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Growth of a ring ripple on a Gaussian electromagnetic beam in a plasma with relativistic - ponderomotive nonlinearity

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Abstract

This paper presents a theoretical model for the propagation/growth of a ring ripple, on a Gaussian electromagnetic beam, propagating in plasma with dominant relativistic-ponderomotive nonlinearity. A paraxial like approach has been invoked to understand the nature of propagation of the ring ripple like instability; in this approach, all the relevant parameters correspond to a narrow range around the irradiance maximum of the ring ripple. The dielectric function is determined by the composite (Gaussian and ripple) electric field profile of the beam. Thus, a unique dielectric function for the beam propagation and a radial field sensitive diffraction term, appropriate to the vicinity of the maximum of the irradiance distribution of the ring ripple has been taken into account. The effect of different parameters on the critical curves has been highlighted and the variation of the beam width parameter with the distance of propagation has been obtained for the three typical cases *viz* of steady divergence, oscillatory divergence and self-focusing of the ripple.

Keywords: Paraxial like approach; Relativistic-ponderomotive nonlinearity; Ring ripple; Self-focusing

1. INTRODUCTION

The introduction of ultra high power laser technology in the field of laser plasma interaction (Kruer, 1988) has led to many theoretical and experimental studies, relevant among other areas to inertial confinement fusion (ICF) (Tabak *et al.*, 1994; Mulser & Bauer, 2004; Chen & Wilks, 2005; Badziak *et al.*, 2005; Hora *et al.*, 2005; Hora, 2005; Winterberg, 2008; Romagnani *et al.*, 2008; Deutsch *et al.*, 2008; Cook *et al.*, 2008; Kline *et al.*, 2009; Liu *et al.*, 2009), charged particle acceleration (Esarey *et al.*, 1988; Wyrtele, 1993; Esarey *et al.*, 1996; Bourdier *et al.*, 2007; Xie *et al.*, 2009), and ionospheric modification (Gurevich, 1978; Perkins & Goldman, 1981; Guzdar *et al.*, 1998; Keskinen & Basu, 2003; Gondarenko *et al.*, 2005). Pukhov and Meyer-ter-Vehn (1996) proposed a three-dimensional simulation model for short-pulse laser propagation in plasma with dominant relativistic nonlinearity. The work shows that the incident laser beam creates a single propagation channel with considerably enhanced irradiance on the axis; however, unstable filamentary propagation of the beam was also predicted.

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The propagation of laser beams through nonlinear media may be affected by plasma instability, characterized by growing electron density on account of irradiance fluctuations, transverse to the direction of the propagation. There are two basic approaches to analyze the growth/propagation of the instabilities in nonlinear media. One of the approaches considers an instability $E_1\exp[i(k_\perp x + k_\parallel z)]$, superposed on an intense electromagnetic laser beam $E_0\exp[i(\omega t + kz)]$, to obtain an expression for the spatial growth rate of the instability i.e., $ik_\parallel$ in terms of relevant parameters, and looks for the condition when $k_\parallel$ is imaginary (Talanov, 1966; Kaw *et al.*, 1973; Perkins & Valeo, 1974; Sodha *et al.*, 1976, 1978; Sodha & Tripathi, 1977; Kruer, 1985; Berger *et al.*, 1993; Ghanshyam & Tripathi, 1993; Wilks *et al.*, 1994; Vidal & Johnston, 1996). However, this approach is limited to the propagation of the instability in one direction in the transverse plane and to idealized beams with uniform irradiance, along the wavefront.

The other approach is based on the direct (Loy & Shen, 1969) and indirect (Abbi & Mahr, 1971) evidence, suggesting that the filamentational instability in nonlinear media is caused by the occurrence of strong irradiance spikes, riding on an incident smooth looking irradiance distribution in the plane, transverse to the direction of propagation. In laser plasma experiments, the filamentary structures created in the underdense plasma undergo self-focusing (Askaryan, 1962; Joshi *et al.*, 1982); this destroys the symmetry of energy deposition and triggers parametric instabilities, which may lead to back and side scattering of the main beam. Thus, the perturbation grows at the cost of the main beam; hence, its study is relevant to the physics of inertial confinement fusion and other heating experiments.

On the basis of the paraxial theory formulated by Akhmanov *et al.* (1968) and elaborated by Sodha *et al.* (1974, 1976), the growth of a Gaussian ripple on a plane uniform beam (Sodha *et al.*, 1979*a*, 2004, 2007; Singh *et al.*, 1979; Sharma *et al.*, 2004), and of a ring ripple on a Gaussian electromagnetic beam (Sodha *et al.*, 1979*b*, 1981, 1992, 2004; Pandey & Tripathi, 1990) in a plasma have been investigated to a significant extent. An interesting critique of the two approaches has been made by Sodha and Sharma (2007*b*). In recent investigations by Misra and Mishra (2008, 2009), a modified approach based on paraxial like approximation has been adopted to analyze the characteristic propagation properties of a ring ripple instability, superposed on a Gaussian electromagnetic beam.

