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WELFARE COST OF MONETARY AND FISCAL POLICY SHOCKS

Lynne Evans
University of Durham

Turalay Kenc
Imperial College

This paper provides estimates of the welfare cost of volatility attributable to monetary and fiscal policy shocks. It uses a continuous-time stochastic dynamic general equilibrium model based on a recursive utility function that disentangles risk aversion from intertemporal substitution. We find that monetary and fiscal policy shocks may lead to opposite welfare effects: negative for monetary growth shocks, but positive for government expenditure shocks. Furthermore, we find that welfare costs are sensitive to the parameter values chosen for risk aversion and intertemporal substitution, and we conclude that welfare costs are potentially much larger than that found by Lucas, forcing some modification of the policy conclusions associated with Lucas's pioneering work.

Keywords: Welfare Cost of Volatility, Recursive Utility, Stochastic General Equilibrium Models

1. INTRODUCTION

Research interest in the welfare cost of macroeconomic volatility began with the pioneering work of Lucas (1987) and centers on the question of how much current consumption society might be willing to give up in return for the elimination of business-cycle fluctuations. However, the literature to date typically has focused on only one source of aggregate volatility, namely, that due to consumption volatility arising from production shocks. However, there are other potential sources of volatility that may be at least as important: For example, monetary and fiscal policies may generate volatility. In recognition of the diverse sources of the welfare costs of business cycles, it is of some significance to disaggregate the total cost into its component parts and, in this paper, we focus particularly on the welfare costs due to monetary and fiscal policy shocks.

The purpose of this paper is to calculate the welfare effects of monetary growth shocks and government expenditure shocks and to analyze the distinct roles played by agents' attitudes toward risk and intertemporal substitution. Obstfeld (1994a)

We are grateful to an anonymous referee, Adrian Darnell, and Malte Krueger for helpful comments and Gulcin Ozkan for discussions related to this paper. The usual disclaimer applies. Address correspondence to: Lynne Evans, Department of Economics and Finance, University of Durham, 23–26 Old Elvet, Durham DH1 3HY, United Kingdom; e-mail: Lynne.Evans@durham.ac.uk.

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has already shown that disentangling these two preference parameters is central to improvements on Lucas's measure of welfare cost, although a limitation of Obstfeld's work is his partial equilibrium approach. A major contribution of this paper is to use a richer approach that captures both portfolio balance and general equilibrium effects. The portfolio effects reveal how policy volatility operates via the portfolio channel; and our incorporation of general equilibrium effects ensures that proper allowance is made for interactions such as variances affecting means in the model. Furthermore, disentangling the two preference parameters allows us to achieve our aim of identifying the separate contributions of risk aversion and intertemporal substitution. In particular, this allows a clean analysis of the comparative statics of degrees of risk aversion. In addition, our approach permits separate examination of the different effects of monetary policy shocks and government expenditure shocks on welfare.

The starting point for our model is the continuous-time general equilibrium stochastic endogenous growth model of Turnovsky and Grinols (1996) in which we relax the parametric restriction on the agent's preferences through "generalized isoelastic preferences." Using this framework we find that eliminating government expenditure shocks from the model is welfare deteriorating whereas removing monetary growth shocks improves welfare for the representative household. One reason for this difference derives from portfolio balance effects. Eliminating government expenditure shocks makes government debt less risky, inducing a portfolio shift away from (productive) capital and into bonds—and this is detrimental to growth and welfare.[1] In contrast, removing the monetary growth shocks makes all assets in the portfolio less risky, encouraging a shift toward the riskier (productive) assets, which, in turn, promotes growth and gives rise to positive welfare effects.

More generally, we find that the magnitude and nature of calculated welfare costs are sensitive to the parameter values chosen for risk aversion and intertemporal substitution. Also, we find welfare effects that are somewhat larger than those typically found in the literature—a consequence of our general (not partial) equilibrium approach and more importantly, our allowing volatility to affect the mean growth rate in our model. This finding prompts modification of the striking conclusion of Lucas's analysis—that the gain from eliminating business-cycle fluctuations is quite small—and has obvious and strong policy implications.[2]

Our paper is organized as follows: Section 2 outlines our development of existing continuous-time stochastic endogenous growth models. In Section 3, we provide numerical estimates for the welfare cost of monetary and fiscal policy. Section 4 concludes the paper.

2. MODEL

We use a representative-agent model that has money in the utility function and assumes a Rebelo (1991) "AK" production function that leads to endogenous growth. The stochastic nature of the model is characterized by four exogenous stochastic shocks. Two of these are policy shocks, namely, government expenditure and monetary growth; the other two are productivity and foreign price shocks. Other shocks could be included but this set is characteristic of the most important exogenous stochastic influences on a small open economy. The economy specializes in the production of a single good and is assumed to be sufficiently small in the world production of this good to have no impact on its market. The importance of having an open-economy model relates to the significance we attach to capturing full quantitative effects of volatility. The behavioral nature of the model is described by the utility-maximizing and portfolio-optimizing behavior of a representative household. The paper deals with only the steady-state stochastic equilibrium, which is separated into deterministic and stochastic components.

2.1. Household Optimization

At each point in time the representative household chooses its consumption, $C(t)$, and allocates its portfolio of wealth, $W(t)$, across four assets: money M, government bonds B, capital K, and foreign bonds B^*. Two of these assets (capital and bonds) are internationally traded.[3] The only source of income for the representative household is the capital income received from holding these assets.

The representative agent's intertemporal utility is given by

$$\begin{aligned} U(t) &= e^{-\rho t}[\tilde{u}(t)^{\zeta} + e^{-\rho dt}V(t+dt)^{\zeta}]^{1/\zeta}, \\ V(t+dt) &= \left[\mathcal{E}_t U(t+dt)^{1-\gamma}\right]^{1/(1-\gamma)}, \\ \tilde{u}(t) &= C(t)^{\theta}(M(t)/P(t))^{1-\theta}, \end{aligned} \tag{1}$$

where $\mathcal{E}$ is the expectation operator, ρ is the time preference rate, $1/(1-\zeta)$ is the usual parameter for intertemporal elasticity of substitution, and γ is the degree of relative risk aversion. The instantaneous utility function, $\tilde{u}$, is defined over consumption C and real money balances M/P. The share of consumption in the instantaneous utility is represented by θ. This structure implies that utility satisfies intertemporal consistency of preferences and removes the restriction that $\zeta + \gamma = 1$.[4]

Utility is maximized subject to the following wealth W constraint, which in real terms is

$$W = \frac{M}{P} + \frac{B}{P} + \frac{EB^*}{P} + K,$$

where E is the exchange rate and P equals the price level; and it is also maximized subject to the stochastic wealth accumulation equation

$$dW = W[n_M\,dR_M + n_B\,dR_B + n_F\,dR_F + n_K\,dR_K] - C(t)\,dt - dT, \tag{2}$$

where n_i refers to the share of the portfolio held in asset i. More specifically, $n_M = (M/P)/W$, $n_B = (B/P)/W$, $n_K = K/W$, and $n_F = (EB^*/P)/W$; dR_i refers to the total (sum of deterministic and stochastic) rates of return on asset i;

and dT is the taxation paid on holdings of wealth. The representative household constructs an optimal portfolio of its total wealth subject to the adding-up condition for portfolio shares,

$$1 = n_M + n_B + n_K + n_F. \tag{3}$$

Households are assumed to purchase output over the instant dt at the nonstochastic rate $C(t)dt$ using the capital income generated from holding assets. To define each asset return, dR_i, requires a description of the dynamics that generate asset prices and asset returns.

2.1.1. Prices and asset returns. There are three prices in the model: the domestic price of the traded good, P; the foreign price level of the traded good, Q; and the exchange rate, E, measured in units of domestic currency per unit of foreign currency. Q is assumed to be exogenous and P and E are endogenously determined. Prices and returns are both generated by geometric Brownian motion processes. Each of the prices P, Q, and E evolves according to

$$dx/x = (\text{drift term})dt + \text{random variable}, \tag{4}$$

where x is either P, Q, or E; and π, π^*, and ϵ are respective drift terms of these price processes. Thus, for example, $\pi\,dt$ is the expected mean rate of change of P. The respective random variables, dp, dq, and de, are assumed to be temporally independent and normally distributed with zero means and variances $\sigma_p^2 dt$, $\sigma_q^2 dt$, $\sigma_e^2 dt$.

