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Drift wave driven zonal flows in electron–positron–ion plasmas

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Abstract. The generation of large-scale zonal flows by small-scale electrostatic drift waves in electron–positron–ion (EPI) plasma is considered. The generation mechanism is based on the parametric excitation of convective cells by finite amplitude drift waves. To describe this process, the Hasegawa–Mima equation generalized for the case of EPI plasma is used. Explicit expressions for the maximum growth rate as well as for the optimal spatial dimensions of the zonal flows are obtained. Dependence of the growth rate on the spectrum purity of the wave packet is also investigated. The relevant instability conditions are determined.

1. Introduction

Because of the long lifetime of positrons, most astrophysical (Lee et al. 2005; Zheleznyakov and Koryagin 2005) and laboratory plasmas (Greaves and Surko 1995; Helander and Ward 2003; Salamin et al. 2006) become an admixture of electrons, positrons and ions. In order to grasp the basic physics of the three-component electron–positron–ion (EPI) plasmas, several theoretical investigations have been carried out (Popel et al. 1995; Jammalamadaka et al. 1996; Pokhotelov et al. 2001; Mahmood and Saleem 2003; Saleem and Mahmood 2003; Saleem et al. 2003; Shukla et al. 2003a, b; Haque and Saleem 2004; Eliasson and Shukla 2005; Hall and Shukla 2005; Haque et al. 2005; Esfandyari-Kalejahi et al. 2006; Kourakis et al. 2007; Moslem et al. 2007; Masood 2008; Mushtaq 2008; Dubinov and Sazonkin 2009; Kaladze et al. 2009) within the framework of multifluid theory which is applied to both astrophysical and laboratory plasmas.

Development of nonlinear theory of waves led to the appearance of new concepts – solitary wave, solitary vortex, soliton, filament, convective cell, jet, double layers, shocks and zonal flow, which are extensively discussed during the last few years in EPI plasma. The study of such self-organized structural formations is of great importance for understanding the macroscopic behavior of the laboratory and space plasmas.

Most papers were devoted to the investigation of ion-acoustic solitons in EPI plasma under different regimes (e.g. Dubinov and Sazonkin 2009 and references therein). Kourakis et al. (2007) have shown that localized envelope solitons and holes occur in EPI plasma. Ion-acoustic drift solitons in EPI plasma were studied by Mushtaq (2008). Solitons of the electrostatic acoustic-like lower hybrid mode and Langmuir-like optic-type upper hybrid mode in EPI plasmas were found by Esfandyar-Kalejahi et al. (2006). Soliton solutions and double layers on electrostatic electron-acoustic waves in EPI plasmas were obtained by Moslem et al. (2007). The formation of light bullets and solitons was discussed by Shukla et al. (2003b) for EPI plasma. Ion-acoustic shock waves in EPI plasma were considered by Masood et al. (2008).

Nonlinear solitary structures of shear Alfvén waves in EPI plasma were studied by Mahmood and Saleem (2003). Shear flow driven solitary vortex structures in an inhomogeneous EPI plasma were studied by Haque et al. (2005). The electrostatic and electromagnetic vortex structures of drift waves in an ideal EPI plasma have been discussed by Saleem et al. (2003). Jammalamadaka et al. (1996) have revealed the vortex formations for low-frequency electrostatic and electromagnetic disturbances in EPI plasma. Haque and Saleem (2004) have shown that low-frequency electromagnetic drift waves might form the dipolar vortices in EPI plasma. Eliasson and Shukla (2005) have found the phase-space holes in a relativistically hot EPI plasma. Vortex structures of the coupled electrostatic drift and ion-acoustic waves in a strongly magnetized EPI plasma in the presence of sheared ion flow were found by Shukla et al. (2003a). Large-scale vortex electrostatic drift structures in EPI plasma were investigated by Kaladze et al. (2009). Pokhotelov et al. (2001) investigated the nonlinear dynamics of drift Alfvén waves in an inhomogeneous electron–positron plasma with a small admixture of heavy ions and two-dimensional dipolar vortices.

However, another very important nonlinear process, viz. the formation of zonal flows in EPI plasma, has not been reported so far.

