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# CLASSICAL COUNTERPOSSIBLES

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**Abstract.** We present four classical theories of counterpossibles that combine modalities and counterfactuals. Two theories are anti-vacuist and forbid vacuously true counterfactuals, two are quasi-vacuist and allow counterfactuals to be vacuously true when their antecedent is not only impossible, but also *inconceivable*. The theories vary on how they restrict the interaction of modalities and counterfactuals. We provide a logical cartography with precise acceptable boundaries, illustrating to what extent nonvacuism about counterpossibles can be reconciled with classical logic.

**§1. Introduction.** Counterpossibles invite us to entertain the impossible. On standard accounts (such as Lewis, 1973; Stalnaker, 1968; Kratzer, 1979), counterfactuals are evaluated at *possible worlds*. Counterfactuals with impossible antecedents (counterpossibles) are accordingly vacuously true, because no world satisfies their antecedent, and so every world that satisfies their antecedent (none!) also satisfies their consequent. Berto *et al*. (2017) call these classical theories of counterpossibles *vacuist*: according to these theories, all counterpossibles are vacuously true.

Motivation against vacuism comes from the fact that at least one of the two following counterpossibles (due to Mares & Fuhrmann, 1995) appears to be false:

1. If someone were to create a square circle, then we would be amazed.
2. If someone were to create a square circle, then we would not be amazed.

Many available theories of counterpossibles resolve this issue by countenancing *impossible* worlds.[1] Impossible worlds let impossible things be true, such as creating a square circle, without allowing *everything* to be true. Having impossible worlds around allows for nonvacuous treatment of counterpossibles. Counterpossibles are

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[1] For examples, see Bjerring (2014), Brogaard & Salerno (2013), Mares & Fuhrmann (1995), Nolan (1997), Bernstein (2016), and Zagzebski (1990). For different kinds of approach, see Kim & Maslen (2006) and Vetter (2016).



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false when their consequent fails in some of the impossible antecedent-worlds. Call this *nonvacuism*. The resulting literature on counterpossibles focusses on questions revolving around the move to *nonclassical* logics involving impossible worlds.

> There is [a] worry often evinced about impossible worlds, and non-trivial reasoning involving the impossible in general: the worry is that allowing such things respectability will bring the evils of nonclassical logic in their train. (Nolan, 1997, p. 543)

This paper is a modest conciliatory logical project. We show that a fine-grained logical analysis of counterpossibles, with a division of labour between modalities and counterfactuals, allows for nonvacuous *classical* theories of counterpossibles. We adopt an orthodox classical mentality. We work with Boolean negation, the material conditional, ex falso quodlibet, excluded middle, disjunctive syllogism, only two truth-values, and no true contradictions. All worlds in our models are classical: they are consistent and respect all classical tautologies. We also use a standard Tarskian consequence relation.

We use conditional modal logic as a framework for analysing counterpossibles. The framework makes use of standard relational models for modal logic, with the accessibility relation understood as modeling relative metaphysical possibility. When two worlds $v, w$ do not stand in this relation, then, $w$ is metaphysically impossible relative to $v$. This allows us to see, within the usual relational models for modal logic, that impossible worlds have been with us all along.

To achieve a fine-grained analysis, we extend the language with a modality that ranges over all worlds. We call it a *universal modality*, with the formula $\blacklozenge\phi$ simply read as '$\phi$ is true in some world'. You can think of the universal modality along the lines of broadly logical possibility in the sense of Hale (1996), or perhaps as *conceivability*. Put in these terms, our framework allows for counterfactuals to reach out to worlds that are impossible, yet conceivable. We acknowledge the required work for a thorough philosophical distinction between metaphysical possibility and what this universal modality stands for, but do not engage further with it. We are here to provide the logical prolegomenon to such future work. Our theoretical commitment is that we can think modally beyond what is metaphysically possible, and we make that commitment explicit in our logical language.