Incorporating money into the model gives rise to a separation of real returns from nominal returns. Returns to the productive asset, capital, is described below, but returns to other assets can be described in terms of the interest rates they pay. Domestic and foreign bonds pay nominal rates of interest i and i^*, respectively. Applying stochastic calculus, we obtain the real rates of return to domestic holders of money, domestic bonds, and foreign bonds as follows:

$$dR_M = r_M dt - dp \qquad r_M = -\pi + \sigma_p^2, \tag{5a}$$

$$dR_B = r_B dt - dp \qquad r_B = i - \pi + \sigma_p^2, \tag{5b}$$

$$dR_F = r_F dt - dq \qquad r_F = i^* - \pi^* + \sigma_q^2. \tag{5c}$$

The real rate of return to equityholders is calculated from the flow of new output dY per capital K. We assume that output is produced from capital by means of the stochastic constant-returns-to-scale technology, and the economywide capital stock is assumed to have a positive external effect on the individual factor capital. We therefore write the aggregate production function as an AK function of the kind discussed by Rebelo (1991) with a stochastic linear coefficient

$$dY(t) = \alpha[dt + dy(t)]K(t), \tag{6}$$

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where α is the marginal physical product of capital and dy is a productivity shock. Technically, dy represents increments to a Brownian motion with zero drift and variance $\sigma_y^2 dt$. Thus, the return to capital before and after separating its deterministic and stochastic terms is, respectively,[5]

$$dR_K = \alpha dt + \alpha dy, \qquad r_K = \alpha dt, \qquad du_K = \alpha dy. \tag{7}$$

This small open economy is linked with the rest of the world through the law of one price. Formally, it means that the exchange rate E relates foreign prices Q to domestic prices P, which is referred to as the purchasing power parity (PPP) relationship

$$P = EQ. \tag{8}$$

Taking the stochastic derivative of this relationship and separating its deterministic and stochastic components while substituting for the price process expressions (4) into the resulting relationship yields

$$\pi = \pi^* + \epsilon + \sigma_{qe}, \tag{9a}$$

$$dp = dq + de, \tag{9b}$$

where σ_{qe} is the instantaneous covariance between dq and de.

Taxes are endogenously determined to satisfy the government budget constraint (see Section 2.2) and include a stochastic component reflecting the changing need for taxes. Because, in a growing economy, taxes and other real variables grow with the size of the economy, measured here by real wealth, we relate total taxes to wealth according to

$$dT = \tau W dt + W dv, \tag{10}$$

where τ is the tax rate and dv is a temporally independent, normally distributed random variable with zero mean and variance $\sigma_v^2 dt$.

As we see below, in a steadily growing economy, the tax rate on the deterministic component of total wealth, τ, is nondistortionary: It operates essentially as a lump-sum tax. However, this is not true of the stochastic component, which will have real effects through the portfolio decision.

The wealth accumulation of the representative consumer can be redefined to separate deterministic and stochastic components as follows:

$$\frac{dW}{W} = \psi dt + dw, \tag{11a}$$

$$\psi = n_M r_M + n_B r_B + n_K r_K + n_F r_F - \tau - C(t)/W(t), \tag{11b}$$

$$dw = -n_M dp - n_B dp + n_K \alpha dy - n_F dq - dv. \tag{11c}$$

The maximization of (1) subject to the stochastic differential equation (11a) represents a continuous-time stochastic dynamic programming problem of the type discussed by Merton (1969) and Turnovsky (1995). The solution method differs

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from theirs due to the time-inseparability feature of the recursive utility function and can be found in Svensson (1989) and/or Grinols (1996).[6] The household maximizes utility by choosing the optimal full (composite) consumption–wealth ratio and the optimal portfolio shares of assets, taking the rates of return on assets, and the relevant variances and covariances as given.[7]

2.2. Government Policy

The government engages in four activities: (i) choosing its expenditure and financing it by (ii) taxation, (iii) printing money, and (iv) issuing bonds. Government expenditure G is determined by the following expression:

$$dG = g\alpha K\,dt + \alpha K\,dz, \tag{12}$$

which means that the instantaneous mean level of public spending is a fraction g of the mean level of economy output, with a stochastic disturbance dz, which is independent of dy.

The government pursues the following monetary and borrowing policy rules:

$$dM/M = \phi dt + dx, \tag{13}$$

$$B/M = \lambda, \tag{14}$$

where ϕ is the mean monetary growth rate; dx is normally distributed and independent over time with zero mean and variance $\sigma_x^2 dt$; and λ is a policy parameter reflecting the choice between monetary expansion and borrowing, set by the government. In addition, dx is not correlated with dy, the random component of production, nor with dz, the random component of government expenditure, but it is correlated with dq, the random component of the world price. Correlation between dx and dq implies that dx may reflect exogenous stochastic failures to meet a monetary growth target set by the monetary authority or may reflect stochastic adjustments in the money supply as the authorities respond to exogenous stochastic movements in an intermediate target, the exchange rate.

Finally, the government budget constraint is

$$d(M/P) + d(B/P) = (B/P)dR_B + (M/P)dR_M + dG - dT. \tag{15}$$

Given the policy rules for monetary growth and borrowing, the stochastic component of taxation must adjust to maintain the government budget constraint.

2.3. Goods Market Equilibrium and Balance of Payments

For goods market equilibrium in our small open-economy model, changes in net exports will equal the excess of changes in production over domestic absorption,

$$dY - dC - dK - dG, \tag{16}$$

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and is related to the balance-of-payments equilibrium condition,

$$d(EB^*/P) - (EB^*/P)dR_F = dY - dC - dK - dG. \quad \mathbf{(17)}$$

Defining a new variable ω, defined as $\omega = n_K/(n_K + n_F)$ to be the portfolio share of equity in tradeable assets, and dividing both sides of equation (17) by K leads to the following expression for the rate of growth of the capital stock:

$$\frac{dK}{K} = \psi dt + \omega\alpha(dy - dz) - (1-\omega)dq, \quad \mathbf{(18)}$$

where ψ is the growth rate and is defined as

$$\psi = \omega\left(\alpha(1-g) - \frac{1}{n_K}\frac{C}{W}\right) + (1-\omega)r_F. \quad \mathbf{(19)}$$

2.4. Macroeconomic Equilibrium

Given the assumption of constant drift (mean) and diffusion (variance) parameters of the geometric Brownian motion, which describe the model variables, risks and returns on assets are unchanging through time. This feature of the model, together with the constant-elasticity utility function, generates a recurring equilibrium, implying that the consumer chooses the same portfolio shares n_M, n_B, n_F, n_K and consumption–wealth ratio, C/W, at each instant of time. Furthermore, the multiplicity of all shocks, that is, stochastic disturbances, are proportional to the current state variables such as the capital stock and wealth; and this leads to an equilibrium in which means and variances of relevant endogenous variables are jointly and consistently determined—a *mean–variance equilibrium*.

The exogenous factors, other than the four stochastic shocks, dy, dz, dx, and dq, explained earlier include (i) the preference and technology parameters γ, ζ, ρ, θ, α; (ii) the policy parameters ϕ (monetary), g (government spending), λ (government borrowing); and (iii) the mean foreign inflation rate π^*. The endogenous variables include (i) the stochastic adjustments in the economy dp (the stochastic adjustment in the domestic price level), de (the stochastic PPP relationship), dv (the stochastic adjustment in taxes), dw (the stochastic component of wealth); (ii) the tax rate τ; (iii) the optimal consumption–wealth ratio and optimal portfolio shares; (iv) the equilibrium prices π (the expected rate of domestic inflation rate), i (nominal domestic interest rate), ϵ (the expected rate of exchange depreciation); and (v) the equilibrium growth rate ψ.

The determination of endogenous variables involves several stages. By using the assumption of constant portfolio shares, we first solve the model for the price level and thereby π and dp. The next stage is to determine stochastic adjustments. Once the stochastic adjustments are obtained, one can then calculate the endogenous variances and covariances that appear in the optimality conditions for the consumption–wealth ratio, portfolio shares, and elsewhere. The final stage is simply to substitute these variances and covariances into the deterministic components of the equilibrium.