In this paper, we will focus our attention on the generation of zonal flows by electrostatic drift waves in EPI plasmas. Zonal flows are associated with azimuthally symmetric band-like shear flows. They are common in planetary atmospheres and laboratory plasmas. Recently, Fujisawa et al. (2004) have presented experimental evidence of zonal flows in a toroidal plasma. Tokamak experiments relevant to zonal flows are widely discussed by Diamond et al. (2005). Note that it is now generally accepted that zonal flows are a key constituent in nearly all regimes of drift wave turbulence so much that this classic problem is now frequently referred to as 'drift wave – zonal-flow turbulence' (e.g. Diamond et al. 2005). That is why zonal flows driven by drift-type turbulence have been intensively investigated theoretically in recent years (e.g. Shukla and Stenflo 2002 and references therein).

In this paper, the problem of zonal-flow generation by electrostatic drift waves in EPI plasma on the basis of parametric instability is investigated. The paper is organized as follows. In Sec. 2, a brief description of the dynamics of electrostatic waves in EPI plasma is presented. A set of coupled equations describing the nonlinear interaction of drift waves and zonal flows is derived in Sec. 3. It is shown that this system admits the excitation of zonal flows. We consider the zonal-flow instability of a monochromatic wave packet in Sec. 4 and the effect of a spectrum broadening of the wave packet in Sec. 5. Summary and conclusions are given in Sec. 6.

2. Hasegawa–Mima equation for drift waves

Let us consider the quasi-two-dimensional motion of a quasi-neutral EPI plasma. We consider a local perturbation (with respect to the unperturbed plasma environment) of the plasma potential $\varphi(t, x, y)$ and assume that plasma is uniform along the z axis which is parallel to the external magnetic field $\mathbf{B}_0$. The unperturbed plasma densities of electrons $n_{e0}(x)$ and positrons $n_{p0}(x)$, and corresponding temperatures T_{e0} and T_{p0} are inhomogeneous and assumed to decrease monotonously along the x axis. The ions are considered to be cold and the quasi-neutrality condition $n_{e0}(x) = Zn_{i0}(x) + n_{p0}(x)$ is satisfied, where Z is the charge number of positive ions. Let us assume that in this system the plasma density perturbation arises (corresponding to the perturbation of plasma potential φ), which generates the drift wave. Assuming that the plasma motion in the (x, y) plane is sufficiently slow, so that electrons and positrons (rapidly moving along the magnetic field) follow the Boltzmann equilibrium. The drift wave regime in plasma takes place when $\omega/\omega_{\rm ci}$, where ω is the characteristic frequency of the perturbation and $\omega_{\rm ci} = ZeB_0/M$ is the ion cyclotron frequency.

The generalized nonlinear Hasegawa–Mima (HM) equation for EPI plasma possessing both vector and scalar nonlinearities, which is valid for arbitrary sizes of vortex structures, was previously obtained by Kaladze et al. (2009). In the case of small-scale vortex structures $a/r_s \ll 1$, where a is the perpendicular size of the structure and $r_s = (T_e/M)^{1/2}/\omega_{\rm ci}$ is the ion-acoustic Larmor radius, respectively, the classical HM equation (containing only vector nonlinearity) can be singled out and takes the form

$$-\frac{Ze}{M\omega_{\rm ci}}\left(1-\frac{n_{p0}}{n_{e0}}\right)\frac{\partial\nabla_\perp^2\varphi}{\partial t}+\frac{e\omega_{\rm ci}}{T_e}\left(1+\frac{n_{p0}}{n_{e0}}\frac{T_e}{T_p}\right)\frac{\partial\varphi}{\partial t}-\frac{Ze}{M}\frac{n'_{e0}-n'_{p0}}{n_{e0}}\frac{\partial\varphi}{\partial y},$$

$$-\frac{Ze}{M\omega_{\rm ci}}\frac{n'_{e0}-n'_{p0}}{n_{e0}}\frac{\partial^2\varphi}{\partial t\partial x}-\frac{Z^2e^2}{M^2\omega_{\rm ci}^2}\left(1-\frac{n_{p0}}{n_{e0}}\right)J(\varphi,\nabla_\perp^2\varphi)=0. \tag{2.1}$$

Here the prime denotes the spatial derivative, $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and the Jacobian $J(a, b) = (\partial a/\partial x)(\partial b/\partial y) - (\partial a/\partial y)(\partial b/\partial x)$ represents the vector nonlinearity. The last linear term that comes from the polarization drift (it is of the order of $\omega/\omega_{\rm ci}$) is retained in order to obtain the special structure of the electrostatic drift waves. Indeed, if we introduce the inverse inhomogeneity length $L^{-1} = |n'_{i0}/n_{i0}| = |n'_{e0} - n'_{p0}|/(n_{e0} - n_{p0})$ and seek for the propagating drift plane waves of the form $\varphi \propto \exp(\pm x/2L)\exp(-i\omega_{\mathbf{k}}t + ik_x x + ik_y y)$, we obtain the expression for the electrostatic drift frequency

$$\omega_{\mathbf{k}} = \frac{\mp k_y v_*}{\beta + r_s^2\left(k_\perp^2 + 1/4L^2\right)}. \tag{2.2}$$

