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RAPID COMMUNICATION

Strain estimation from flattened parallel folds: application of the Wellman method and Mohr circle

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Abstract – Parallel folds exhibit a characteristic orthogonal relationship between the tangent and the corresponding isogon drawn at any point on folded surface. Modification of parallel fold to flattened parallel fold by superimposition of homogeneous strain introduces an angular shear along the tangents at different points. The angular shears in different directions, obtained by measuring angles between the tangents and the corresponding isogons, can be used for estimation of flattening strain by a variety of geometrical and numerical methods. We show that several simple geometrical techniques, such as the Wellman method and the Mohr circle method, can rapidly decipher the strain from flattened parallel folds. These methods, in contrast to most of the existing methods of strain estimation, are independent of the assumption that one of the principal strain directions parallels the axial trace on the profile plane of fold.

Keywords: flattened parallel fold, oblique flattening, angular shear, Wellman method, Mohr circle.

1. Introduction

Structural geologists seek ways of deciphering ductile strain in orogenic belts, but there are few reliable ways of measuring ductile strain in rocks that do not contain classic 'strain markers', or are folded. In such situations, flattened parallel folds, developed by superimposition of a homogeneous strain on parallel folds, can be used as a potential indicator for revealing the flattening component of strain history (Campbell, 1951; Ramsay, 1962; Hudleston, 1973a,b,c; Mukhopadhyay, 1965; Naha & Halyburton, 1977; Gray & Durney, 1979; Ramsay & Huber, 1987, pp. 349–60).

Amongst several methods that are available for estimation of the flattening strain in the flattened parallel folds, the t'_α–α method of Ramsay (1962; 1967, p. 413) and the ϕ–α method of Hudleston (1973a) have been most widely used. These methods provide estimates of strain by visual matching of observed t'_α–α or ϕ–α plots with standard t'_α–α or ϕ–α curves designed for various amounts of flattening strain (figure 7–29 in Ramsay, 1967, p. 413; fig. 19 in Hudleston, 1973a). Advantage of the visual curve matching in these methods is, however, offset by the fact that the standard curves tend to merge with each other, particularly at low angles of limb dip (α) and high amounts of flattening strain. In addition, the application of these methods is subject to the satisfaction of two main conditions, namely (1) the profile plane represents one of the principal planes of strain ellipsoid, and (2) on the profile plane, the axial trace of fold parallels one of the principal strain directions.

Since the above two conditions are not satisfied by many natural folds, we need methods that do not assume parallelism between the geometrical elements of folds and principal strain directions. This article describes two simple geometrical techniques, namely the Wellman method and the Mohr circle method, that can estimate flattening strain in any flattened parallel fold regardless of the angle between the principal strain directions and the axial trace of the fold.

2. The rationale

Parallel (class 1B) folds are characterized by the orthogonal relationship between the dip isogon, a line joining the points of equal dips on the outer and inner arcs, and the corresponding tangents on the profile section (Fig. 1a). The tangents and the isogons deform as passive lines as the superimposition of homogeneous strain on a parallel fold modifies its geometry into a flattened parallel (class 1C) fold. An important geometrical consequence of such a deformation is that the orthogonality between the tangents and the corresponding isogon is lost at every point, except in the principal strain directions (Fig.1b). The angular shears obtained by measuring the changes in orthogonality between the tangents and the respective isogons at different points on the folded surface form the basis for deciphering the flattening strain by a variety of methods. These include: (i) geometrical techniques, such as the Breddin's graph method (Breddin, 1956; Ramsay & Huber, 1983, pp. 129–32), the maximum shear strain method (Ramsay, 1967, pp. 197–9), the Wellman method (Wellman, 1962; Ramsay, 1967, p. 242; Ragan, 1985, pp. 175–7) and the Mohr circle method (Ramsay, 1967, pp. 235–7; Treagus, 1987; Lisle, 1991); (ii) numerical solutions, e.g. solutions to equations 5.39a and 5.39b in Ramsay (1967, p. 237) and