Many analyses of filamentation and self-focusing instability have been reported by taking into account the relativistic nonlinearity (Sprangle & Esarey, 1991, 1992; Tabak *et al.*, 1994; Borisov *et al.*, 1995; Purohit *et al.*, 2005). These investigations are characterized by taking into account only the relativistic nonlinearity, while different kinds of nonlinearities are in fact operative, depending on the time scale of the pulse *viz.* (1) $\tau < \tau_{pe}$ and (2) $\tau_{pe} < \tau < \tau_{pi}$ where τ is the pulse duration, τ_{pi} is the ion plasma period, and τ_{pe} is the electron plasma period. Case 1 corresponds to dominant relativistic nonlinearity while case 2 refers to the situation when the relativistic (Hora, 1975) and ponderomotive (Hora, 1969) nonlinearities are simultaneously operative. Few studies on self-focusing (Brandi *et al.* 1993; Osman *et al.*, 1999) and cross focusing (Gupta *et al.*, 2005) of the laser beams have made, incorporating the combined effect of relativistic and ponderomotive nonlinearities. Further, the effect of ultra intense laser pulse on the propagation of an electron plasma wave has been analyzed by Kumar *et al.* (2006) in the relativistic-ponderomotive regime. A critical look into the literature reveals that all three kinds of basic nonlinearities have been discussed separately, but their combined effect on the propagation of the ring ripple has not yet been considered. In this context, the present investigation aims at exploring the propagation of a ring ripple, riding on a Gaussian electromagnetic beam in a plasma by taking into account relativistic-ponderomotive nonlinearity (*viz.* $\tau_{pe} < \tau < \tau_{pi}$).

The present work is based on the modified approach followed by Misra and Mishra (2008) and represents the extension of the theory to plasmas in which the relativistic and ponderomotive nonlinearities are simultaneously operative. This investigation is inclusive of the following considerations: (1) The radial field distribution profile of the electromagnetic beam has been taken as that of the composite electric field of the Gaussian beam and the ring ripple, and treated as such throughout the analysis. This assumption leads to the same dielectric function for both ring ripple part as well as Gaussian part of the electromagnetic beam, and hence the same focusing factors (valid in the vicinity of the maximum of the ring ripple). This is in contrast to the earlier analyses (Sodha *et al.*, 1979*a*, 1981*a*, 2004*a*; Pandey & Tripathi, 1990; Purohit *et al.*, 2004, 2005; Gill & Saini, 2007), which separately consider the beam and the ripple, leading to incorrect results. (2) The diffraction term derived in the present analysis is appropriate for the vicinity of the maximum of the irradiance of the ring ripple, occurring away from the beam axis ($r = 0$). (3) The r independent term in the eikonal of the beam is taken into account; it is seen that the phase difference between the electric field vectors associated with the Gaussian part and the ring ripple part change continuously. (4) All the relevant parameters have been expanded in terms of the radial distance from the maximum of the ring ripple, which is away from the axis $r = 0$. (5) The plasma is electrically neutral everywhere. (6) The pulse duration τ (with condition $\tau_{pe} < \tau < \tau_{pi}$) has been chosen so that both the nonlinearities *viz.* relativistic and ponderomotive are operative.

This paper investigates some interesting aspects associated with the propagation of the electromagnetic beam with ring ripple kind of instability in a paraxial like approximation and the results are appreciated through the critical curves, and the dependence of the beam width parameter associated with the vicinity of the maximum of the ring ripple part on various factors. The results have been discussed in Section 2.6 and a short summary (an outcome of the theory) concludes the paper.

2. PROPAGATION OF THE RING RIPPLE ON A GAUSSIAN ELECTROMAGNETIC BEAM

2.1. Propagation

Consider the propagation along the z-axis of a linearly polarized Gaussian electromagnetic beam with a small coaxial perturbation (the ring ripple) having its electric vector along the y-axis, in a homogeneous plasma. The effective electric field vector E of the Gaussian electromagnetic beam with the coaxial ripple can be expressed as

$$E = \hat{j} F_0 \exp(i\omega t), \tag{1}$$

where

$$(F_0)_{z=0} = E_{00} \exp\left(-\frac{r^2}{2r_0^2}\right) + E_{10}\left(\frac{r^2}{r_1^2} - \delta\right)^{n/2} \exp\left(-\frac{r^2}{2r_1^2}\right) \exp(i\phi_p), \tag{2}$$

F_0 refers to the complex amplitude of the electromagnetic beam, E_{00} and E_{10} correspond to the initial amplitude of the Gaussian beam (with initial beam width r_0) and the ring ripple (with initial beam width r_1) components respectively, n and δ are positive numbers, characterizing the position of the ring ripple on the wave front of the electromagnetic beam, ϕ_p is the initial phase difference between the electric field vectors of the Gaussian beam and the ring ripple, ω is the wave frequency, and $\hat{j}$ is the unit vector along the y-axis. The first term on the right-hand-side of Eq. (2) corresponds to the Gaussian profile while the second term represents the radial distribution of the coaxial perturbation in the form of the ring ripple, having its maximum at $r = r_{\max} = r_1\sqrt{(n+\delta)}$.

The effective electric field vector E satisfies the wave equation,

$$\nabla^2 E - \nabla(\nabla \cdot E) + (\omega^2/c^2)\varepsilon(r, z)E = 0. \tag{3}$$

where ε is the effective dielectric function of the plasma and c is the speed of light in free space.

Under the JWKB approximation i.e., $k^{-2}\nabla^2(\ln \varepsilon) \ll 1$, the second term of Eq. (3) may be neglected, where k is the wave number of propagation. One can thus write the wave equation, as

$$\nabla^2 E + (\omega^2/c^2)\varepsilon(r, z)E = 0$$

or

$$\nabla^2 F_0 + (\omega^2/c^2)\varepsilon(r, z)F_0 = 0. \tag{4}$$

Following Akhmanov *et al.* (1968) and Sodha *et al.* (1974, 1976), the solution of Eq. (4) in the cylindrical coordinate system may be chosen as

$$F_0(r, z) = A(r, z)\exp\left[-i\int k(z)dz\right], \tag{5}$$

where $A(r,z)$ is the complex amplitude of the electric field F_0 and $k(z)$ is the wave number that for convenience has been defined as $k(z) = (\omega/c)\sqrt{\varepsilon_0(z)}$, $\varepsilon_0(z)$ is the dielectric function, corresponding to the maximum of the electric field, of the ripple part of the electromagnetic beam, far from the axis $r = 0$. Substituting for $F_0(r, z)$ from Eq. (5) and neglecting the term $((\partial^2 A)/(\partial z^2))$ (assuming $A(r, z)$ to be a slowly varying function of z), one obtains

$$2ik\frac{\partial A}{\partial z} + iA\frac{\partial k}{\partial z} = \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\omega^2}{c^2}(\varepsilon - \varepsilon_0). \tag{6}$$

The complex amplitude $A(r, z)$ of the electric field $F_0(r, z)$ may be expressed as,

$$A(r, z) = A_0(r, z)\exp(-ik(z)S(r, z)), \tag{7}$$

where $S(r, z)$ is termed as the eikonal associated with the electromagnetic beam.

