

The large-moon hypothesis: can it be tested?

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Abstract: The large-moon hypothesis states that planetary habitability is enhanced by the presence of a large satellite. This controversial proposal is linked to the equally controversial idea that the axial stability of a planet also enhances habitability. Previous work has shown that, far from encouraging axial stability, large moons actually destabilize obliquity when the effects of tidal drag on the planetary spin rate are taken into account. However, our Moon's mass is remarkably close to the upper limit allowed by axial stability, suggesting that the large-moon hypothesis is actually correct but constrained by an independent requirement for axial stability. This conclusion can be tested by looking at the typical separations between large exo-planets because axial stability is more likely in planetary systems with widely spaced gas giants. Thus, if axial stability really does enhance habitability, Jupiter and Saturn should be unusually widely spaced. This can be tested using data expected from gravitational microlensing planet-search surveys. Unfortunately, in the event of a negative result from this test, it is not possible to distinguish whether this results from large moons and stable axes being unimportant for life or results from large moons and stable axes being widespread.

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Introduction

It is generally acknowledged that the mass of a planet and the distance to its parent star are important factors in planetary habitability. A reasonably massive planet is necessary for a substantial atmosphere to exist (Stoney 1900) whilst the planet–star separation influences surface temperature (Kasting *et al.* 1993). Both of these factors directly affect the likelihood of liquid water being present, a widely accepted precondition for the emergence and/or survival of life (see Schulze-Makuch & Irwin (2004) for a recent review). However, beyond these two attributes, there is little consensus on which other planetary characteristics, if any, influence a planet's habitability. This is a serious impediment to obtaining sensible estimates for the probability of life in other planetary systems as even a fairly small number of such factors significantly reduce the chances of life-bearing exoplanets in our stellar neighbourhood. It is therefore important to learn whether there are any planetary attributes beyond planet size and star separation that enhance or reduce habitability.

One possible factor is that the large size of our planet's satellite may have been an important precondition for life, or at least for intelligent life, to have emerged on the Earth. This is the large-moon hypothesis and it is undoubtedly controversial. Critics point out that the large-moon hypothesis is based upon a number of questionable assumptions. For example, it is far from clear that the Earth is even unusual in

having a large satellite. A larger sample of terrestrial planets may well show that Earth–Moon-like systems are rather common. Another assumption inherent in many versions of the large-moon hypothesis is that the axial stability of the Earth is essential for maintaining a climate suitable for the emergence of complex organisms (Laskar *et al.* 1993). Thus the additional hypothesis is being advanced that axial stability enhances the habitability of a planet. This hypothesis is itself debatable and the large-moon hypothesis is therefore built on rather insecure foundations.

The link between a large moon and the axial stability of the Earth arises because increased tidal forces, resulting from the presence of the Moon, produce a relatively rapid precession rate for the Earth's axis (Ward 1982). As a consequence of this rapid precession, the Earth is immune to chaos-inducing resonant interactions with the significantly slower orbital variations of the other planets. However, Ward (1982) himself pointed out that the presence of the Moon has an additional destabilizing influence on axial stability. Tidal drag slows the Earth's rotation, which, in turn decreases the precession rate. Waltham (2004) showed that, over geological time, tidal drag was more important than increased tidal strength and so axial instability is more likely for a planet with a large satellite than it is for a planet with a small moon or even no moon at all.

Waltham (2004) also demonstrated that our planet is remarkably close to having an unstable obliquity. If the Moon resulting from the Earth–Moon forming collision

event 4.5 Gyr ago (Canup & Asphaug 2001) had had a radius just 11 km greater (but with identical Earth–Moon angular momentum) the slightly increased tidal drag would have slowed the Earth’s rotation down to a 36 hour day today and would have given an Earth–Moon separation approximately 10% larger than actually observed. The decreased rotation rate and decreased Lunar tidal force would then have given a modern precession rate of $26'' \text{ y}^{-1}$ instead of the $50'' \text{ y}^{-1}$ we actually enjoy. That precession rate of $26'' \text{ y}^{-1}$ would result in resonance with the nodal-precession rate of Saturn’s orbit (also at $26'' \text{ y}^{-1}$) giving rise to variations in obliquity which Laskar *et al.* (1993) estimated as more than 50° over a few million years. In summary, had the Moon’s radius been just 11 km larger, the Earth’s axis would be becoming unstable today.