Here v_* is the drift velocity, $\beta = n_{e0}\left(1 + n_{p0}T_e/n_{e0}T_p\right)/Z(n_{e0} - n_{p0})$, $k_\perp^2 = k_x^2 + k_y^2$ and $\mp$ sign corresponds to positive and negative signs of n'_{i0}/n_{i0} (the latter can change the sign depending on the difference between n_{e0} and n_{p0}). Thus, the drift wave potential in EPI plasma is either spatially increasing or decreasing in the direction of the inhomogeneity. The drift wave structure given by (2.2) is similar to acoustic-gravity waves propagating in an inhomogeneous atmosphere embedded in gravitational field.

Let us rewrite (2.1) in the form relevant for investigation of zonal-flow generation. We represent the perturbed potential as

$$\varphi(x, y, t) = \bar{\varphi}(x, y, t)e^{\mp\frac{x}{2L}}. \tag{2.3}$$

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Then from (2.1) we obtain the following form of the HM equation for EPI plasma

$$\beta\frac{\omega_{\mathrm{ci}}}{T_e}\frac{\partial\varphi}{\partial t}-\frac{1}{M\omega_{\mathrm{ci}}}\frac{\partial}{\partial t}\left(\nabla_\perp^2-\frac{1}{4L^2}\right)\varphi\mp\frac{1}{ML}\frac{\partial\varphi}{\partial y}-\frac{Ze}{M^2\omega_{\mathrm{ci}}^2}J(\varphi,\nabla_\perp^2\varphi)=0. \tag{2.4}$$

Here, in the nonlinear terms, the factor $\exp(\mp x/2L)$, as well as the bar over φ, has been omitted.

3. Nonlinear interaction of drift waves and zonal flows in EPI plasmas

We consider a standard three-wave coupling process in EPI plasma, where the coupling between the pump drift waves and sideband modes drives the low-frequency large-scale one-dimensional modes propagating along the $x$axis, i.e. the zonal flows.

We decompose the perturbed electric potential into three parts:

$$\varphi=\tilde{\varphi}+\hat{\varphi}+\bar{\varphi}. \tag{3.1}$$

Here $\tilde{\varphi}$, $\hat{\varphi}$ and $\bar{\varphi}$ correspond to the primary mode, the secondary small-scale mode and the zonal flow, respectively. The function $\bar{\varphi}$ is taken in the form:

$$\bar{\varphi}=\bar{\varphi}_0\exp\left(-i\Omega t+iq_x x\right)+c.c, \tag{3.2}$$

where Ω and q_x are the frequency and the wave number of the zonal flow, respectively, and $c.c.$ is the complex conjugate. The amplitude of the zonal-flow mode is assumed to be constant within the local approximation. The function $\tilde{\varphi}$ is

$$\tilde{\varphi}=\sum_{\mathbf{k}}\left[\tilde{\varphi}_+\left(\mathbf{k}\right)\exp\left(i\mathbf{k}\cdot\mathbf{r}-\mathbf{i}\omega_{\mathbf{k}}t\right)+\tilde{\varphi}_-\left(\mathbf{k}\right)\exp\left(-i\mathbf{k}\cdot\mathbf{r}+\mathbf{i}\omega_{\mathbf{k}}t\right)\right], \tag{3.3}$$

where $\omega_{\mathbf{k}}$ and $\mathbf{k}$ are the frequencies and wave vectors of the primary modes. Summation is performed over the whole totality of the primary modes. Finally, the function $\hat{\varphi}$ is

$$\hat{\varphi}=\sum_{\mathbf{k}}\left[\hat{\varphi}_+\left(\mathbf{k}\right)\exp\left(i\mathbf{k}\cdot\mathbf{r}-\mathbf{i}\omega_{\mathbf{k}}t\right)+\hat{\varphi}_-\left(\mathbf{k}\right)\exp\left(-i\mathbf{k}\cdot\mathbf{r}+\mathbf{i}\omega_{\mathbf{k}}t\right)\right], \tag{3.4}$$

where $\hat{\varphi}_\pm\left(\mathbf{k}\right)$ is the sideband amplitude.