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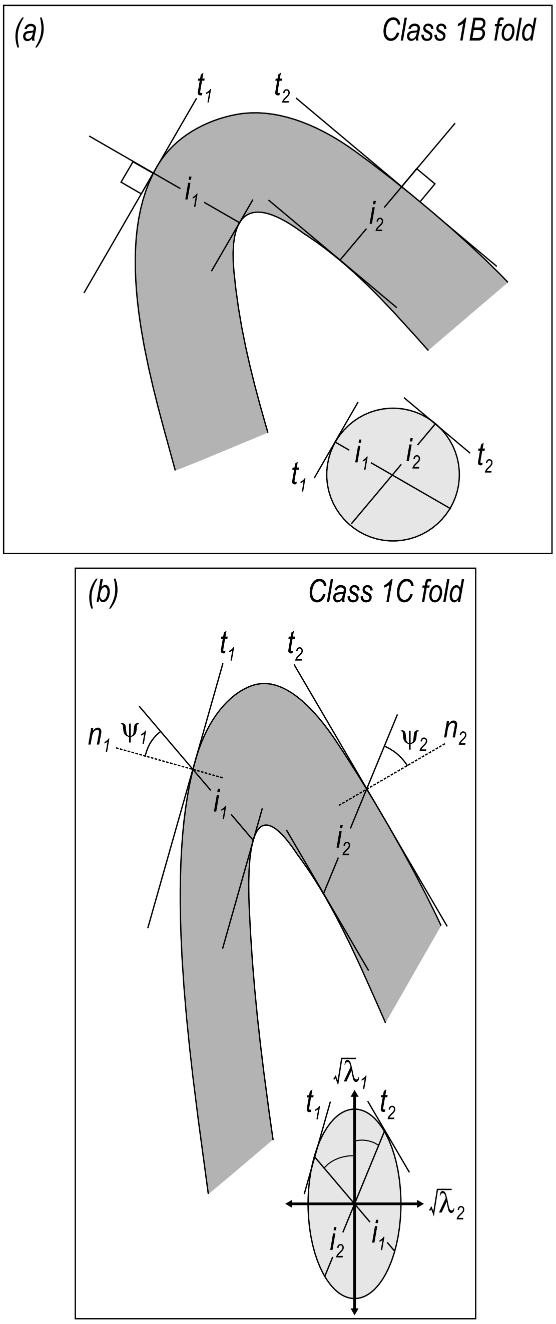

Figure 1. (a) Orthogonal relationship between the isogons i_1 and i_2, and the corresponding tangents t_1 and t_2, on the profile section of a parallel (class 1B) fold. A reference circle is drawn below the fold. (b) Profile section of the class 1C fold developed due to flattening of the class 1B fold in (a). Note that the axial trace of the fold does not parallel principal strain directions, $\sqrt{\lambda_1}$ and $\sqrt{\lambda_2}$, along the major and minor axes of the stain ellipse. ψ_1 and ψ_2 are respectively the angles of shear along the tangents t_1 and t_2. Orientation of strain ellipse, representing flattening strain, can be measured in terms of the angle between the major axis and the tangents or isogons.