Paradoxically, this large-moon-generated near-instability is evidence in favour of the large-moon hypothesis. This follows if we assume that a large moon is useful for life for some reason that is *independent* of obliquity stability. Thus, if large moons are anthropically selected but their size is limited by the *additional* need for obliquity stability, then a lunar-size approaching the stability limit is precisely what would be expected. This conclusion is independent of the reason why a large moon is beneficial to life. That makes the conclusion robust but, on the other hand, it also means that this result sheds little light on the mechanism.

This new argument in favour of the large-moon hypothesis can be weakly criticized because it is a *post hoc* explanation of already known facts. It would be far more convincing if independent support were obtained based upon a successful prediction of data not yet collected. Thus, the aim of this paper is to move the anthropic-selection debate forward by proposing a way in which attainable data can be used to support or discredit the large-moon hypothesis. In particular, it is shown that major-planet separation data should be able to settle this question within the next decade. In other words, the large-moon hypothesis is a well-defined (although implausible in some views) hypothesis that can and should be scientifically tested against data.

The next section investigates a previously neglected aspect of the Earth’s axial stability, namely the effect of the separation between Jupiter and Saturn. The analysis shows that, if the large-moon hypothesis really is correct, the separation between Jupiter and Saturn should be unusually large compared to typical separations between planets in extra-solar planetary systems. This is a prediction which gravitational microlensing planet-search surveys should be able to test within the next decade. In microlensing events, a distant star is temporarily brightened as a result of gravitational focusing by a closer star passing between the observer and the distant star. Mao & Paczynski (1991) showed that deviations from the expected light-curve could, in principle, be used to detect planets in orbit around the lensing star. This technique has now detected four exoplanets including the first known terrestrial exoplanet (Beaulieu *et al.* 2006). Microlensing is particularly well suited to discovering multiple-planet systems with major-planet spacings comparable to those in our Solar

System (Gaudi *et al.* 1998) and so this paper concludes with clear criteria for accepting or rejecting the large-moon hypothesis using such data.

Obliquity stability and planetary separations

A key consideration in the analysis of axial stability is that instability will occur if the Earth’s precession rate were small enough to resonate with the orbital variations of any major planet. The maximum such variation happens to be $26'' \text{ y}^{-1}$ owing to the nodal precession of Saturn’s orbit and below this rate there are many possible resonant interactions (Laskar *et al.* 1993). This $26'' \text{ y}^{-1}$ cut-off produces an angular-momentum-dependent upper limit to the mass of the Moon, but that limit would be higher if the precession rate of Saturn were lower. Thus if life, or intelligent life, really is favoured by having as large a moon as possible consistent with axial stability, it follows that life is favoured by planetary systems in which orbital precession rates are low (Waltham 2004). This section investigates the main planetary system properties that control orbital precession rates.

For such a study it is vital to use a highly simplified ‘toy’ solar system. Realistic planetary systems consisting of, say, 10 planets with half a dozen time-varying orbital elements for each planet would require weeks of CPU time to model and would have an impossibly large parameter space to explore. Here, a planetary system consisting of one star and two identical-mass planets in circular but mutually inclined orbits is considered. This simplified solar system, therefore, has only five parameters (two masses, two orbital radii and the angle of inclination). An analytic treatment of such systems has recently been published by Veras & Armitage (2004) and they demonstrated that the two orbits precess, at identical rates given by

$$\Omega = \frac{\sqrt{G}}{4} b_{3/2}^{(1)} \frac{m}{\sqrt{m+M}} a_2^{-3/2} (\sqrt{\alpha} + \alpha) \quad (1)$$

where m is the planetary mass, M the stellar mass, $\alpha = a_1/a_2$ with a_1 and a_2 the orbital radii and b is a Laplace integral defined by

$$b_s^{(j)}(\alpha) = \frac{1}{\pi} \int_0^{2\pi} \frac{\cos(j\psi) d\psi}{(1 - 2\alpha \cos(\psi) + \alpha^2)^s}. \quad (2)$$

Note that the angle at which the orbits are inclined to one another does not appear in these expressions and so only four parameters are left. As a test of Eq. (1) the semi-major axes of Jupiter and Saturn in our own Solar System were input along with $m = (m_J + m_S)/2$ and M given by the mass of our Sun. The resulting prediction is an orbital precession rate of $26'' \text{ y}^{-1}$, i.e. exactly that observed, and so this toy solar system appears to approximate our real, much more complex, Solar System remarkably well.

