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Thermal defocusing of intense hollow Gaussian laser beams in atmosphere

ASHUTOSH SHARMA,[1] MAHENDRA SINGH SODHA,[1] SHIKHA MISRA,[2] AND S.K. MISHRA[3]

[1]Department of Education Building, Lucknow University, Lucknow, India
[2]Centre of Energy Studies, Indian Institute of Technology, New Delhi, India
[3]Institute for Plasma Research (IPR), BHAT, Gandhinagar, India

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Abstract

In this paper, the authors have presented a paraxial theory for propagation of (1) Gaussian (2) dark hollow Gaussian high power laser beams in the atmosphere, considering the nonlinearity arising from the temperature variation along the wavefront. Specifically, the focusing parameter for both beams has been evaluated as a function of distance and initial beam power and width (corresponding to radiation of wavelengths 1.045 μ, 1.625 μ, and 2.141 μ in the water absorption window) for the maritime, desert, rural, and urban environments as modeled at NRL; the results have been presented in the dimensionless form. It is seen that in all four environments a dark hollow beam defocuses less than the corresponding Gaussian beam of same radius and power. It is suggested that this conclusion based on the paraxial theory be verified by numerical simulation.

Keywords: Atmosphere; Paraxial approach; Thermal defocusing

INTRODUCTION

Many directed energy systems are based on long distance propagation of high power laser beams through the atmosphere in a variety of environments like maritime, desert, rural, and urban. The efficiency of propagation, i.e., the ratio of the power of the laser beam incident on a target to the initial power of the beam is significantly affected by the phenomenon of thermal blooming; it is of interest to consider the basics of this phenomenon. When a Gaussian beam with maximum irradiance on the axis of the beam propagates through an absorbing medium, the temperature are maximum on the axis and falls off radially. Such a temperature profile corresponds to radially increasing refractive index in air and consequent defocusing, known as thermal blooming (Akhmanov *et al*., 1968*a*; 1968*b*; Brown & Smith, 1975; Schmitt, 2003).

The scattering and absorption of laser radiation in the water vapor absorption windows, specifically corresponding to the wavelengths 1.045 μ, 1.625 μ, and 2.141 μ has been reviewed at length by Sprangle *et al*. (2005); the gross scattering and absorption coefficients, evaluated by these authors, which are in agreement (Dos Hammel *et al*., 2004; Bodhaine, 1995) with in situ observations have been used in computations, reported in this paper.

A steady-state paraxial theory of thermal blooming was given by Akhmanov *et al*. (1968*a*; 1968*b*) and seen to be in agreement with experiments on laser propagation in water and acetone. Brown and Smith (1975) pointed out that aerosol absorption is mainly responsible for thermal blooming in air and that the steady-state gets established in a time on the order of a millisecond. Kaushik *et al*. (1975) evaluated thermal distortion of the beam due to wind. Weiss and Me Innis (1980) concluded from computer simulations that contrary to the suggestion by Fried (1974), a round beam in all cases of interest suffers less thermal blooming than a square beam. Armstrong (1984) has studied the dynamics of laser aerosol interaction including vaporization, breakdown, etc. Sprangle *et al*. (2002) and Penano *et al*. (2004) have studied the propagation of a very high energy laser pulse in air, causing a host of nonlinear phenomena on a time scale, much shorter than that for the onset of thermal blooming. On the basis of numerical simulation, Schmitt (2003) has concluded that thermal blooming can be mitigated by the use of short pulses of very intense laser beams with associated nonlinear effects (causing self-focusing).

Possibly the most advanced tool for the study of high energy laser propagation in the atmosphere *viz* HELCAP

Address correspondence and reprint requests to: Mahendra Singh Sodha, Department of Education Building, Lucknow University, Lucknow – 226 007, India. E-mail: msodha@rediffmail.com

was developed at NRL (Sprangle *et al.*, 2003). The model incorporates (1) aerosol and molecular absorption and scattering, (2) aerosol heating and vaporization, (3) thermal blooming, (4) atmospheric turbulence, and (5) laser beam quality. However, all these studies to date are confined to Gaussian beams and little if any attention has been given to other types of beams.

In this paper, the authors have investigated the thermal blooming of laser beams with central shadow, usually called dark hollow beams (DHB) for possible applications; such beams have found attractive applications in modern optics, atomic optics, and plasmas (Soding *et al.*, 1995; Kuga *et al.*, 1997; Ovchinnikov *et al.*, 1997; Yin *et al.*, 2003; Song *et al.*, 1999; Xu *et al.*, 2002; Cai *et al.*, 2003; York *et al.*, 2008). A number of experimental methods have been developed for production of hollow Gaussian beams (Herman & Wiggins, 1991; Wang & Littman, 1993; Lee *et al.*, 1994). The focusing of such beams in plasmas has been extensively studied (Sodha *et al.*, 2009*a*; 2009*b*; Misra & Mishra, 2009).

