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THE INSTABILITY OF OPEN PARETO EFFICIENT ECONOMIES

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This paper analyzes the impact of trade on the stability properties of trading countries and on stationary welfare. We consider a two-country two-good two-factor overlapping generations model where countries differ in terms of their technology. In the autarky equilibrium and the free-trade equilibrium, indeterminacy relies, under dynamic efficiency, on a capital intensive consumption good and intermediate values of the elasticity of intertemporal substitution in consumption. Opening the borders to trade can be a source of a global destabilizing effect. Indeed, considering a free-trade equilibrium in which one country is an exporter of the consumption good and the other country is an exporter of the investment good, indeterminacy can occur with trade even though the two countries are determinate in autarky. Finally, opening to trade increases the stationary welfare of the country that exports the investment good and deteriorates the one of the other country.

Keywords: Two-Sector Model, Two-Country, Local Indeterminacy, Stationary Welfare

1. INTRODUCTION

The effects of international trade and globalization on welfare are one of the main concerns of economic theory. The traditional trade theory shows that there exist gains from free-trade. However, dynamic considerations are usually not considered.[1] When one takes it into account, the dynamic effect of opening the borders to free-trade may overcome the static effects.[2] In top of that, if the free-trade equilibrium may be Pareto superior than the former one obtained, under autarky, it may be possible that it occurs at the cost of higher instability, given rise to the relevancy of the gains. Indeed, when considering empirical work, ambiguous results

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are obtained in terms of stability. di Giovanni and Levchenko (2009) show that countries more exposed to trade are those with higher macroeconomic volatility; meanwhile, Kose et al. (2003) show that an important trade is associated with less macroeconomic volatility. The aim of this paper is, then, to provide some answers on the link between welfare, stability, and globalization. The increasing integration of financial markets and the internationalization of trade have marked the evolution of international relations among countries in the past decades. World international trade grew almost three times the world production from 1960 to 2010. The share of world international trade as a percentage of the world's gross domestic product increased from 19% in 1980 to 24% in 2010.[3] These stylized facts show that the internationalization of trade grows quickly. As a consequence, understanding the effect of trade on the stability and on welfare of economies is more and more important.

While the interlinkage of business cycles has extensively been studied in the literature on real business cycle, see, for example, Obstfeld (1994), the analysis of indeterminacy and sunspot equilibria is more scarce. The seminal paper, Nishimura and Yano (1993), investigates the dynamic properties of a general equilibrium two-sector, two-country infinitely lived agent model. In their model, the utility associated with consumption is linear. They compare the correlation of the endogenous real business cycles in trading countries before and after the opening up of trade and show that trade can be destabilizing. After, few studies have analyzed the effect of trade on stability. Such studies have found that opening to trade may have different impacts on the stability properties of trading countries and has mainly considered two trade structures. On the one hand, there are papers considering that there is international immobility of inputs.[4] In that case, for example, Nishimura and Shimomura (2002) consider a model where countries only differ with respect to their initial factor endowments and show that international trade does not bring any instability.[5] In contrast, Sim and Ho (2007) consider different technologies across countries and prove that opening the border to trade is stabilizing. More recently, Hu and Mino (2013) consider different trade structure with lending and borrowing and show that international trade brings instability. The second subset of studies includes contributions which deal with free-trade, international capital mobility and international labor immobility. Nishimura et al. (2009) consider an infinitely lived agent model with asymmetric technologies across countries and sector-specific externalities. They show that trade creates a contagion of sunspot cycles from one country to the other.[6] In a second paper, Nishimura et al. (2014) consider again an infinitely lived agent model with asymmetric technologies across countries, assuming now Cobb–Douglas decreasing returns to scale technologies. They analyze the existence of a flip bifurcation and deterministic cycles and prove that the destabilizing effect of international trade and international capital mobility arises under certain parameter configurations.

This literature focuses on the infinitely lived agent model. However, local indeterminacy and sunspot fluctuations require the presence of market imperfections. It implies that any equilibrium is Pareto inefficient. In the overlapping generations

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(hereafter *OLG*) model, the coexistence of local indeterminacy and dynamic efficiency, that is, Pareto-optimal equilibrium paths, can occur without any market imperfections. Pareto efficiency is associated with under-accumulation of capital stock with respect to the Golden Rule. Reichlin (1986) shows that if local indeterminacy occurs with a Pareto-optimal equilibrium path, the introduction of a fiscal policy based on taxes and transfers could simultaneously stabilize the economy and reach the Pareto-optimal steady state on which all generations have a same level of welfare. Few exceptions address the issue of a destabilizing effect of international trade in the *OLG* model. Le Riche (2017) considers a two-factor (capital and labor), two-good (consumption and investment), two-country *OLG* model where countries differ only with respect to their discount rates and shows that international integration may induce a propagation of indeterminacy from one country to another.[7]

Nowadays a significant and increasing percentage of trade between developed countries is the trade of goods that belong to the same industry, see, for example, Krugman (2009).[8] However, according to Autor *et al.* (2013), there is an important increase of trade between advanced economies and low-wage economies, for example, China, in the last 15 years. Then, comparative advantage has recently experienced an important renewed interest from a theoretical point of view.[9] Moreover, empirical studies such as Fadinger and Fleiss (2011) show that differences in sectoral total factor productivity across countries are quantitatively important to explain trade flows. Accordingly, in this paper, countries have asymmetric technologies and thus different sectoral total factor productivities. A two-factor, two-good, two-period two-country *OLG* model with *CES* preferences, a nonincreasing returns to scale *CES* technology in the consumption good and a constant returns to scale Leontief technology in the investment good, is considered. These assumptions allow to obtain a non-degenerate social production function in the free-trade equilibrium and to characterize the local stability properties of the free-trade equilibrium. In the trade regime, lending and borrowing are not permitted.

We consider a sufficiently high propensity to current consumption ensuring gross rate of return higher than one at steady state in accordance with empirical evidence and analyze autarkic equilibria and then consider equilibria with trade and capital mobility between the two countries, studying the corresponding changes in steady-state welfare and in the local stability properties when economies change from an autarkic environment to free-trade. First, we assume that the two countries are in the autarky regime meaning that goods and factors are traded only on their respective domestic market. Second, both countries are in a trade regime implying that goods are traded on the international market, capital moves freely across countries whereas labor is immobile across countries.

The results of the paper are the following. The existence of indeterminacy under dynamic efficiency in the autarky regime requires a sufficiently capital intensive consumption good and an intermediate value of the elasticity of intertemporal substitution in consumption. Second, we analyze the pattern of trade. We prove that there exist a free-trade allocation such that one country exports the consumption good and imports the investment good meanwhile the other country exports the investment good and imports the consumption good. Third, we show that the existence of local indeterminacy under dynamic efficiency in the free-trade equilibrium depends on, as in the autarky equilibrium, a sufficiently capital intensive consumption good and an intermediate value of the elasticity of intertemporal substitution in consumption. The main result of this paper is to show the existence of a global destabilizing effect of international trade. In other words, when countries differ only with respect to their technologies, the opening to international trade and international capital mobility can induce indeterminacy in both countries, even if before the opening to international trade saddle-point stability existed in the two countries.

This result is also related to two other contributions that, however, consider an infinitely lived agent framework. First, Nishimura et al. (2014) consider two-factor, two-good, two-country, infinitely lived agent model with asymmetric technologies across countries, capital mobility across countries. This paper proves the existence of a global destabilization effect of international trade. In other words, the opening of trade can create persistent endogenous fluctuations at the world level meanwhile the closed-economy equilibrium in each country is saddle-point stable. Second, Hu and Mino (2013) consider a different trade structure of the current paper and of Nishimura et al. (2014) by assuming four main differences: (1) international immobility of inputs, (2) lending and borrowing, (3) non-tradable goods, and (4) difference in initial capital endowment. The mechanism at the ground of this effect is totally different. In the closed economy, two necessary conditions for indeterminacy are required: (1) an intertemporal elasticity of substitution in consumption high enough and (2) a capital intensive consumption good. After the opening to trade, indeterminacy conditions are independent on the preference parameters. In this open economy, facing lending and borrowing, if the agent wants to invest more to jump on a new equilibrium trajectory that agent does not need to curtail his consumption.

The impact of international trade on the stationary welfare is done by analyzing the change of the indirect stationary utility of agents in each countries. We show that international trade improves the stationary welfare of the country that exports the consumption good and deteriorates the stationary welfare of the country that exports the investment good. This result is similar to the one of Cremers (2005). Althought, Cremers (2005) consider a Cobb-Douglas economy where there is immobility of capital and labor across countries and country differs with respect to their discount rate, she shows that welfare gains and losses are possible in the transition path and also at the stationary equilibrium.

The remainder of the paper is organized as follows. Section 2 describes the behavior of firms and households, the perfect foresight, and the dynamic efficiency of the closed economy meanwhile Section 3 provides the setup of the two-country model. Section 4 introduces the analysis of the stability properties of the closed economy and the two-country model. Section 5 presents the effect of international trade and capital movements on indeterminacy and on the stationary welfare. Section 6 contains the concluding remarks. Finally, proofs are gathered in the Appendix.

2. THE AUTARKY MODEL

We consider two closed economies, which can be A or B, that have two goods (consumption and investment), two factors (capital and labor), and two generations (young and old) at each period of time. Both countries have perfect competition in the output market, in the capital market, and in the labor market. Countries only differ with respect to their technology. The structure of the autarky model is similar to Nourry and Venditti (2011) extended to possibly decreasing returns to scale technologies in the consumption good sector as in Le Riche (2017). In the present section, to simplify the exposition we do not consider any superscripts for country A and country B. However, when the two countries are considered at the same time we will add superscripts $\{A, B\}$ to distinguish them.

2.1. Technology

Consider a competitive economy in which there are two sectors, one representative firm for each sector and each firm producing one good. In this economy, there exists two goods: one consumption good produced in quantity, $Y_{0,t}$, and one investment good produced in quantity, Y_t. The consumption good is taken as the numéraire. Each sector uses two factors, capital, K_t, and labor, L_t, both factors being mobile between sectors. Depreciation of capital is complete within one period: $K_{t+1} = Y_t$, where K_{t+1} is the total amount of capital in period $t + 1$.[10]

The consumption good, $Y_{0,t}$, is assumed to be produced with a CES technology and the investment good, Y_t, is assumed to be produced with a Leontief technology:

$$Y_{0,t} = F^0\left(K_t^0, L_t^0\right) = \Theta\left\{\mu\left(K_t^0\right)^{-\rho} + (1-\mu)\left(L_t^0\right)^{-\rho}\right\}^{-\frac{\nu}{\rho}},$$
$$Y_t = F^1\left(K_t^1, L_t^1\right) = \min\left\{\frac{K_t^1}{\eta}, L_t^1\right\}, \quad (1)$$

where $\mu \in (0, 1)$ reflects the capital intensity in production, $\sigma = 1/(1+\rho) \geq 0$ is the sectoral elasticity of capital–labor substitution in the consumption good sector, $\nu > 0$ is the degree of returns to scale in the consumption good sector, $\eta \in (0, 1)$ is the capital intensity in the investment good sector, and $\Theta > 0$ is the total factor productivity used as a normalization constant. We assume a nonincreasing returns to scale in the consumption good sector, that is, $\nu \leq 1$. In other words, we incorporate a fixed factor of production, normalized to one, in the consumption good sector such that the production function in that sector exhibits diminishing

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returns to capital and labor. Labor is normalized to one and given by $L = L_t^0 + L_t^1 = 1$, and the capital stock is given by $K_t = K_t^0 + K_t^1$.

The set of admissible for the two-uple (K_t, Y_t) is now determined. By definition it holds that $Y \leq F^1(K, 1)$. Let $\overline{K} > 0$ be a positive threshold of the capital stock such that $K - F^1(K, 1) = 0$. Such a threshold implies that $F^1(K, 1) > K$ when $K < \overline{K}$ and $F^1(K, 1) < K$ when $K > \overline{K}$. All of these indicate that it is not possible to maintain stock beyond $\overline{K}$. Then, we define the set of admissible for the two-uple (K_t, Y_t) as follows:

$$\tilde{K} = \left\{ (K_t, Y_t) \in \mathbb{R}_+^2 \mid K_t \leq \overline{K},\ Y_t \leq F^1(K_t, 1) \right\}. \tag{2}$$

For any $(K_t, Y_t) \in \tilde{K}$, the profit maximization of each firm is similar to solving the following problem of optimal allocation of factors between sectors:

$$\begin{aligned} T(K_t, Y_t, L) = \max_{K_t^j, L_t^j, j\in\{0,1\}} \quad Y_{0,t} \\ s.t.\ \ Y_t \leq F^1(K_t^1, L_t^1),\ K_t^0 + K_t^1 \leq K_t,\ L_t^0 + L_t^1 \leq L \end{aligned}. \tag{3}$$

The function $T(K_t, Y_t, L)$ is called the social production function, which describes the frontier of the production possibility set and gives the maximal output of the consumption good.[11] Using the production function used in each sector given by (1) and the resource constraints on capital and labor, the social production function is written as follows:

$$T(K_t, Y_t, L) = \Theta\left[\mu (K_t - \eta Y_t)^{-\rho} + (1-\mu)(L - Y_t)^{-\rho}\right]^{-\frac{\nu}{\rho}}. \tag{4}$$

Each of the two firms operates in a perfectly competitive market and thus takes the price of its produced good as well as the prices of the productive inputs as given. Let us denote r_t the rental rate of capital, w_t the wage rate, and p_t the price of the investment good all in terms of the price of the consumption good. Each firm chooses the quantities of physical capital and labor to employ in order to maximize her profits. Then, it is possible to formulate the aggregate profit maximization problem as follows:

$$\max_{(K_t, Y_t)\in\tilde{K}} T(K_t, Y_t, L) + p_t Y_t - r_t K_t - w_t L. \tag{5}$$

The associated first-order conditions are

$$\begin{aligned} r(K_t, Y_t, L) = T_1(K_t, Y_t, L), p(K_t, Y_t, L) = -T_2(K_t, Y_t, L), w(K_t, Y_t, L) \\ = T_3(K_t, Y_t, L), \end{aligned} \tag{6}$$

where $T_1 = \partial T/\partial K_t$, $T_2 = \partial T/\partial Y_t$, and $T_3 = \partial T/\partial L$.[12] From the production function used in each sector defined by (1) and the resource constraints of the economy, it is possible to derive the relative capital intensity difference, b_t, and the capital intensity in the consumption good sector, a_t:

$$b(K_t, Y_t, L) = \frac{L_t^1}{Y_t}\left(\frac{K_t^1}{L_t^1} - \frac{K_t^0}{L_t^0}\right) = \frac{\eta - K_t}{L - Y_t},\ a(K_t, Y_t, L) = \frac{K_t^0}{L_t^0} = \frac{K_t - \eta Y_t}{L - Y_t}. \tag{7}$$

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The sign of b is positive (resp. negative) if and only if the consumption good is labor (resp. capital) intensive. The Stolper–Samuelson effect $(dr/dp, dw/dp)$ and the Rybczynski effect $(dY_0/dK, dY/dK)$ are determined, respectively, by the factor-price frontier and the full employment condition and given by

$$\frac{dr}{dp} = \frac{dY}{dK} = b^{-1}, \frac{dw}{dp} = \frac{dY_0}{dK} = -ab^{-1}. \tag{8}$$

Under a labor (resp. capital) intensive consumption good, the Stolper–Samuelson effect states that an increase (resp. decrease) of the relative price decreases (resp. increases) the rental rate of capital and raises (resp. decreases) the wage rate whereas the Rybczynski effect specifies that an increase (resp. decrease) of the capital–labor ratio decreases (resp. increases) the production of the consumption good and increases (resp. decreases) the production of the capital good.