One immediate implication of Eq. (1) is that, for $m \ll M$, the precession rate is proportional to planetary mass. Hence, if slowly varying planetary systems are anthropically selected,

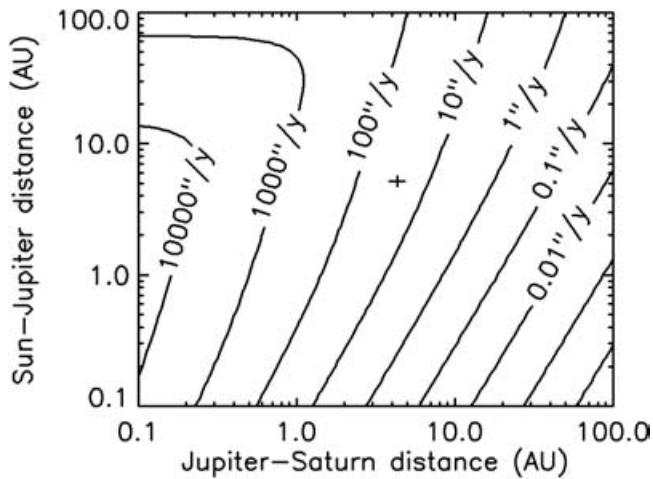


Fig. 1. Orbital precession rates for a planetary system consisting of two identical-mass planets in inclined circular orbits. Planetary masses here equal the average mass of Jupiter and Saturn. The central star's mass equals the Sun's mass. The cross shows the true distances to Jupiter and Saturn. Note that the main control on precession rate is the separation between 'Jupiter' and 'Saturn'.

there should be selection for low-mass systems. Similarly, for $m \ll M$, the precession rate is inversely proportional to the square root of solar mass, which implies that large solar masses are anthropically favoured. However, as discussed below, giant-planet mass and stellar mass both have other important consequences and so are not suitable indicators of support for the large-moon hypothesis.

This leaves the effect of orbital radii to be investigated. Figure 1 shows how the precession rate is affected by changes in the Sun–Jupiter and Jupiter–Saturn separations. Note that the predicted precession rate varies by nine orders of magnitude when these distances are varied over three orders of magnitude and so there is plenty of scope for producing planetary systems with very different precession rates to our own. From Fig. 1 it is clear that slow precession rates will occur in planetary systems where the equivalent of Jupiter is close to its star whilst the equivalent of Saturn is as far out as possible.

In summary, planetary systems are more habitable, from the point of view of enhanced axial stability of terrestrial planets, if the giant planets have relatively low masses, the central star has a relatively high mass, the equivalent of Jupiter is close to its star and the equivalent of Saturn is far from its star. However, most of these factors also affect other aspects of habitability. Massive stars have relatively short main-sequence lifetimes and relatively narrow continuously habitable zones (Kasting *et al.* 1993). Low-mass gas giants may be unable to act as shields against asteroid collision (Wetherill 1994) and a close-in 'Jupiter' may directly disrupt the orbits of otherwise habitable inner planets. Thus, all of these factors have potentially complex effects upon habitability, making them unsuitable as indicators of anthropic selection.

However, the 'Jupiter' to 'Saturn' separation does not appear to suffer from this problem and is therefore the

extra-solar planetary system attribute most likely to give us a clear signature of selection for axial stability. The next section investigates the statistics of identifying such a signature.

Statistical testing

The concept of anthropic selection may be expressed, statistically, by saying that the probability distribution of a property (e.g. planetary separations) is different for inhabited worlds than it is for planets in general. Bayes' theorem links these probability distributions with

$$p(x/L) = \frac{p(x)p(L/x)}{p(L)} \quad (3)$$

where $p(x/L)$ is the distribution of property x for inhabited planets, $p(x)$ is the probability distribution for all planets, $p(L/x)$ is the probability of life on a planet with property equal to x (i.e. this term expresses the influence that property x has on the likelihood of life) and $p(L)$ is a (presumably small) constant expressing the probability of life on any given planet (i.e. it is the number of planets in the Universe with life divided by the total number of planets).