Substitution for $A(r, z)$ from Eq. (7) in Eq. (6) and separation of the real and imaginary parts, yields

$$\frac{2S}{k}\frac{\partial k}{\partial z} + 2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\omega^2}{k^2 c^2}(\varepsilon - \varepsilon_0) \tag{8}$$

and

$$\frac{\partial A_0^2}{\partial z} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + \frac{A_0^2}{k}\frac{\partial k}{\partial z} = 0. \tag{9}$$

In view of the interest of the current study (the ring ripple part, far from the axis $r = 0$), one can use a paraxial like approximation, which is valid around $r = r_{\max}$, the position of the maximum irradiance of the ring ripple; this is analogous to the usual paraxial approach. One can thus express Eqs. (8) and (9) in terms of z and a new variable χ, where χ is a parameter introduced for algebraic convenience, which may be defined as

$$\chi^2 = \left(\frac{r^2}{r_1^2 f^2} - \lambda\right), \tag{10}$$

$\lambda = (n + \delta)$, $r_1 f(z)$ is the width of the ring ripple and $r_{\max}^2 = \lambda r_1^2 f^2$ indicates the position of the maximum irradiance for the ring ripple; it is shown later that Eqs. (8) and (9) lead to retention of the original profile of the beam during propagation in the paraxial like approximation, i.e., when $\chi^2 \ll n$.

The present paraxial like theory is valid, when $\chi^2 \ll n$ (Eq. 10). This condition defines the range of r^2/r_1^2 (around $\lambda = (n + \delta)$) for which the theory is valid. Such a transformation leads to the form

$$\frac{\partial}{\partial z} \rightarrow \frac{\partial}{\partial z} - \frac{(\lambda + \chi^2)}{\chi f}\frac{df}{dz}\frac{\partial}{\partial \chi} \tag{11}$$

and

$$\frac{\partial}{\partial r} \rightarrow \frac{1}{r_1 f}\frac{\partial}{\partial \chi} \tag{12}$$

Thus, with help of Eqs. (11) and (12), the set of focusing

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equations (i.e., Eqs. (8) and (9)) reduces to

$$\frac{2S}{k}\frac{\partial k}{\partial z}+2\left(\frac{\partial S}{\partial z}-\frac{(\lambda+\chi^2)}{\chi f}\frac{df}{dz}\frac{\partial S}{\partial \chi}\right)+\frac{1}{r_1^2f^2}\frac{(\lambda+\chi^2)}{\chi^2}\left(\frac{\partial S}{\partial \chi}\right)^2=\frac{1}{k^2A_0r_1^2f^2}\left[\frac{\lambda}{\chi^2}\left(\frac{\partial^2 A_0}{\partial \chi^2}-\frac{1}{\chi}\frac{\partial A_0}{\partial \chi}\right)+\left(\frac{\partial^2 A_0}{\partial \chi^2}+\frac{1}{\chi}\frac{\partial A_0}{\partial \chi}\right)\right]+\frac{\omega^2}{k^2c^2}(\varepsilon-\varepsilon_0) \quad (13)$$

and

$$\frac{A_0^2}{k}\frac{\partial k}{\partial z}+\left(\frac{\partial A_0^2}{\partial z}-\frac{(\lambda+\chi^2)}{\chi f}\frac{df}{dz}\frac{\partial A_0^2}{\partial \chi}\right)+\frac{1A_0^2}{r_1^2f^2}\left[\frac{\lambda}{\chi^2}\left(\frac{\partial^2 S}{\partial \chi^2}-\frac{1}{\chi}\frac{\partial S}{\partial \chi}\right)+\left(\frac{\partial^2 S}{\partial \chi^2}+\frac{1}{\chi}\frac{\partial S}{\partial \chi}\right)\right]+\frac{1}{r_1^2f^2}\frac{(\lambda+\chi^2)}{\chi^2}\frac{\partial A_0^2}{\partial \chi}\frac{\partial S}{\partial \chi}=0. \quad (14)$$

In the paraxial like approximation, the relevant parameters (i.e., dielectric function $\varepsilon(r, z)$, eikonal, and irradiance) may be expanded around the maximum of the ring ripple, i.e., around $\chi^2 = 0$. Thus the dielectric function $\varepsilon(\chi, z)$ can be expressed around $\chi^2 = 0$ as

$$\varepsilon(\chi, z)=\varepsilon_0(z)-\chi^2\varepsilon_2(z). \quad (15)$$

where $\varepsilon_0(z)$ and $\varepsilon_2(z)$ are the coefficients associated with χ^0 and χ^2 in the expansion of $\varepsilon(\chi, z)$ around $\chi^2 = 0$. The expressions for these coefficients have been derived later.