Since there may exist decreasing returns to scale in the consumption good sector, the representative firm in that sector earns positive profit, π_c, which is given by $\pi_c (K_t, Y_t, L) = T (K_t, Y_t, L) (1 - \nu)$. In the following, we suppose that the owner of the representative firm spend all the profit by purchasing the consumption good, that is, $\pi_c (K_t, Y_t, L) = E_t$. Finally, we define the gross domestic product function as $T (K_t, Y_t, L) + p (K_t, Y_t, L) Y_t = w (K_t, Y_t, L) L + r (K_t, Y_t, L) K_t + \pi_c (K_t, Y_t, L)$. It follows that the share of capital in the economy, s, is given by

$$s (K_t, Y_t, L) = \frac{r (K_t, Y_t, L) K_t}{T (K_t, Y_t, L) + p (K_t, Y_t, L) Y_t - \pi_c (K_t, Y_t, L)}. \tag{9}$$

2.2. Preferences

Consider an infinite-horizon discrete time economy that is populated by *OLG* of agents who live for two periods: young and old. It is assumed that there is no population growth and that the population is normalized to one. In the first period, young agents inelastically supply one unit of labor and receive an income, w_t. They assign this income between the saving, ϕ_t, and the first period consumption, C_t. In the second period, old agents are retired. The return on saving, $R_{t+1}\phi_t$, give their income which they spend entirely in the second period consumption, D_{t+1}. An agent born in period t has preferences defined over consumption of first and second period consumption. Intertemporal preferences of agent are described by the following *CES* utility function:

$$U (C_t, D_{t+1}) = \left[C_t^{\frac{\gamma-1}{\gamma}} + \delta \left(\frac{D_{t+1}}{\Gamma} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{10}$$

where δ denotes the discount factor, γ represents the elasticity of intertemporal substitution in consumption, and Γ is a scaling constant parameter. Under perfect-foresight and perfect competition w_t and R_{t+1} are considered as given. A young agent who is born at period t solves the following dynamic program:

$$\max_{C_t, D_{t+1}, \phi_t} \{U (C_t, D_{t+1}) \mid C_t + \phi_t = w_t, D_{t+1} = R_{t+1}\phi_t\}. \tag{11}$$

Solving for the first-order condition gives

$$C_t = \alpha\left(\frac{R_{t+1}}{\Gamma}\right)w_t, \quad \alpha\left(\frac{R_{t+1}}{\Gamma}\right) = \frac{1}{1+\delta^{\gamma}\left(\frac{R_{t+1}}{\Gamma}\right)^{\gamma-1}}, \tag{12}$$

where $\alpha\,(R_{t+1}/\,\Gamma) \in (0, 1)$ is the propensity to consume of young agent at period t. From the budget constraint given in (11), the saving function and the demand for second period consumption are given by:

$$\phi_t = \left[1-\alpha\left(\frac{R_{t+1}}{\Gamma}\right)\right]w_t, D_{t+1} = \left[1-\alpha\left(\frac{R_{t+1}}{\Gamma}\right)\right]R_{t+1}w_t. \tag{13}$$

It is assumed that the saving is increasing with respect to the gross rate of return, R_{t+1}.

ASSUMPTION 1. $\gamma > 1$.

This assumption states that the substitution effect following an increase in the gross rate of return R_{t+1} is greater than the income effect. Such restriction implies that the saving function is increasing with respect to the gross rate of return R_{t+1}. There is no consensus in the literature about the elasticity of intertemporal substitution in consumption. However, recent estimate of Vissing-Jorgensen and Attanasio (2003) provides estimate higher than one.

Total savings equal the production of the investment good: $\phi_t = p_tY_t = p_tK_{t+1}$. The utility function given in (10) and the demand for first and second period consumption, respectively, given in (12) and (13) give together the indirect utility function of a young agent at period t:

$$V\,(w_t, R_{t+1}) = \left\{\left[\alpha\left(\frac{R_{t+1}}{\Gamma}\right)w_t\right]^{\frac{\gamma-1}{\gamma}} + \delta\left[\left[1-\alpha\left(\frac{R_{t+1}}{\Gamma}\right)\right]R_{t+1}w_t\right]^{\frac{\gamma-1}{\gamma}}\right\}^{\frac{\gamma}{\gamma-1}}. \tag{14}$$

From this expression, it can be seen that the welfare of young agent at period t relies on the discount rate, the propensity to consume of young agent, the current wage rate, and the future gross rate of return. Utility of the first old agent is derived from consumption at period $t = 0$, purchased from their initial capital endowment. It is assumed that the utility of the first old agent is increasing in consumption and is given by their budget constraint $D_1 = R_1\phi_0 = r_1K_1$.

2.3. Intertemporal Equilibrium

At intertemporal equilibrium all markets clear in each period t. There exist three markets, respectively, the investment good one, the consumption good one, and the labor one. Total labor is given by the number of young agents which is normalized to one. From now on, let $T(K_t, Y_t, 1) = \tau(K_t, Y_t)$ and $T_g(K_t, Y_t, 1) = \tau_g(K_t, Y_t)$, with $g \in \{1, 2, 3\}$. It is, then, possible to provide the following definition for a perfect-foresight competitive equilibrium.

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DEFINITION 1. *A sequence* $\{K_t, Y_t\}_{t=0}^{\infty}$, *with* $K_{t=0}$ *given, is a perfect-foresight competitive equilibrium if:*
(i) *Producers and households are at their optimum: the first-order conditions given in (6) and (12)-(13) are satisfied and* $R_{t+1} = r_{t+1}/p_t$;
(ii) *The capital accumulation is determined by* $p_t Y_t = \phi_t$ *with* $Y_t = K_{t+1}$;
(iii) *The market clearing condition for the consumption good is given by* $E_t + C_t + D_t = \tau\ (K_t, K_{t+1})$.

By exploiting the Walras law and considering the capital accumulation equation, it follows from Definition 1 that an equilibrium path satisfies the following difference equation of order two:

$$K_{t+1} + \frac{\tau_3(K_t,K_{t+1})\left\{1-\alpha\left[-\frac{\tau_1(K_{t+1},K_{t+2})}{\Gamma\tau_2(K_t,K_{t+1})}\right]\right\}}{\tau_2(K_t,K_{t+1})} = 0. \quad (\mathbf{15})$$

Equation (15) defines implicitly a two-dimensional system that describes the deterministic equilibrium trajectories of capital, a predetermined variable whose value in period t is fixed by past saving.

2.4. Normalized Steady State

In this section, conditions for the existence of the stationary solutions of the dynamic system given in (15) are provided. Actually, a steady state of this system is a constant sequence for the relevant variable, that is, $\{K_t\}_{t=0}^{+\infty} = K^*$ for every t, where K^* solves the following:

$$K^* + \frac{\tau_3(K^*,K^*)\left\{1-\alpha\left[-\frac{\tau_1(K^*,K^*)}{\Gamma\tau_2(K^*,K^*)}\right]\right\}}{\tau_2(K^*,K^*)} = 0. \quad (\mathbf{16})$$

A set of system parametrized by the elasticity of intertemporal substitution in consumption, γ, is considered. The procedure used in Drugeon et al. (2010) is followed: Building on the homogeneity property of the utility function defined by (10), the scaling parameter Γ is used in order to give conditions for the existence of a normalized steady state (hereafter *NSS*) which remain unaltered as γ is varied. However, it is needed also to ensure that the value of the propensity to consume of young agent $\alpha(-\tau_1(K^*, K^*)/\,\Gamma\tau_2(K^*, K^*))$, when evaluated at the *NSS*, does not depend on γ. This characteristic will be derived by choosing appropriately the value of δ. Let express $-\tau_1(K^*, K^*)/\,\Gamma\tau_2(K^*, K^*)$ by ξ. Under Assumption 1, $\alpha(\xi)$ is a monotone decreasing function with $\lim_{\xi\to 0} \alpha(\xi) = \alpha_{sup}$, $\lim_{\xi\to +\infty} \alpha(\xi) = \alpha_{inf}$ and $(\alpha_{inf}, \alpha_{sup}) \subseteq (0, 1)$. It is possible to define the inverse function of $\alpha(\xi)$ as follows:

$$\Phi_{K^*} = 1 + \frac{K^*\tau_2(K^*,K^*)}{\tau_3(K^*,K^*)}. \quad (\mathbf{17})$$

By adopting a proper value for K^*, one may find a corresponding value for $\Phi_{K^*} \in (\alpha_{inf}, \alpha_{sup})$. Then, the following proposition holds:

PROPOSITION 1. *Suppose that Assumption 1 is satisfied and let* $K^* \in (0, \overline{K})$ *be such that* $\Phi_{K^*} \in (\alpha_{inf}, \alpha_{sup})$. *There exists a unique positive value* $\Gamma(K^*)$ *solution of (16) such that* K^* *is a steady state if and only if* $\Gamma = \Gamma(K^*)$.

Proof. See Appendix A.2. ■

This proposition allows, for a given set of parameters describing the consumption and production behavior, to use the elasticity of intertemporal substitution in consumption, γ, on the analysis of the local stability of competitive equilibria. In the rest of the paper, the following assumption is made so that the existence of an *NSS* is ensured.

ASSUMPTION 2. $\Gamma = \Gamma(K^*)$.

When $\Gamma = \Gamma(K^*)$ the share of capital in total income, s, and the propensity to consume of young agent, α, remain constant as the elasticity of intertemporal substitution in consumption, γ, is made to vary.

2.5. Dynamic Efficiency

This section presents the dynamic efficiency properties of the competitive two-sector equilibrium around the *NSS*. In a one-sector *OLG* model, competitive equilibrium can be not Pareto optimal since intertemporal exchanges are restricted in view of agents' limited planning horizon.[13] As matter of fact, if too much capital is accumulated, the economy turns out to be dynamically inefficient. This occurs when the population growth factor exceeds the steady-state marginal product of capital and the capital–labor ratio exceeds the Golden Rule level, denoted $\hat{K}$. The Golden Rule level is the steady-state allocation chosen by a central planner that maximizes the utility of each individual at the steady state. The highest utility is defined as the maximum of the utility function $u(C, D)$ subject to the total stationary consumption $C + D + E = \tau(K, K)$.

Denote $\hat{R} = R(\hat{K}, \hat{K}, 1) = -\tau_1(\hat{K}, \hat{K})/\tau_2(\hat{K}, \hat{K})$ where $\hat{K}$ satisfies $\hat{R} = 1$. From equation (9) it is possible to get the following equality $\tau_3/\tau_1 K = (1 - s)/s$. This last expression together with the capital accumulation evaluated at the steady state yields to the stationary gross rate of return $R^* = -\tau_1(K^*, K^*)/\tau_2(K^*, K^*)$:

$$R^* = \frac{s}{(1-\alpha)\,(1-s)}. \tag{18}$$

If $R^* > 1$ (resp. $R^* < 1$), the *NSS* is lower (resp. higher) than the Golden Rule level, that is, under- (resp. over-) accumulation of capital. Using the Golden Rule level $\hat{R} = 1$ and equation (18), a condition on the propensity to consume of the young agent α to obtain an *NSS* lower than the Golden Rule level $\hat{K}$ is obtained. Note that it also ensures the dynamic efficiency of equilibria. From the Proof of Propositions 2 and 3 in Drugeon et al. (2010), the following holds:

PROPOSITION 2. *Suppose that Assumptions 1 and 2 are satisfied and let* $\underline{\alpha} = (1 - 2s)/(1 - s)$. *Then, the following generally holds:*

(*i*) *The NSS is characterized by an under-accumulation of capital if and only if the propensity to consume of the young agent is sufficiently large,* $\alpha > \underline{\alpha}$*, that is,* $R^* > \hat{R} = 1$*;*
(*ii*) *An intertemporal competitive equilibrium converging towards the NSS is dynamically efficient if* $\alpha \in (\underline{\alpha}, 1)$*, that is,* $R^* < \hat{R} = 1$.

Proposition 2 establishes that if the labor income of agent is relatively lower than the capital income, that is, $s > 1/2$, then a young agent does not have enough wage to provide a large amount of savings so that an under-accumulation of capital is obtained without additional restriction, that is, $\underline{\alpha} < 0$. On the contrary, if labor income of agents is higher than capital income, that is, $s < 1/2$, then a young agent receives enough wage to be able to save a large amount. Under-accumulation of capital can be attained provided that the share of consumption of young agents is high enough, that is, $\alpha > \underline{\alpha}(> 0)$.

In the sequel only a dynamically efficient paths is considered. Moreover, the share of capital in the economy s is restricted in order to get positive value for the bound $\underline{\alpha}$ and to focus on realistic empirical values. Indeed, Cecchi and Garcia-Peñalosa (2010) show that over the period 1960-2003, *OECD* countries were characterized by a share of capital between 0.35 and 0.5. Then, the following assumption is made:

ASSUMPTION 3. $\alpha \in (\underline{\alpha}, 1)$ *and* $s \in (1/3, 1/2)$.

As a matter of fact, such an assumption restricts the gross rate of return to be higher than one.

3. THE TWO-COUNTRY MODEL

In this section, a competitive two-sector (capital and labor) two-good (consumption and investment) model composed of country *A* and country *B* having asymmetric technologies is considered. As in the closed setting, the perfect-foresight equilibrium and the existence of the *NSS* are discussed. Finally, the pattern of trade is presented.

3.1. Intertemporal Equilibrium

It is assumed that both goods are freely tradable between countries and that the gross rate of return of capital is higher than one such that international capital mobility is possible.[14] There does not exist any trade cost.[15] Moreover, lending and borrowing are not allowed.[16] Under free-trade, it holds that countries have the same relative price of the investment good, that is, $p_t = p_t^A = p_t^B$. The mobility of capital across countries implies that the rental rate of capital is the same in the two countries, that is, $r_t = r_t^A = r_t^B$. It follows that the rental rate of capital in each country will be determined in the international market of capital. However, labor is internationally immobile implying that the wage rate is different across

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countries, that is, $w_t^A \neq w_t^B$. It implies that the wage rate in each country will be characterized in the national market of labor. As already mentioned, the population in both countries is normalized to one, that is, $L = L^A = L^B = 1$. Let denote $K_t^W = K_t^A + K_t^B$ the world capital stock and $Y_t^W = Y_t^A + Y_t^B$ the world production of investment good at period t.