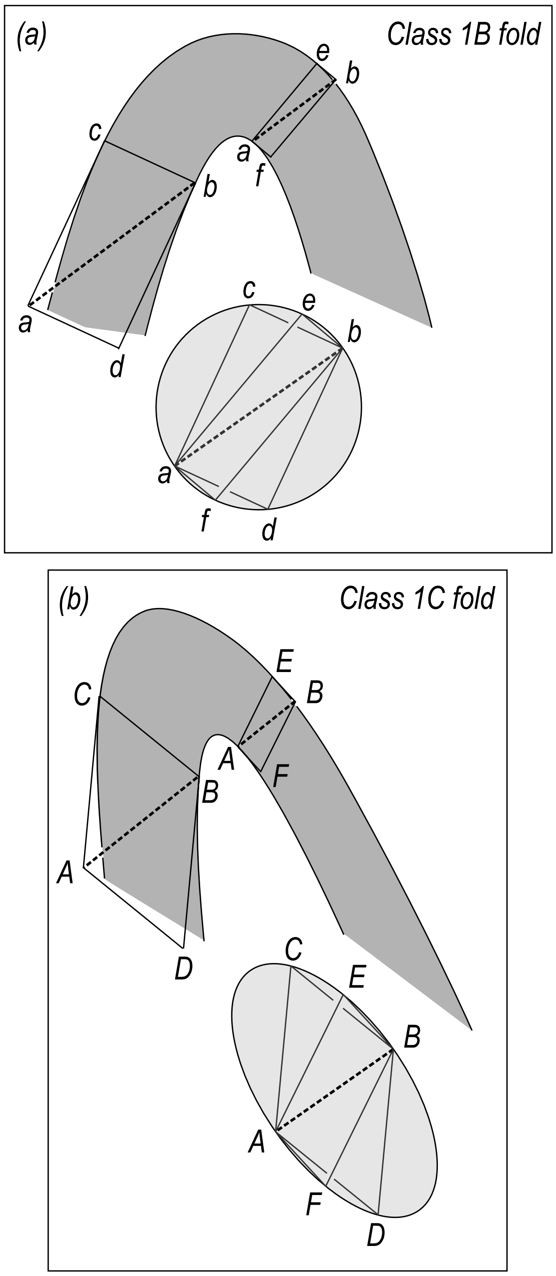

Figure 2. Principle for application of the Wellman method. (a) *acbd* and *aebf* are two typical rectangles obtained by drawing lines paralleling the isogons and the corresponding tangents at two points on the profile section of a parallel fold. By scaling the diagonals of rectangles to a common length *ab*, the locus of vertices of the rectangles is obtained as a circle of diameter *ab*. (b) Flattened parallel fold developed due to superimposition of homogeneous strain on the parallel fold in (a). *ACBD* and *AEBF* are parallelograms with sides paralleling the isogons and the corresponding tangents. The diagonals of parallelograms are scaled to a common length *AB*, so that the locus of vertices of the parallelograms is the required strain ellipse.

(iii) computer-based techniques, such as the retrodeformational method (Srivastava & Shah, 2006). Herein, we briefly describe the procedures of the application of the Wellman method and the Mohr circle method, which involve simple geometrical constructions but produce rapid and accurate results.

3. The Wellman method

On the profile section of a class 1B fold, it is possible to draw a set of rectangles such that the two adjacent sides of each rectangle parallel the isogon and the corresponding tangents (Fig. 2a). These rectangles can be represented in a common

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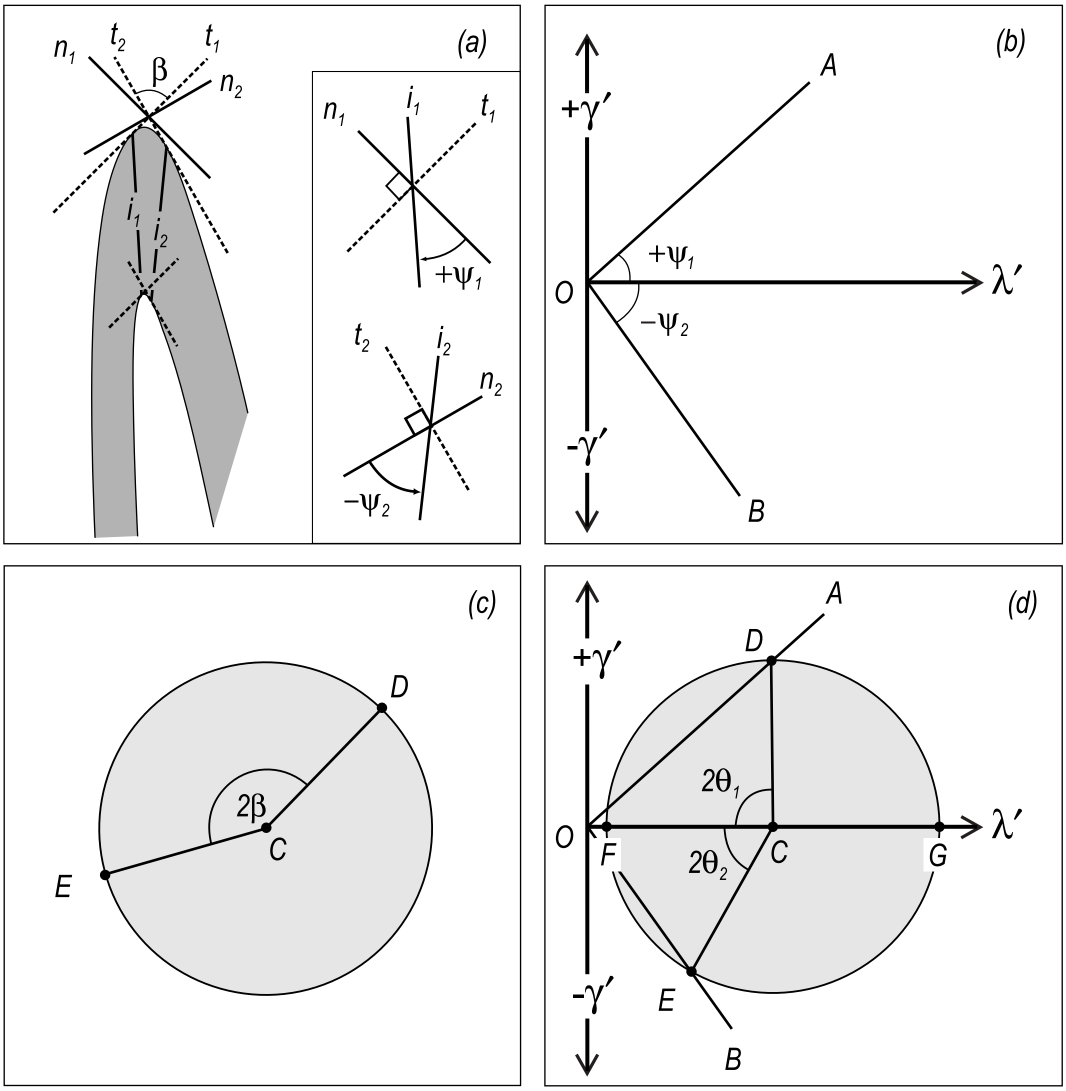