Substitution for $\varepsilon(\chi, z)$ from Eq. (15) in Eqs. (13) and (14) leads to,

$$\frac{2S}{k}\frac{\partial k}{\partial z}+2\left(\frac{\partial S}{\partial z}-\frac{(\lambda+\chi^2)}{\chi f}\frac{df}{dz}\frac{\partial S}{\partial \chi}\right)+\frac{\delta_m}{r_1^2f^2}\frac{(\lambda+\chi^2)}{\chi^2}\left(\frac{\partial S}{\partial \chi}\right)^2=\frac{1}{k^2A_0r_1^2f^2}\left[\frac{\lambda}{\chi^2}\left(\frac{\partial^2 A_0}{\partial \chi^2}-\frac{1}{\chi}\frac{\partial A_0}{\partial \chi}\right)+\left(\frac{\partial^2 A_0}{\partial \chi^2}+\frac{1}{\chi}\frac{\partial A_0}{\partial \chi}\right)\right]-\chi^2\frac{\omega^2}{k^2c^2}\varepsilon_2 \quad (16)$$

and

$$\frac{A_0^2}{k}\frac{\partial k}{\partial z}+\left(\frac{\partial A_0^2}{\partial z}-\frac{(\lambda+\chi^2)}{\chi f}\frac{df}{dz}\frac{\partial A_0^2}{\partial \chi}\right)+\frac{\delta_m A_0^2}{r_1^2f^2}\left[\frac{\lambda}{\chi^2}\left(\frac{\partial^2 S}{\partial \chi^2}-\frac{1}{\chi}\frac{\partial S}{\partial \chi}\right)+\left(\frac{\partial^2 S}{\partial \chi^2}+\frac{1}{\chi}\frac{\partial S}{\partial \chi}\right)\right]+\frac{\delta_m}{r_1^2f^2}\frac{(\lambda+\chi^2)}{\chi^2}\frac{\partial A_0^2}{\partial \chi}\frac{\partial S}{\partial \chi}=0. \quad (17)$$

In the paraxial like approximation ($\chi^2 \ll n$), the solution of Eq. (17) may be chosen as,

$$A_0^2=\frac{E_0^2}{f^2}\exp[-m(\lambda+\chi^2)]+\frac{E_1^2}{f^2}(n+\chi^2)^n\exp[-(\lambda+\chi^2)]+\frac{E_0E_1}{f^2}(n+\chi^2)^{n/2}\exp\left[-\frac{1}{2}(1+m)(\lambda+\chi^2)\right]Cos\phi_p, \quad (18)$$

where

$$S(\chi, z)=\frac{\chi^2}{2}\beta(z)+\varphi(z), \quad (19)$$

$$\beta(z)=r_1^2f\,\frac{df}{dz},$$

$$E_0^2=E_{00}^2\left(\frac{k(0)}{k(z)}\right)=E_{00}^2\left(\frac{\varepsilon_0(0)}{\varepsilon_0(z)}\right)^{1/2},$$

$$E_1^2=E_{10}^2\left(\frac{k(0)}{k(z)}\right)=E_{10}^2\left(\frac{\varepsilon_0(0)}{\varepsilon_0(z)}\right)^{1/2},$$

$$m=(r_1^2/r_0^2),$$

$\varphi(z)$ is an arbitrary function of z, and $f(z)$ is the beam width parameter.

For further algebraic analysis, it is convenient to expand the solution for A_0^2 as a polynomial in χ^2; thus

$$A_0^2=g_0+g_2\chi^2+g_4\chi^4+g_6\chi^6, \quad (20)$$

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where

$$g_0 = \frac{E_0^2}{f^2}\left(e^{-m\lambda} + p^2 n^n e^{-\lambda} + 2pn^{n/2} e^{-(m+1)\lambda/2} Cos\phi_p\right), \quad (21)$$

$$g_2 = -\frac{E_0^2}{f^2}\left(me^{-m\lambda} + pmn^{n/2} e^{-(m+1)\lambda/2} Cos\phi_p\right), \quad (22)$$

$$g_4 = \frac{E_0^2}{f^2}\left(\frac{m^2}{2} e^{-m\lambda} - \frac{1}{2n} p^2 n^n e^{-\lambda} + pn^{n/2} e^{-(m+1)\lambda/2} Cos\phi_p \left(\frac{m^2}{4} - \frac{1}{2n}\right)\right), \quad (23)$$

$$g_6 = \frac{E_0^2}{f^2}\left(-\frac{m^3}{6} e^{-m\lambda} + \frac{1}{3n^2} p^2 n^n e^{-\lambda} + pn^{n/2} e^{-(m+1)\lambda/2} Cos\phi_p \left(\frac{m}{4n} + \frac{1}{3n^2} - \frac{m^3}{24}\right)\right) \quad (24)$$

and $p = (E_1/E_0)$.

On substituting for A_0^2 and S from Eqs. (20) and (19) in Eq. (13) and equating the coefficients of χ^0 and χ^2 on both sides of the resulting equation, one obtains

$$f\left(\varepsilon_0 \frac{d^2 f}{d\xi^2} + \frac{1}{2}\frac{df}{d\xi}\frac{d\varepsilon_0}{d\xi}\right) g_0^2 + 2g_2 g_0 \times \left(\Phi \frac{d\varepsilon_0}{d\xi} + \varepsilon_0\left(2\frac{d\Phi}{d\xi} - \lambda\left(\frac{df}{d\xi}\right)^2\right)\right) = \frac{1}{f^2}[4g_0(2g_4 + 3\lambda g_6) + g_2^2] - \rho^2 g_0^2 \varepsilon_2 \quad (25)$$

and

$$g_0^2\left[\Phi \frac{d\varepsilon_0}{d\xi} + \varepsilon_0\left(2\frac{d\Phi}{d\xi} - \lambda\left(\frac{df}{d\xi}\right)^2\right)\right] = \frac{1}{f^2}[2g_0(g_2 + 2\lambda g_4) - \lambda g_2^2] \quad (26)$$

where $\xi = (c/r_1^2\omega)z$ is the dimensionless distance of propagation, $\rho = (r_1\omega/c)$ is the dimensionless initial width of the ring ripple and $\Phi = (\omega/c)\varphi$ is the dimensionless function associated with the eikonal.