Note that energy and momentum conservation is imposed on sideband frequencies $\omega_\pm$ and wave vector $k_\pm$ by the condition that $\omega_\pm=\Omega\pm\omega_{\mathbf{k}}$ and $k_\pm=q_x\mathbf{e}_x\pm\mathbf{k}$. For the problem of zonal-flow generation, the following conditions $|\Omega/\omega_{\mathbf{k}}|\sim|q_x/\mathbf{k}_\perp|\ll 1$ are satisfied.

To obtain the equations describing the turbulence and amplitude evolution of the zonal flow modes, we substitute (3.1)–(3.4) into (2.4). The averaging over the fast small-scale fluctuations gives the equation for the zonal-flow evolution:

$$\frac{\partial}{\partial t}\left[\beta\frac{\omega_{\mathrm{ci}}}{T_e}-\frac{1}{M\omega_{\mathrm{ci}}}\left(\nabla_\perp^2-\frac{1}{4L^2}\right)\right]\bar{\varphi}=\frac{Ze}{M^2\omega_{\mathrm{ci}}^2}\left\langle J(\varphi,\nabla_\perp^2\varphi\right\rangle. \tag{3.5}$$

This equation can be rewritten for the appropriate Fourier components as

$$i\Omega\left[\beta\frac{\omega_{\mathrm{ci}}}{T_e}+\frac{1}{M\omega_{\mathrm{ci}}}\left(q_x^2+\frac{1}{4L^2}\right)\right]\bar{\varphi}_0=-\frac{Zeq_x^2}{M^2\omega_{\mathrm{ci}}^2}\sum_{\mathbf{k}}k_y[2k_x\left(\tilde{\varphi}_+\hat{\varphi}_-+\tilde{\varphi}_-\hat{\varphi}_+\right)$$
$$+q_x\left(\tilde{\varphi}_-\hat{\varphi}_+-\tilde{\varphi}_+\hat{\varphi}_-\right)]. \tag{3.6}$$

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The right-hand side of (3.6) corresponds to the driving force of the zonal flow which is known as Reynolds stresses.

In order to calculate the Reynolds stresses, we should find the sideband amplitudes $\hat{\varphi}_{\pm}$. From (2.4), we obtain for the turbulent part contributions the following equation:

$$\frac{\partial}{\partial t}\left[\beta\frac{\omega_{\mathrm{ci}}}{T_e}-\frac{1}{M\omega_{\mathrm{ci}}}\left(\nabla_{\perp}^2-\frac{1}{4L^2}\right)\right]\bar{\varphi}_{\pm}\mp\frac{1}{ML}\frac{\partial}{\partial y}\bar{\varphi}_{\pm}$$
$$=\frac{Ze}{M^2\omega_{\mathrm{ci}}^2}\left[J(\tilde{\varphi}_{\pm},\nabla_{\perp}^2\bar{\varphi}_0)+J(\bar{\varphi}_0,\nabla_{\perp}^2\tilde{\varphi}_{\pm})\right]. \tag{3.7}$$

Using the expansion over small parameters Ω and q_x the solution of this equation can be represented as

$$\hat{\varphi}_{\pm}\approx i\frac{Zek_yq_xk_{\perp}^2}{M^2\omega_{\mathrm{ci}}^2CD}\left[\pm 1+\frac{q_x^2V_g'}{2D}-\frac{2k_xq_x}{M\omega_{\mathrm{ci}}C}\right]\bar{\varphi}_0\tilde{\varphi}_{\pm}. \tag{3.8}$$

Here $D=\Omega-q_xV_g(\mathbf{k})$, $C=\beta\omega_{\mathrm{ci}}/T_e+M^{-1}\omega_{\mathrm{ci}}^{-1}\left(k_{\perp}^2+(2L)^{-1}\right)$, and the group velocity of the pump wave and its derivative are defined as

$$V_g(\mathbf{k})=\frac{\partial\omega_{\mathbf{k}}}{\partial k_x}=-2\frac{k_x\omega_{\mathbf{k}}}{M\omega_{\mathrm{ci}}C}\quad\text{and}\quad V_g'(\mathbf{k})=\frac{\partial V_g}{\partial k_x}=-2\frac{\omega_{\mathbf{k}}}{M\omega_{\mathrm{ci}}C}\left(1-\frac{4k_x^2}{M\omega_{\mathrm{ci}}C}\right). \tag{3.9}$$