In central portion of the dark hollow beam $0 < r < r_m$, the temperature increases radially, while in the peripheral region $r > r_m$ the temperature falls radially (the maximum of irradiance occurs at $r = r_m$) Hence, the refractive index is maximum on the axis and it decrease radially in the central region; for $r = r_m$, the refractive index increases radially. Hence, the central portion tends to focus while the peripheral one defocuses. As a result, such a beam defocuses less than that experienced by a corresponding Gaussian beam.

In this paper, the authors have analyzed the thermal blooming for a Gaussian and a dark hollow beam in the atmosphere, characterized by specific absorption and scattering coefficients in the paraxial and near paraxial approximation as outlined by Akhmanov *et al.* (1968*a*; 1968*b*) and Sodha *et al.* (2009*a*). The theory has been used to study the beam width parameter for DHB and Gaussian beam in the four environments, characterized by Sprangle (2005).

In a preliminary study like the present one, the use of paraxial approximation indicates the relative merits of the Gaussian and DHB beams. However, it is indicated that in all environments the DHB defocuses less than a corresponding Gaussian beam; it is suggested that this conclusion be verified by a more rigorous numerical simulation (like HELCAP).

PROPAGATION OF ELECTROMAGNETIC BEAM

The Dielectric Function

Following Sodha *et al.* (1976), the variation of the dielectric function of air due to thermal self-action at a temperature T may be expressed as,

$$\varepsilon(r,z) = \varepsilon_r(T_0) + [T(r,z) - T_0]\frac{d\varepsilon_r}{dT}\bigg|_{T=0} - i\varepsilon_i, \tag{1a}$$

where ε_r is the real part of the dielectric function, T_0 is the maximum temperature on the wave-front of the beam, and ε_i is the imaginary part of the dielectric function and is practically independent of temperature; for air $d\varepsilon_r/dT$ is negative. Since T is a function of r and z, Eq. (1a) may be written as

$$\varepsilon(r,z) = \varepsilon_R(r,z) - i\varepsilon_i, \tag{1b}$$

where $R = g$, n for Gaussian and Hollow Gaussian beam, respectively, in this analysis.

Propagation

Consider the propagation of a linearly polarized radially symmetric electromagnetic beam with its electric vector polarized along the y-axis, propagating in air along the z-axis. The amplitude of the electric field vector E satisfies the scalar wave equation for such a beam, which may be expressed in a cylindrical coordinate system with azimuthal symmetry as

$$\frac{\partial^2 E}{\partial z^2} + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)E + \frac{\omega^2}{c^2}\varepsilon(r,z)\,E = 0, \tag{2}$$

where c is the speed of light in vacuum. Eq. (2) can be solved in the paraxial approximation by following the analysis of Akhmanov *et al.* (1968*a*; 1968*b*) and its extension by Sodha *et al.* (1974; 1976) for Gaussian beam and the work of Sodha *et al.* (2009*a*; 2009*b*) and Misra and Mishra (2009) on nonlinear propagation of hollow Gaussian beams in plasmas.

The starting point is a solution of the form

$$E(r,z) = A(r,z)\exp\left(-i\int_0^z k(z)dz\right), \tag{3}$$

where $A(r,z)$ is the complex amplitude of the electric field, $k(z) = \omega/c\sqrt{\varepsilon_{R0}(z)}$, ε_{R0} (z) is the dielectric function, corresponding to the maximum electric field on the wave-front of the electromagnetic beam. Substituting for $E(r,z)$ from Eq. (3) in Eq. (2) and neglecting $\partial^2 A/\partial z^2$ (in the Jeffreys-Wentzel-Kramers-Brillouin approximation) one obtains

$$2ik\frac{\partial A}{\partial z} + iA\frac{\partial k}{\partial z} + k^2 A = \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\omega^2}{c^2}\varepsilon(r,z)A. \tag{4}$$

For a nearly spherical wave front (a valid assumption in the paraxial approximation), the complex amplitude $A(r,z)$ may be expressed as,

$$A(r,z) = A_0(r,z)\exp(-ik(z)S(r,z)), \tag{5}$$

where $S(r,z)$ is the eikonal associated with the electromagnetic beam and $A_0(r,z)$ is the real amplitude of the electromagnetic beam. The propagation of the Gaussian beam and various orders HGBs in air has been analyzed separately as follows.