Our goal is to determine the impact of international trade with international capital mobility on the dynamical properties of the two countries. This is done by comparing the dynamics of a free-trade equilibrium with those of an economy in autarky. To do so, we characterize the dynamic free-trade equilibrium and thus we first define the world social production function $\tau^W\left(K_t^W, Y_t^W\right)$ as:

$$\begin{aligned} \tau^W\left(K_t^W, Y_t^W\right) = \max_{K_t^i, Y_t^i, i \in \{A,B\}} \quad & \tau^A\left(K_t^A, Y_t^A\right) + \tau^B\left(K_t^B, Y_t^B\right) \\ s.t. \quad & K_t^A + K_t^B \leq K_t^W \\ & Y_t^A + Y_t^B \geq Y_t^W \end{aligned} \quad . \tag{19}$$

Solving, the associated first-order conditions give the optimal demand function for capital and production of the investment good: $K^A\left(K_t^W, Y_t^W\right)$, $K^B\left(K_t^W, Y_t^W\right)$, $Y^A\left(K_t^W, Y_t^W\right)$ and $Y^B\left(K_t^W, Y_t^W\right)$. From the Envelope theorem, we obtain

$$\begin{aligned} \tau_1^W\left(K_t^W, Y_t^W\right) &= \tau_1^A\left(K^A\left(K_t^W, Y_t^W\right), Y^A\left(K_t^W, Y_t^W\right)\right) \\ &= \tau_1^B\left(K^B\left(K_t^W, Y_t^W\right), Y^B\left(K_t^W, Y_t^W\right)\right), \\ \tau_2^W\left(K_t^W, Y_t^W\right) &= \tau_2^A\left(K^A\left(K_t^W, Y_t^W\right), Y^A\left(K_t^W, Y_t^W\right)\right) \\ &= \tau_2^B\left(K^B\left(K_t^W, Y_t^W\right), Y^B\left(K_t^W, Y_t^W\right)\right). \end{aligned} \tag{20}$$

It is, then, possible to give a definition of the perfect-foresight competitive equilibrium:

DEFINITION 2. *A sequence* $\left\{K_t^W, Y_t^W\right\}_{t=0}^{\infty}$ *with* $K_0^W = K_0^A + K_0^B$ *given, is a world perfect-foresight competitive equilibrium if:*

(i) *Producers and households are at their optimum: the FOC (6), (12), and (13) are satisfied and* $R_{t+1} = r_{t+1}/p_t$;

(ii) *The capital accumulation is determined by* $p_t\left(Y_t^A + Y_t^B\right) = \phi_t^A + \phi_t^B$ *with* $Y_t^A + Y_t^B = K_{t+1}^A + K_{t+1}^B$;

(iii) *The consumption level is given by* $E_t^A + C_t^A + D_t^A + E_t^B + C_t^B + D_t^B = \tau^W\left(K_t^W, Y_t^W\right)$.

For simplicity, let us denote $\tau_3^i(K_t^W, K_{t+1}^W) = \tau_3^i(K^i(K_t^W, K_{t+1}^W), Y^i(K_t^W, K_{t+1}^W))$ with $i \in \{A, B\}$, then, we establish from Definition 2 that a perfect-foresight competitive equilibrium satisfies the following difference equation:

$$K_{t+1}^W + \frac{\tau_3^A\left(K_t^W, K_{t+1}^W\right)\left[1 - \alpha^A\left(-\frac{\tau_1^W\left(K_{t+1}^W, K_{t+2}^W\right)}{\Gamma^A \tau_2^W\left(K_t^W, K_{t+1}^W\right)}\right)\right] + \tau_3^B\left(K_t^W, K_{t+1}^W\right)\left[1 - \alpha^B\left(-\frac{\tau_1^W\left(K_{t+1}^W, K_{t+2}^W\right)}{\Gamma^B \tau_2^W\left(K_t^W, K_{t+1}^W\right)}\right)\right]}{\tau_2^W\left(K_t^W, K_{t+1}^W\right)} = 0. \tag{21}$$

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This expression defines an implicit two-dimensional dynamical system with one predetermined variable, the current world capital stock, and one forward variable, the world capital stock of the next period. The set of admissible paths is defined as follows:

$$\tilde{K}^W = \left\{ \left(K_t^W, K_{t+1}^W\right) \in \mathbb{R}_+^2 \mid K_t^W \leq \overline{K}^W,\ K_{t+1}^W \leq \left[F^1\left(K_t^A, 1\right) + F^1\left(K_t^B, 1\right)\right] \right\}. \quad (22)$$

where $\overline{K}^W$ is solution of $K^W - \left[F^1\left(K^A, 1\right) + F^1\left(K_t^B, 1\right)\right] = 0$.

3.2. Normalized Steady State and Dynamic Efficiency

A steady state $K_t^W = K_{t+1}^W = K_{t+2}^W = K^{W*}$ is defined by:

$$K^{W*} + \frac{\tau_3^A\left(K^{W*}, K^{W*}\right)\left[1 - \alpha^A\left(-\frac{\tau_1^W(K^{W*},K^{W*})}{\Gamma^A \tau_2^W(K^{W*},K^{W*})}\right)\right] + \tau_3^B\left(K^{W*}, K^{W*}\right)\left[1 - \alpha^B\left(-\frac{\tau_1^W(K^{W*},K^{W*})}{\Gamma^B \tau_2^W(K^{W*},K^{W*})}\right)\right]}{\tau_2^W\left(K^{W*}, K^{W*}\right)} = 0. \quad (23)$$

As in Section 2.4, the existence of an *NSS* is proved using the same procedure. Building on the homogeneity property of the utility function, the scaling parameter Γ^A is used in order to give conditions for the existence of an *NSS*, $K^{W*} \in (0, \overline{K}^W)$, in the world economy which remain unaltered as γ is varied. Let us express $\xi^A = -\tau_1^W(K^{W*}, K^{W*}) / \Gamma^A \tau_2^W(K^{W*}, K^{W*})$. Under Assumption 1, $\alpha(\xi^A)$ is a monotone decreasing function with $\lim_{\xi^A \to 0} \alpha^A(\xi^A) = \alpha^A_{sup}$, $\lim_{\xi^A \to +\infty} \alpha^A(\xi^A) = \alpha^A_{inf}$ and $(\alpha^A_{inf}, \alpha^A_{sup}) \subseteq (0, 1)$. We define the inverse function of $\alpha^A(\xi^A)$:

$$\Phi_{K^{W*}} = 1 + \frac{K^* \tau_2(K^*, K^*)}{\tau_3(K^*, K^*)}. \quad (24)$$

By adopting a proper value for K^{W*}, we may find a corresponding value for $\Phi_{K^*} \in (\alpha^A_{inf}, \alpha^A_{sup})$. Then, the following proposition holds:

PROPOSITION 3. *Assume that Assumption 1 is satisfied and let* $K^{W*} \in (0, \overline{K}^W)$ *be such that* $\Phi_{K^{W*}} \in (\alpha^A_{inf}, \alpha^A_{sup})$. *Then, there exists a unique value* $\Gamma\left(K^{W*}\right) > 0$ *solution of (23) such that* K^{W*} *is a steady state if and only if* $\Gamma = \Gamma^A\left(K^{W*}\right)$.

Proof. See Appendix A.3. ∎

The next assumption is introduced to guarantee the existence of an *NSS* in the world economy.

ASSUMPTION 4. $\Gamma = \Gamma^A\left(K^{W*}\right)$.

When $\Gamma = \Gamma^A(K^{W*})$ the share of capital in total income in both countries s^i, $i \in \{A, B\}$, and the propensity to consume of young agent in each countries α^i, $i \in \{A, B\}$ remain constant as γ is made to vary.

Since we assume free-trade in goods, that is, $\tau_2^W = \tau_2^A = \tau_2^B$, and international mobility of capital across countries, that is, $\tau_1^W = \tau_1^A = \tau_1^B$, we get that the gross rate of return is the same between the two countries, that is, $R^{W*} = R^{A*} = R^{B*}$. It follows that it is possible to apply Proposition 2, and that the *NSS* is dynamically efficient if $\alpha^A \in (\underline{\alpha}^A, 1)$ and $\alpha^B \in (\underline{\alpha}^B, 1)$. The consideration of sufficiently high propensity to consume of young agents ensures that the gross rates of return are higher than one at steady state in accordance with empirical evidence. Such an assumption also ensures that international capital mobility is relevant.

3.3. Pattern of Trade

In a dynamic setting, when considering the pattern of trade due to comparative advantage, two dimensions are important. The first dimension refers to the fact that one country, or even both countries, might completely specialize in the production of one of the two goods during the dynamic transition as well as at the long-run equilibrium.[17] The second dimension concerns the good/sector in which the country will imperfectly specialized.

Under a consumption good capital intensive, mentioned in Assumption 3, the capital–labor ratio in the investment sector is lower than the one in the consumption sector, that is, $\eta^i < a_t^i$. In our model, a_t^i evolves over time since K_t^i changes over time. Countries may specialize in the consumption if $K_t^i > a_t^i$ or the investment good if $K_t^i < \eta^i$. Then, during the dynamic transition the pattern of specialization may change over time since the capital–labor ratio, K_t^i, and the capital–labor ratio in the consumption good sector, a_t^i, evolve. As a matter of fact, if η^i is not too high, we get that both countries produce both goods at the *NSS* as shown by the following proposition.

PROPOSITION 4. *Assume that Assumptions 1 and 3-6 are satisfied and let* $\eta^i \in (0, 1)$, *with* $i \in \{A, B\}$. *Then, each country produces both goods at the NSS.*

Proof. See Appendix A.4. ■

This proposition shows that both countries produce both goods at the stationary solution and thus, by continuity, in a small neighborhood of it.

In the remaining of the paper, it is considered that country A has a comparative advantage in the production of the consumption good allowing to export that good meanwhile country B has a comparative advantage in the investment good sector such that country A imports it. In order for each country to produce according to their comparative advantage, a specific allocation of capital across countries is chosen by using some normalization constant. In fact, several allocation of capital are possible depending if there is net trade or not in the long-run.

On the one hand, there exists a free-trade allocation (K^{A*}, K^{B*}) such that each country produces in the long-run the amount of capital required to produce the consumption good and the investment good ($K^{i*} = Y^{i*}$). In this case, international trade may occur during the dynamics transition but the stationary equilibrium is characterized with no net trade. On the other hand, there is a free-trade allocation (K^{A*}, K^{B*}) such that one country imports the investment good ($K^{i*} > Y^{i*}$) and the other country exports the investment good ($K^{i*} < Y^{i*}$) can occur. It implies that both countries trade together during the dynamics transition and at the

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steady-state equilibrium. In this situation, there exists net trade even at the long-run equilibrium.

Building on Nishimura et al. (2014), a particular free-trade allocation is considered such that there exists net trade and countries export and import according to their comparative advantage. In other words, country A exports the consumption good meanwhile country B exports the investment good. Then, the following holds:

PROPOSITION 5. *Suppose that Assumptions 1, 3, and 4 are satisfied and let* $\Theta^A = 1$. *Then there exist* $\overline{\psi} > 1$ *and* $\Theta^{B*} > 0$ *such that the free-trade allocation* $K^{W*} = K^{A*} + K^{B*}$ *with* $K^{A*} = \psi Y^{A*} > Y^{A*}$ *and* $K^{B*} = Y^{B*}/\psi < Y^{B*}$ *is a solution of (21) if and only if* $\psi \in (1, \overline{\psi})$ *and* $\Theta^B = \Theta^{B*} < 1$. *Moreover, the associated free-trade allocation of consumption is such that* $E^A + C^A + D^A = \tau^B$ *and* $E^B + C^B + D^B = \tau^A$.

Proof. See Appendix A.5. ■

The parameter ψ represents the degree of openness of trade in the investment good sector. When $\psi = 1$, no net trade does not take place while if $\psi > 1$ there is net trade between country A and country B. The greater is ψ the more countries exchange good. Proposition 5 shows that by changing Θ^{A*}, we can construct economies satisfying the symmetry property for different values of $\psi \in (1, \overline{\psi})$.

In the rest of the paper, we assume that the restrictions of Proposition 5 are satisfied in order to ensure the existence of the free-trade allocation in which the country A exports the consumption good and the country B exports the investment good.

ASSUMPTION 5. $\Theta^A = 1$, $\psi \in (1, \overline{\psi})$ *and* $\Theta^B = \Theta^{B*} < 1$.

4. LOCAL DYNAMICS

In this section, the local stability of an economy in autarky and those of the world economy are derived.

4.1. Characteristic Polynomial and Local Stability in Autarky

For analytical tractability, the elasticity of the rental rate of capital, ε_{rk}, is introduced

$$\varepsilon_{rk} = -\frac{\tau_{11}(K^*,K^*)K^*}{\tau_1(K^*,K^*)} \in (0, +\infty). \quad \textbf{(25)}$$

This elasticity is a decreasing function of the elasticity of capital–labor substitution in the consumption good sector.[18] We now turn to the analysis of the transitional dynamics of the system in the neighborhood of the *NSS*. The first step is to linearize the implicit two-dimensional dynamical system defined by (15) around the *NSS* using equations (4), (7), and (25). Then, the characteristic polynomial is:

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PROPOSITION 6. *Assume that Assumptions 1 and 2 are satisfied. Then, the characteristic polynomial is defined by* $\mathscr{P}(\lambda) = \lambda^2 - \lambda\mathscr{T}(\gamma) + \mathscr{D}(\gamma)$ *with the trace* $\mathscr{T}(\gamma)$ *and the determinant* $\mathscr{D}(\gamma)$*:*

$$\mathscr{T}(\gamma) = \frac{1+\varepsilon_{rk}\left\{R^*\left[\frac{\tau_{32}(1-\alpha)}{\tau_{11}K^*}+\frac{\tau_{22}}{\tau_{11}}\right]+\alpha(\gamma-1)\left[1+\frac{\tau_{22}R^*}{\tau_{11}}\right]\right\}}{\alpha(\gamma-1)\left(-\frac{\tau_{21}\varepsilon_{rk}}{\tau_{11}}\right)},\ \mathscr{D}(\gamma) = \frac{R^*\left[1+\alpha(\gamma-1)+(1-\alpha)\frac{\tau_{31}}{\tau_{21}K^*}\right]}{\alpha(\gamma-1)}, \tag{26}$$

where the second partial derivatives of $\tau(K^*, K^*)$ *are given in Lemma 2.*

Since there is one predetermined variable, the current capital stock, the dimension of the stable manifold is required to be two in order to obtain local indeterminacy. Local indeterminacy occurs when there exists a continuum of equilibrium paths converging to one steady state from the same initial value of the capital stock whereas local determinacy occurs when there is a unique converging equilibrium path for a given initial capital stock. In our setting, the existence of local indeterminacy occurs if the two characteristic roots associated with the linearization of the dynamical system defined by (15) around the *NSS* have a modulus less than one.

In view of the complicated form of the trace, $\mathscr{T}$, and the determinant, $\mathscr{D}$, given in (26), it may seem that the study of the local dynamics of system (15) requires long and tedious computations. However, by applying the geometrical method adopted in Grandmont et al. (1998), it is possible to analyze qualitatively the (in)stability of the characteristic roots of the Jacobian evaluated at the steady state of system defined by (15) and their bifurcations (changes in stability) by locating the point $(\mathscr{T}, \mathscr{D})$ in the plane and studying how $(\mathscr{T}, \mathscr{D})$ varies when the value of a bifurcation parameter changes continuously. The elasticity of intertemporal substitution in consumption γ is used as the bifurcation parameter.

Proposition 3 shows that the scaling parameter satisfies $\Gamma = \Gamma(K^*)$, and the *NSS*, the share of capital in total income s, and the propensity to consume of young agent α remain constant as γ is made to vary between 0 and $+\infty$. As in Grandmont et al. (1998), the variation of the trace $\mathscr{T}$ and the determinant $\mathscr{D}$ in the $(\mathscr{T}, \mathscr{D})$ plane as γ varies continuously within $(1, +\infty)$ is studied.

Under the elasticity of intertemporal substitution in consumption higher than one, the *NSS* is locally determinate as soon as the consumption good sector is labor intensive, that is, $b > 0$, since in this configuration the determinant, $\mathscr{D}$, is higher than one. However, local indeterminacy requires that the determinant, $\mathscr{D}$, to be lower than one. Then, in order to investigate the occurrence of local indeterminacy, we shall focus on a capital intensive consumption good sector, that is, $b < 0$.[19]

ASSUMPTION 6. $b < 0$.