Now we can substitute (3.8) into (3.6) to obtain the zonal-flow dispersion equation

$$1-\sum_{\mathbf{k}}\frac{F(\mathbf{k})}{[\Omega-q_xV_g(\mathbf{k})]^2}=0, \tag{3.10}$$

with

$$F(\mathbf{k})=\frac{Z^2e^2q_x^4k_y^2k_{\perp}^2V_g'(\mathbf{k})}{M^3\omega_{\mathrm{ci}}^3\omega A}\,|\tilde{\varphi}_+|^2\quad\text{and}\quad A=\beta\frac{\omega_{\mathrm{ci}}}{T_e}+\frac{1}{M\omega_{\mathrm{ci}}}\left(q_x^2+\frac{1}{4L^2}\right). \tag{3.11}$$

4. Zonal-flow instabilities in case of monochromatic wave packet

In the case of the monochromatic wave packet, one has $F(\mathbf{k})\sim\delta(\mathbf{k}-\mathbf{k}_0)$ and (3.10) reduces to a hydrodynamic-type coherent instability $(\Omega-q_xV_g)^2=F(\mathbf{k}_0)=-\Gamma^2$, where Γ^2 denotes the squared zonal-flow growth rate given by

$$\Gamma^2=-\frac{Z^2e^2q_x^4k_{y0}^2k_{\perp 0}^2V_g'(\mathbf{k}_0)}{2M^3\omega_{\mathrm{ci}}^3A\omega_{\mathbf{k}_0}}I_{\mathbf{k}_0}, \tag{4.1}$$

and $I_{\mathbf{k}0}=2\tilde{\varphi}_+\tilde{\varphi}_-=2\,|\tilde{\varphi}_+|^2$.

The necessary condition for instability is $V_g'/\omega_{\mathbf{k}}<0$. This condition is similar to the Lighthill criterion for modulation instability in nonlinear optics. Taking into account (3.9), the instability condition becomes

$$\beta\frac{M\omega_{\mathrm{ci}}^2}{T_e}+k_y^2-3k_x^2>0. \tag{4.2}$$

The most simple case corresponds to the short wavelength limit, when $k_{\perp}r_s\gg 1$. In this approximation, the instability condition reduces to

$$\frac{V_g'}{\omega_{\mathbf{k}_0}}=-\frac{2}{k_{\perp}^4}\left(k_y^2-3k_x^2\right)>0. \tag{4.3}$$

Thus, the instability condition applies to drift waves with the wave vectors in the cone $-k_y/\sqrt{3} < k_y/\sqrt{3}$. The maximum growth rate is obtained at the axis of the cone when $k_x = 0$. In this case, the mode is purely growing with the growth rate

$$\Gamma = \frac{q_x^2 \omega_{\mathrm{ci}} \left| k_{y0} \right| r_s^3}{\left[\beta + r_s \left(q_x^2 + 1/4L^2 \right) \right]^{1/2}} I_{\mathbf{k}_0}^{1/2}. \tag{4.4}$$

Here in the expression for $I_{\mathbf{k}_0}$, the electrostatic potential $\tilde{\varphi}_+$ of the pump wave is normalized to the value T_e/Ze. This equation describes the initial (linear) stage of zonal-flow growth due to the parametric instability of small-scale drift waves. For $q_x r_s \sim 1$, we can estimate the growth rate as

$$\Gamma \approx \omega_{\mathrm{ci}} \left| k_{y0} \right| r_s I_{\mathbf{k}_0}^{1/2}. \tag{4.5}$$

This is the maximum growth rate. **This estimation shows that Γ increases as k in the short wavelength limit ($k_\perp r_s \gg 1$). Physically, this instability is the manifestation of an inverse cascade.

5. Zonal-flow instabilities in case of non-monochromatic wave packet

Now we consider the effects of non-monochromaticity of the wave packets on the generation of zonal flows. Let us take the function $I_{\mathbf{k}}$ in the Gaussian form

$$I_{\mathbf{k}} = \frac{1}{\pi^{1/2} \left| \Delta k_x \right|} \exp\left(-\frac{\left(k_x - k_{x0} \right)^2}{\left(\Delta k_x \right)^2} \right) I_{\mathbf{k0}}. \tag{5.1}$$

Here k_{x0} is the centered wave vector of the wave packet and $\Delta k_x > 0$ is the characteristic width of the wave packet. The component of the wave vector k_y is assumed to be the same for all modes of the wave packet, $k_y = k_{y0}$. The summation over $\mathbf{k}$ in (3.10) is now understood as the integral over k_x. Then, we allow for the primary mode frequency $\omega = \omega_{\mathbf{k}}$ and the group velocity V_g to be functions of k_x, $\omega = \omega(k_x)$ and $V_g = V_g(k_x)$.