The parameter Φ can be eliminated between Eqs. (25) and (26); thus

$$f\left(\varepsilon_0 \frac{d^2 f}{d\xi^2} + \frac{1}{2}\frac{df}{d\xi}\frac{d\varepsilon_0}{d\xi}\right) g_0^3 + \frac{2g_2}{f^2}[2g_0(g_2 + 2\lambda g_4) - \lambda g_2^2] = \frac{1}{f^2}[4g_0(2g_4 + 3\lambda g_6) + g_2^2] g_0 - \rho^2 g_0^3 \varepsilon_2. \quad (27)$$

The dependence of the beam width parameter f on the dimensionless distance of propagation ξ can be obtained by the simultaneous numerical integration of Eqs. (25) and (26) after putting suitable expressions for ε_0 and ε_2, with the initial boundary conditions $f = 1$, $(df/d\xi) = 0$ and $\Phi = 0$ at $\xi = 0$.

2.2. Dielectric Function

Following Sodha *et al.* (1974, 1976), the effective dielectric function of the plasma can be expressed as

$$\varepsilon(r, z) = 1 - \Omega^2 (N_{0e}/N_0), \quad (28)$$

where $\Omega = (\omega_{pe}/\omega)$, $\omega_{pe} = (4\pi N_0 e^2/m)^{1/2}$, is the electron plasma frequency, N_0 is the undisturbed electron density of the plasma, N_{0e} is the electron density of the plasma in the presence of the electromagnetic field, m is the mass of the electron, and e is the electronic charge.

Following the paraxial like approximation (i.e., $\chi^2 \ll n$) one can expand the dielectric function $\varepsilon(\chi, z)$ in axial and radial parts around the maximum of the ring ripple $\chi^2 = 0$. Thus, one obtains from Eq. (15) and Eq. (28),

$$\varepsilon_0(z) = \varepsilon(\chi, z)_{\chi^2=0}, \quad (29)$$

and

$$\varepsilon_2(z) = -\left(\frac{\partial \varepsilon(\chi, z)}{\partial(\chi^2)}\right)_{\chi^2=0}. \quad (30)$$

2.3. Evaluation of Effective Dielectric Function

The present study considers plasma, characterized by simultaneously operative relativistic, and ponderomotive nonlinearities, because of the relativistic change in the mass of electron and the modification of the background electron density due to ponderomotive nonlinearity. The relativistic ponderomotive force on an electron in the presence of an intense electromagnetic beam may be represented as (Borishov *et al.*, 1992; Brandi *et al.*, 1993; Gupta *et al.*, 2005) as,

$$F_p = -m_0 c^2 \nabla(\gamma - 1). \quad (31)$$

Where γ is the relativistic factor given by

$$\gamma = [1 + \alpha E E^*]^{1/2} \quad (32)$$

and $\alpha = (e^2/m_0^2 c^2 \omega^2)$.

Using the electron continuity equation and the current density equation, for the second order correction in the electron density equation (with the help of ponderomotive force), the total electron density may be represented by (Brandi *et al.*, 1993)

$$(N_{0e}/N_0) = 1 + (c^2/\omega_{p0}^2)\left(\nabla^2\gamma - \frac{(\nabla\gamma)^2}{\gamma}\right) \quad (33)$$

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The effective dielectric function in the case of relativistic ponderomotive nonlinearity may be given by

$$\varepsilon(r, z) = 1 - \Omega_0^2(N_{0e}/\gamma N_0) \tag{34}$$

where $\Omega_0 = (\omega_{p0}/\omega)$, $\omega_{p0} = (4\pi N_0 e^2/m_0)^{1/2}$, and m_0 is the rest mass of the electron.

With the help of Eq. (33), Eq. (34) reduces to the form

$$\begin{aligned}\varepsilon(r, z) &= 1 - (\Omega_0^2/\gamma)\left[1 + (c^2/\omega_{p0}^2)\left(\nabla^2\gamma - \frac{(\nabla\gamma)^2}{\gamma}\right)\right] \\ &= 1 - (\Omega_0^2/\gamma) - (c^2/\omega^2)\nabla\left(\frac{\nabla\gamma}{\gamma}\right).\end{aligned} \tag{35}$$

Following the paraxial like approximation (i.e., $\chi^2 \ll n$) one can expand the dielectric function $\varepsilon(\chi, z)$ in axial and radial parts around the maximum of the ring ripple $\chi^2 = 0$. Thus, one obtains from Eqs. (15) and (35),

$$\begin{aligned}\varepsilon_0(z) = 1 &- \left(\frac{\Omega_0^2}{(1+g_0)^{1/2}}\right) \\ &- \frac{1}{\rho^2 f^2}\left[\frac{(4\lambda g_4 + 2g_2)}{(1+g_0)} - \frac{2\lambda g_2^2}{(1+g_0)^2}\right]\end{aligned} \tag{36}$$

and

$$\begin{aligned}\varepsilon_2(z) = &-\left(\frac{\Omega_0^2}{2(1+g_0)^{3/2}}\right)g_2 \\ &+ \frac{1}{\rho^2 f^2}\left[\frac{(12g_6\lambda + 8g_4)}{(1+g_0)} - \frac{(12\lambda g_2 g_4 + 4g_2^2)}{(1+g_0)^2}\right. \\ &\left. + \frac{4\lambda g_2^3}{(1+g_0)^3}\right]\end{aligned} \tag{37}$$

2.4. Critical Condition for Focusing: Critical Curves

With initially ($\xi = 0$) plane wave front $[(df/d\xi) = 0]$ of the beam and $f = 1$ at $\xi = 0$; the condition $(d^2f/d\xi^2)_{\xi=0} = 0$ leads to $f(\xi) = 1$ or propagation of the ripple without convergence or divergence; this condition is known as the critical condition. Thus, putting $(d^2f/d\xi^2)_{\xi=0} = 0$ in Eq. (25), one obtains a relation between dimensionless initial width of the ripple $\rho[= r_1\omega/c]$ and $p_0[= E_{10}/E_{00}]$ (i.e., critical curve), ensuring the propagation of the ring ripple in the self trapped mode. Further, for $(d^2f/d\xi^2)_{\xi=0} < 0$, the ripple displays self-focusing, while for $(d^2f/d\xi^2)_{\xi=0} > 0$ ripple undergoes either oscillatory or steady divergence.