2.4.1. Solution. The full solution is given in the Appendix to this paper, but key results are reproduced here for convenience.

The solution of consumption is

$$\frac{C}{W} = \frac{\theta}{1-\theta\zeta}\left[\rho - \zeta\left(\beta - \frac{1}{2}\gamma\sigma_w^2\right)\right], \tag{20a}$$

where $\beta = n_M r_M + n_B r_B + n_K r_K + n_F r_F - \tau$ and $\sigma_w^2 = \alpha^2\omega^2(\sigma_y^2 + \sigma_z^2) + (1-\omega)^2\sigma_q^2$. We define $\beta - \frac{1}{2}\gamma\sigma_w^2$ as the risk-adjusted rate of return. Expression (20a) reveals that the optimal consumption and saving decision depends on both the intertemporal elasticity of substitution and the coefficient of risk aversion. The optimal portfolio share of each of the four assets is given by

$$n_M = \left[\frac{\theta}{(1-\theta)}\right]\left[\frac{C/W}{i}\right], \tag{20b}$$

$$n_K = \omega[1-(1+\lambda)n_M], \tag{20c}$$

$$n_F = \frac{1-\omega}{\omega} n_K, \tag{20d}$$

$$n_B = \lambda n_M. \tag{20e}$$

The solutions for the exchange rate, inflation rate, and the interest rate are as follows:

$$\epsilon = \pi - \pi^* + \omega\sigma_q^2 - \sigma_{xq}, \tag{21}$$

$$\pi = \phi - \psi + \sigma_w^2 - \sigma_{xw}, \tag{22}$$

$$i = \alpha + \pi - \sigma_p^2 - \gamma\left[\alpha^2\omega(1-\omega)\sigma_y^2 - \alpha^2\omega^2\sigma_z^2 - (1-\omega)^2\sigma_q^2 - \sigma_{xw}\right], \tag{23}$$

where σ_p^2

$$\sigma_p^2 = \sigma_x^2 + \alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + (1-\omega)^2\sigma_q^2 + (1-\omega)\sigma_{xq}. \tag{24}$$

3. WELFARE CALCULATIONS

To measure the welfare effects of volatility, we focus on the welfare of the representative agent and first evaluate the expected lifetime utility associated with the optimal consumption path,

$$\mathcal{E}_0(U) = \delta\frac{W(0)^{1-\gamma}}{1-\gamma}, \tag{25}$$

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where

$$\delta = (C/W)^{-\frac{(1-\zeta\theta)(1-\gamma)}{\zeta}} \theta^{\frac{(1-\gamma)}{\zeta}} \left[\frac{1-\theta}{\theta}\right]^{(1-\theta)(1-\gamma)} i^{-(1-\theta)(1-\gamma)}. \quad \textbf{(26)}$$

Then, following the work of Obstfeld (1994b) and Epaulard and Pommeret (1998), we utilize a definition of the welfare cost as follows.

DEFINITION 1. *The welfare cost is defined as the percentage of initial wealth the representative agent is ready to give up at period zero to be as well off under* $(\psi(\tilde{\Omega}), \tilde{\Omega})$ *as under* $(\psi(\Omega), \Omega)$ (*i.e., it is an* "*equivalent variation measure*"). Ω *is the usual variance–covariance matrix* (*after the policy change*) *and* $\tilde{\Omega}$ *represents the benchmark case for certain shocks with zeros for variance and covariances in the corresponding row and column*; ψ *summarizes the expected mean growth rate*, *which in this model is affected by changes in volatility*, *unlike many of the earlier models used in the literature*.

Thus, denoting the cost by κ,

$$\mathcal{E}_0[U(W(0), \mu(\Omega), \Omega)] = \mathcal{E}_0[U((1-\kappa)W(0), \mu(\tilde{\Omega}), \tilde{\Omega})]. \quad \textbf{(27)}$$

Using (25), we can write the cost of the volatility as

$$\kappa = 1 - \left(\frac{\delta(\mu(\Omega), \Omega)}{\delta(\mu(\tilde{\Omega}), \tilde{\Omega})}\right)^{1/(1-\gamma)}. \quad \textbf{(28)}$$

Equation (28) can be used to quantify the effects of policy changes on economic welfare. In this model, the government's policy parameters relate to monetary growth, government expenditure, and the government's borrowing policy. The exogenous stochastic processes include monetary growth (dx), government expenditure (dz), foreign price (dq), and productivity (dy). However, although equation (28) provides the basis for quantification of the welfare effects, the generation of numerical estimates requires the specification of a number of baseline parameters and variables. Table 1 sets out the values used in the numerical exercises carried out here.

No particular claim is made for the precision of these numerical values; rather, the intention is to utilize plausible values. To achieve this, we choose parameter values in line with those used by Obstfeld (1994b) and Asea and Turnovsky (1998). Nevertheless, particular mention should be made of the values assigned to the risk aversion parameter and the intertemporal elasticity of substitution. The risk aversion parameter is assigned the value 4 as in Obstfeld (1994b), and which is the midpoint of the range of conventional estimates (2–6) referred to by Obstfeld (1994a). However, we are mindful that some authors suggest that values of unity or values as high as 30 cannot be ruled out [see Epstein and Zin (1991) and Kandel and Stambough (1991), respectively]. The intertemporal elasticity of substitution is set to 0.5, which is the value used by Obstfeld (1994b) and is consistent with what Epstein and Zin (1991) describe as "a reasonable inference." However, smaller

TABLE 1. Baseline parameters and variables

Variables	Symbol	Value
	Parameters	
Marginal product of capital	α	0.050
Risk aversion parameter	γ	4.000
Intertemporal elasticity of substitution	$1/(1-\zeta)$	0.500
Rate of time preference	ρ	0.020
Debt policy parameter	λ	0.240
Government size	g	0.250
Foreign interest rate	i^*	0.068
Foreign inflation rate	π^*	0.002
Variance of output	σ_y^2	0.020
Variance of government outlay	σ_z^2	0.015
Variance of money supply	σ_x^2	0.015
Variance of foreign price	σ_q^2	0.015
Covariance of money and world price	σ_{xq}	0.010
	Variables	
Consumption–wealth ratio	$\frac{C}{W}$	0.015
Beta	β	0.048
Mean equilibrium growth rate	ψ	0.033
Variance of growth rate	σ_w^2	0.001
Variance of price level	σ_p^2	0.019
Inflation rate	π	0.046
Interest rate	i	0.094
Exchange rate	ϵ	0.045
Rate of return on money	r_M	−0.027
Rate of return on government bonds	r_B	0.067
Rate of return on capital	r_K	0.050
Rate of return on foreign bond	r_F	0.068
Risk-adjusted rate of return	$\beta - \frac{1}{2}\gamma\sigma_w^2$	0.045
Portfolio share of equity in tradables	ω	0.700
Portfolio share of money	n_M	0.127
Portfolio share of bonds	n_B	0.031
Portfolio share of equity	n_K	0.589
Portfolio share of foreign money	n_F	0.253

values cannot be ruled out: For example, Hall (1988) and Campbell and Mankiw (1989) suggest an intertemporal elasticity of substitution of 0.10, and Ogaki and Reinhart (1998) refer to the range 0.32–0.45. Later in this paper, we calculate welfare costs for a range of values for these preference parameters and carry out a sensitivity analysis of how these preference parameters influence the numerical estimates of welfare costs.

TABLE 2. Welfare cost of policy shocks

Policies	Growth effects	Welfare cost[a]
Elimination of government expenditure shock	−0.037	0.00133
Elimination of monetary growth shock	0.209	−0.01419
Elimination of both policy shocks	−0.031	−0.01285

[a] Welfare cost is measured by the present discounted value of expected utility over the horizon of the representative household along the path realized by the economy, and it is defined as a proportion of initial wealth. It is an equivalent variation measure and is presented as the equivalent variation in the bottom row of Table 3.