The Gaussian Beam

The electric field distribution for the Gaussian beam may be expressed as

$$(E_{0g})_{z=0} = E_{00g}\exp\left(-\frac{r^2}{2r_{0g}^2}\right), \tag{6}$$

where r_{0g} is the initial beam width of the Gaussian beam.

Following the paraxial approach the relevant parameters (i.e., dielectric function $\varepsilon(r,z)$, eikonal and irradiance) may be expanded around the maximum of the Gaussian electromagnetic beam, i.e., around $r = 0$. Thus, one can express the dielectric function $\varepsilon_g(r,z)$ as

$$\varepsilon_g(r,z) = \varepsilon_{0g}(z) - (r^2/r_{0g}^2)\varepsilon_{2g}(z) - i\varepsilon_i, \tag{7}$$

where $\varepsilon_{0g}(z)$ and $\varepsilon_{2g}(z)$ are the coefficients associated with r^0 and r^2 in the expansion of $\varepsilon_g(r,z)$ around $r = 0$. The expressions for these coefficients (all real) have been derived later.

Substitution for $A(r,z)$ from Eq. (5) and $\varepsilon(r,z)$ from Eqs. (1b) and (7) in Eq. (4) and equating the real and imaginary parts on both sides of the resulting equation one obtains

$$\frac{2S_g}{k}\frac{\partial k}{\partial z} + 2\frac{\partial S_g}{\partial z} + \left(\frac{\partial S_g}{\partial r}\right)^2 = \frac{1}{k^2 A_{0g}}\left(\frac{\partial^2 A_{0g}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{0g}}{\partial r}\right) - \frac{r^2}{r_{0g}^2}\frac{\varepsilon_{2g}(z)}{\varepsilon_{0g}(z)}, \tag{8a}$$

and

$$\frac{\partial A_{0g}^2}{\partial z} + A_{0g}^2\left(\frac{\partial^2 S_g}{\partial r^2} + \frac{1}{r}\frac{\partial S_g}{\partial r}\right) + \frac{\partial A_{0g}^2}{\partial r}\frac{\partial S_g}{\partial r} + A_{0g}^2\left(\frac{1}{k(z)}\frac{\partial k(z)}{\partial z} + k(z)\frac{\varepsilon_i(z)}{\varepsilon_{0g}(z)}\right) = 0, \tag{8b}$$

where $\varepsilon_i(z)$ is the imaginary part of the dielectric function.

One can express the solution of Eq. (8b) (in the paraxial approximation $r/r_0 \ll 1$) as

$$\begin{aligned} A_{0g}^2 &= \frac{E_{0g}^2}{f_0^2}\left(\frac{\varepsilon_{0g}(0)}{\varepsilon_{0g}(z)}\right)^{1/2}\exp\left(-\frac{r^2}{r_{0g}^2 f_0^2}\right) \\ &\quad\times\exp\left[-\int_0^z \frac{\varepsilon_i(\omega/c)dz}{\sqrt{\varepsilon_{0g}(z)}}\right], \\ &\approx \frac{E_{0g}^2}{f_0^2}\left(\frac{\varepsilon_{0g}(0)}{\varepsilon_{0g}(z)}\right)^{1/2}\exp\left(-\frac{r^2}{r_{0g}^2 f_0^2}\right)\times\exp(-\delta z), \end{aligned} \tag{9a}$$

where

$$\begin{aligned} &S_g(r,z) = \frac{r^2}{2}\beta_g(z) + \varphi_g(z), \\ &\beta_g(z) = \frac{1}{f_0}\frac{df_0}{dz}, \\ &\varepsilon_{0g}(z) \approx 1 \text{ for air}, \\ &\delta = \varepsilon_i(\omega/c) \text{ may be recognized as the irradiation} \\ &\text{attenuation constant in the medium}, \\ &\varphi_g(z) \text{ is an arbitrary function of } z, \end{aligned} \tag{9b}$$

and

$f_0(z)$ is the beam width parameter, associated with the Gaussian beam.

On substituting for A_{0g}^2 and S_g from Eqs. (9a) and (9b) in Eq. (8a) and equating the coefficient of r^2 on both sides of the resulting equation, one obtains

$$f_0\frac{d^2 f_0}{dz^2} = \frac{1}{k^2 r_{0g}^2 f_0^2} - \frac{1}{r_{0g}^2}\frac{\varepsilon_{2g}}{\varepsilon_{0g}}. \tag{10}$$

The dependence of the beam width parameter f_0 on z can be obtained by the numerical solution of Eq. (10) after putting suitable expressions for ε_{0g} and ε_{2g}, with the initial boundary conditions $f_0 = 1$, $\frac{df_0}{dz} = B^{-1}$ at $z = 0$, where B is the initial radius of curvature of the beam.