Thus, the critical curve can be represented as,

$$\rho^2\varepsilon_2(0) = \left[\frac{1}{g_0^2}[4g_0(2g_4 + 3\lambda g_6) + g_2^2]\right]_{\xi=0}. \tag{38}$$

Using the appropriate expression for $\varepsilon_2(z)$ at $z = 0$ from Eq. (37) for relativistic-ponderomotive nonlinearity, Eq. (38) reduces to

$$\begin{aligned}g_2\rho^2 = &-\frac{2(1+g_0)^{3/2}}{\Omega_0^2}\left[\frac{1}{g_0^2}[4g_0(2g_4 + 3\lambda g_6) + g_2^2]\right. \\ &-\left(\frac{(12g_6\lambda + 8g_4)}{(1+g_0)} - \frac{(12\lambda g_2 g_4 + 4g_2^2)}{(1+g_0)^2}\right. \\ &\left.\left. + \frac{4\lambda g_2^3}{(1+g_0)^3}\right)\right]_{\xi=0;p=p_0}\end{aligned} \tag{39}$$

On substitution for the coefficients g_0, g_2, g_4, and g_6 from Eqs. (21), (22), (23), and (24), Eq. (39) represents the critical power curve ρ *versus* p_0 and separates the self-focusing region from the rest. The critical curves, which exhibit a relationship between the initial dimensionless amplitude $p_0[=E_{10}/E_{00}]$ and the width $\rho[=r_1\omega/c]$, correspond to the propagation of the ripple without convergence or divergence. Points below the curve correspond to divergence (or dissipation) of the ripple, while points above the curves refer to continued self-focusing (or filamentation) of the ripple.

2.5. Computational Scheme

For the physical understanding of the phenomenon and numerical appreciation of the results, the critical curves and the dependence of the beam-width parameter f (near the maximum of the ring ripple) on ξ for a chosen set of parameters for ponderomotive-relativistic nonlinearity has been computed.

The critical curves for the propagation of the ring ripple, between $p_0[=E_{10}/E_{00}]$ i.e., $p(\xi = 0)$ and the initial dimensionless width of the ring ripple $\rho[=r_1\omega/c]$, have been plotted with the help of Eq. (39), by using the expressions for the coefficients g_0, g_2, g_4, and g_6 from Eqs. (21), (22), (23), and (24), corresponding to the applicable plasma nonlinearity and chosen sets of parameters E_0^2, δ, ϕ_p, m, n, and Ω_0. Further, the computations have also been made to investigate the variation of the ripple width parameter f, associated with the propagation of the ring ripple on the dimensionless distance of propagation ξ in homogeneous plasmas. Starting with a combination of parameters α E_{00}^2, p_0, ρ, and Ω_0, one can obtain the solution for the beam width parameter f by simultaneous numerical integration of Eqs. (25) and (26) using appropriate expressions for the parameters ϵ_0 and ϵ_2; appropriate boundary conditions *viz.* $f = 1$, $df/d\xi = 0$ at $\xi = 0$ have been used.

2.6. Numerical Results and Discussion

The present study investigates the propagation of a coaxial ring ripple superposed on a Gaussian electromagnetic beam in a homogeneous plasma; the electric field profile of the propagating beam is assumed to be composed of the radial electric field distributions of the Gaussian beam as well as that of the ring ripple. A paraxial like approach has