Figure 3. The Mohr circle method. (a) Profile section of the given class 1C fold. t_1 and t_2 – tangents at any two points; n_1 and n_2 – normals to the tangents t_1 and t_2 respectively; i_1 and i_2 – dip isogons; β – the angle between t_1 and t_2. Inset shows the criterion for sense of angular shear. (b) λ'–γ' cartesian frame showing two lines *OA* and *OB* that pass through the origin and make angles $+\psi_1$ and $-\psi_2$ with respect to the λ' axis. (c) A circle of arbitrary radius. 2β is the angle between radii *CD* and *CE*. (d) Mohr diagram showing a unique solution. $\sqrt{OG/OF}$ – axial ratio of strain ellipse. Major axis of strain ellipse is inclined at angles θ_1 and θ_2 with respect to the tangents t_1 and t_2, respectively.

reference frame by drawing the lines parallel to the isogons and the corresponding tangents from the ends *a* and *b* of a line *ab* of arbitrary length. As shown in Figure 2a, such a construction results in a geometrical condition where the locus of the vertices of the rectangles is a circle of diameter *ab*.

As the superimposition of homogeneous strain flattens a class 1B fold into a class 1C fold, the rectangles on the class 1B fold transform into parallelograms such that the two adjacent sides of each parallelogram parallel the isogon and the corresponding tangents on class 1C fold (Fig. 2b). These parallelograms can be transferred into a common reference frame by drawing the lines that pass through the ends *A* and *B* of an arbitrarily chosen reference line *AB*, and parallel the isogon and the corresponding tangents at different points on the given class 1C fold. The locus of the vertices of such parallelograms is the required strain ellipse that represents the flattening strain (Fig. 2b).

4. The Mohr circle method

Different techniques for construction of the Mohr circles for determining the strain ratio and the principal strain directions are described by Ramsay (1967, p. 78), Treagus (1987) and Lisle (1991). We use the angular shears ψ_1 and ψ_2 along the tangents t_1 and t_2 drawn at any two points on the flattened parallel fold and adopt the procedures explained by Ramsay (1967, p. 237) and Ramsay & Huber (1983, pp. 132–4) for drawing the Mohr circle in a λ'–γ' cartesian frame (Fig. 3).

The Mohr circle intersects the λ' axis at two points *F* $(k\lambda_1', 0)$ and *G* $(k\lambda_2', 0)$, respectively (Fig. 3d). The square root of ratio *OG/OF* equals to $\sqrt{(\lambda_2'/\lambda_1')}$, which is the axial ratio of the finite strain ellipse for the flattening component in the given class 1C fold. The direction of maximum stretching is located at angles θ_1 or θ_2 from the tangents t_1 or t_2, respectively, measured according to the standard conventions