The Hollow Gaussian Beam

The amplitude of the electric field associated with a hollow Gaussian electromagnetic beam (HGB) having zero irradiance along the axis $r = 0$ and a maximum away from the axis, can be expressed as

$$(E_{0n})_{z=0} = E_{00n}\left(\frac{r^2}{2r_{0n}^2}\right)^n \exp\left(-\frac{r^2}{2r_{0n}^2}\right), \tag{11}$$

where r_{0n} is the initial beam width of the HGB, n is the order of the HGB and a positive integer, characterizing the shape of the HGB and position of its maximum and $|E_0|$ is maximum at $r^2 = r_{max}^2 = 2nr_{0n}^2$.

To proceed further one can adopt a paraxial like approach (Sodha *et al.*, 2009*a*; 2009*b*; Misra & Mishra, 2009), analogous to the paraxial approximation. Thus, one may start by expressing Eq. (4) in terms of variables η and z, where η is defined as

$$\eta^2 = \left[(r/r_{0n}f_n)^2 - 2n\right], \tag{12}$$

$r_{0n}\ f_n\ (z)$ is the width of the beam, and $r^2 = 2nr_{0n}^2\ f_n^2$ is the position of the maximum irradiance of the

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propagating beam. It is shown later that in the paraxial like approximation, i.e., when $\eta^2 \ll 2n$, the beam retains its profile.

In the paraxial like approximation the relevant parameters (i.e., dielectric function $\varepsilon(r,z)$, eikonal and irradiance) may be expanded around the maximum of the HGB, i.e., around $\eta = 0$. Thus, one can express the dielectric function $\varepsilon_n(\eta,z)$ around the maximum ($\eta = 0$) of the HGB as

$$\varepsilon_n(\eta, z) = \varepsilon_{0n}(z) - \eta^2\varepsilon_{2n}(z) - i\varepsilon_i, \tag{13}$$

where $\varepsilon_{0n}(z)$ and $\varepsilon_{2n}(z)$ are the coefficients (both real) associated with η^0 and η^2 in the expansion of $\varepsilon_n(\eta\ ,z)$ around $\eta = 0$. The expressions for these coefficients have been derived later.

Substitution for $A(r,z)$ from Eq. (5) and $\varepsilon(r,z)$ from Eq. (1b) and $\varepsilon_n(\eta,z)$ from Eq. (13) in Eq. (4) by using the transformation Eq. (12) and the separation of the real and imaginary parts on both side of the resulting equation leads to

$$\begin{aligned}&\frac{2S_g}{k}\frac{\partial k}{\partial z} + 2\left(\frac{\partial S_n}{\partial z} - \frac{(\sqrt{2n}+\eta)}{f}\frac{\partial f}{\partial z}\frac{\partial S_n}{\partial \eta}\right)\\ &+ \frac{1}{r_{0n}^2 f_n^2}\left(\frac{2n+\eta^2}{\eta^2}\right)\left(\frac{\partial S_n}{\partial \eta}\right)^2\\ &= \left[\left(\frac{2}{\eta} - \frac{2n+\eta^2}{\eta^3}\right)\frac{\partial A_{0n}}{\partial \eta} + \frac{2n+\eta^2}{\eta^2}\frac{\partial^2 A_{0n}}{\partial \eta^2}\right]\\ &- \eta^2\frac{\varepsilon_{2n}(z)}{\varepsilon_{0n}(z)},\end{aligned} \tag{14a}$$

and

$$\begin{aligned}&\left(\frac{\partial A_{0n}^2}{\partial z} - \frac{(\sqrt{2n}+\eta)}{f}\frac{\partial f}{\partial z}\frac{\partial A_{0n}^2}{\partial \eta}\right) + \frac{A_{0n}^2}{r_{0n}^2 f_n^2}\\ &\times\left[\left(\frac{\partial^2 S_n}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial S_n}{\partial \eta}\right) + \frac{2n}{\eta^2}\left(\frac{\partial^2 S_n}{\partial \eta^2} - \frac{1}{\eta}\frac{\partial S_n}{\partial \eta}\right)\right]\\ &+ \frac{1}{r_{0n}^2 f_n^2}\frac{(2n+\eta^2)}{\eta^2}\frac{\partial A_{0n}^2}{\partial \eta}\frac{\partial S_n}{\partial \eta}\\ &+ A_{0n}^2\left(\frac{1}{k(z)}\frac{\partial k(z)}{\partial z} + k(z)\frac{\varepsilon_{i0}(z)}{\varepsilon_{0n}(z)}\right) = 0.\end{aligned} \tag{14b}$$