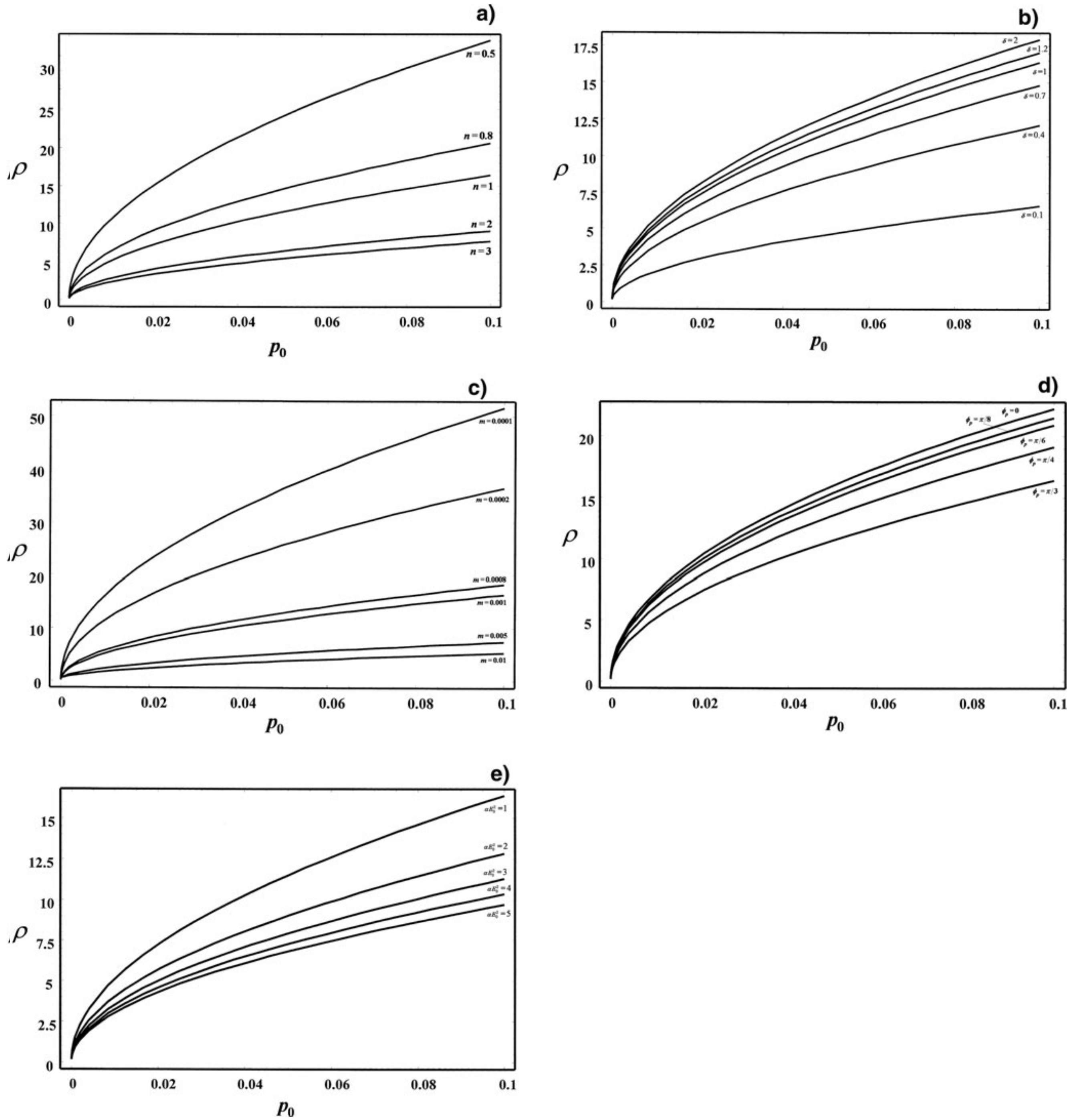

Fig. 1. (**a**–**e**) Critical curves for the ring ripple (the dependence of the initial beam width ρ $[=r_1\omega/c]$ of the ring ripple on p_0 $(=E_{10}/E_{00})$), corresponding to relativistic-ponderomotive nonlinearity, around the standard set of the parameters $\Omega_0^2 = 0.8$, $m = 0.001$, $\delta = 1.0$, $n = 1$, $\phi_p = \pi/3$, and $\alpha E_{00}^2 = 1$; which refer to the effect on the critical width by varying the parameter n, δ, m, ϕ_p and, αE_{00}^2 respectively, keeping rest of these same constant (the magnitude of the varying parameter is indicated with the curve).

been adopted to analyze the characteristics of the propagation. The nature of propagation of such a beam is characterized by the effective dielectric function which gets modified by the effective electric field of the electromagnetic beam (i.e., composite field of both the Gaussian and ring ripple parts).

In view of the interest of the present investigation (the ring ripple), all the characteristic parameters (*i.e.* the dielectric function $\varepsilon(r, z)$, eikonal S and irradiance) have been evaluated around the maximum of the electric field of the ring ripple part (i.e., at $r^2 = \lambda r_1^2 f^2$) on the basis of a paraxial like approach. Such a modification is a significant departure

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from many earlier studies (Sodha *et al.*, 1979*a*, 1981*a*, 2004*a*; Pandey & Tripathi, 1990; Purohit *et al.*, 2004, 2005; Gill and Saini, 2007) in which the Gaussian beam and the ring ripple have been treated as separate entities having a separate dielectric function for each part. In the current study the modified diffraction term is appropriate for the region around the maximum of the ring ripple, far from the axis $r = 0$, it also depends on other parameters like irradiance, δ, ϕ_p, μ, n, and p_0, in contrast to the earlier investigations. Further the inclusion of the parameter Φ (associated phase term with the eikonal) affects the propagation significantly; the second term on the left hand side of Eq. (27) is a consequence of the inclusion of Φ in the analysis.

It is interesting to point out that the third term on the right hand side of the expression of dielectric function $\varepsilon(r,z)$ [from Eq. (35)] is independent of the background electron density and if one ignores this term the expression for the dielectric function gets reduced to the simpler form of relativistic nonlinearity. Thus the third term describes the combined effect of relativistic and ponderomotive forces and strongly depends on the width and irradiance of the electromagnetic beam.

It is instructive to have a numerical appreciation of the results and hence the critical curves and the plot of beam width parameter f as a function of dimensionless distance of propagation ξ has been computed for a chosen set of parameters E_0^2, δ, ϕ_p, m, n, Ω_0, and p_0. The critical curve for the ring ripple characterizes the self-focusing region in the ρ (dimensionless width)-p_0 (ratio of the initial amplitude of the ripple and the Gaussian beam) space. The points (ρ, p) above the critical curve display self-focusing, while points lying below the critical curve lead to oscillatory divergence or steady divergence. The set of curves in Figure 1 illustrate the dependence of the initial dimensionless amplitude of the electric field p_0 (associated with the ring ripple) on the dimensionless initial ring ripple width $\rho(= r_1\omega/c)$ for self trapping corresponding to the relativistic-ponderomotive nonlinearity. The curves have been drawn around a standard set of parameter $\alpha\ E_{00}^2 = 1.0$, $\delta = 1.0$, $\phi_p = \pi/3$, $m = 0.001$, $n = 1.0$, and $\Omega_0^2 = 0.8$; the effect of the variation of these parameters (one at a time) on the critical curves of the ring ripple (keeping rest of them the same), have been computed.