When the gross rate of return is higher than one, that is, $\alpha \in (\underline{\alpha}, 1/2)$, the following Figure 1 is obtained. Such a figure gives a complete picture of the local stability properties of the *NSS*.

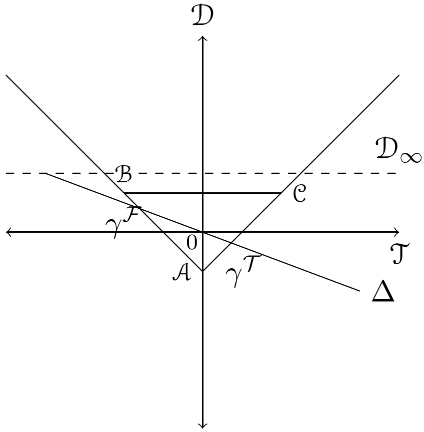

FIGURE 1. Local indeterminacy in the autarky equilibrium.

In this figure, $\gamma^{\mathcal{T}}$ is generically a transcritical bifurcation leading to the existence of a second steady state which is locally unstable (resp. saddle-point stable) in a right (resp. left) neighborhood of $\gamma^{\mathcal{T}}$, meanwhile $\gamma^{\mathcal{F}}$ is generically a flip bifurcation value giving rise to period-two cycles which are locally indeterminate (resp. unstable) in a right (resp. left) neighborhood of $\gamma^{\mathcal{F}}$. Nourry and Venditti (2011) first show that local indeterminacy arises under dynamic efficiency. Then they present the following:

PROPOSITION 7. *Suppose that Assumptions 1-2 and 6 are satisfied. Then, there exist* $\underline{b} < \overline{b} < 0$, $\underline{\varepsilon}_{rk} > 0$, *and* $\gamma^{\mathcal{F}} > \gamma^{\mathcal{T}} > 1$ *such that for* $b \in (\underline{b}, \overline{b})$, $\alpha \in (\underline{\alpha}, 1/2)$ *and* $\varepsilon_{rk} > \underline{\varepsilon}_{rk}$, *the following generally holds:*

(i) the NSS is a sink, that is, a locally indeterminate NSS, when $\gamma \in (\gamma^{\mathcal{T}}, \gamma^{\mathcal{F}})$;
(ii) the NSS is a saddle, that is, a locally determinate NSS, when $\gamma \in (1, \gamma^{\mathcal{T}}) \cup (\gamma^{\mathcal{F}}, +\infty)$.

Proof. See Proof of Theorem 1 in Nourry and Venditti (2011) and Proof of Proposition 2 in Le Riche (2017). ■

Proposition 7 implies that endogenous fluctuations and local indeterminacy are based on a high enough ε_{rk}, that is, a low level of elasticity of capital–labor substitution in the consumption good sector. As in Le Riche (2017), the result of Nourry and Venditti (2011) is extended to a decreasing returns to scale technology in the consumption good sector. Nourry and Venditti (2011), as in Proposition 7, show the coexistence of indeterminacy and dynamic efficiency when the elasticity of capital–labor substitution in each sector is enough low; the consumption sector is sufficiently capital intensive and the intertemporal elasticity of substitution in consumption has intermediate values.[20] The steady state is a sink when there is intermediate values of the elasticity of intertemporal substitution in consumption.

The intuition of Proposition 7 is the following. Suppose that agents expect that the rate of investment will rise at period t inducing a higher capital stock at period

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$t+1$. This expectation will be self-fulfilling provided that there is enough saving at period t, see equation (15). For agents to save a sufficient amount, they must first reduce their consumption at period t. This decrease of the current consumption lowers their level of utility and to compensate agents must increase their consumption at period $t+1$. This configuration is obtained provided that the intertemporal elasticity of substitution in consumption is sufficiently high, that is, $\gamma > \gamma^{\mathcal{T}}$.

By contrast, if the latter is too high, that is, $\gamma > \gamma^{\mathcal{F}}$, the intertemporal substitution effect is large and thus the expected increase in the rate of investment produces a relatively high amount of savings while the present consumption decreases. Meanwhile, the capital stock in the next period will rise at an important level. Since the consumption good sector is the most capital intensive sector, it implies through the Rybczynski effect that there is a large increase in the production of the consumption good at period $t+1$. This rise in consumption good production may exceed the increase in future consumption demand. As a result, the initial expectation can be realized provided that the intertemporal elasticity of substitution in consumption has intermediate values, that is, $\gamma \in \left(\gamma^{\mathcal{T}}, \gamma^{\mathcal{F}}\right)$.

4.2. Local Stability in the Free-Trade Equilibrium

In the remaining part of the section, the local stability of the two-country model is provided. In order to do so, it is needed to determine the characteristic roots associated with the linearization of (21) around the *NSS*. Let linearize the implicit two-dimensional dynamical system defined by (21) around the *NSS* by using the fact that $K^{W*} = [(1+\psi)/\psi]K^{A*}$ and $K^{W*} = (1+\psi)K^{B*}$ and equations (4), (7), and (25). Then, the following holds:

PROPOSITION 8. *Suppose that Assumptions 1 and 4 are satisfied. Then, the characteristic polynomial is defined by* $\mathscr{P}^W(\lambda^W) = (\lambda^W)^2 - \lambda^W \mathscr{T}^W(\gamma) + \mathscr{D}^W(\gamma)$ *with the trace* $\mathscr{T}^W(\gamma)$ *and the determinant* $\mathscr{D}^W(\gamma)$*:*

$$\mathscr{T}^W(\gamma) = \frac{1 - \frac{\tau_{11}^W K^{W*}}{\tau_1^W}\left\{\Psi(\gamma-1)\left(1+\frac{\tau_{22}^W R^{W*}}{\tau_{11}^W}\right) + \frac{R^{W*}\left[K^{W*}\tau_{22}^W + \frac{\partial \tau_3^A}{\partial Y^{W*}}\left(1-\alpha^A\right) + \frac{\partial \tau_3^B}{\partial Y^{W*}}\left(1-\alpha^B\right)\right]}{K^{W*}\tau_{11}^W}\right\}}{\Psi(\gamma-1)\left(\frac{\tau_{12}^W K^{W*}}{\tau_1^W}\right)}, \tag{27}$$

$$\mathscr{D}^W(\gamma) = \frac{R^{W*}\left[1+\Psi(\gamma-1) + \frac{\frac{\partial \tau_3^A}{\partial K^{W*}}\left(1-\alpha^A\right) + \frac{\partial \tau_3^B}{\partial K^{W*}}\left(1-\alpha^B\right)}{\tau_{21}^W K^{W*}}\right]}{\Psi(\gamma-1)}, \tag{28}$$

with

$$\Psi = \frac{\alpha^A \psi + \alpha^B}{1+\psi} > 0,$$

where the first and second partial derivatives of $\tau^W\left(K^W, Y^W\right)$ *are given by Lemma 3 and the partial derivatives of* $\tau_3^i\left(K^W, Y^W\right)$ *are given by Lemma 4.*

As a matter of fact, when the consumption sector is labor intensive, the determinant, $\mathscr{D}^W$, is higher than one implying that any equilibrium is locally determinate. Therefore, in the remaining of the section, a capital intensive consumption sector is considered. In such a case, a gross rate of return higher than one is considered and the local stability analysis is performed on the ground of the geometrical method of Grandmont et al. (1998). Then, the following results hold:

PROPOSITION 9. *Suppose that Assumptions 1, 3–6 are satisfied. Then, there exist* $\underline{b}^A < \overline{b}^A < 0$, $\underline{b}^B < \overline{b}^B < 0$ $\underline{\varepsilon}_{rk}^A > 0$, $\underline{\varepsilon}_{rk}^B > 0$ *and* $\gamma^{W,\mathcal{F}} > \gamma^{W,\mathcal{T}} > 1$ *such that for* $b^A \in (\underline{b}^A, \overline{b}^A)$, $b^B \in (\underline{b}^B, \overline{b}^B)$, $\alpha^A \in (\underline{\alpha}^A, 1/2)$, $\alpha^B \in (\underline{\alpha}^B, 1/2)$, $\varepsilon_{rk}^A > \underline{\varepsilon}_{rk}^A$ *and* $\varepsilon_{rk}^B > \underline{\varepsilon}_{rk}^B$*, the following generally holds:*

(i) the NSS is a sink, that is, a locally indeterminate NSS, when $\gamma \in (\gamma^{W,\mathcal{T}}, \gamma^{W,\mathcal{F}})$*;*

(ii) the NSS is a saddle, that is, a locally determinate NSS, when $\gamma \in (1, \gamma^{W,\mathcal{T}}) \cup (\gamma^{W,\mathcal{F}}, +\infty)$[21]

Proof. See Appendix A.6. ■

Proposition 9 provides conditions on the technologies and preferences for a locally indeterminate *NSS* at the world level. As in the autarly, local indeterminacy (sink) requires intermediate values of the elasticity of intertemporal substitution in consumption, otherwise the steady state is saddle-point stable. The intuition of this result is similar to the one given in Proposition 7. Note that the indeterminacy conditions are similar as the one of Proposition 7. However, now, any share and elasticities will depend on the fundamental of both countries. Moreover, even if the results are similar, it does not imply that a country moving from autarky to trade exhibits the same stability property. The aim of the next section is precisely to look at this change in the indeterminacy conditions.

5. IMPACT OF FREE-TRADE ON LOCAL STABILITY AND ON WELFARE

In this section, we present the effects of opening to international trade and international capital mobility on the local stability and on stationary welfare.

5.1. Global Destabilizing Effect of Trade

We introduce a numerical exercise in order to show that by opening the border to free-trade and international capital movement can generate indeterminacy (sink) at the free-trade equilibrium although the closed equilibrium in both countries is a saddle. First, we expose the numerical example in the autarky regime such that Proposition 7 is satisfied. Second, based on the same set of parameters as previously used we present numerical conditions such that Proposition 9 holds. Finally,

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TABLE 1. Parameters values

	μ^i	η^i	ρ^i	ν^i	K^i	γ	Θ^i
Country A	0.999961	0.0995	9.05	0.99	0.7499	1.568	1
Country B	0.9999585	0.1	9	1	0.7501	1.568	0.9943

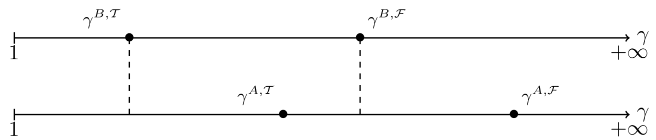

FIGURE 2. Bifurcation parameters of country A and of country B.

the effect of opening the borders on indeterminacy is obtained by comparing the conditions on elasticities and shares of both countries before and after trade. In order to illustrate this effect, let us consider the following set of parameters.

We, now, derive the conditions in correspondence which Proposition 7 is satisfied: Gross rate of return is higher than one, holds for $\alpha^A \approx 0.1617 \in (\underline{\alpha}^A, 1/2)$ and $\alpha^B \simeq 0.1625 \in (\underline{\alpha}^B, 1/2)$ with $\underline{\alpha}^A \approx 0.1115$ and $\underline{\alpha}^B \approx 0.1319$; Restrictions on technology are satisfied in both countries for any $s^A \approx 0.4704 \in (1/3, 1/2)$, $s^B \approx 0.4646 \in (1/3, 1/2)$, $b^A \approx -2.601 \in (-5.1842, -1.9416)$, $b^B \approx -2.601 \in (-5.1507, -1)$, $\varepsilon^A_{rk} \approx 2.6664 > 2.5034$ and $\varepsilon^B_{rk} \approx 2.608 > 2.6492$; Conditions on γ are ensured in country A for any $\gamma \in (1.5569, 1.7283)$ and country B for any $\gamma \in (1.5425, 1.71)$. The conditions on the intertemporal elasticity of substitution in consumption are represented in Figure 2.

Figure 2 shows that the bifurcation parameters of country A and of country B are not the same, implying that the stability properties of the two countries before opening the borders are different. As an example, it is possible to see from direct inspection of Figure 2 that when $\gamma \in (\gamma^{B,\mathcal{T}}, \gamma^{B,\mathcal{F}})$, the *NSS* of the country B is locally indeterminate (sink) meanwhile the *NSS* of the country A can be either locally determinate (saddle) or indeterminate (sink).

Let us consider the case when the borders are open to free-trade and international capital movement. Proposition 5 is satisfied under $\psi \approx 1.0002 \in (1, 1.334)$, $\Theta^A = 1$ and $\Theta^B = 0.9943$ such that each country exports according to its comparative advantage.

Using Table 1, the conditions on preferences in Proposition 9 are satisfied in the world economy for any $\gamma \in (1.5587, 1.7795)$. Figure 3 gathers the bifurcation parameters of country A and country B in the autarky regime and the world economy.

As mentioned above, the stability properties of the two countries are not the same before opening to free-trade. Moreover, the bifurcation parameters of the world economy are different from the bifurcation parameters of the two countries as these depend now on the characteristics of both countries. In particular, when $\gamma \in (\gamma^{W,\mathcal{T}}, \gamma^{W,\mathcal{F}})$, the *NSS* of the world economy is locally indeterminate (sink)

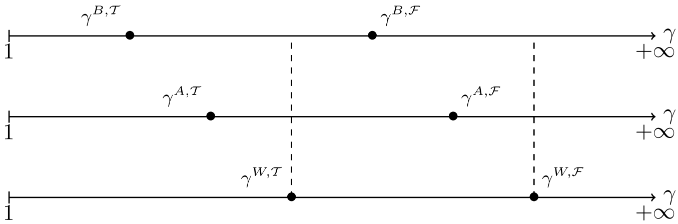

FIGURE 3. Bifurcation parameters of country A, country B, and the world economy.

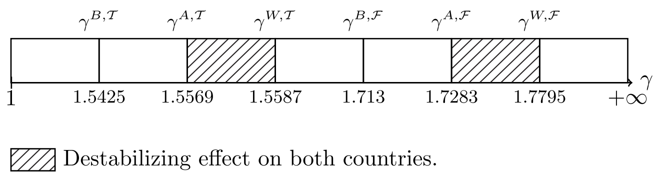

FIGURE 4. Trade effect on the occurrence of endogenous fluctuations.

meanwhile the *NSS* of country A and country B can be either locally determinate (saddle) or indeterminate (sink).

In Figure 4, we represent the different values of γ defined in Figure 3 in only one line. Then, for a given value of γ, we can deduce the destabilizing effect of trade.

Let us now depart from a situation where in autarky both countries are saddle-point stable and that opening to free-trade and international capital movement leads to indeterminacy (sink). Such a case is depicted in Figure 4. The following proposition represents the main result of the impact of international trade on the local stability properties of the two countries.

PROPOSITION 10. *Under Assumptions 1 and 3, there exists a set of parameters* $(\eta^i, \mu^i, \rho^i, \nu^i, \gamma, \gamma^{A,\mathcal{F}}, \gamma^{W,\mathcal{F}})$ *and allocations* (K^{A*}, K^{B*}) *such that the NSS of both countries is locally determinate (saddle) before trade meanwhile local indeterminacy (sink) holds for both countries in the trade regime if* $\gamma \in (\gamma^{A,\mathcal{F}}, \gamma^{W,\mathcal{F}})$.

This result shows that a sink can occur in the free-trade equilibrium even though the two countries are characterized by saddle-point stability in the autarky equilibrium for any degree of openness satisfying Assumption 5. The intuition of this result is the following.