Equation (3) can be used in a number of slightly different contexts. For some properties it may be more useful to regard the probabilities as applying to planetary systems rather than individual planets (i.e. $p(x)$ could be the probability distribution for a planetary system property such as separation between a star and its largest planet). Similarly, it may be useful to think of the unbiased distribution, $p(x)$, as coming from the set of terrestrial planets rather than from the set of all planets. It may also be useful, depending on context, to regard condition L as representing planets with life, planets with complex life or even planets with intelligent life. In general, we might expect the distributions $p(x)$ and $p(x/L)$ to differ more if L refers to intelligent life compared to the case where L refers to life of any kind.

Bayes' theorem is schematically illustrated for the case of Jupiter–Saturn separation in Fig. 2. It is important to emphasize that the distributions in Fig. 2 are for illustrative purposes only. We do not yet have the data to say what such distributions really look like but, for the purposes of this section, this is not important. In this figure, $p(s)$ is the probability distribution of separations between the two largest planets in a planetary system and it could be estimated once a large number of high-magnification microlensing events have been detected (Gaudi *et al.* 1998). The distribution $p(L/s)$ describes how Jupiter–Saturn separation affects habitability. Here it is assumed that the foregoing analysis is correct so that planetary systems with large separations are more habitable than planetary systems with small s . The distribution $p(L/s)$ therefore increases from a low value for small separations and becomes large for wide separations. The final distribution, $p(s/L)$, shows how s is distributed within the subset of those planets that happen to be inhabited. In principle, it could be measured once a large number of inhabited systems are known but has been constructed here using Bayes' theorem, which tells us that,

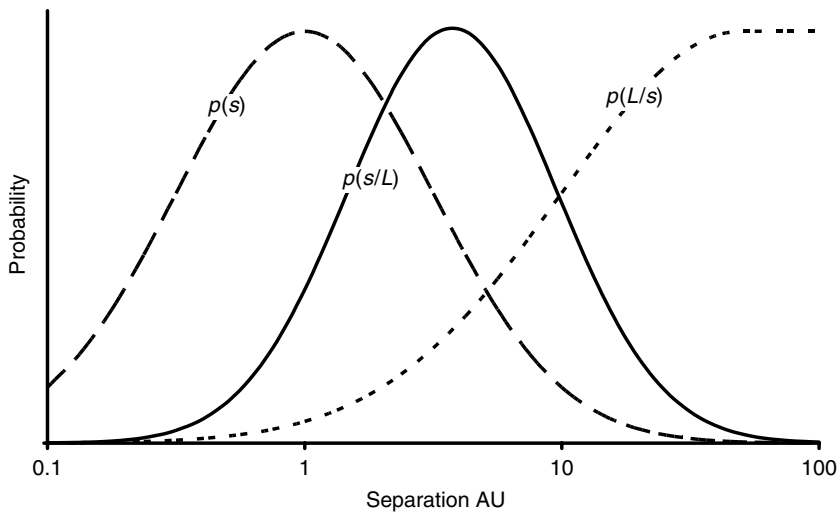


Fig. 2. Bayes' theorem and anthropic selection: schematic planetary separation example. The probability distribution for planets in general ($p(s)$) is different to that for planets with life ($p(s/L)$) because habitability is enhanced by large planetary separations (i.e. $p(L/s)$ increases with separation).

ignoring a scale factor, it is given by the product of the other two distributions.

Note that, given these assumed distributions, the most probable planetary separation in an inhabited planetary system is significantly greater than the most probable separation for planetary systems in general. However, also note that the most probable separation in inhabited systems is still significantly smaller than the optimum separation implied by the peak of $p(L/s)$. In general, the peak of an anthropically-selected distribution, $p(x/L)$, lies between the peak of the unbiased distribution, $p(x)$, and the peak of the selecting distribution, $p(L/x)$.