One can express the solution of Eq. (14b) (in the paraxial like approximation $\eta^2 \ll 2n$) as

$$\begin{aligned}A_{0n}^2 = &\frac{E_{0n}^2}{2^{2n}f_n^2}\left(\frac{\varepsilon_{0n}(0)}{\varepsilon_{0n}(z)}\right)^{1/2}\left(2n+\eta^2\right)^{2n}\\ &\times\exp\left(-\left(2n+\eta^2\right)\right)\times\exp(-\delta z),\end{aligned} \tag{15a}$$

where

$$\begin{aligned}&S_n(\eta, z) = \frac{\eta^2}{2}\beta_n(z) + \varphi_n(z) \text{ is the eikonal term}\\ &\qquad\text{corresponding to HGB,}\\ &\beta_n(z) = r_{0n}^2 f_n\frac{df_n}{dz},\\ &\varphi_n(z) \text{ is an arbitrary function of z,}\end{aligned} \tag{15b}$$

and

$f_n(z)$ is the beam width parameter for the HGB.

On substituting for A_{0n}^2 and S_n from Eqs. (15a) and (15b) in Eq. (14a) and equating the coefficients of η^2 on both sides of the resulting equation, one obtains

$$\varepsilon_{0n}f_n\frac{d^2 f_n}{dz^2} = \frac{(12n^2 - 2n - 13/4)}{k^2 r_{0n}^2 f_n^2} - \frac{\varepsilon_{2n}}{r_{0n}^2}. \tag{16}$$

The variation of the beam width parameter f_n with z can be obtained by the numerical integration of Eq. (16) after putting suitable expressions for ε_{0n} and ε_{2n}, with appropriate initial boundary conditions *viz.* $f_n = 1$, $\frac{df_n}{dz} = B^{-1}$ at $z = 0$.

The Nonlinear Term

Consider a cylindrical shell of thickness dr (with radius r), whose axis is coincident with the beam. Neglecting convection effects the steady state thermal balance may be expressed as

$$\chi\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = r\frac{dI(r, z)}{dz}, \tag{17a}$$

where

$$\begin{aligned}&I(r, z) = (c/4\pi)A_0^2(r, z),\\ &A_0^2 \text{ is given by Eqs. (9a) and (15a) with } \varepsilon_{0r} \approx 1,\end{aligned} \tag{17b}$$

and

χ is the thermal conductivity of the medium.

The attenuation coefficient δ is the sum of the absorption (α_a) and scattering (α_s) coefficients, i.e.,

$$\delta = \alpha_a + \alpha_s. \tag{17c}$$

Gaussian Beam

In the paraxial approximation ($r/r_0 \ll 1$), one can expand $T(r,z)$ around the maximum *viz.* $r = 0$, as

$$T(r, z) = T_{0g} - (r^2/r_0^2)T_{2g}. \tag{18}$$

Substituting for $T(r,z)$ from Eq. (18) in Eq. (1) and comparing the coefficient of r^0 and r^2 in the resulting equation with Eq. (7) one obtains,

$$\varepsilon_{0g} = \varepsilon_0 + \left[T_{0g} - T_0\right]\frac{d\varepsilon}{dT}, \tag{18a}$$

and

$$\varepsilon_{2g} = T_{2g}\frac{d\varepsilon}{dT}. \tag{18b}$$

On substituting for $T(r,z)$ from Eq. (18) in Eq. (17a) and comparing the r independent terms on both side of the resulting equation one obtains,

$$T_{2g} = \frac{\alpha_a c r_{0g}^2}{16\pi\chi}\frac{E_{0g}^2}{f_0^2}\exp(-(\alpha_a + \alpha_s)z) \tag{19}$$

Substituting for T_{2g} from Eq. (19) in Eq. (7)

$$\varepsilon_{2g} = \frac{\alpha_a c r_{0g}^2}{16\pi\chi}\frac{E_{0g}^2}{f_0^2}\exp(-(\alpha_a + \alpha_s)z)\frac{d\varepsilon}{dT}$$

or

$$\varepsilon_{2g} = \frac{\alpha_a}{4\pi\chi}\frac{P_{0g}}{f_0^2}\exp(-(\alpha_a + \alpha_s)z)\frac{d\varepsilon}{dT}, \tag{20a}$$

where

$P_{0g} = c r_0^2 E_{0g}^2/4$ is the initial power of the Gaussian electromagnetic beam. (20b)