Assume that all agents anticipate a higher rate of investment inducing an increase of the future capital stock. This expectation will be self-fulfilling provided that the amount of saving is sufficiently high. Agents save enough if they reduce their current consumption. To compensate this, agents must increase their future consumption.

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Let us first consider the expectation of agents in the autarky equilibrium. Since γ is high, that is, $\gamma > \underline{\gamma}^{\mathcal{F}}$, the intertemporal substitution effect is relatively important and thus the expected rise in the rate of investment produces a relatively high amount of savings meanwhile present consumption decreases. It follows that there is a large increase of the future capital stock. Since the consumption sector is capital intensive, this increase in the future capital stock induces an important raise in the future consumption production. This rise in the consumption good production exceeds the increase of the future demand. Then the initial expectation is not fulfilled.

Suppose now that countries move from an autarky to a free-trade with international capital movement regime.

Let us now consider the free-trade equilibrium. The initial anticipation can be realized in each country if:

- The production of the future consumption decreases, or (and)
- The demand for future consumption increases.

In country B, the relative capital intensity difference is greater than the one in the autarky regime.[22] It implies that for a same level of capital stock, the production is less favorable for the consumption good. Thus, there is a decrease of the production of the future consumption good. Moreover, country B imports the consumption good. It follows that the opening to international trade decreases the relative price of the consumption good. Thus, agents are more willing to purchase this good (by importing or by own production). It implies that there is an equalization of the future consumption production and future consumption demand in country B. Since agents in country B import the future consumption, there is less production of the future consumption that remains in country A. Then the initial expectation is fulfilled for both countries in the trade regime.

In Le Riche (2017), the autarkic economy is the same as in the current paper. However, the two-country model has two key differences: (1) the comparative advantage is the discount rate; (2) international mobility of both inputs (capital and labor). The difference in discount rate allows for each country to have a different capital accumulation path and thus a different stationary capital–labor ratio, meanwhile the international mobility of both inputs will equalize the wage rate and the gross rate of return in each country in the transition path and the stationary equilibrium. When one opens the borders to free-trade and international mobility of inputs, the gross rate will increase and the wage rate will decrease in the most patient country until they equalize the ones of the less patient country. Since the future capital stock is determined by present saving, the equalization of wage, due to the mobility of inputs, will entail some change in the world capital accumulation and thus there is limited difference on the indeterminacy condition of the intertemporal elasticity of substitution in consumption before and after opening the borders to free-trade and international capital mobility.

On the contrary, in the current paper, countries have a different technology sets and there is international mobility of capital and not of labor. Such an assumption implies that the gross rate will equalize across countries meanwhile the wage rate will not equalize since they are determined on the ground of the national labor market. Since the future capital stock is determined by present saving of the two countries, the fact that wage is not equalized will lead to important difference on the intertemporal elasticity of substitution in consumption conditions for indeterminacy before and after the opening to international trade and international capital mobility.

Notice that our result relies on the fact that there is difference of technology and international capital mobility. Based on this argument, Nishimura et al. (2014) show also a global destabilizing effect of trade in an infinitely lived agent model. As in our paper, they emphasize that their result is based on the fact that countries differ with respect to technology. As it is well known in two-sector models, see, for example, Benhabib and Nishimura (1998), indeterminacy relies on preferences condition through the elasticity of intertemporal substitution in consumption and on technologies condition and the allocation of labor–capital across sector through the Rybczynski effect and the Stolper–Samuelson effect. In Nishimura et al. (2014) a similar argument holds in autarky. When the borders are open to free-trade and international capital mobility, the indeterminacy conditions on technology are weaker and thus such an effect may emerge.

5.2. Stationary Welfare

In the remaining of the section, we examine how opening to international trade impacts the welfare of agents in each countries. Agents of each generation observe a change in welfare from a comparison of their utility levels before and after trade.[23] The welfare of the first old at period $t = 0$ in each country is given by $D_1^i = R_1^i \phi_0^i = r_1^i K_1^i$. It follows that the effect of trade on the utility of the first old in each country will result from the impact of free-trade on the rental rate of capital. The welfare of agents in each country at period $t > 0$ is characterized from the indirect utility defined by (14). This indirect utility is determined by the discount rate, the wage rate, and the gross rate of return. Totally differentiating (14) gives the changes in the indirect utility in terms of the different prices. Using the fact that $R^{*i} = r^{i*}/p^{i*}$, we derive that $dR_{t+1}^i/dp_t^i = (dr_{t+1}^i/dp_t^i - r_{t+1}^i/p_t^i)/p_t^i$. Then, it follows that:

$$dV_t^i = \Lambda^i \left[\frac{dw_t^i}{dp_t^i} + \frac{\left(1-\alpha^i\left(\frac{R_{t+1}^i}{\Gamma^i}\right)\right)w_t^i}{r_{t+1}^i} \left(\frac{dr_{t+1}^i}{dp_t^i} - \frac{r_{t+1}^i}{p_t^i} \right) \right] dp_t^i, \tag{29}$$

where

$$\Lambda^i = \left(C_t^i\right)^{-\frac{1}{\gamma}} \left[\left(C_t^i\right)^{\frac{\gamma-1}{\gamma}} + \delta^i \left(\frac{D_{t+1}^i}{\Lambda^i} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma-1}} > 0.$$

In our model, any agent has two roles as factor suppliers in his lifetime. Indeed, when young, agents are workers meanwhile, when old, agents are capitalists. Because of these dual functions, the impact of free-trade through price change has opposite implications on the utility of agents through the Stolper–Samuelson effect (dr/dp and dw/dp) as seen in the change of the indirect utility given in (29). Let us finally consider (29) evaluated at the *NSS*. From $\tau^i + p^i Y^{i*} = w^i + r^i K^{i*} + \pi_c^i$ and the Envelope theorem, we get the following formulation:

$$dV^i = \Lambda^i \left[(Y^{i*} - K^{i*}) + \frac{K^{i*}}{R^{i*}} \frac{dr^i}{dp^i} (1 - R^{i*}) \right] dp^i. \tag{30}$$

Two factors contribute to welfare at the steady state. First, $Y^{i*} - K^{i*}$ corresponds to the steady-state export of investment good. Second, $1 - R^{i*}$ corresponds to the efficiency properties of the steady state. This term is always negative since we consider dynamic efficiency. The following proposition provides the impact of free-trade on welfare of agents at the steady state in both countries:

PROPOSITION 11. *Suppose that Assumptions 1, 3, and 5 are satisfied. Then, the welfare of all generations at steady state in country A decreases meanwhile the welfare of all generations at steady state in country B increases.*

Proof. See Appendix A.7. ■

Other papers have studied the impacts of opening to free-trade on welfare. Cremers (2005), Kemp and Wong (1995), and Naito and Zhao (2009) show that welfare does not always increase with trade. This result is due to the fact that, as already mentioned, each agent has dual functions as suppliers of inputs. It implies that the effect of free-trade on stationary welfare may improve or worsen it. In our model, a similar result holds. The intuition of this result is the following. Country *A* exports the consumption good which implies that there is less production of the consumption good that remains in country *A*. Then at the opening to international trade agents in country *A* decrease their consumption of C^A and of D^A; thus, their level of utility decreases. Thus, all generations at steady state in country *A* loss from trade. An opposite reasoning can apply for agents in country *B*.

On the basis of Proposition 10, we derive a relationship between the welfare losses and gains, and the global destabilizing effect of international trade. Considering Proposition 11, we conclude that opening to international trade will bring at a same time indeterminacy for both countries, a deterioration of the welfare in country *A* and a raise of the welfare in country *B*.

Few papers in the literature have simultaneously addressed the effect of free-trade and mobility of inputs (capital and labor) on welfare and on stability properties. In an infinitely lived agent model, Nishimura et al. (2009) find that opening the borders to free-trade and international capital movement might be destabilizing and, at a same time, one country will gain in terms of stationary welfare meanwhile the other country always losses. In an *OLG* framework with international capital and labor mobility, Aloi and Lloyd-Braga (2010) consider the effect of globalization when there is involuntary unemployment in one country. They show that unemployment decreases and world output expands, when workers migrate to the country with the competitive labor market.

6. CONCLUDING REMARKS

In a two-country two-good (consumption and investment) two-factor (capital and labor) *OLG* model with a *CES* life cycle utility, a *CES* nonincreasing returns to scale technology in the consumption good sector and a Leontief constant returns to scale technology in the investment good sector, we have examined the impact of international trade on the existence of sunspot cycles in a two-country model and on welfare. We have assumed that countries have asymmetric technologies. The main contribution of this paper is to show that opening to international trade can generate instability and welfare losses. Indeed, period-two cycles can occur for both countries in the trade equilibrium although the two countries are characterized by saddle-point stability in the autarky equilibrium.

It is interesting to note that if this model is placed in a voting context, there exists external conflict that has profound impact on the viability of opening the border to free-trade and international capital movement. As shown by Kemp and Wong (1995), several compensation scheme can be implemented allowing that welfare evaluated under free-trade is Pareto superior than the one in autarky.

NOTES

1. One of the exception is Kemp and Wong (1995).
2. See, for example, Cremers (2005).
3. The source of the data is "World Economic Outlook Database."
4. On what regards poverty trap, see Bond et al. (2013) in dynamic Heckscher–Ohlin framework.
5. Iwasa and Nishimura (2014) extend Nishimura and Shimomura (2002) by introducing a consumption capital good. They show that international trade can create sunspot fluctuations in the world economy.
6. See also Nishimura et al. (2006).
7. See also Aloi and Lloyd-Braga (2010) and Aloi et al. (2000) for a similar issue in a one sector *OLG* model.
8. See Le Riche et al. (2019) who analyze the welfare and stability properties of an *OLG* model with intra-industry trade only.
9. See, for instance, Eaton et al. (2002) and Matsuyama (2013).
10. In a two-period OLG model, full depreciation of capital is justified by the fact that one period is about 30 years.
11. See Benhabib and Nishimura (1985).
12. The expressions of the first partial derivatives of the social production function $T(K_t, Y_t, L)$ are given in Lemma 2.
13. See Diamond (1965).

14. The reader is referred to Parello (2019) for the configuration in which there is only international mobility of labor.

15. The reader is referred to Chen and Zhang (2011) and Zhang and Chen (2012) for the impact of tariff on stability.

16. Hu and Mino (2013) consider, although in a different framework, the effect of lending and borrowing with different trade structure.

17. See for a discussion in the infinitely lived agent framework Stiglitz (1971) and Baxter (1992) and in the *OLG* framework Bianconi (1995).

18. From the production functions defined in (1) and Drugeon (2004), it holds that $\varepsilon_{rk} = L^0 wK/Y_0K^0\sigma$.

19. Using national accounting data (aggregate input–output tables) on the most developed countries, Takahashi et al. (2012) show that the aggregate consumption good sector is more capital intensive than the investment good sector.

20. See also Nourry and Venditti (2012) for a generalization of that result in an *OLG* framework with multiple consumption good.

21. When $\gamma = \gamma^{W,\mathcal{T}}$, the *NSS* undergoes a transcritical bifurcation leading to the existence of a second steady state which is locally unstable (resp. saddle-point stable) in a right (resp. left) neighborhood of $\gamma^{W\mathcal{T}}$, whereas when $\gamma = \gamma^{W,\mathcal{F}}$ the *NSS* undergoes a flip bifurcation value giving rise to period-two cycles which are locally indeterminate (resp. unstable) in a right (resp. left) neighborhood of $\gamma^{W,\mathcal{F}}$.

22. It is obtained from equation (7), the fact that $K^A = Y^A$ in the autarky regime and $K^A = Y^A\psi$ in the trade regime.

23. See also Cremers (2005), Kemp and Wong (1995) and Naito and Zhao (2009) for a similar issue.

REFERENCES

Aloi, M. and T. Lloyd-Braga (2010) National labor markets, international factor mobility and macroeconomic instability. *Economic Theory* 43(3), 431–456.

Aloi, M., D. Dixon Huw and T. Lloyd-Braga (2000) “Endogenous fluctuations in an open economy with increasing returns to scale. *Journal of Economic Dynamics & Control* 24(1), 97–125.

Autor, D., D. Dorn and G. Hanson (2013) The China syndrome: Local labor market effects of import competition in the United States. *American Economic Review* 103(6), 2121–2168.

Baxter, M. (1992) Fiscal policy, specialization, and trade in the two-sector model: The return of Ricardo? *Journal of Political Economy* 10(4), 713–744.

Bianconi, M. (1995) On dynamic real trade models. *Economics Letters* 47(1), 47–52.

Benhabib, J. and K. Nishimura (1985) Competitive equilibrium cycles. *Journal of Economic Theory* 35(2), 284–306.

Benhabib, J. and K. Nishimura (1998) Indeterminacy and sunspots with constant returns. *Journal of Economic Theory* 81(1), 58–96.

Bond, E. W., K. Iwasa and K. Nishimura (2013) Poverty traps and inferior goods in a dynamic Heckscher-Ohlin model. *Macroeconomic Dynamics* 17(6), 1227–1251.

Cecchi, D. and C. Garcia-Peñalosa (2010) Labour market institutions and the personal distribution of income in the OECD. *Economica* 77(307), 413–450.

Y. Chen and Y. Zhang (2011) A note on Tariff policy, increasing returns, and endogenous fluctuations and equilibrium indeterminacy: A global analysis. *Macroeconomic Dynamics* 15(2), 279–291.

Cremers, E. T. (2005) Intergenerational welfare and trade. *Macroeconomic Dynamics* 9(5), 585–611.

di Giovanni, J. and A. Levchenko (2009) Trade openness and volatility. *Review of Economics and Statistics* 9(3), 558–585.

Diamond, P. A. (1965) National debt in a neoclassical growth model. *American Economic Review* 55(5), 1126–1150.

Drugeon, J.-P. (2004) On consumptions, inputs and outputs subsitutabilities and the evanescence of optimal cycles. *Journal of Difference Equations and Applications* 10(5), 473–487.

Drugeon, J.-P., Nourry, C. and A. Venditti (2010) On efficiency and local uniqueness in two-sector OLG economies. *Mathematical Social Sciences* 59(1), 120–144.

Eaton, J. and S. Kortum (2002) Technology, geography and trade. *Econometrica* 70(5), 1741–1779.

Fadinger, H. and P. Fleiss (2011) Trade and sectoral productivity. *The Economic Journal* 121(555), 120–144.

Grandmont, J.-M., Pintus, P. and R. de Vilder (1998) Capital-labor substitution and competitive nonlinear endogenous business cycles. *Journal of Economic Theory* 80(1), 14–59.

Hu, Y. and K. Mino (2013) Trade structure and belief-driven fluctuations in a global economy. *Journal of International Economics* 90(2), 414–424.

Iwasa, K. and K. Nishimura (2014) Dynamic two-country Heckscher-Ohlin model with externality. *International Journal of Economic Theory* 10(1), 53–74.

Kemp, M. C. and K.-Y. Wong (1995) Gains from trade with overlapping generations. *Economic Theory* 6(2), 283–303.

Kose, A., E. Prasad and M. Terrones (2003) Financial integration and macroeconomic volatility. *IMF Staff Papers* 50, 119–141.

Krugman, P. (2009) The increasing returns revolution in trade and geography. *American Economic Review* 99(3), 561–571.

Le Riche, A. (2017) Macroeconomic volatility and trade in OLG economies. *International Journal of Economic Theory* 13(4), 401–425.