Anthropic selection may be defined as occurring when the anthropically-selected distribution, $p(x/L)$, differs significantly from the unbiased distribution, $p(x)$. This will happen whenever the x most likely to occur is substantially different from the x most likely to produce life. Hence, there are two ways in which planet-level anthropic selection might be unimportant. First, the most likely value of x may be very close to the optimum value of x for life. This would be strong evidence for cosmological anthropic selection, i.e. selection for universes having properties conducive to life (see Barrow & Tipler (1986) for an exhaustive review of this topic). Second, the selecting distribution, $p(L/x)$, may have a very broad peak covering a wide range of x values, i.e. property x does not significantly effect the likelihood of complex life. This is presumably the view of those who are sceptical about the importance of anthropic selection. This distinction between two ways in which anthropic selection fails to occur will be important later in this paper.

Given this background, it should be clear that anthropic selection could, in principle, be tested if we knew the probability distribution of a proposed anthropically-constrained property, both for planets in general and for planets having complex life. The two distributions could then be compared using a χ^2 test. Space-based planet-search programmes

together with microlensing surveys should provide enough data to allow the general distributions to be estimated, but even those most optimistic about the chances of extra-terrestrial life would be surprised if we found enough inhabited extra-solar planets to estimate property distributions for this small sub-set of all planets. Thus, for the foreseeable future, it is unlikely that we could perform this most obvious of tests for anthropic selection.

Instead, we need to test using only one example of a planet with life, i.e. the Earth. The null hypothesis to test is that the Earth is extracted from a distribution which does not differ from the unbiased distribution. If we again look at the hypothetical case of Fig. 2, it is quite easy to see how this should be achieved. The actual separation of Jupiter and Saturn in our Solar System is 4.34 AU, i.e. higher than the most likely separation given by the peak in the hypothetical $p(s)$. Is it significantly higher? This can be tested simply by calculating the percentage of $p(s)$ which lies even higher than 4.34 AU. To reject the null hypothesis, the resulting value should fall below a pre-set significance level. For statistical studies in the Earth sciences it is common to set the significance level at 5%. However, for more critical problems (e.g. medical statistics) a more conservative threshold of 1% is common. Given that anthropic selection is controversial, and following the convention that extraordinary theories require extraordinary evidence, the more conservative significance threshold is more appropriate for anthropic testing.

Discussion

The procedure suggested above is a direct, and independent, test of the axial-stability hypothesis. If planets are more habitable when they have stable rotation axes then planetary systems with large separations between their two largest gas giants will be more habitable than systems with more closely spaced planets. Hence, if the axial-stability hypothesis is

correct, our Solar System should be significantly more widely spaced than average. Furthermore, the axial-stability and large-moon hypotheses taken together predict that the Earth's Moon should be just below the stability limit as observed. Clearly, this argument is greatly strengthened if one of the underlying hypotheses is independently supported. Thus, data demonstrating that Saturn and Jupiter are unusually far apart would be evidence in favour of the large-moon hypothesis.

In conclusion, a positive result (Jupiter–Saturn separation unusually large at the 1% significance level) would support the large-moon hypothesis. What criteria should we use for rejecting the hypothesis? There is clearly a grey area (say, a significance between 1% and 25%) where the results are suggestive but not conclusive. However, if the Jupiter–Saturn separation is not significantly larger than average at the 25% level, this suggests that the large moon and axial stability of the Earth do not contribute significantly to the Earth's habitability. This could be due to two very different reasons linked to the two previously discussed ways in which anthropic selection fails to occur. First, the typical moon size and typical gas-giant separation may be ideal for life, i.e. large moons and axial stability enhance habitability but are the norm. As discussed earlier, this would be evidence for cosmological anthropic selection. The second possible explanation for a negative result to the proposed test is that moon size and gas-giant separation have little effect on habitability. The only way to distinguish between these alternate explanations would be to obtain accurate estimates of both $p(s)$ and $p(s/L)$, thus allowing $p(L/s)$ to be calculated from Bayes' theorem. It is likely to be some considerable time before this can be achieved.

Thus, planetary-level anthropic selection for a large moon can be tested, i.e. there are clear criteria for accepting or rejecting the large-moon hypothesis using data on planetary

distributions expected over the next decade or so. However, a rejection of the hypothesis by this test leaves the possibility open that there is still cosmological-level anthropic selection for large moons. If true, this latter possibility would imply that large moons are conducive to life but sufficiently common that their occasional absence does not drastically change the probability of life in other planetary systems. Rejecting both planet-level and cosmological-level anthropic selection requires data from large numbers of inhabited systems, data we are unlikely to gather in the near future.

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