However, first we focus on the calculated welfare costs from different experiments, using Table 1 values. The calculated welfare costs are set out in Table 2. Positive values indicate that the welfare cost is positive, and the numerical value measures the cost as a proportion of initial wealth. Negative values indicate that there is a gain in welfare. The benchmark for the experiments assigns a zero welfare cost to the situation in which all four stochastic shocks may take nonzero values and both the inflation rate and the interest rate are free to vary. Each row of the table represents a different set of restrictions on the two exogenous stochastic components, dz (government expenditure shocks) and dx (monetary growth shocks). In the benchmark case, both shocks are present alongside the productivity shock dy and the foreign price shock dq. As one reads down the table, it can be seen that elimination of government expenditure shocks from the benchmark situation results in a calculated welfare loss, whereas removing monetary growth shocks (and retaining government expenditure shocks) yields a welfare gain. Elimination of both policy shocks yields a welfare gain.[8] The inference is that households would be willing to give up 1.3% of their initial wealth to live in a world with neither monetary growth nor government expenditure shocks; yet it is monetary growth shocks that are damaging to welfare. Indeed households are better off with than without government expenditure shocks.

To see how these welfare effects come about, we focus on key features of the model. Central is the (representative) household-maximizing utility by choosing the optimal consumption–wealth ratio and the optimal portfolio shares of assets, taking the rates of return on assets and the relevant variances and covariances as given. Risk aversion leads the household to reduce demand for relatively high-risk (high-return) assets. If this involves a reduction in demand for the productive asset, K, this results in the mean growth rate being sacrificed to avoid risk. However, the household's dislike of fluctuations in the economy—characterized by the coefficient of intertemporal substitution—is also important [see equation (20a)]. If households are fluctuations-averse, they respond to such fluctuations by increasing saving and, *ceteris paribus*, the consumption–wealth ratio falls. Thus, changes in the variance of the growth rate are important.

Table 3 summarizes changes in the values of key variables in the model as a result of the different experiments.

TABLE 3. Effects of policy shocks

		% of change	
Variable	Symbol	Government expenditure shock	Monetary growth shock
Consumption–wealth ratio	$\frac{C}{W}$	0.049	0.138
Beta	β	−0.011	0.188
Mean equilibrium growth rate	ψ	−0.037	0.209
Variance of growth rate	σ_w^2	−1.319	0.000
Variance of price level	σ_p^2	−0.095	−92.818
Inflation rate	π	−0.013	−6.654
Interest rate	i	−0.065	3.112
Exchange rate	ϵ	−0.014	15.355
Rate of return on money	r_M	0.046	55.831
Rate of return on government bonds	r_B	−0.109	−17.796
Risk-adjusted rate of return	$\beta - \frac{1}{2}\gamma\sigma_w^2$	0.070	0.199
Portfolio share of money	n_M	0.114	−2.884
Portfolio share of bonds	n_B	0.114	−2.884
Portfolio share of equity	n_K	−0.021	0.541
Portfolio share of foreign money	n_F	−0.021	0.541
Equivalent variation	EV	0.001	−0.014

3.1. Elimination of Government Expenditure Shocks

As has been noted, the overall effect of removing volatility in government expenditure is to worsen the welfare of the representative household. One main reason for this can be traced to the fact that volatility in government expenditure is reflected in volatility in tax revenues. As a result, the elimination of government expenditure shocks removes tax revenue shocks, which in turn increases volatility in the household's wealth and damages welfare.[9] Admittedly, this increase in volatility encourages precautionary savings, as in Leland (1974) and these savings enhance growth and thus welfare. Yet it is clear from our experiment (see the percentage reduction in the mean equilibrium growth rate in Table 3) that the net effect is a welfare loss.

A clear reason for this depressing effect on the mean equilibrium growth rate can be traced back to our allowance of government borrowing in the model: Less volatility in government expenditure makes government debt less risky, making government bonds more attractive relative to other assets, *including* the productive asset, equity. This reduces the portfolio share of equity, resulting in a fall in the mean equilibrium growth rate—a "crowding out" of private assets (equity) as found by Eaton (1981) and Turnovsky (1995). This impact on the mean growth rate is in striking contrast to the models of Lucas (1987) and Obstfeld (1994a), where the mean growth rate is restricted to be unaffected by volatility.

3.2. Elimination of Monetary Growth Shocks

Turning to the elimination of monetary growth shocks, we can see from Table 3 that this reduction in monetary growth risk gives rise to declines in the variance of the price level [see also equation (24)] and the mean inflation rate (22) and a rise in the domestic nominal interest rate (23). This latter effect is attributable to the fall in monetary growth risk having increased the volatility of the interest rate, requiring a higher interest rate to compensate for the risk.[10] This rise in the nominal interest rate pushes down the portfolio share of money (20b), which in turn directs more funds toward the productive asset (i.e., increases n_K) (20c), resulting in an increase in the mean growth rate (19) by about 0.2% and thus the consumption–wealth ratio (20a).

The impact on welfare cannot be determined unambiguously. Of the three elements from which utility is derived, money balances have fallen, as has initial wealth,[11] both *welfare deteriorating*; meanwhile, the mean growth rate has risen, *welfare improving*. What is evident from the results in Tables 2 and 3 is that *welfare improvement* dominates in this case.

The intuition behind our result is as follows: With all assets in the portfolio now less risky, even risk-averse households can shift toward the riskier assets in their portfolio and maintain the overall risk level of the portfolio. The portfolio rate of return rises, an adjustment fully reflected in the risk-adjusted rate of return; and there is a balancing of the portfolio in favor of the riskier and productive asset, promoting growth and improving welfare.

3.3. Elimination of Both Government Expenditure and Monetary Growth Shocks

Evidence from the experiments conducted here suggests that the welfare effect of eliminating government expenditure shocks is outweighed by that attributable to the elimination of monetary growth shocks. However, it may be inappropriate to place too much weight on the particular welfare costs presented in Tables 2 and 3. It is well known that the degree of risk aversion embedded in consumer preferences is a key parameter in welfare evaluations of economic policies conducted in dynamic stochastic models; and since Obstfeld (1994a), there has been an awareness that it is misleading to assume that risk aversion and intertemporal substitutability cannot vary independently. Indeed, the timing of risk resolution is important for the evaluation of welfare: Kreps and Porteus (1978) have shown that (independent) coefficients for risk aversion and intertemporal substitution have a straightforward meaning. In particular, high values of risk aversion relative to intertemporal substitution reflect agents who dislike risk more than they dislike fluctuations in the economy and, as a result, will prefer an early resolution of risk uncertainty (i.e., will prefer to consume sooner rather than later); low values of risk aversion relative to intertemporal substitution reflect a dislike of fluctuations that is greater than the dislike of risk and thus agents prefer a late resolution of risk uncertainty (i.e., prefer to save now). We calculate the welfare costs for a range of

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values for these preference parameters, and in Figure 1, present the results of this sensitivity analysis with respect to parameters for risk aversion and intertemporal elasticity of substitution.

3.4. Sensitivity Analysis

In Figure 1A we again make inference about the welfare cost of government expenditure shocks. In our previous analysis, we found that government expenditure shocks enhanced welfare because their elimination resulted in a welfare loss for the representative household. This result is now found to be robust to the range of parameter values considered here for risk aversion and the elasticity of substitution, although higher risk aversion and higher values for the elasticity of substitution lead to higher values of the welfare effect of government expenditure shocks. This is not surprising: higher risk aversion strengthens the effect of removing some insurance from households and of removing risk from government debt. Similarly, the more households dislike fluctuations in the economy, the greater is the effect of reducing the variability of the growth rate on saving.

However, note that the range of calculated values of welfare cost is itself small (all values are positive and less than 1% of initial wealth). This is in sharp contrast to the evidence presented in Figure 1B, where the range of calculated values for the welfare effects of eliminating monetary growth shocks is markedly wider (ranging from -6% to $+18\%$ of initial wealth); and the switch in sign from negative to positive is important. The previous result that monetary growth shocks damage welfare now needs to be qualified: This result could be reversed with a high enough risk aversion parameter. This is because an important part of the process by which the welfare effects come about is a rebalancing of the portfolio in favor of riskier assets. With regard to the elasticity-of-substitution parameter, it can be seen from Figure 1B that the calculated welfare cost is largely insensitive to the range of values considered here.

4. CONCLUSION

This paper has calculated the welfare cost attributable to government expenditure shocks and monetary growth shocks in a continuous-time, general equilibrium model of a small open economy that is subject to a variety of stochastic disturbances. We find that eliminating government expenditure shocks from the model is welfare deteriorating, whereas removing monetary growth shocks improves welfare for the representative household.