On substituting for ε_{2g} from Eq. (20a) in Eq. (10) and remembering that $\varepsilon_{0g} \approx 1$ one obtains,

$$f_0\frac{d^2 f_0}{dz^2} = \frac{1}{r_{0g}^4 k^2 f_0^2} - \frac{\alpha_a}{r_{0g}^2 4\pi\chi}P_{0g}\exp(-(\alpha_a + \alpha_s)z)\frac{d\varepsilon}{dT},$$

or

$$f_0\frac{d^2 f_0}{d\xi^2} = \frac{1}{f_0^2\rho_{0g}^2} + \Pi_{0g}\exp(-\beta\xi), \tag{10a}$$

where $\Pi_{0g} = \Lambda = -\left(\alpha_a P_{0g}/4\pi\chi d\varepsilon/dT\right)$ is proportional to the product of the initial power of the Gaussian electromagnetic beam and the absorption coefficient, $\rho_{0g} = (r_{0g}\omega/c)$ is the dimensionless beam width, $\beta = r_{0g}\,(\alpha_a + \alpha_s)$ and $\xi = z/r_0$.

Hollow Gaussian Beam

Following the paraxial like approach (i.e., $\eta^2 \ll 2n$), the radial dependence of the electron temperature $T(\eta,z)$, can be expanded around the maximum of the irradiance of the beam; thus,

$$T(\eta, z) = T_{0n} - \eta^2 T_{2n}. \tag{21}$$

Substituting for $T(\eta,z)$ from Eq. (21) in Eq. (1) and comparing the coefficient of r^0 and r^2 in the resulting equation with Eq. (13) one obtains,

$$\varepsilon_{0n} = \varepsilon_0 + [T_{0n} - T_0]\frac{d\varepsilon}{dT}, \tag{22a}$$

and

$$\varepsilon_{2n} = T_{2n}\frac{d\varepsilon}{dT}. \tag{22b}$$

The thermal balance (Eq. (17a)) may be expressed in terms of variables (η,z) as follows,

$$\frac{\chi}{r_{0n}^2 f_n^2}\left[\left(\frac{\partial^2 T}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial T}{\partial\eta}\right) + \frac{2n}{\eta^2}\left(\frac{\partial^2 T}{\partial\eta^2} - \frac{1}{\eta}\frac{\partial T}{\partial\eta}\right)\right] = \frac{dI}{dz}, \tag{23}$$

where

$$\beta = r_{0g}(\alpha_a + \alpha_s).$$

Substituting for $T(\eta,z)$ from Eq. (21) in Eq. (23) and comparing the η independent terms on both sides in the resulting equation one gets,

$$T_{2n} = \frac{c}{4\pi}\frac{\alpha_a r_{0n}^2}{4\chi f_n^2}E_{0n}^2\exp(-2n)n^{2n}\exp(-(\alpha_a + \alpha_s)z). \tag{24}$$

From Eqs. (22b) and (24) one obtains,

$$\varepsilon_{2n} = \frac{c}{4\pi}\frac{\alpha_a r_{0n}^2}{4\chi f_n^2}E_{0n}^2\exp(-2n)n^{2n}\exp(-(\alpha_a + \alpha_s)z)\frac{d\varepsilon}{dT},$$

or

$$\varepsilon_{2n} = \frac{\alpha_a}{4\pi\chi f_n^2}\left(\frac{2^{2n}}{2n!}\right)P_{0n}\exp(-2n)n^{2n}\exp(-(\alpha_a + \alpha_s)z)\frac{d\varepsilon}{dT}, \tag{25a}$$

where

$$P_{0n} = \frac{c r_{0n}^2 E_{0n}^2}{4}\left(\frac{2n!}{2^{2n}}\right) \text{ is the initial power of the HGB.} \tag{25b}$$

On substituting for ε_{2n} from Eq. (25a) in Eq. (16) and remembering that $\varepsilon_{0n} \approx 1$ the equation for beam width parameter f_n is

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$$f_n \frac{d^2 f_n}{dz^2} = \frac{(12n^2 - 2n - 13/4)}{k^2 r_{0n}^4 f_n^2} - \frac{\alpha_a}{4\pi\chi} \frac{P_{0n}}{r_{0n}^2} \frac{d\varepsilon}{dT} \left(\frac{2^{2n}}{2n!}\right) n^{2n} \times \exp(-2n) \exp(-(\alpha_a + \alpha_s) z),$$

or

$$f_n \frac{d^2 f_n}{d\xi^2} = \frac{(12n^2 - 2n - 13/4)}{f_n^2 \rho_{0n}^2} + \Pi_{0n} \exp(-\beta\xi), \qquad (16a)$$

where

$$\Pi_{0n} = -\left(\frac{\alpha_a}{4\pi\chi_e} \frac{d\varepsilon}{dT} P_{0n} \left(\frac{2^{2n}}{2n!}\right) n^{2n} \exp(-2n)\right) = \left(\left(\frac{2^{2n}}{2n!}\right) n^{2n} \exp(-2n)\right) \Lambda \text{ (with } P_{og} = P_{on}\text{)},$$

corresponds to initial dimensionless power of the HGB, $\rho_{0n} = (r_{0n}\ \omega/c)$ is the dimensionless beam width and $\beta_n = r_{0n}\ (\alpha_a + \alpha_s)$; for $n = 1$, $\Pi_{on} = 2\exp(-2)\Lambda$.