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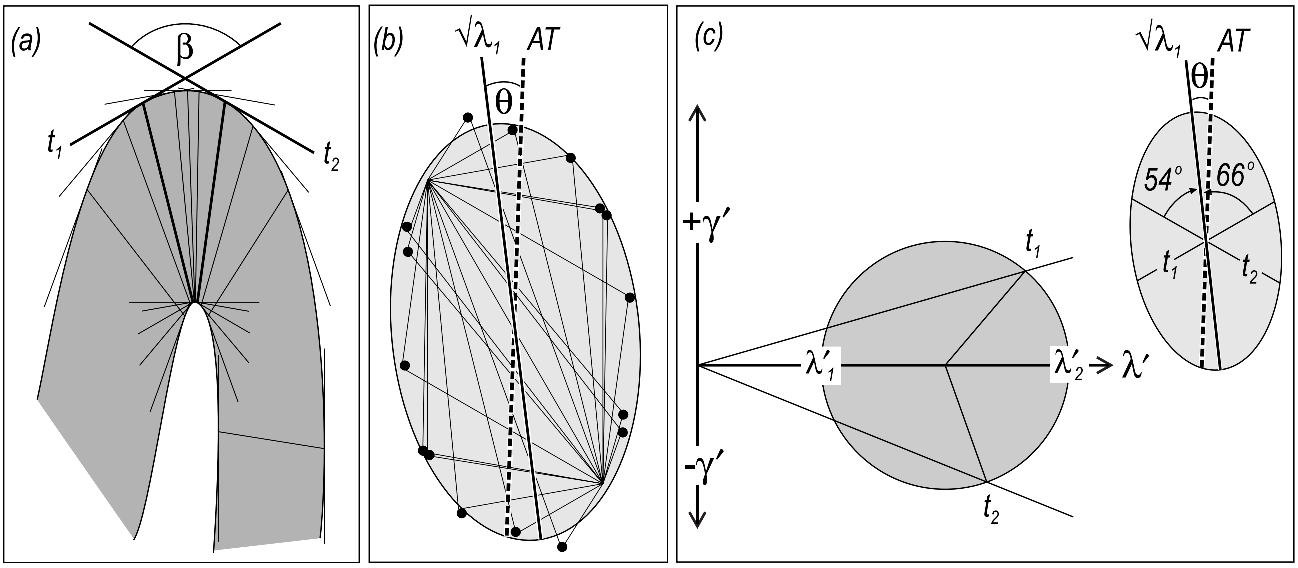

Figure 4. (a) Profile section of a flattened parallel fold (taken from figure 19.11 in Ramsay & Huber, 1987, p. 393), along with the isogons and corresponding tangents. Tangents t_1 and t_2 and the corresponding isogons are highlighted. (b) Wellman's construction for fold in (a). θ – angle between axial trace (AT) and major axis ($\sqrt{\lambda_1}$) of the strain ellipse. (c) Mohr circle solution on λ'–γ' graph. Mohr circle intersects λ' axis at λ'_1 and λ'_2. Orientation of strain ellipse with respect to tangents t_1 and t_2 and axial trace of the fold (AT) is shown in the strain ellipse.

Table 1. Results of strain analysis on the natural example of a flattened parallel fold

	Wellman method		Mohr circle method	
Example	$\sqrt{\lambda_1/\lambda_2}$	θ	$\sqrt{\lambda_1/\lambda_2}$	θ
Figure 4	1.69	8°	1.73	7°

$\sqrt{\lambda_1/\lambda_2}$ – axial ratio of strain ellipse representing flattening strain.
θ – angle between the direction of maximum stretching and the axial trace.

of the Mohr circle (Ramsay, 1967, p. 73). Alternatively, the same solution can be obtained by drawing the Mohr circle through the pole to the Mohr circle (Lisle, 1991).

5. Example

We demonstrate application of the Wellman method and the Mohr circle method on a natural example of the flattened parallel fold taken from figure 19.11 in Ramsay & Huber (1987, p. 393). This example represents a ptygmatic fold in the siltstone layer within shale beds exposed near Hope Cove, South Devon, England (Fig. 4a). The results of strain analysis on this fold show that the axial ratios and the orientations of strain ellipses obtained by the Wellman method and the Mohr circle method are quite consistent (Figure 4b,c and Table 1).

6. Discussion and conclusions

One implicit assumption in these methods is that flattening follows the process of buckling, albeit the class 1C folds can also develop by simultaneous buckling and flattening, particularly at low viscosity contrasts (Hudleston & Stephansson, 1973). The latter type of class 1C folds is not discussed in this paper. The other assumption is that of a homogeneous nature of the strain during the flattening process (Flinn, 1962; Treagus & Treagus, 1981).

The condition of orthogonality between the isogon and the corresponding tangents on the profile section of a class 1B fold opens up the possibility of the application of a large number of simple techniques for estimation of flattening strain in class 1C folds. Of these, the Wellman and the Mohr circle methods, proposed by us, have several distinct merits over the commonly used t'_α–α and the ϕ–α methods. In particular, on account of being independent of the condition of parallelism between principal strain direction and axial trace of the fold, these methods are capable of providing estimates of strain from obliquely-flattened parallel folds. In this regard, these approaches are comparable to the inverse thickness method, which is free from the assumption of parallelism between axial trace and principal strain direction (Lisle, 1992).

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