Le Riche, A., T. Lloyd-Braga and L. Modesto (2019) Intra-industry trade, involuntary unemployment and indeterminacy. *Submitted*.

Matsuyama, K. (2013) Endogenous ranking and equilibrium lorenz curve across (ex-ante) identical countries. *Econometrica* 81(5), 2009–2031.

Naito, T. and L. Zhao (2009) Aging, transitional dynamic, and gains from trade. *Journal of Economic Dynamics and Control* 33(8), 1531–1542.

Nishimura, K. and Shimomura, K. (2002) Trade and indeterminacy in a dynamic general equilibrium model. *Journal of Economic Theory* 105(1), 244–260.

Nishimura, K, A. Venditti and M. Yano (2006) Endogenous fluctuations in two-country models. *The Japanese Economic Review* 57(4), 516–532.

Nishimura, K, A. Venditti and M. Yano (2009) Optimal growth an competitive equilibrium business cycles under decreasing returns in two-country models. *Review of International Economics* 17(2), 371–391.

Nishimura, K, A. Venditti and M. Yano (2014) Destabilization effect of international trade in a perfect foresight dynamic general equilibrium model. *Economic Theory* 55(2), 357–392.

Nishimura, K. and M. Yano (1993) Interlinkage in the endogenous real business cycles of international economies. *Economic Theory* 3(1), 151–168.

Nourry, C. and A. Venditti (2011) Local indeterminacy under dynamic efficiency in a two-sector overlapping generations economy. *Journal of Mathematical Economics* 47(2), 164–169.

Nourry, C. and A. Venditti (2012) Endogenous business cycles in overlapping-generations economies with multiple consumption goods. *Macroeconomic Dynamics* 16(1), 86–102.

Obstfeld, M. (1994) Risk-taking, global diversification and growth. *American Economic Review* 84(5), 1310–1329.

Parello, C. P. (2019) Equilibrium indeterminacy in one-sector small open economies: the role of international labor migration. *Macroeconomic Dynamics* 23(4), 1528–1562.

Reichlin, P. (1986) Equilibrium cycles in overlapping generations economy with production. *Journal of Economic Theory* 40(1), 89–102.

Stiglitz, J. E. (1971) Factor price equalization in a dynamic economy. *Journal of Political Economy* 78(3), 456–485.

Sim, N., and K.-W. Ho (2007) Autarky indeterminacy and trade determinacy. *International Journal of Economic Theory* 3(4), 151–168.

Takahashi, H., Mashiyama, K. and R. Sakagami (2012) Does the capital intensity matter? Evidence from the postwar Japanese economy and other OECD countries. *Macroeconomic Dynamics* 16(1), 103–116.

Vissing-Jorgensen, A. and O. Attanasio (2003) Stock-market participation, intertemporal substitution and risk aversion. *American Economic Review* 93(2), 383–391.

Zhang, Y. and Y. Chen (2011) Tariff and equilibrium indeterminacy: A global analysis. *Macroeconomic Dynamics* 16(3), 394–410.

A: APPENDIX

A.1. Preliminary Results

For the reader convenience, we provide preliminary results necessary for the proof. We start by recalling that $\tau_g = \tau_g\,(K_t, Y_t)$, $\tau_{gh} = \tau_{gh}\,(K_t, Y_t)$, with $g \in \{1, 2, 3\}$ and $h \in \{1, 2\}$. The first partial derivatives of $\tau\,(K_t, Y_t)$ directly follow from computations of (4) and given by:

LEMMA 1. *The first partial derivatives of* $\tau\,(K_t, Y_t)$ *satisfy the following:*

$$\tau_1 = \Theta\mu\nu\,(K_t - \eta Y_t)^{-(1+\rho)}\left[\mu\,(K_t - \eta Y_t)^{-\rho} + (1-\mu)\,(1-Y_t)^{-\rho}\right]^{-\frac{\nu+\rho}{\rho}},$$

$$\tau_2 = -\tau_1\left[\eta + \left(\frac{1-\mu}{\mu}\right)\left(\frac{K_t - \eta Y_t}{1-Y_t}\right)^{1+\rho}\right],$$

$$\tau_3 = \left(\frac{1-\mu}{\mu}\right)\left(\frac{K_t - \eta Y_t}{1-Y_t}\right)^{1+\rho}\tau_1.$$

Using a_t and b_t defined in (7) and computations of Lemma 1, we derive the second partial derivatives of $\tau\,(K_t, Y_t)$.

LEMMA 2. *The second partial derivatives of* $\tau\,(K_t, Y_t)$ *satisfy the following:*

$$\tau_{11} = \frac{\tau_1\left[(\nu-1) - (1+\rho)\,\frac{\tau_3}{\tau_1 a_t}\right]}{(K_t - \eta Y_t)\left(1 + \frac{\tau_3}{\tau_1 a_t}\right)},$$

$$\tau_{21} = \frac{\tau_{11}\left[(\nu-1)\,\frac{\tau_2}{\tau_1} + (1+\rho)\,\frac{\tau_3 b_t}{\tau_1 a_t}\right]}{\nu - 1 - (1+\rho)\,\frac{\tau_3}{\tau_1 a_t}},$$

$$\tau_{22} = -\frac{\tau_{11}\left[-(\nu-1) + (1+\rho)\,\frac{\tau_3 b_t^2}{\tau_1 a_t}\right]}{\nu - 1 - (1+\rho)\,\frac{\tau_3}{\tau_1 a_t}},$$

$$\tau_{31} = \frac{\tau_{11}\,(\nu+\rho)\,\frac{\tau_3}{\tau_1}}{\nu - 1 - (1+\rho)\,\frac{\tau_3}{\tau_1 a_t}},$$

$$\tau_{32} = -\frac{\tau_{11}\frac{\tau_3}{\tau_1}\left[-(\nu-1)\frac{\tau_2}{\tau_1} + (1+\rho)b\right]}{\nu - 1 - (1+\rho)\,\frac{\tau_3}{\tau_1 a_t}}.$$

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A.2. Proof of Proposition 1

From the set of admissible paths defined by (2), we have $K^* \in \left(0, \overline{K}\right)$. K^* is a solution of (16) if:

$$\frac{1}{1+\delta^{\gamma}\left(-\frac{\tau_1(K^*,K^*)}{\Gamma\tau_2(K^*,K^*)}\right)^{\gamma-1}} = 1 + \frac{K^*\tau_2\left(K^*,K^*\right)}{\tau_3\left(K^*,K^*\right)} \equiv \Phi_{K^*} \in (0,1)\,.$$

Let us express $\xi = \xi = -\tau_1(K^*,K^*)/\,\Gamma\tau_2(K^*,K^*)$. Under Assumption 1, $\alpha(\xi)$ is a monotone decreasing function with $\lim_{\xi\to 0}\alpha(\xi) = \alpha_{sup}$, $\lim_{\xi\to+\infty}\alpha(\xi) = \alpha_{inf}$ and $(\alpha_{inf},\alpha_{sup}) \subseteq (0,1)$. It follows that $\alpha(\xi)$ admits an inverse function defined over $(\alpha_{inf},\alpha_{sup})$. Let $K^* \in \left(0,\overline{K}\right)$ be such that $\Phi_{K^*} \in (\alpha_{inf},\alpha_{sup})$. Solving equation (16) with respect to Γ gives

$$\Gamma(K^*) = R^*\left\{\frac{\delta^{\gamma}\left[(1-s)a(1-K^*) - s\eta(1-b)K^*\right]}{K^*\left[s\eta(1-b)+(1-s)a\right]}\right\}^{\frac{1}{\gamma-1}}. \qquad \textbf{(A1)}$$

Thus, K^* is an *NSS* if and only if $\Gamma = \Gamma(K^*)$.

A.3. Proof of Proposition 3

From the set of admissible paths defined by (22), we have $K^{W*} \in (0, \overline{K}^W)$. K^{W*} is a solution of (23) if:

$$\frac{1}{1+(\delta)^{\gamma}\left(-\frac{\tau_1^W\left(K^{W*},K^{W*}\right)}{\Gamma^A\tau_2^W\left(K^{W*},K^{W*}\right)}\right)^{\gamma-1}} = 1 + \frac{\tau_3^B\left(K^{W*},K^{W*},1\right)(1-\alpha^B)}{\tau_3^A\left(K^{W*},K^{W*}\right)} + \frac{K^{W*}\tau_2^W\left(K^{W*},K^{W*}\right)}{\tau_3^A\left(K^{W*},K^{W*}\right)} \equiv \Phi_{K^{W*}} \in (0,1)\,. \qquad \textbf{(A2)}$$

Let us express $\xi^A = R^W/\,\Gamma^A$. Under Assumption 1, $\alpha^A(\xi^A)$ is a monotone decreasing function with $\lim_{\xi^A\to 0}\alpha^A(\xi^A) = \alpha^A_{sup}$, $\lim_{\xi^A\to+\infty}\alpha^A(\xi^A) = \alpha^A_{inf}$ and $(\alpha^A_{inf},\alpha^A_{sup}) \subseteq (0,1)$. It follows that $\alpha^A(\xi^A)$ admits an inverse function defined over $(\alpha^A_{inf},\alpha^A_{sup})$. Let $K^{W*} \in (0,\overline{K}^W)$ be such that $\Phi_{K^{W*}} \in (\alpha^A_{inf},\alpha^A_{sup})$. Solving equation (23) with respect to Γ^A gives

$$\Gamma(K^{W*}) = R^{W*}\left\{\frac{\delta^{\gamma}\left[(1-K^{W*})\left[(1-s^A)a^A+s^A\eta^A(1-b^A)\right]-s^A(1-b^A)(\alpha^B\eta^A+(1-\alpha^B)\eta^B)-(1-s^A)a^A(1-\alpha^B)\right]}{K^{W*}\left[(1-s^A)a^A+s^A\eta^A(1-b^A)\right]+a^A(1-\alpha^B)(1-s^A)-(1-\alpha^B)(\eta^A-\eta^B)}\right\}^{\frac{1}{\gamma-1}}. \qquad \textbf{(A3)}$$

Thus, K^{W*} is an *NSS* if and only if $\Gamma^A = \Gamma(K^{W*})$.

A.4. Proof of Proposition 4

Consider the resource constraints on capital $K^i = K^{i0} + K^{i1}$ and on labor $1 = L^{i0} + L^{i1}$ in a country $i \in \{A, B\}$. From these resource constraints, we are able to write the capital–labor ratio in a country i as

$$K^i = L^{i0}k^{i0} + (1-L^{i0})k^{i1},$$

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where $k^{ij} = K^{ij}/L^{ij}$ with $i \in \{A, B\}$ and $j \in \{0, 1\}$. L^{i0} is thus given by

$$L^{i0} = \frac{K^i - k^{i1}}{k^{i0} - k^{i1}}.$$

Note that from the capital intensity difference defined by (7), we can obtain the two following expressions $k^{i0} - k^{i1} = -b^i$ and $K^i = (\eta^i - b^i)/(1 - b^i)$. Substituting them in L^{i0} yields

$$L^{i0} = \frac{1 - \eta^i}{1 - b^i}.$$

Country $i \in \{A, B\}$ produces both goods at the *NSS* when $L^{i0} \in (0, 1)$. Since $b^i < 0$ and $\eta^i \in (0, 1)$, straightforward computations show that $L^{i0} \in (0, 1)$.

A.5. Proof of Proposition 5

Let us compute the stationary consumption levels associated with this particular free-trade allocation. We suppose that country A exports the consumption good, $\mathscr{X}_0^A > 0$, while country B imports the consumption good, $\mathscr{M}_0^B > 0$. We characterize the exports of the consumption good of country A and imports of the consumption good of country B. Let a constant ξ, the openness of the consumption good sector, be higher than one such that consumption good market in each country rewrites as $C^A + D^A + E^A = \tau^A/\xi < \tau^A$ and $C^B + D^B + E^B = \xi\tau^B > \tau^B$. It is immediate to verify that $\mathscr{X}_0^A = (\xi - 1)\tau^A/\xi$ and $\mathscr{M}_0^B = (\xi - 1)\tau^B$.

We assume that the exports of the investment good of country B are given by $\mathscr{X}_1^B > 0$ and imports of the investment good of country A are given by $\mathscr{M}_1^A > 0$. In top of that, let us define a constant ψ higher than one, the openness in the investment good sector where ψ satisfying $K^{A*} = \psi Y^{A*}$ and $K^{B*} = Y^{B*}/\psi$. Using the fact that $K^{W*} = K^{A*} + K^{B*} = Y^{A*} + Y^{B*}$ we obtain: $K^{A*} = \psi K^{B*}$, $Y^{B*} = \psi Y^{A*}$, $K^{W*} = [(1 + \psi)/\psi]K^{A*}$, and $K^{W*} = (1 + \psi)K^{B*}$. It follows that: $\mathscr{X}_1^B = (\psi - 1)Y^{B*}/\psi$ and $\mathscr{M}_1^A = (\psi - 1)Y^{A*}$.

Finally, we assume a balance of trade in equilibrium: $\mathcal{BT}^A = \mathscr{X}_0^A - p\mathscr{M}_1^A = 0$, $\mathcal{BT}^B = p\mathscr{X}_1^B - \mathscr{M}_0^B = 0$, which implies $(\xi - 1)\tau^A/\xi = p(\psi - 1)Y^{A*}$ and $(\xi - 1)\tau^B = p(\psi - 1)Y^{B*}/\psi$.

Taking the ratio of the last two expressions we get: $\tau^A/\tau^B = \xi$. We now use the normalization constant Θ^i which represent the comparative advantage of each country in the consumption good sector to show that this free-trade allocation occurs. Indeed, when $\Theta^A > \Theta^B$ the productivity in the consumption good sector is sufficiently high in country A relative to country B such that country A has a comparative advantage to produce the consumption good.