NUMERICAL RESULTS AND DISCUSSION

Atmospheric Parameters

The absorption (α_a) and scattering (α_s) coefficients for four environments, arrived at by Spangle *et al.* (2005) are given in Table 1.

Thermal conductivity of air $\chi = 2.5 \times 10^3\ erg/s\ cmK$

Rate of change of dielectric function with temperature $\frac{d\varepsilon}{dT} = -8.7 \times 10^{-7}/K$

Laser Parameters

$r_0\ (r_{0g} = r_{0n}) = 50\ to\ 100\ cm$, initial power $P_0 = 0.5$ to $5.0\ MW$ and wavelengths $1.045\ \mu m$, $1.625\ \mu m$ and $2.141\ \mu m$ (water absorption windows).

Dimensionless Parameters

With the above data the relevant dimensionless parameters lie in the ranges indicated as follows;

$$\Lambda = \frac{\alpha_a}{4\pi\chi} \frac{P_0}{r_{0n}^2} \frac{d\varepsilon}{dT};\ 10^{-6}\ to\ 5 \times 10^{-4},$$

$$\beta = r_0(\alpha_a + \alpha_s);\ 2.5 \times 10^{-5}\ to\ 2 \times 10^{-4},$$

and

$$\rho_0 = (r_0\omega/c) = (2\pi r_0/\lambda);\ 1.5 \times 10^6\ to\ 6 \times 10^6.$$

Table 1. *Absorption and scattering coefficients of laser beams in air (Sprangle et al., 2005)*

Atmospheric Environment	Maritime			Desert			Rural			Urban		
Laser Wavelength (in μm)	1.045	1.625	2.141	1.045	1.625	2.141	1.045	1.625	2.141	1.045	1.625	2.141
Absorption Coefficient (α_a) (in km^{-1})	2×10^{-3}	2×10^{-3}	3×10^{-3}	7×10^{-4}	5×10^{-4}	6×10^{-4}	0.016	0.012	0.006	0.05	0.036	0.028
Scattering Coefficient (α_s) (in km^{-1})	1.2×10^{-1}	7×10^{-2}	5×10^{-2}	0.17	0.097	0.072	0.15	0.076	0.053	0.13	0.065	0.044

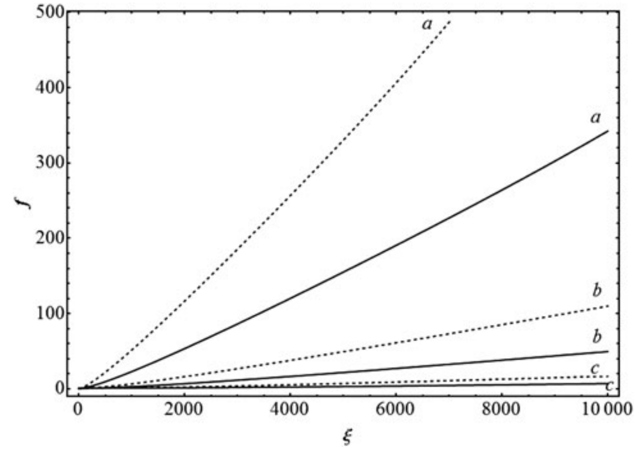

Fig. 1. Dependence of beam width parameter f on dimensionless distance of propagation ξ for different values of Λ (proportional to the product of initial beam power and the absorption coefficient) for $\beta = 10^{-4}$ and $\rho = 4.5 \times 10^6$; where *a*, *b* and *c* refer to $\Lambda = 5 \times 10^{-4}$, 2×10^{-5} and 10^{-5} respectively. The broken and full curves correspond to Gaussian and hollow Gaussian beams, respectively.

Figure 1 illustrates the dependence of the focusing parameter f on dimensionless distance of propagation ξ for different values of Λ (proportional to the product of initial beam power and the absorption coefficient). It is seen that f (or defocusing) increases with increasing Λ and that it is very much less for a hollow Gaussian beam as compared to that for a Gaussian beam; the physical reasoning for this fact is rather simple. In the central portion of a dark hollow beam $0 < r < r_m$ the temperature increases radially, while in the peripheral region the temperature decreases radially; the maximum irradiance of the beam occurs at $r = r_m$. Hence the dielectric function decreases radially in the central $r < r_m$ portion of beam which tends to focus the beam, while the peripheral portion of the beam (in which ε increases with r) defocuses.