In addition, we conduct some sensitivity analyses, varying the coefficients for risk aversion and intertemporal substitution and find that the magnitude of calculated welfare costs is sensitive to the parameter values chosen. Also, there are important differences between the effects attributable to the two shocks. The results for the elimination of government expenditure shocks are straightforward: Removal of these shocks is always welfare deteriorating, the estimates of welfare

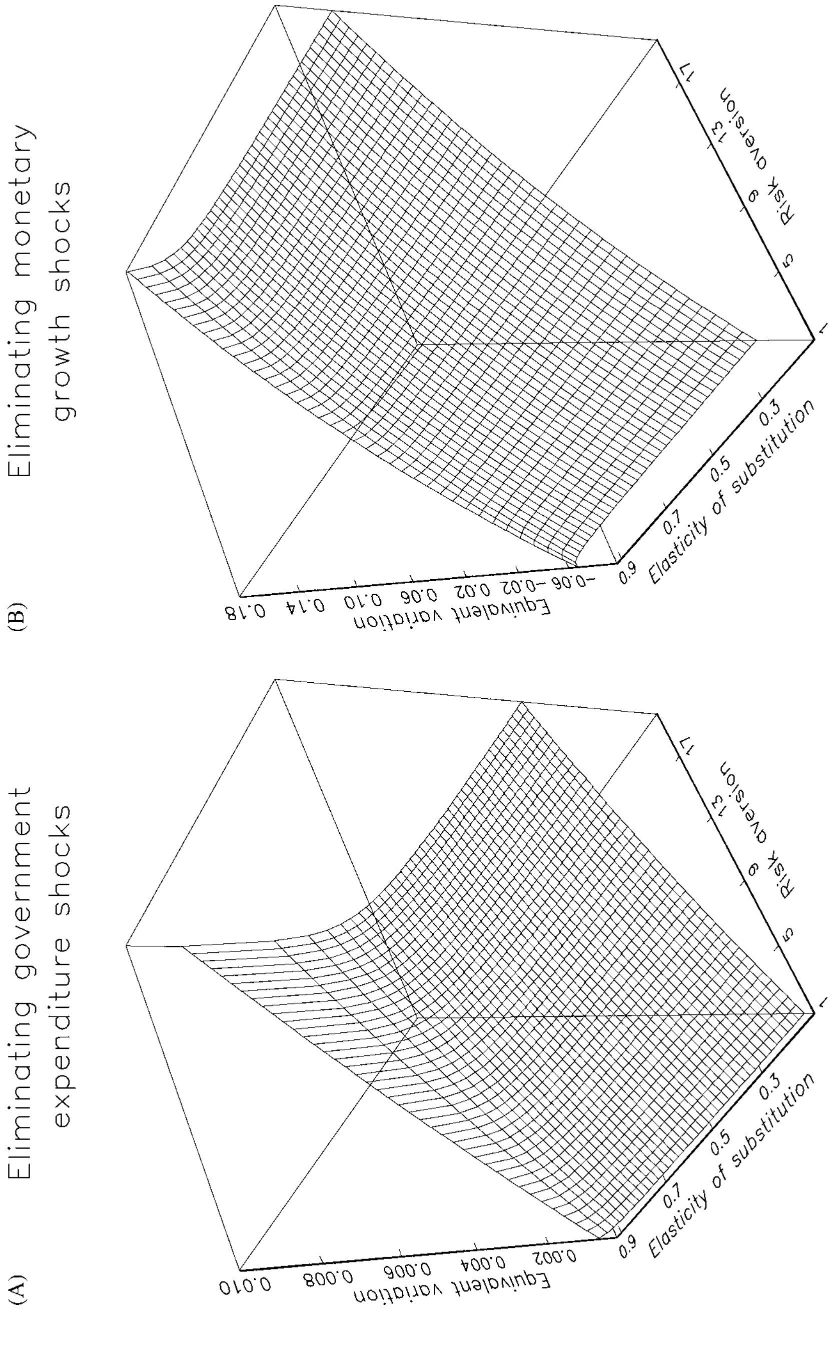

FIGURE 1. Welfare costs of eliminating policy shocks.

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cost are positively related to the values of these parameters, and the range of calculated values is small (less than 1% of initial wealth). However, elimination of monetary growth shocks is found to have a wide range of welfare effects associated with it, which may be either positive or negative. The calculated values for the welfare effects suggest that the elimination of monetary growth shocks may be either welfare enhancing (as large as 6% of initial wealth) or welfare deteriorating and it is the degree of risk aversion that drives this outcome. It seems therefore that although there is unambiguous evidence that households are better off with than without government expenditure shocks (a consequence of a portfolio shift across more or less risky government bonds and equity capital), no such conclusion can be made for monetary growth shocks. There is some evidence that households are better off without monetary growth shocks, but with high risk aversion, this result may be reversed.

This paper makes two key contributions to the literature. First, it has highlighted the distinct and separate contributions of the preference parameters to the evaluation of welfare effects of shocks and has thus signaled the need for further empirical work to establish bounds for these deep parameters. Second, it finds estimates of the welfare costs that are larger than those typically found in the literature—a consequence of our general (not partial) equilibrium approach and, more importantly, our allowing volatility to affect the mean growth rate in our model. The policy implications of this are important. The finding that the welfare cost of business-cycle fluctuations is potentially much larger than that found by Lucas forces modification of Lucas's conclusion—*that the gain from eliminating business-cycle fluctuations is quite small*—and forces some modification of its attendant strong policy conclusions.

NOTES

1. It is perhaps worth noting here that shocks in government expenditure could be attributed to active government behavior—in which case, government behavior may itself insure against aggregate risk for the economy. Furthermore, there may be analogies with the stochastic taxation literature [see Domar and Musgrave (1944), Stiglitz (1969), and Sandmo (1989)], which suggests that taxes on stochastic components of income create insurance effects (risk shared by the government) and thus lead to positive effects on risk taking, growth, and welfare. In models in which nondistortionary taxes are used to finance government expenditures, the presence of government expenditure shocks reduces the volatility of households' income and thus may increase risk taking, growth, and welfare. Thus, elimination of government expenditure shocks increases the risk faced by households, and this is detrimental to welfare.

2. There is an obvious cautionary note here: In this paper, we focus on policy shocks as the source of volatility, whereas much of the literature focuses on productivity shocks. Nevertheless, our paper is a conscious development of a literature that has been finding that the welfare cost of business-cycle fluctuations is potentially much larger than that found by Lucas [see, e.g., Imrohoroglu (1989) and Dolmas (1998)], forcing some modification of the Lucas conclusion and its attendant policy conclusions.

3. The assumption that some assets are nontraded does not pose any problem as long as the risk characteristics of these nontraded assets can be replicated with those of the traded assets; i.e., nontraded assets are spanned. In other words, markets are complete in the sense that the number of stochastic processes equals the number of traded assets.

4. The utility function employed here disentangles risk aversion from intertemporal substitution as proposed by Epstein and Zin (1989) and Weil (1990). There are two reasons for choosing a utility function with this property: (i) It has been shown that dynamic welfare comparisons that conflate risk aversion and intertemporal substitutability can be misleading [Obstfeld (1994a)], and (ii) one would like to answer questions about how preference parameters influence the numerical estimates of welfare costs. There is a range of possible formulations for the recursive utility function [see Dolmas (1998)]: The particular formulation used in this paper was chosen by reference to a criterion of requiring tractability of the model [and differs slightly from that used by Grinols (1996)]. Our formulation resembles that of Svensson (1989) and Epaulard and Pommeret (1998).

5. To derive equation (7), we assume a linear investment technology for capital.

6. The solution is available from the authors on request. It follows the solution method of Svensson (1989).

7. However, the general equilibrium conditions, i.e., market-clearing conditions, of the model will determine these rates of return, variances, and covariances.

8. The outcome of monetary growth shocks is consistent with the findings of Carlstrom and Fuerst (1995) and the analytical results obtained by Turnovsky (1993). The outcome of government expenditure shocks is consistent with the analytical results obtained by Eaton (1981).