We need to show that the allocations $K^{A*} = \psi Y^{A*}$, $K^{B*} = Y^{B*}/\psi$ and $\tau^A/\tau^B = \xi$ are feasible. First, let us consider the allocations $K^{A*} = \psi Y^{A*}$ and $K^{B*} = Y^{B*}/\psi$. From the capital accumulation evaluated at autarky steady state, we have

$$K^{i*} + \frac{\tau_3^i\left(K^{i*}, K^{i*}\right)}{\tau_2^i\left(K^{i*}, K^{i*}\right)}\left(1 - \alpha^i\right) = 0 \quad i \in \{A, B\}.$$

The ratio of capital–labor of the two countries gives

$$\frac{K^{A*}}{K^{B*}} = \frac{\tau_3^A\left(K^{A*}, K^{A*}\right)\left(1 - \alpha^A\right)}{\tau_3^B\left(K^{B*}, K^{B*}\right)\left(1 - \alpha^B\right)} = \psi.$$

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From equation (12) and $\Gamma^B = 1$ we derive α^B. α^A is defined in equation (A2):

$$\alpha^A = 1 + \frac{\tau_3^B\left(K^{W*}, K^{W*}, 1\right)(1-\alpha^B)}{\tau_3^A\left(K^{W*}, K^{W*}\right)} + \frac{K^{W*}\tau_2^W\left(K^{W*}, K^{W*}\right)}{\tau_3^A\left(K^{W*}, K^{W*}\right)}.$$

Using the two last expressions, Lemma 1, equation (12), and $\Theta^A = 1$, we get

$$\tau_3^B = -\frac{K^{W*}\tau_2^A}{(1+\psi)(1-\alpha^B)}.$$

Using $K^{B*} = Y^{A*}$ and $Y^{B*} = \psi Y^{A*}$, the allocations $K^{A*} = \psi Y^{A*}$ and $K^{B*} = Y^{B*}/\psi$ are feasible if and only if

$$\Theta^{B*} = \frac{K^{A*}\nu^A(1-\mu^A)(a^A)^{1+\rho^A}(K^{A*}-\eta^A Y^{A*})\left[\mu^B + (1-\mu^B)(a^B)^{\rho^B}\right]^{\frac{\nu^B+\rho^B}{\rho^B}}\left[\eta^A + \left(\frac{1-\mu^A}{\mu^A}\right)(a^A)^{1+\rho^A}\right]}{\psi\nu^B(1-\alpha^B)(1-\mu^B)(a^B)^{1+\rho^B}(K^{B*}-\eta^B Y^{B*})\left[\mu^A + (1-\mu^A)(a^A)^{\rho^A}\right]^{\frac{\nu^A+\rho^A}{\rho^A}}}.$$

We get $\Theta^{B*} < 1$ if $\psi \in (1, \overline{\psi})$ with $\overline{\psi}$ given by:

$$\overline{\psi} = \frac{\nu^B(1-\alpha^B)(1-\mu^B)(a^B)^{1+\rho^B}(K^{B*}-\eta^B Y^{B*})\left[\mu^A + (1-\mu^A)(a^A)^{\rho^A}\right]^{\frac{\nu^A+\rho^A}{\rho^A}}}{K^{A*}\nu^A(1-\mu^A)(a^A)^{1+\rho^A}(K^{A*}-\eta^A Y^{A*})\left[\mu^B + (1-\mu^B)(a^B)^{\rho^B}\right]^{\frac{\nu^B+\rho^B}{\rho^B}}\left[\eta^A + \left(\frac{1-\mu^A}{\mu^A}\right)(a^A)^{1+\rho^A}\right]}.$$

Second let us consider the following allocation $\tau^A/\tau^B = \xi$. From the social production function defined by (4), we get

$$\frac{\left(\mu^B\right)^{\frac{\nu^B}{\rho^B}}\left(K^{A*}-\eta^A Y^{A*}\right)^{\nu^A}\left[1+\left(\frac{1-\mu^B}{\mu^B}\right)\left(a^B\right)^{\rho^B}\right]^{\frac{\nu^B}{\rho^B}}}{\Theta^B\left(\mu^A\right)^{\frac{\nu^A}{\rho^A}}\left(K^{B*}-\eta^B Y^{B*}\right)^{\nu^B}\left[1+\left(\frac{1-\mu^A}{\mu^A}\right)\left(a^A\right)^{\rho^A}\right]^{\frac{\nu^A}{\rho^A}}} = \xi.$$

Using the value of Θ^{B*}, we get

$$\xi = \frac{\psi\nu^B(1-\alpha^B)(1-\mu^B)(a^B)^{1+\rho^B}(K^{B*}-\eta^B Y^{B*})\left[\mu^A+(1-\mu^A)(a^A)^{\rho^A}\right]^{\frac{\nu^A+\rho^A}{\rho^A}}\left(\mu^B\right)^{\frac{\nu^B}{\rho^B}}\left(K^{A*}-\eta^A Y^{A*}\right)^{\nu^A}\left[1+\left(\frac{1-\mu^B}{\mu^B}\right)\left(a^B\right)^{\rho^B}\right]^{\frac{\nu^B}{\rho^B}}}{K^{A*}\nu^A(1-\mu^A)(a^A)^{1+\rho^A}(K^{A*}-\eta^A Y^{A*})\left[\mu^B+(1-\mu^B)(a^B)^{\rho^B}\right]^{\frac{\nu^B+\rho^B}{\rho^B}}\left[\eta^A+\left(\frac{1-\mu^A}{\mu^A}\right)(a^A)^{1+\rho^A}\right]\left(\mu^A\right)^{\frac{\nu^A}{\rho^A}}\left(K^{B*}-\eta^B Y^{B*}\right)^{\nu^B}\left[1+\left(\frac{1-\mu^A}{\mu^A}\right)\left(a^A\right)^{\rho^A}\right]^{\frac{\nu^A}{\rho^A}}}.$$

Result follows.

A.6. Proof of Proposition 9

In order to simplify the exposition, we denote

$$\tau^W\left(K^W, Y^W\right) = \tau^A\left(K^A\left(K^W, Y^W\right), Y^A\left(K^W, Y^W\right)\right) + \tau^B\left(K^B\left(K^W, Y^W\right), Y^B\left(K^W, Y^W\right)\right),$$
$$\tau_{jh}^W = \tau_{jh}^W\left(K^W, Y^W\right), \quad j, h \in \{1, 2\},$$
$$\tau_{jh}^i = \tau_{jh}^i\left(K^i\left(K^W, Y^W\right), Y^i\left(K^W, Y^W\right)\right), \quad i \in \{A, B\}, \quad j, h \in \{1, 2\},$$
$$\tau_3^i\left(K^W, Y^W\right) = \tau_3^i\left(K^i\left(K^W, Y^W\right), Y^i\left(K^W, Y^W\right)\right), \quad i \in \{A, B\},$$
$$| H^i | = | H^i\left(K_t^i, Y_t^i\right) |, \quad i \in \{A, B\}.$$

To derive a tractable expressions for the trace $\mathscr{T}^W$ and the determinant $\mathscr{D}^W$, we need to determine the second partial derivatives of the social production $\tau^W\left(K_t^W, Y_t^W\right)$ and the

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partial derivatives of $\tau_3^i\left(K_t^W, Y_t^W\right)$, $i \in \{A, B\}$. First, as stated in Nishimura and Yano (1993), the second partial derivatives are given by:

LEMMA 3. *The second partial derivatives of* $\tau^W\left(K_t^W, Y_t^W\right)$ *satisfy the following:*

$$\tau_{11}^W = \frac{1}{\Xi}\left[\tau_{11}^A \mid H^B \mid + \tau_{11}^B \mid H^A \mid\right],$$

$$\tau_{12}^W = \frac{1}{\Xi}\left[\tau_{12}^A \mid H^B \mid + \tau_{12}^B \mid H^A \mid\right],$$

$$\tau_{22}^W = \frac{1}{\Xi}\left[\tau_{22}^A \mid H^B \mid + \tau_{22}^B \mid H^A \mid\right],$$

where

$$\mid H^i \mid = \tau_{11}^i \tau_{22}^i - \left(\tau_{12}^i\right)^2 > 0,$$

$$\Xi = \left[\tau_{11}^A + \tau_{11}^B\right]\left[\tau_{22}^A + \tau_{22}^B\right] - \left[\tau_{12}^A + \tau_{12}^B\right]^2 > 0.$$

Second, the partial derivatives of the function τ_3^i are derived in the following lemma.

LEMMA 4. *The partial derivatives of* $\tau_3^i\left(K^W, Y^W\right)$, $i \in \{A, B\}$, *satisfy the following:*

$$\frac{\partial \tau_3^i}{\partial K^W} = \tau_{31}^i \frac{\partial K^i}{\partial K^W} + \tau_{32}^i \frac{\partial Y^i}{\partial K^W},$$

$$\frac{\partial \tau_3^i}{\partial Y^W} = \tau_{31}^i \frac{\partial K^i}{\partial Y^W} + \tau_{32}^i \frac{\partial Y^i}{\partial Y^W},$$

where

$$\begin{pmatrix} \frac{\partial K^A}{\partial K^W} & \frac{\partial K^A}{\partial Y^W} \\ \frac{\partial Y^A}{\partial K^W} & \frac{\partial Y^A}{\partial Y^W} \end{pmatrix} = \frac{1}{\Xi}\begin{pmatrix} \tau_{11}^B \tau_{22}^A - \tau_{12}^A \tau_{12}^B + \mid H^B \mid & \tau_{22}^A \tau_{12}^B - \tau_{12}^A \tau_{22}^B \\ \tau_{11}^A \tau_{12}^B - \tau_{12}^A \tau_{11}^B & \tau_{11}^A \tau_{22}^B - \tau_{12}^A \tau_{12}^B + \mid H^B \mid \end{pmatrix}, \tag{A4}$$

$$\begin{pmatrix} \frac{\partial K^B}{\partial K^W} & \frac{\partial K^B}{\partial Y^W} \\ \frac{\partial Y^B}{\partial K^W} & \frac{\partial Y^B}{\partial Y^W} \end{pmatrix} = \frac{1}{\Xi}\begin{pmatrix} \tau_{11}^A \tau_{22}^B - \tau_{12}^A \tau_{12}^B + \mid H^A \mid & \tau_{12}^A \tau_{22}^B - \tau_{22}^A \tau_{12}^B \\ \tau_{12}^A \tau_{11}^B - \tau_{11}^A \tau_{12}^B & \tau_{22}^A \tau_{11}^B - \tau_{12}^A \tau_{12}^B + \mid H^A \mid \end{pmatrix}, \tag{A5}$$

$$\Xi = \left[\tau_{11}^A + \tau_{11}^B\right]\left[\tau_{22}^A + \tau_{22}^B\right] - \left[\tau_{12}^A + \tau_{12}^B\right] > 0.$$

Proof. Totally differentiate $\tau_3^i\left(K^i\left(K^W, Y^W\right), Y^i\left(K^W, Y^W\right)\right)$, $i \in \{A, B\}$, we obtain ■

$$\frac{\partial \tau_3^i}{\partial K^W} = \tau_{31}^i \frac{\partial K^i}{\partial K^W} + \tau_{32}^i \frac{\partial Y^i}{\partial K^W},$$

$$\frac{\partial \tau_3^i}{\partial Y^W} = \tau_{31}^i \frac{\partial K^i}{\partial Y^W} + \tau_{32}^i \frac{\partial Y^i}{\partial Y^W}.$$

Using the world social production program defined in (19), we can rewrite the program as:

$$\tau^W\left(K_t^W, Y_t^W\right) = \max_{K_t^A, Y_t^A} \tau^A\left(K_t^A, Y_t^A\right) + \tau^B\left(K_t^W - K_t^A, Y_t^W - Y_t^A\right). \tag{A6}$$

We get the following first-order conditions:

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$$\frac{\partial \tau}{\partial K^A} = 0, \qquad \tau_1^A - \tau_1^B = 0,$$
$$\frac{\partial \tau}{\partial Y^A} = 0, \qquad \tau_2^A - \tau_2^B = 0,$$

and the second-order conditions:

$$Z_N = \begin{pmatrix} \frac{\partial (\tau^W)^2}{\partial^2 K^A} & \frac{\partial (\tau^W)^2}{\partial K^A \partial Y^A} \\ \frac{\partial (\tau^W)^2}{\partial K^A \partial Y^A} & \frac{\partial (\tau^W)^2}{\partial^2 Y^A} \end{pmatrix} = \begin{pmatrix} \tau_{11}^A + \tau_{11}^B & \tau_{21}^A + \tau_{21}^B \\ \tau_{21}^A + \tau_{21}^B & \tau_{22}^A + \tau_{22}^B \end{pmatrix},$$

$$B_N = \begin{pmatrix} \frac{\partial (\tau^W)^2}{\partial^2 K^W} & \frac{\partial (\tau^W)^2}{\partial Y^W \partial K^W} \\ \frac{\partial (\tau^W)^2}{\partial K^W \partial Y^W} & \frac{\partial (\tau^W)^2}{\partial^2 Y^W} \end{pmatrix} = \begin{pmatrix} \tau_{11}^B & \tau_{21}^B \\ \tau_{21}^B & \tau_{22}^B \end{pmatrix}. \tag{A7}$$

We assume that Z_N is non singular. Equation (A4) follows from $Z_N^{-1} B_N$. Equation (A5) is obtained in a similar way.

Result follows.

The result of this proposition is obtained using the geometrical method developed in Grandmont et al. (1998) which allows us to determine the occurrence of local indeterminacy in terms of one parameter. Under Assumption 4, the *NSS* remains constant as γ varies continuously within $(1, +\infty)$, we can analyze the variation of the trace $\mathscr{T}^W(\gamma)$ and the determinant $\mathscr{D}^W(\gamma)$ in the $(\mathscr{T}^W(\gamma), \mathscr{D}^W(\gamma))$ plane. The relationship between $\mathscr{T}^W(\gamma)$ and $\mathscr{D}^W(\gamma)$ is given by a half-line $\Delta^W\left(\mathscr{T}^W\right)$ which is characterized from the consideration of its extremities. The starting point is the couple $(\lim_{\gamma \to +\infty} \mathscr{T}^W \equiv \mathscr{T}_\infty^W, \lim_{\gamma \to +\infty} \mathscr{D}^W \equiv \mathscr{D}_\infty^W)$, while the end point is the couple $(\lim_{\gamma \to 1} \mathscr{T}^W \equiv \mathscr{T}_1^W, \lim_{\gamma \to 1} \mathscr{D}^W \equiv \mathscr{D}_1^W)$. Solving $\mathscr{T}^W$ and $\mathscr{D}^W$ with respect to $\Psi\,(\gamma - 1)$ yields the following linear relationship:

$$\mathscr{D}^W = \Delta\left(\mathscr{T}^W\right) = \mathscr{S}^W \mathscr{T}^W + \mathscr{D}_\infty^W - \mathscr{S}^W \mathscr{T}_\infty^W,$$

where the slope $\mathscr{S}^W$, $\mathscr{D}_\infty^W$, and $\mathscr{T}_\infty^W$ are given

$$\mathscr{S}^W = \frac{R^{W*}\left[1 + \left(\frac{1}{\tau_{21} K^{W*}}\right)\left(\frac{\partial T_3^A}{\partial K^{W*}}\left(1 - \alpha^A\right) + \frac{\partial T_3^B}{\partial K^{W*}}\left(1 - \alpha^B\right)\right)\right]}{1 + R^{W*} \varepsilon_{rk}^W \left\{\frac{\tau_{22}}{\tau_{11}} + \frac{1}{K^{W*}\tau_{11}}\left[\frac{\partial T_3^A}{\partial Y^{W*}}\left(1 - \alpha^A\right) + \frac{\partial T_3^B}{\partial Y^{W*}}\left(1 - \alpha^B\right)\right]\right\}},$$

$$\mathscr{D}_\infty^W = R^{W*}, \;\; \mathscr{T}_\infty^W = -\frac{1 + \frac{\tau_{22}^W R^{W*}}{\tau_{11}^W}}{\frac{\tau_{21}^W}{\tau_{11}^W}}. \tag{A8}$$

Assume that $b^A < 0$, $b^B < 0$, $\nu^A \leq 1$, $\nu^B \leq 1$, $\alpha^A \in (\underline{\alpha}, 1/2)$, $\alpha^B \in (\underline{\alpha}, 1/2)$ $s^A \in (1/3, 1/2)$ and $s^B \in (1/3, 1/2)$. Let us first consider the starting point $(\mathscr{T}_\infty^W, \mathscr{D}_\infty^W)$. Under dynamic efficiency, we get from (A8) that $\mathscr{D}_\infty^W > 1$. To establish the precise location of the starting point $(\mathscr{T}_\infty^W, \mathscr{D}_\infty^W)$, we need to determine the sign of $\mathscr{T}_\infty^W$, $\mathscr{P}_\infty^W(1) = 1 - \mathscr{T}_\infty^W + \mathscr{D}_\infty^W$ and of $\mathscr{P}_\infty^W(-1) = 1 + \mathscr{T}_\infty^W + \mathscr{D}_\infty^W$. Using Lemmas 2 and 3, we get

$$\mathscr{T}_\infty^W = \frac{\tau_{11}^W + R^W\left[\frac{\left(1-\nu^A\right)\left(1-b^A\right)s^A + \left(1+\rho^A\right)\left(1-s^A\right)\left(b^A\right)^2}{\left(1-\nu^A\right)\left(1-b^A\right)s^A + \left(1+\rho^A\right)\left(1-s^A\right)}\tau_{11}^A |H^B| + \frac{\left(1-\nu^B\right)\left(1-b^B\right)s^B + \left(1+\rho^B\right)\left(1-s^B\right)\left(b^B\right)^2}{\left(1-\nu^B\right)\left(1-b^B\right)s^B + \left(1+\rho^B\right)\left(1-s^B\right)}\tau_{11}^B |H^A|\right]}{\frac{\left(1-s^A\right)\left[\left(1-\nu^A\right)\left(1-b^A\right)\left(1-s^A\right) + \left(1+\rho^A\right)b^A\right]}{\left(1-\nu^A\right)\left(1-b^A\right)s^A + \left(1+\rho^A\right)\left(1-s^A\right)}\tau_{11}^A |H^B| + \frac{\left(1-s^A\right)\left[\left(1-\nu^B\right)\left(1-b^B\right)\left(1-s^B\right) + \left(1+\rho^B\right)b^B\right]}{\left(1-\nu^B\right)\left(1-b^B\right)s^B + \left(1+\rho^B\right)\left(1-s^B\right)}\tau_{11}^B |H^A|}.$$

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When $b^A < \overline{b}^A$ and $b^B < \overline{b}^B$, with $\overline{b}^A = -(1-\alpha^A)(1-\nu^A)/(\nu^A + \rho^A + (1-\nu^A)\alpha^A)$ and $\overline{b}^B = -(1-\alpha^B)(1-\nu^B)/(\nu^B + \rho^B + (1-\nu^B)\alpha^B)$, we get $\mathscr{T}_\infty^W < 0$.