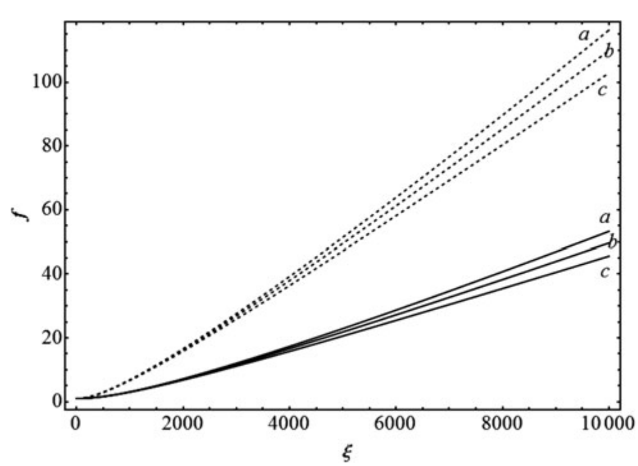

Fig. 2. Dependence of beam width parameter f on dimensionless distance of propagation ξ for different values of attenuation constant $\beta = r_0(\alpha_a + \alpha_s)$ for $\Lambda = 2 \times 10^{-5}$ and $\rho = 4.5 \times 10^6$; where *a*, *b* and *c* refer to $\beta = 5 \times 10^{-4}$, 2×10^{-5} *and* 10^{-5} respectively. The broken and full curves correspond to Gaussian and hollow Gaussian beams, respectively.

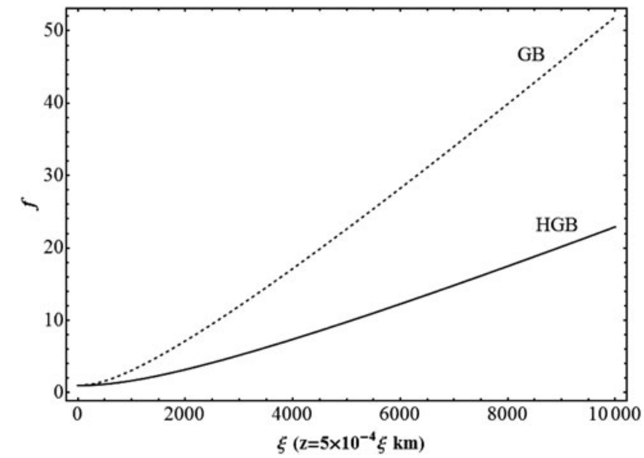

Fig. 3. Dependence of beam width parameter f on dimensionless distance of propagation $\xi(=z/r_0)$ in maritime environment; $\lambda = 1.045$ μ*m*, $P_0 = 1$ *MW*, $\alpha_a = 2 \times 10^{-3}\ km^{-1}$, $\alpha_s = 1.2 \times 10^{-1}\ km^{-1}$ and $r_0 = 50$ *cm*. The broken and full curves correspond to Gaussian and hollow Gaussian beams, respectively.

Hence in contrast to a Gaussian beam the central $r < r_m$ portion of HGB tends to focus the beam, while the peripheral portion of the beam defocuses. As a result the defocusing in case of HGB is much less than that in case of Gaussian beams, where all portions of the beam defocus. Fig. 2 indicates the dependence of f on ξ for different values of β. It is seen that f or defocusing decreases with increasing β this is due to the fact that the thermal effect decreases as the beam propagates due to decrease in the beam power, corresponding to enhanced absorption. Fig. 2 also indicates much reduced defocusing for the hollow Gaussian beam, as compared to a Gaussian beam. Computations also lead to an interesting conclusion that $f - \xi$ relationship is independent of ρ_0 and to a good approximation $f \propto \xi\ (=z/r_0)$ and hence inversely proportional to r_0 at a given position (value of z).

In addition to the dimensionless discussion (Figs. 1 and 2), it will also be of general interest to consider a specific case of propagation (in maritime environment) *viz.* $\lambda = 1.045$ μ*m*, $P_0 = 1$ *MW*, $\alpha_a = 2 \times 10^{-3}\ \text{km}^{-1}$, $\alpha_s = 1.2 \times 10^{-1}\ \text{km}^{-1}$ and $r_0 = 50$ *cm*; the dependence of the beam width parameter f on the distance of propagation z in km is shown in Figure 3. This figure also highlights the fact that an HGB defocuses significantly less than a corresponding Gaussian beam of same radius and power.

CONCLUSION

The thermal defocusing of a hollow Gaussian beam in the atmosphere is much less than that of a corresponding Gaussian beams with the same radius and power.

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