9. From an institutive perspective, one might think of the government bearing some of the impact of exogenous shocks. When we eliminate government expenditure shocks from the model we restrict the extent to which risk is shared between the government and households. In this sense, government expenditure shocks could be seen as providing insurance to households and this contributes to their welfare. The extent to which government expenditure shocks reflect active or passive government behavior is important here.

10. It can be seen from (23) that there are three influences on the interest rate: π, σ_p^2, and σ_{xw}. For the parameter values assumed here, the σ_p^2 effect dominates. However, as we shall see later, different values of the risk aversion parameter will change this.

11. Initial real wealth falls as a result of the initial price jump required to maintain portfolio balance in stock terms [see Turnovsky (1993)].

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APPENDIX. SOLUTION TO THE HOUSEHOLD'S OPTIMIZATION PROBLEM

The representative household's optimization problem is to find the solution to

$$\lim_{dt\to 0^+} \max_{\{C,\vec{n}\}} e^{-\rho t}\left\{[C(t)^{\theta}(M(t)/P(t))^{1-\theta}]^{\zeta}dt + e^{-\rho dt}\left[\mathcal{E}_t U(t+dt)^{1-\gamma}\right]^{\zeta/(1-\gamma)}\right\}^{1/\zeta} \tag{A.1a}$$

subject to

$$\frac{dW}{W} = \psi dt + dw, \tag{A.1b}$$

$$\vec{n}'i = 1, \tag{A.1c}$$

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$$\vec{n} = \begin{pmatrix} n_M \\ n_B \\ n_K \\ n_F \end{pmatrix} \qquad i = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \tag{A.2}$$

$$\psi = \vec{n}'\vec{r} - C/W - v, \qquad r = \begin{pmatrix} r_M \\ r_B \\ r_K \\ r_F \end{pmatrix}, \tag{A.3}$$

$$dw = \vec{n}'\vec{du} - dv, \qquad du = \begin{pmatrix} -dp \\ -dp \\ \alpha dy \\ -dq \end{pmatrix}, \tag{A.4}$$

$$\sigma_w^2 = \vec{n}'\Omega\vec{n} - 2\vec{n}'\vec{\sigma}_{vu} + \sigma_v^2, \tag{A.5}$$

where Ω is the variance–covariance matrix of $\vec{du}$.

The Bellman function associated with the problem is then defined as

$$\begin{aligned}(1-\gamma)X(W(t)) = \lim_{dt\to 0^+} \max_{\{C,\vec{n}\}} e^{-\rho t}\Big\{[C^\theta (M/P)^{1-\theta}]^\zeta dt \\ + e^{-\rho dt}[(1-\gamma)\mathcal{E}_t X(W(t+dt))]^{\zeta/(1-\gamma)}\Big\}^{(1-\gamma)/\zeta}\end{aligned} \tag{A.6}$$

Postulate a value function $X(W(t), t)$ for some constant δ of the form

$$X(W(t), t) = e^{-\rho t}\frac{\delta W(t)^{1-\gamma}}{1-\gamma}.$$

Its current value version is given by

$$V(W(t)) = \frac{\delta W(t)^{1-\gamma}}{1-\gamma}. \tag{A.7}$$

The expression $\mathcal{E}_t V(W(t+dt))$ can be calculated from the following relationship:

$$\mathcal{E}_t V(W(t+dt)) - \mathcal{E}_t V(W(t), t) = \mathcal{E}_t(dV). \tag{A.8}$$

Using Ito's formula, we calculate $\mathcal{E}_t(dV)$ as

$$\mathcal{E}_t(dV) = \frac{\partial V}{\partial W}\mathcal{E}(dW) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\mathcal{E}(dW)^2 \tag{A.9}$$

Applying methods of stochastic calculus, the preceding expression is rewritten as

$$\mathcal{E}_t(dV) = \psi W\frac{\partial V}{\partial W}dt + \frac{1}{2}\sigma_w^2 W^2\frac{\partial^2 V}{\partial W^2}dt. \tag{A.10}$$

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After substituting and simplifying, from (A.10) we obtain an expression for $\mathcal{E}_t V(W(t+dt))$:

$$\mathcal{E}_t V(W(t+dt)) = \psi W \frac{\partial V}{\partial W} dt + \frac{1}{2}\sigma_w^2 W^2 \frac{\partial^2 V}{\partial W^2} dt + \frac{\delta W(t)^{1-\gamma}}{1-\gamma}. \tag{A.11}$$

Using the definition of the current value function $V(W)$, we compute the partial derivatives as

$$V_W = \delta W^{-\gamma}, \tag{A.12}$$

$$V_{WW} = -\gamma \delta W^{-\gamma-1}, \tag{A.13}$$

Finally, we get

$$\mathcal{E}_t V(W(t+dt)) = \left[(1-\gamma)\psi dt + \frac{1}{2}(\gamma-1)\gamma\sigma_w^2 dt + 1\right] \frac{\delta W^{1-\gamma}}{1-\gamma}. \tag{A.14}$$

Substituting (A.14) into (A.6)

$$\delta W^{1-\gamma} = \lim_{dt\to 0^+} \max_{\{C,\vec{n}\}} \left\{ [C^\theta (M/P)^{1-\theta}]^\zeta dt + e^{-\rho dt}\left[(1-\gamma)\left[\psi dt - \frac{1}{2}\sigma_w^2\gamma dt + \frac{1}{1-\gamma}\right]\delta W^{1-\gamma}\right]^{\zeta/(1-\gamma)} \right\}^{1/\zeta}. \tag{A.15}$$

We may conjecture that the consumption function is linear in wealth:

$$C = ZW \qquad Z > 0,$$

where Z is a constant to be determined.

Substituting the definitions of consumption and real balances and rewriting the resulting expression, we obtain

$$\delta^{\frac{\zeta}{1-\gamma}} W(t)^\zeta = \lim_{dt\to 0^+} \max_{\{C,\vec{n}\}} \left\{ \left[Z^\theta n_M^{1-\theta}\right]^\zeta W^\zeta dt + e^{-\rho dt}\left[(1-\gamma)\left[\psi dt - \frac{1}{2}\sigma_w^2\gamma dt + \frac{1}{1-\gamma}\right]\delta W^{1-\gamma}\right]^{\zeta/(1-\gamma)} \right\}.$$

The expression in curly brackets on the right-hand side can then be written as

$$\lim_{dt\to 0^+} \max_{\{C,\vec{n}\}} \left\{ \left[Z^\theta n_M^{1-\theta}\right]^\zeta W^\zeta dt + \left[\zeta\left(\psi - \frac{1}{2}\gamma\sigma_w^2\right)dt - \rho dt + 1\right]\delta^{\frac{\zeta}{1-\gamma}} W^\zeta \right\}.$$

Dividing by $\delta^{\frac{\zeta}{1-\gamma}} W(t)^\zeta$, subtracting 1, dividing by dt, and taking the limit yields

$$0 \equiv \max_{\{C,n\}} \left\{ \left[Z^\theta n_M^{1-\theta}\right]^\zeta / \left[\delta^{\frac{\zeta}{1-\gamma}}\right] + \zeta\left(\psi - \frac{1}{2}\gamma\sigma_w^2 - \rho\right) \right\} \tag{A.16}$$

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[This follows Obstfeld (1994b)]. First-order conditions are

$$\frac{\partial}{\partial Z}\colon \frac{\zeta\theta\left[Z^{\theta}n_M^{1-\theta}\right]^{\zeta}}{\left[Z\delta^{\frac{\zeta}{1-\gamma}}\right]} + \zeta = 0, \tag{A.17}$$

and

$$\partial/\partial\vec{n}\colon \begin{bmatrix} \zeta(1-\theta)\left[Z^{\theta}n_M^{1-\theta}\right]^{\zeta}/\left[n_M\delta^{\frac{\zeta}{1-\gamma}}\right] \\ 0 \\ 0 \\ 0 \end{bmatrix} + \zeta\left(\frac{\partial\psi}{\partial\vec{n}} - (1/2)\gamma\left(\frac{\partial\sigma_w^2}{\partial\vec{n}}\right)\right) - \zeta\xi i = 0, \tag{A.18}$$

where ξ is the Lagrange multiplier for (A.1c). Equation (A.17) implies that

$$\theta\left[Z^{\theta}n_M^{1-\theta}\right]^{\zeta} = \left[Z\delta^{\frac{\zeta}{1-\gamma}}\right], \tag{A.19}$$

which, after substituting and simplifying, we obtain

$$Z = \frac{\theta}{1-\zeta}\left[\rho - \zeta\left(\beta - \frac{1}{2}\gamma\sigma_w^2\right)\right], \tag{A.20}$$

and

$$\begin{bmatrix} (1-\theta)C/\theta n_M \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} r_M \\ r_B \\ r_K \\ r_F \end{bmatrix} - \gamma \begin{bmatrix} \mathrm{cov}(dw, -dp) \\ \mathrm{cov}(dw, -dp) \\ \mathrm{cov}(dw, \alpha dy) \\ \mathrm{cov}(dw, -dq) \end{bmatrix} - \begin{bmatrix} \xi \\ \xi \\ \xi \\ \xi \end{bmatrix} = 0, \tag{A.21}$$

where $\beta = n_M r_M + n_B r_B + n_K r_K + n_F r_F - \tau$.