Since $\mathscr{T}_\infty^W < 0$ when $b^A < \overline{b}^A$ and $b^B < \overline{b}^B$, it follows that $\mathscr{P}_\infty^W(1) > 0$. Using equation (A8), we derive that $\mathscr{P}_\infty^W(-1) = [(1+R^{W*})\tau_{21}^W - \tau_{11}^W - \tau_{22}^W]/\tau_{21}^W$. Using Lemmas 2 and 3, we get

$$\mathscr{P}_\infty^W(-1) = \frac{\frac{2(1-\nu^A)(1-b^A)\left(\frac{1+R^{W*}}{R^{W*}}\right)s^A+(1+\rho^A)(1-s^A)(1+bR^{W*})(1+b^A)}{(1-s^A)[(1-\nu^A)(1-\alpha^A)+b^A(\rho^A+\alpha^A+\nu^A(1-\alpha^A))]}\tau_{11}^A \mid H^B \mid + \frac{2(1-\nu^B)(1-b^B)\left(\frac{1+R^{W*}}{R^{W*}}\right)s^B+(1+\rho^B)(1-s^B)(1+b^B R^{W*})(1+b^B)}{(1-s^B)[(1-\nu^B)(1-\alpha^B)+b^B(\rho^B+\alpha^B+\nu^B(1-\alpha^B))]}\tau_{11}^B \mid H^A \mid}{\frac{(1-s^A)[(1-\nu^A)(1-b^A)(1-\alpha^A)+(1+\rho^A)b^A]}{(1-\nu^A)(1-b^A)s^A+(1+\rho^A)(1-s^A)}\tau_{11}^A \mid H^B \mid + \frac{(1-s^B)[(1-\nu^B)(1-b^B)(1-\alpha^B)+(1+\rho^B)b^B]}{(1-\nu^B)(1-b^B)s^B+(1+\rho^B)(1-s^B)}\tau_{11}^B \mid H^A \mid}.$$

Under $b^A < \overline{b}^A$ and $b^B < \overline{b}^B$, we derive that $\mathscr{P}_\infty^W(-1) < 0$ since $\overline{b}^A < -1/R^{W*} < -1$ and $\overline{b}^B < -1/R^{W*} < -1$. As a result, the starting point is in the left area outside the triangle $\mathscr{A}\mathscr{B}\mathscr{C}$. Let us now consider the end point. To determine the precise location of the end point $(\mathscr{T}_1^W, \mathscr{D}_1^W)$, it is sufficient to determine that $\Delta(\mathscr{T}^W)$ is pointing upward or downward. We thus study the sign of $\mathscr{D}^{W'}(\gamma)$. Using Lemma 4, we get

$$\mathscr{D}^{W'}(\gamma) = -\frac{R^{W*}\left\{\tau_{21}^A \mid H^B \mid \left[1 + \frac{\tau_{31}^A(1-\alpha^A)}{\tau_{21}^A K^{W*}}\right] + \tau_{21}^B \mid H^A \mid \left[1 + \frac{\tau_{31}^B(1-\alpha^B)}{\tau_{21}^B K^{W*}}\right] + X^A + X^B\right\}}{\Psi\,\Xi\,\tau_{21}^W(\gamma-1)^2},$$

with:

$$X^A = \frac{\tau_{31}^A(1-\alpha^A)\left[\tau_{11}^B\tau_{22}^A - \tau_{12}^B\tau_{12}^A\right]}{K^{W*}} + \frac{\tau_{32}^A(1-\alpha^A)\left[\tau_{11}^A\tau_{12}^B - \tau_{12}^A\tau_{11}^B\right]}{K^{W*}},$$
$$X^B = \frac{\tau_{31}^B(1-\alpha^B)\left[\tau_{11}^A\tau_{22}^B - \tau_{12}^A\tau_{12}^B\right]}{K^{W*}} + \frac{\tau_{32}^B(1-\alpha^B)\left[\tau_{11}^B\tau_{12}^A - \tau_{12}^B\tau_{11}^A\right]}{K^{W*}}.$$

From the fact that $K^{W*} = [(1+\psi)/\psi]K^{A*}$ and $K^{W*} = (1+\psi)K^{B*}$, Lemmas 2 and 3, we get that:

$$\tau_{21}^A \mid H^B \mid \left[1 + \frac{\tau_{31}^A(1-\alpha^A)}{\tau_{21}^A K^{W*}}\right] = -\frac{\tau_{11}^A(1-s^A)|H^B|\left[(1-\alpha^A)\left[1-\nu^A+\psi(1+\rho^A)\right]+b^A\left[(1+\rho^A)(1+\alpha^A\psi)+(1-\alpha^A)(\nu^A-1)\right]\right]}{(1+\psi)\left[(1-\nu^A)(1-b^A)s^A+(1+\rho^A)(1-s^A)\right]},$$

$$\tau_{21}^B \mid H^A \mid \left[1 + \frac{\tau_{31}^B(1-\alpha^B)}{\tau_{21}^B K^{W*}}\right]$$
$$= -\frac{\tau_{11}^B(1-s^B)|H^A|\left[(1-\alpha^B)\left[(1-\nu^B)\psi+(1+\rho^B)\right]+b^B\left[(1+\rho^B)(1+\alpha^B)+\psi(1-\alpha^B)(\nu^B-1)\right]\right]}{(1+\psi)\left[(1-\nu^B)(1-b^B)s^B+(1+\rho^B)(1-s^B)\right]},$$

$$X^A = \frac{\psi\tau_{11}^B\left(\tau_{11}^A\right)^2(1-\alpha^A)(1-s^A)(1-b^A)(\nu^A-1)(1+\rho^A)(1-s^B)(1-s^A b^A)(b^A R^A-1)\left[(1-\nu^B)(1-\alpha^B)(1-b^B)+(1+\rho^B)b^B\right]}{R^A(1+\psi)\left[(1-\nu^A)(1-b^A)s^A+(1+\rho^A)(1-s^A)\right]^2\left[(1-\nu^B)(1-b^B)s^B+(1+\rho^B)(1-s^B)\right]}$$
$$-\frac{\psi\tau_{11}^B\left(\tau_{11}^A\right)^2(1-\alpha^A)(1-s^A)(1-b^A)(\nu^A-1)\left[(b^A R^A-1)(1-s^A)\left[(1-\nu^A)(1-\alpha^A)(1-b^A)+(1+\rho^A)b^A\right]-(\nu^A+\rho^A)(1-b^A)(R^A-b^A)s^A\right]}{R^A(1+\psi)\left[(1-\nu^A)(1-b^A)s^A+(1+\rho^A)(1-s^A)\right]^2},$$

$$X^B = \frac{\psi\tau_{11}^A\left(\tau_{11}^B\right)^2(1-\alpha^B)(1-s^B)(1-b^B)(\nu^B-1)(1+\rho^B)(1-s^A)(1-s^B b^B)(b^B R^B-1)\left[(1-\nu^A)(1-\alpha^A)(1-b^A)+(1+\rho^A)b^A\right]}{R^B(1+\psi)\left[(1-\nu^B)(1-b^B)s^B+(1+\rho^B)(1-s^B)\right]^2\left[(1-\nu^A)(1-b^A)s^A+(1+\rho^A)(1-s^A)\right]}$$
$$-\frac{\psi\tau_{11}^A\left(\tau_{11}^B\right)^2(1-\alpha^B)(1-s^B)(1-b^B)(\nu^B-1)\left[(b^B R^B-1)(1-s^B)\left[(1-\nu^B)(1-\alpha^B)(1-b^B)+(1+\rho^B)b^B\right]-(\nu^B+\rho^B)(1-b^B)(R^B-b^B)s^B\right]}{R^B(1+\psi)\left[(1-\nu^B)(1-b^B)s^B+(1+\rho^B)(1-s^B)\right]^2}.$$

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We obtain $\mathscr{D}^{W'}(\gamma) > 0$ if and only if $b^A \in (\underline{b}^A, \overline{b}^A)$ and $b^A \in (\underline{b}^A, \overline{b}^A)$ with $\underline{b}^A$ and $\underline{b}^B$ defined as:

$$\underline{b}^A = -\frac{(1-\alpha^A)\left[1-\nu^A+\psi(1+\rho^A)\right]}{(1+\rho^A)(1+\alpha^A\psi)+(1-\alpha^A)(\nu^A-1)},$$
$$\underline{b}^B = -\frac{(1-\alpha^B)\left[1+\rho^B+\psi(1-\nu^B\right]}{(1+\rho^B)(1+\alpha^B)-(1-\alpha^B)(1-\nu^B)\psi}.$$

Local indeterminacy may arise if and only if $\mathscr{T}^W(\gamma) > 0$ when $\mathscr{D}^W(\gamma) = -1$. Using equations (27)-(28) allows to show that when $\mathscr{D}^W(\gamma) = -1$, $\mathscr{T}^W(\gamma) > 0$ if and only if:

$$\varepsilon_{rk}^W > \underline{\varepsilon}_{rk}^W = \frac{1+R^{W*}}{R^{W*}\left\{\left(1+\frac{\tau_{22}^W R^{W*}}{\tau_{11}^W}\right)\left[1+\frac{\frac{\partial \tau_3^A}{\partial K^{W*}}\left(1-\alpha^A\right)+\frac{\partial \tau_3^B}{\partial K^{W*}}\left(1-\alpha^B\right)}{\tau_{21}^W K^{W*}}\right]-\frac{\left[K^{W*}\tau_{22}^W+\frac{\partial \tau_3^A}{\partial Y^{W*}}\left(1-\alpha^A\right)+\frac{\partial \tau_3^B}{\partial Y^{W*}}\left(1-\alpha^B\right)\right]}{K^{W*}\tau_{11}^W}\right\}}.$$

It follows that if $\alpha^A \in (\underline{\alpha}^A, 1/2)$, $\alpha^B \in (\underline{\alpha}^B, 1/2)$, $s^A \in (1/3, 1/2)$, $s^B \in (1/3, 1/2)$, $b^A \in (\underline{b}^A, \overline{b}^A)$, $b^B \in (\underline{b}^B, \overline{b}^B)$ and $\varepsilon_{rk}^W > \underline{\varepsilon}_{rk}^W$, $\mathscr{D}^W(\gamma) = -1$ when $\mathscr{T}^W(\gamma) > 0$, which prove the result.

The bifurcation values $\gamma^{W,\mathcal{T}}$ and $\gamma^{W,\mathcal{F}}$ are, respectively, defined as the solutions of $\mathscr{P}^W(1) = 1 - \mathscr{T}^W + \mathscr{D}^W = 0$ and $\mathscr{P}^W(-1) = 1 + \mathscr{T}^W + \mathscr{D}^W = 0$, where $\mathscr{T}^W$ and $\mathscr{D}^W$ are, respectively, defined in equations (27) and (28) and given by

$$\gamma^{W,\mathcal{T}} = 1 - \frac{1+\varepsilon_{rk}^W\left\{\frac{\tau_{22}}{\tau_{11}}R^W+\frac{R^W}{2\tau_{11}K^{W*}}\left[\left(1-\alpha^A\right)\left(\frac{\partial T_3^A}{K^{W*}}+\frac{\partial T_3^A}{Y^{W*}}\right)+\left(1-\alpha^B\right)\left(\frac{\partial T_3^B}{K^{W*}}+\frac{\partial T_3^B}{Y^{W*}}\right)\right]\right\}}{\Psi\varepsilon_{rk}^W\left[1+\frac{\tau_{22}}{\tau_{11}}R^W+\frac{\tau_{21}}{\tau_{11}}\left(1+R^W\right)\right]}, \tag{A9}$$

$$\gamma^{W,\mathcal{F}} = 1 + \frac{1+\varepsilon_{rk}^W\left\{-\frac{\tau_{22}}{\tau_{11}}R^W+\frac{R^W}{2\tau_{11}K^{W*}}\left[\left(1-\alpha^A\right)\left(\frac{\partial T_3^A}{K^{W*}}-\frac{\partial T_3^A}{Y^{W*}}\right)+\left(1-\alpha^B\right)\left(\frac{\partial T_3^B}{K^{W*}}-\frac{\partial T_3^B}{Y^{W*}}\right)\right]\right\}}{\Psi\varepsilon_{rk}^W\left[1+\frac{\tau_{22}}{\tau_{11}}R^W-\frac{\tau_{21}}{\tau_{11}}\left(1+R^W\right)\right]}. \tag{A10}$$

Results follow.

A.7. Proof of Proposition 11

Consider equation (30) for country A. Using the fact that $Y^{A*} = K^{A*}/\psi$ and equation (8), it holds that:

$$dV^A = \frac{\Lambda^A K^{A*}}{\psi}\left[1+\frac{\psi[1-R^A(1+b^A)]}{R^A b^A}\right]dp^A.$$

Under dynamic efficiency, that is, $R^A > 1$, a capital intensive consumption good, that is, $b^A < 0$, and $\psi > 1$, we get the term in bracket is positive. Moreover, country A is a steady-state importer of the investment good and thus the relative price of the investment good decreases, that is, $dp^A < 0$. Then all generations at steady state from country A losses from free-trade.

Let us now consider (30) for country B. Using the fact that $Y^{B*} = \psi K^{B*}$ and equation (8), we derive

$$dV^B = K^{B*}\left[\psi + \frac{1-R^B(1+b^B)}{R^B b^B}\right]dp^B.$$

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Under dynamic efficiency, that is, $R^B > 1$, a capital intensive consumption good, that is, $b^B < 0$, and $\psi > 1$, we get the term in bracket is positive. Moreover, country B is a steady-state exporter of the investment good and thus the relative price of the investment good increases, that is, $dp^B > 0$. Then agent from country B gains from free-trade.