Let us consider the case when the broadening of the wave packet is relatively small, i.e. $\Delta k_x / k_{x0} \ll 1$. Then we expand V_g in a series in the vicinity of k_{x0}, obtaining $V_g = V_{g0} + V_{g0}^{'}(k_x - k_{x0})$. Then instead of (4.1), we have

$$\hat{\Omega}^2 = \left(\Omega - q_x V_{g0} \right)^2 = -\Gamma^2 \left(1 + \frac{3}{2} \frac{\left(q_x V_{g0}^{'} \right)^2}{\hat{\Omega}^2} \left(\Delta k_x \right)^2 \right). \tag{5.2}$$

Considering the second term in the parentheses of (5.2) as a small correction, one can see that weak spectrum broadening leads to decrease of the growth rate of hydrodynamic instability. It follows from (5.2) that the spectrum broadening can be neglected only if $\left| \Delta k_x / k_{x0} \right| < \left| \Gamma / q_x V_{g0} \right|$.

Let us now consider the case when the broadening of the wave packet is arbitrary. In this case, the zonal-flow instability has a resonant character and we get the following zonal-flow dispersion relation (cf. Kaladze et al. 2007):

$$\hat{\Omega} = i \frac{\left| q_x V_{g0}^{'} \Delta k_x \right|}{\sqrt{\pi}} \left(1 - \frac{\left(q_x V_{g0}^{'} \Delta k_x \right)^2}{2\Gamma^2} \right). \tag{5.3}$$

Then we can find the instability condition $\Gamma^2 > (q_x V_{g0}^{'} \Delta k_x)^2/2$. The growth rate obtained from (5.3) attains the maximum when

$$|\Delta k_x| = \left(\frac{2}{3}\right)^{\frac{1}{2}} \left|\frac{\Gamma}{q_x V_{g0}^{'}}\right|. \tag{5.4}$$

By the order of magnitude, this kinetic growth rate is equal to the hydrodynamic one, i.e. $\gamma \approx \Gamma$.

6. Summary

In this paper, the generation of the zonal flow by small-scale electrostatic drift waves in EPI plasma is investigated. The generation mechanism is based on the parametric excitation of convective cells by finite amplitude drift waves and the spectrum of primary modes assumed to be arbitrary. Our investigation provides an important nonlinear mechanism for the transfer of spectral energy from small-scale drift waves to the large-scale enhanced zonal flows in EPI plasmas. To describe this process extended for EPI plasma, HM equation is used. A set of coupled equations describing the nonlinear interaction of drift waves and zonal flows is deduced. The driving force (Reynolds stresses) in the equation governing the evolution of zonal flows (see (3.6)) is represented as a summation over the spectrum of the primary modes. We have made such a generalization and thereby obtain the zonal-flow dispersion relation given by (3.10) for an arbitrary spectrum of drift pump waves. Explicit expressions for the maximum growth rate as well as for the optimal spatial dimensions of the zonal flows are obtained. It is shown that in contrast to usual electron–ion plasma, the existence of positrons in the plasma causes modification of both, the zonal-flow growth rate and instability conditions (see (3.10)–(4.2)). In addition, the temperature non-homogeneity of electrons and positrons in the case of small-scale drift waves has no influence on the generation of zonal flows in EPI plasmas. The dependence of the growth rate on the spectrum purity of the wave packet is also investigated (see (5.2)–(5.4)). It is shown that the sufficient broadening of the wave packet gives resonant-type instability with the growth rate of the same as for hydrodynamic instability. The relevant instability conditions are found. The unstable branch obtained in this paper has the zonal-flow growth rate proportional to the small value of q_x^2. If we choose for the typical tokamak-type devices $\tilde{\varphi}_+ \sim 10^{-2}$, $k_\perp r_s \sim 10$ and $\omega_{\mathrm{ci}} \sim 10^8\ \mathrm{s}^{-1}$, we obtain the value of the maximum growth rate $\Gamma \sim 10^7\ \mathrm{s}^{-1}$. It is found that the wave vector of the fastest growing mode is perpendicular to that of the pump drift wave.

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