Subtracting the second relation in equation (A.21) from the first one, we obtain an expression for n_M as follows:

$$n_M = \left[\frac{\theta}{(1-\theta)}\right]\left[\frac{C/W}{i}\right]. \tag{A.22a}$$

Similarly, subtracting from the remaining rows (3 and 4) yields

$$(r_K - r_B)dt = \gamma\,\mathrm{cov}(dw, \alpha dy + dp), \tag{A.22b}$$

$$(r_F - r_B)dt = \gamma\,\mathrm{cov}(dw, -dq + dp). \tag{A.22c}$$

A.1. DERIVATION OF THE RATE OF GROWTH OF THE CAPITAL STOCK

Substituting for dR_F, dY, and dG in (17) and noting that $dC = C\,dt$ yields

$$dK + d(B^*/Q) = [\alpha(1-g)K - C + r_F(B^*/Q)]dt + \alpha K(dy - dz) - (B^*/Q)dq. \tag{A.23}$$

Dividing (A.23) by $K + B^*/Q$ and noting that $\omega = n_K/(n_K + n_F)$ yields

$$\omega \frac{dK}{K} + (1-\theta\omega)\frac{d(B^*/Q)}{B^*/Q} = \left[\omega\left(\alpha(1-g) - \frac{1}{n_K}\frac{C}{W}\right) + (1-\omega)r_F\right]dt + \omega\alpha(dy - dz) - (1-\omega)dq. \tag{A.24}$$

Using the asset market equilibrium condition

$$\frac{d(B^*/Q)}{B^*/Q} = \frac{dK}{K},$$

equation (A.24) is expressed as the rate of growth of the capital stock by the relationship

$$\frac{dK}{K} = \left[\omega\left(\alpha(1-g) - \frac{1}{n_K}\frac{C}{W}\right) + (1-\omega)r_F\right]dt + \omega\alpha(dy - dz) - (1-\omega)dq \tag{A.25}$$

A.2. DERIVATION OF THE PRICE LEVEL

From the constant portfolio shares assumption, we can write

$$\frac{M/P}{K + B^*/Q} = \frac{n_M}{n_K + n_F}. \tag{A.26}$$

The price level can then be written as

$$P = \left[\frac{n_K + n_F}{n_M}\right]\left[\frac{K + B^*/Q}{M}\right]. \tag{A.27}$$

Taking the stochastic differential of (A.27) (noting that portfolio shares are constant through time) leads to

$$\frac{dP}{P} = \pi dt + dp = \frac{dM}{M} - \frac{d[K + B^*/Q]}{K + B^*/Q} - \left(\frac{dM}{M}\right)\left(\frac{d[K + B^*/Q]}{K + B^*/Q}\right) + \left(\frac{d[K + B^*/Q]}{K + B^*/Q}\right)^2. \tag{A.28}$$

Using (13) and (A.23), noting that the variances are of order dt, the right-hand side of this equation can be expressed as

$$\left\{\phi - \left(\omega\left[\alpha(1-g) - \frac{1}{n_K}\frac{C}{W}\right] + (1-\omega)r_F\right) + \alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + (1-\omega)^2\sigma_q^2 - \alpha\omega(\sigma_{xy} - \sigma_{xz}) + (1-\omega)\sigma_{xq}\right\}dt + dx - \alpha\omega(dy - dz) + (1-\omega)dq. \tag{A.29}$$

Equating the deterministic and stochastic components of (A.28) implies

$$\pi = \phi - \left(\omega\left[\alpha(1-g) - \frac{1}{n_K}\frac{C}{W}\right] + (1-\omega)r_F\right) + \alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + (1-\omega)^2\sigma_q^2 - \alpha\omega(\sigma_{xy} - \sigma_{xz}) + (1-\omega)\sigma_{xq}, \tag{A.30}$$

$$dp = dx - \alpha\omega(dy - dz) + (1-\omega)dq. \tag{A.31}$$

A.3. DETERMINATION OF TAX ADJUSTMENTS

To determine the tax adjustments, we use the following government budget constraint:

$$d(M/P) + d(B/P) = (B/P)dR_B + (M/P)dR_M + dG - dT. \qquad \textbf{(A.32)}$$

Dividing both sides by W, we can rewrite this equation as

$$n_M \frac{d(dM/P)}{(dM/P)} + n_B \frac{d(dB/P)}{(dB/P)} = \frac{dG - dT}{W} + n_M dR_M + n_B dR_B.$$

By substitution for debt policy (14) into the above equation, this equation becomes

$$n_M \frac{d(dM/P)}{(dM/P)} + n_B \frac{d(\lambda dM/P)}{(dM/P)} = \frac{dG - dT}{W} + n_M dR_M + n_B dR_B.$$

By substitution for government expenditure policy (12), monetary policy (13), tax collection (10) and the price evolution (A.28) into the above equation, while noting the stochastic derivatives of $d(M/P)$ and $d(B/P)$, this equation becomes

$$(n_M + n_B)\left(\phi - \pi - \sigma_{xp} + \sigma_p^2\right)dt + (n_M + n_B)(dx - dp) = \Big[\alpha n_K g - \tau$$
$$+ n_M\left(-\pi + \sigma_p^2\right) + n_B\left(i - \pi + \sigma_p^2\right)\Big]dt + (n_M + n_B)dp + \alpha n_K dz - dv. \qquad \textbf{(A.33)}$$

Equating deterministic and stochastic parts of this equation leads to the following:

$$\tau = \alpha n_K g - (n_M + n_B)\phi + n_B i + (n_M + n_B)\sigma_{xp}, \qquad \textbf{(A.34)}$$

$$dv = \alpha n_K dz - (n_M + n_B)dx. \qquad \textbf{(A.35)}$$

A.4. MACROECONOMIC EQUILIBRIUM

The derivation of macroeconomic equilibrium takes place in two stages. The first stage involves the determination of stochastic components. The second stage involves substitution of these solutions into the deterministic components of the equilibrium.

A.4.1. Calculation of the Stochastic Components

The stochastic adjustments in the economy include (i) the stochastic adjustment in the domestic price level; (ii) the PPP relationship; (iii) the definition of the stochastic component of wealth; and (iv) the stochastic adjustment in taxes:

$$dp = dx - \alpha\omega(dy - dz) + (1 - \omega)dq, \qquad \textbf{(A.36a)}$$

$$de = dx - \alpha\omega(dy - dz) - \omega dq, \qquad \textbf{(A.36b)}$$

$$dw = \alpha\omega(dy - dz) - (1 - \omega)dq, \qquad \textbf{(A.36c)}$$

$$dv = \alpha n_K dz - (n_M + n_B)dx. \qquad \textbf{(A.36d)}$$

The first equation is determined from the price-level-determination equation. The second equation follows the stochastic PPP relationship equation (9b). The third equation is derived

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from the stochastic component of the capital accumulation equation (A.25). Finally, the last one is derived by the determination of tax adjustments.

