $\mathcal{L}^r(E_1, \dots, E_n; F)$ and $\mathcal{P}^r({}^nE; F)$ are AM-spaces.*

* The authors thank A. W. Wickstead for pointing out this error in Theorem 3.1 in [**2**] to which this note is a correction.

Proof.

Case 1. To see that (2) implies (1), take $T, S \in \mathcal{L}(E_1, \ldots, E_n; F)^+$, $x_1 \in E_1^+, \ldots, x_n \in E_n^+$ and use the estimates in [**2**, Theorem 3.1] to show that

$$\Bigg\| \sum_{i_1,\ldots,i_n} T(u_{1,i_1}, \ldots, u_{n,i_n}) \vee S(u_{1,i_1}, \ldots, u_{n,i_n}) \Bigg\| \leqslant (\|T\| \vee \|S\|) \cdot \|x_1\| \cdots \|x_n\|.$$

It follows from [**2**, Proposition 2.1] that

$$\sum_{i_1,\ldots,i_n} T(u_{1,i_1}, \ldots, u_{n,i_n}) \vee S(u_{1,i_1}, \ldots, u_{n,i_n}) \uparrow (T \vee S)(x_1, \ldots, x_n).$$

So the Fatou property tells us that

$$\Bigg\| \sum_{i_1,\ldots,i_n} T(u_{1,i_1}, \ldots, u_{n,i_n}) \vee S(u_{1,i_1}, \ldots, u_{n,i_n}) \Bigg\| \uparrow \|(T \vee S)(x_1, \ldots, x_n)\|,$$

from which

$$\|(T \vee S)(x_1, \ldots, x_n)\| \leqslant (\|T\| \vee \|S\|) \cdot \|x_1\| \cdots \|x_n\|$$

and the result follows as in [**2**].

For the converse, suppose that (1) holds. Then it follows that $\mathcal{L}^r(E; F)$ is an AM-space for any AL-space E, which implies, by [**6**, Theorem 2.3], that F has a Fatou norm.

Case 2. In this case, we need no assumption on F. Indeed, $E, E_1, \ldots, E_n$ may all be identified with ℓ_1. It follows from [**3**] that the Fremlin projective tensor product $E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n$ is lattice isomorphic with ℓ_1 as well. Now note that ℓ_1 is a prime space, that is, every infinite-dimensional closed complemented subspace of ℓ_1 is isomorphic to ℓ_1. Thus, the Fremlin projective symmetric tensor product $\hat{\otimes}_{n,s,|\pi|} E$ is also lattice isomorphic to ℓ_1. It is essentially well known (as well as explicitly proved in [**5**, Theorem 10.2]) that for the domain space $E = \ell_1$ and any Banach lattice F we have that $\mathcal{L}^r(E, F)$ is a vector lattice, which, in addition, is a Banach space under the regular norm by [**4**, Proposition 1.3.6]. Again, by the last part of [**5**, Theorem 10.2], the lattice operations in $\mathcal{L}^r(E, F)$ can be calculated by the Kantorovich formulae as in [**4**, Corollary 1.3.4]. It then follows from the last part of [**4**, Corollary 1.3.4 (i)] that $\mathcal{L}^r(E, F)$ is a Banach lattice under the regular norm. Thus, $\mathcal{L}^r(E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n; F)$ and $\mathcal{L}^r(\hat{\otimes}_{n,s,\pi} E; F)$ are Banach lattices. Finally, by similar reasoning to that in [**1**, Propositions 3.3 and 3.4], $\mathcal{L}^r(E_1, \ldots, E_n; F)$ and $\mathcal{P}^r({}^n E; F)$ are Banach lattices as well, and from [**6**, Theorem 2.2] it follows that these spaces are AM-spaces. □

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https://doi.org/10.1017/S0013091516000468 Published online by Cambridge University Press

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