Figure 1a indicates that the self-focusing region increases with increasing order of the ring ripple i.e., n. This can be understood in terms of radial part of dielectric function $\varepsilon_2(z)$, which falls sharply with increasing irradiance of the ring ripple (p). The ring ripple experiences weaker interaction with the Gaussian beam as its position shifts away from the axis of the main Gaussian beam *viz.* as δ increases and thus the area of self-focusing decreases; this behavior has been shown in Figure 1b. Larger widths of the ripple (m) lead to larger region of self-focusing as expressed in Figure 1c. Figure 1d suggests that the focusing occurs at lower initial ring ripple width ρ with increase in the initial phase difference ϕ_p between the electric fields of the Gaussian beam and the ring ripple; this behavior is characteristic of saturating nonlinearity and high values of axial irradiance (for the ring ripple). Further, it is seen that the region of self-focusing increases with increasing initial axial irradiance of the main Gaussian beam due to strong interaction with the field of the ring ripple. This nature has been expressed in Figure 1e.

Figure 2a expresses the dependence of the beam width parameter f on the dimensionless distance of propagation ξ in plasma with relativistic-ponderomotive nonlinearity. It may be remembered that $r_{\max}^2 = \lambda r_1^2\, f^2$; thus the parameter f characterizes both the width of the ripple (r, f) and position of the maximum *viz.* $r_{\max}^2 = \lambda r_1^2 f^2$. The figure describes the characteristic propagation of the ring ripple on Gaussian beam in the three regions for chosen points (ρ,p) corresponding to the self-focusing, oscillatory divergence, and steady divergence for the relevant parameters $\alpha\ E_{00}^2 = 5$, $\delta = 1.0$, $\phi_p = \pi/3$, $n = 1.0$, $m = 0.001$, and $\Omega_0^2 = 0.8$. The curves are in conformance with the critical curves. Further the variation of dimensionless phase Φ on the dimensionless

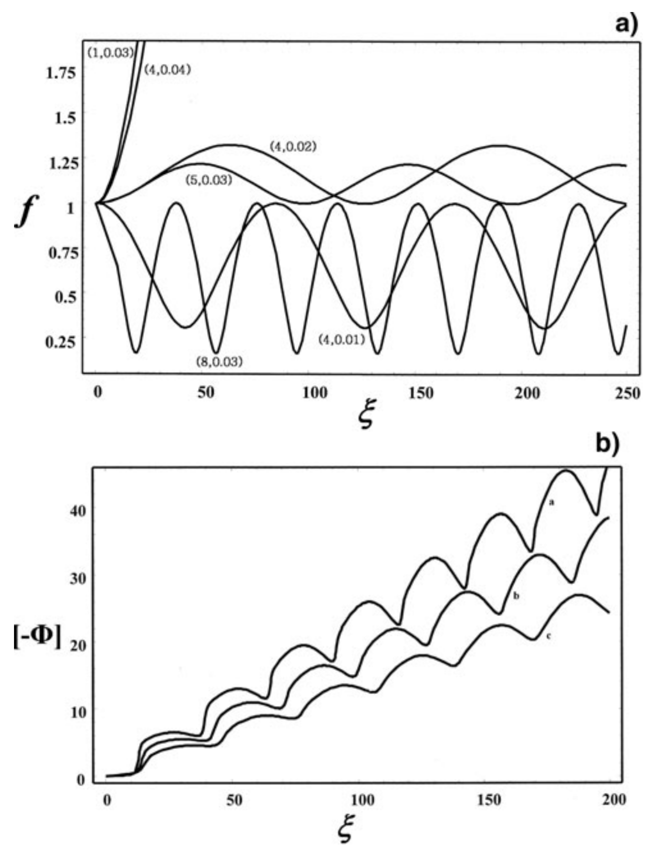

Fig. 2. (**a**) Variation of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, in a collisionless plasma with dominant relativistic-ponderomotive nonlinearity, for the parameters $\Omega_0^2 = 0.8$, $m = 0.001$, $\delta = 1.0$, $n = 1$, $\phi_p = \pi/3$, and $\alpha E_{00}^2 = 5$; the curves refer to an arbitrarily chosen set (ρ, p_0) as indicated over the curves. (**b**) Variation of the dimensionless phase associated with the eikonal Φ on the dimensionless distance of propagation ξ, in a collisionless plasma with dominant ponderomotive nonlinearity, for the parameters $\Omega_0^2 = 0.8$, $m = 0.001$, $\delta = 1.0$, $n = 1$, $\phi_p = \pi/3$ and $\alpha\ E_{00}^2 = 5$, and $\rho = 10$; the curves a, b, and c refer to $p_0 = 0.03$, 0.04 and 0.05, respectively.

distance of propagation ξ in Figure 2b, displays its significance in the characteristic propagation of the ring ripple.

3. CONCLUSIONS

This paper presents an analysis and discussion of the propagation of a coaxial ring ripple, superposed on a high irradiance electromagnetic Gaussian beam in a homogeneous plasma, taking into account the simultaneous occurrence of ponderomotive and relativistic nonlinearities. A paraxial like approach, valid near the position of the maximum irradiance of the ring ripple has been followed to obtain the critical curves corresponding to the ripple. Variation of the beam width parameter *f*, characterizing the width and magnitude/position of the maximum of the ring ripple with the distance of the propagation has been studied for the case of self-focusing, propagation without convergence or divergence (self-trapped mode) and steady divergence. In contrast to most of the investigations an expression for a composite dielectric function, valid for the main beam and the ripple has been derived and used in the analysis.

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