Using equations (A.36c) and (A.36d), we can calculate the endogenous variances and covariances that appear in the optimality conditions (A.20), (A.20b), (A.22b), (22c), and elsewhere:

$$\sigma_w^2 = \alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + (1-\omega)^2\sigma_q^2, \tag{A.37a}$$

$$\sigma_p^2 = \sigma_x^2 + \alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + (1-\omega)^2\sigma_q^2 + (1-\omega)\sigma_{xq}, \tag{A.37b}$$

$$\mathrm{cov}(dw, \alpha dy + dp) = \left[\alpha^2\omega(1-\omega)\sigma_y^2 - \alpha^2\omega^2\sigma_z^2 - (1-\omega)^2\sigma_q^2 + \sigma_{xw}\right]dt, \tag{A.37c}$$

$$\mathrm{cov}(dw, dq) = -(1-\omega)\sigma_q^2 dt, \tag{A.37d}$$

$$\mathrm{cov}(dw, -dq + dp) = \left[-\alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + \omega(1-\omega)\sigma_q^2 + \sigma_{xw}\right]dt. \tag{A.37e}$$

A.4.2. Calculation of the Deterministic Components

Substituting for r_B, r_K, and r_F from (5b), (7), and (5c), respectively, together with expressions (A.37a)–(A.37e), equations (A.22b) and (A.22c) can be rewritten as

$$\alpha - i + \pi - \sigma_p^2 = \gamma\left[\alpha^2\omega(1-\omega)\sigma_y^2 - \alpha^2\omega^2\sigma_z^2 - (1-\omega)^2\sigma_q^2 + \sigma_{xw}\right], \tag{A.38a}$$

$$i^* - \pi^* + \sigma_q^2 - i + \pi - \sigma_p^2 = \gamma\left[-\alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + \omega(1-\omega)\sigma_q^2 + \sigma_{xw}\right]. \tag{A.38b}$$

Next, subtracting (A.38b) from (A.38a) yields the following solution for ω:

$$\omega = \frac{\alpha - \left(i^* - \pi^* + \sigma_q^2\right)}{\gamma\left(\alpha^2\sigma_y^2 + \sigma_q^2\right)} + \frac{\sigma_q^2}{\alpha^2\sigma_y^2 + \sigma_q^2}. \tag{A.39}$$

With portfolio shares remaining constant over time, all real components of wealth must grow at the same stochastic rate as follows:

$$\frac{d(M/P)}{M/P} = \frac{d(B/P)}{B/P} = \frac{d(B^*/P)}{B^*/P} = \frac{dK}{K} = \frac{dW}{W} = \psi dt + dw. \tag{A.40}$$

Taking expectations of the accumulation equation (A.25), using (A.40) and the definition of ω, the real rate of growth is given by the expression

$$\psi = \omega\left(\alpha(1-g) - \frac{1}{n_K}\frac{C}{W}\right) + (1-\omega)\left(i^* - \pi^* + \sigma_q^2\right). \tag{A.41a}$$

Substituting for (A.37a) and (A.37d) in (A.20) yields an expression for C/W:

$$\frac{C}{W} = \frac{\theta}{1-\theta\zeta}\left[\rho - \zeta\left(\beta - \tfrac{1}{2}\gamma\sigma_w^2\right)\right]. \tag{A.41b}$$

From (A.1b) and (A.40), an expression for β is obtained:

$$\beta = \psi + \frac{C}{W}. \tag{A.41c}$$

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The optimal portfolio share of money is given by

$$n_M = \left[\frac{\theta}{(1-\theta)}\right]\left[\frac{C/W}{i}\right]. \tag{A.41d}$$

Combining the debt policy (14) and the portfolio shares adding up condition (3), we obtain an expression for n_K:

$$n_K = \omega[1-(1+\lambda)n_M]. \tag{A.41e}$$

Once n_K is known, an expression for n_F is obtained from the definition of ω:

$$n_F = \frac{1-\omega}{\omega}n_K. \tag{A.41f}$$

Given n_M, equation (14) gives an expression for n_B

$$n_B = \lambda n_M. \tag{A.41g}$$

Equation (9a) gives an expression for ϵ, the exchange rate:

$$\epsilon = \pi - \pi^* + \omega\sigma_q^2 - \sigma_{xq}. \tag{A.42}$$

This is referred to as the "risk–adjusted" PPP equation:

$$\pi = \phi - \psi + \sigma_w^2 - \sigma_{xw}. \tag{A.43}$$

From (A.38a), we obtain i:

$$i = \alpha + \pi - \sigma_p^2 - \gamma\left[\alpha^2\omega(1-\omega)\sigma_y^2 - \alpha^2\omega^2\sigma_z^2 - (1-\omega)^2\sigma_q^2 + \sigma_{xw}\right]. \tag{A.44}$$

A.4.3. Derivation of Closed-Form Solution

To show the existence of a closed-form solution, we rewrite our key equations by collecting endogenous variables to the left-hand sides and collecting exogenous variables to the right-hand sides. These equations are (A.30), (A.38a), (9a), (A.38b) together with (9a), (A.41b) and (A.41c), (A.41d), and (A.41e):

$$\pi - Z/X = R1, \tag{A.45a}$$

where

$$R1 = \phi - \omega\alpha(1-g) + \omega(1-\omega)r_F + \alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + (1-\omega)^2\sigma_q^2$$
$$-\alpha\omega(\sigma_{xy} - \sigma_{xz}) + (1-\omega)\sigma_{xq}$$

and

$$X = \frac{n_K}{\omega},$$
$$i - \pi = R2, \tag{A.45b}$$

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where

$$R2 = \alpha - \sigma_p^2 - \gamma\left[\alpha^2\omega(1-\omega)\sigma_y^2 - \alpha^2\omega^2\sigma_z^2 - (1-\omega)^2\sigma_q^2 + \sigma_{xw}\right],$$
$$\pi - \epsilon = R3, \tag{A.45c}$$

where

$$R3 = \pi^* + \sigma_{qe},$$
$$i - \epsilon = R4, \tag{A.45d}$$

where

$$R4 = i^* + \sigma_q^2 - \sigma_p^2 - \gamma\left[-\alpha^2\omega^2\left(\sigma_y^2 + \sigma_z^2\right) + \omega(1-\omega)\sigma_q^2 + \sigma_{xw}\right] + \sigma_{qe},$$
$$Z + \theta\zeta\psi = R5, \tag{A.45e}$$

where

$$R5 = \theta\left(\rho + \tfrac{1}{2}\gamma\sigma_w^2\right),$$
$$\frac{n_M i}{Z} = R6, \tag{A.45f}$$

where

$$R6 = \frac{\theta}{(1-\theta)},$$
$$X + (1+\lambda)n_M = R7, \tag{A.45g}$$

where

$$R7 = 1.$$

Finally, the deterministic part of the balanced-growth-rate equation (A.41a) yields

$$\psi + Z/X = R8, \tag{A.45h}$$

where

$$R8 = \omega\alpha(1-g) + (1-\omega)\left(i^* - \pi^* + \sigma_q^2\right).$$

As in Grinols (1996), we formed a seven-equation system (A.45a)–(A.45g) in the seven unknowns $(Z, \pi, n_K, i, \epsilon, n_M, \psi)$.

The first two equations—(A.45a) and (A.45b)—can be solved for i for a given Z/X:

$$i - Z/X = R1 + R2. \tag{A.46a}$$

Substitute (A.45h) for Z/X into (A.46a) to obtain

$$i + \psi = R1 + R2 + R8. \tag{A.46b}$$

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Use (A.45e) to eliminate ψ:

$$i + \frac{Z}{\theta\zeta} = R1 + R2 + R8 + \frac{R5}{\theta\zeta} \tag{A.46c}$$

Equations (A.45f) and (A.45g) can be used to eliminate n_M:

$$X + [(1+\lambda)\theta Z]/[(1-\theta)i] = R7 \tag{A.46d}$$

Similarly, eliminating X (A.46d) yields another expression in two unknowns, Z and i:

$$-i + [(1+\lambda)\theta Z]/(1-\theta) + Zi/[i - R1 - R2] = 0. \tag{A.46e}$$

We now have two equations, (A.46c) and (A.46e), in two unknowns, Z and i. One way to solve this equation system is to solve (A.46c) for Z and substitute into (A.46e). This produces a quadratic equation in i, which can be solved for i by taking the larger value. Once i is solved, one can recursively solve the model: Equation (A.46c) gives Z and (A.46a) then gives X. The variables n_M, ϕ, π, and ϵ can then be recursively solved from (A.45). The remaining variables can be solved by exploiting the definitions used: Definition X gives n_K, and then ω gives n_F.