We propose four theories, divided into two categories: *anti-vacuism* and *quasi-vacuism*. Anti-Vacuism forbids vacuous counterfactuals altogether, whereas quasi-vacuism allows for vacuous counterfactuals of sorts. With these categories, we consider *successful* and *necessary* versions, depending on a trade-off between entertainments that are successful and those constrained by necessities. A philosophical outcome of our studies is that nonvacuism is compatible with classical logic. The dichotomy between vacuism and nonvacuism isn't a logical one.[2]

We have one more announcement to make before the start of the show, as we don't want to mislead our audience. Our theories of counterpossibles have a *ceteris paribus* rider: we keep classical logic *fixed*. Our theories do not cover *counterlogicals*, such as "if intuitionistic logic were correct, excluded middle would be invalid," but they do cover counterpossibles such as "if water was xyz, then it wouldn't be drinkable" (would it?)

[2] We acknowledge a radical nonvacuist view alluded to in Lewis (1973) in which all counterpossibles are made false, but won't take it into serious consideration.

and *countermathematicals* such as “if 2 and 6 were factors of 13, then 13 wouldn’t be prime.” This is already quite an achievement for classical logic! We come back to the case of counterlogicals in the conclusion and suggest a way to extend our framework to analyse them. The idea is that we work within a classical universe of worlds, which allows for impossible things, but not illogical ones. Counterlogicals would open portals to different logical universes. We have conjectures on how to create those portals, and hopefully you will see how that might go after understanding our classical theories.

After a brief exposition of the formal language and models for our investigation in §2, we lay down and discuss particular principles regulating the interaction of the modalities and counterfactuals in §3, and present crucial logical relations between them in §4. In §5, we use these logical relations to identify four *acceptable* nonvacuist theories. We conclude in §6 by discussing the different worldviews each theory yields for counterpossibles. We reserve the wildest claims and conjectures for the conclusion.

**§2. Language and models.** Our language is a standard propositional language with a set of atoms PROP, propositional connectives $\neg$ and $\vee$, a metaphysical modality $\Diamond$, a universal modality $\blacklozenge$, and a counterfactual operator $\Box\!\!\rightarrow$. Other connectives and modalities ($\wedge, \supset, \equiv, \Box, \blacksquare, \Diamond\!\!\rightarrow$) are defined in the usual way. Modalities (including $\Box\!\!\rightarrow$) bind more tightly than extensional connectives, and unary connectives tighter than binary; so for example $\Diamond\phi \Box\!\!\rightarrow \psi \supset \theta$ is $((\Diamond\phi) \Box\!\!\rightarrow \psi) \supset \theta$.

Models $M$ have a domain of worlds $W$, a propositional valuation $V$ assigning sets of worlds to atoms, and two selection functions:

$$R : W \longrightarrow \wp(W)$$
$$S : W \times \wp(W) \longrightarrow \wp(W).$$

The selection function $R$ represents a relative notion of metaphysical possibility and can be read as saying that $v$ is *metaphysically accessible* from $w$ when $Rwv$.[3] We suppose that the evaluation of a counterfactual $\phi \Box\!\!\rightarrow \psi$ at a world $w$ involves selecting a set of worlds based on $\phi$ and $w$, and checking whether $\psi$ holds throughout this set; we refer to this set of worlds as the worlds *entertained*. The selection function $S$ selects the entertained worlds for every world-proposition pair. We make no assumptions on the selection functions for the time being. Our theories are about interactions between the various modalities, rather than about their respective desirable restrictions.

$$\begin{array}{lcl}
w \in [\![p]\!]^M & \Leftrightarrow & w \in V(p) \\
w \in [\![\neg\phi]\!]^M & \Leftrightarrow & w \in W[\![\backslash\phi]\!]^M \\
w \in [\![\phi \vee \psi]\!]^M & \Leftrightarrow & w \in [\![\phi]\!]^M \cup [\![\psi]\!]^M \\
w \in [\![\Diamond\phi]\!]^M & \Leftrightarrow & R(w) \cap [\![\phi]\!]^M \neq \emptyset \\
w \in [\![\blacklozenge\phi]\!]^M & \Leftrightarrow & [\![\phi]\!]^M \neq \emptyset \\
w \in [\![\phi \Box\!\!\rightarrow \psi]\!]^M & \Leftrightarrow & S(w, [\![\phi]\!]^M) \subseteq [\![\psi]\!]^M.
\end{array}$$

When a model $M$ is clear from context, we omit the superscript $M$ and write $[\![\phi]\!]$ instead of $[\![\phi]\!]^M$. A formula $\phi$ is *valid in a model* if $w \in [\![\phi]\!]$ for every $w \in W$, and it is *valid in a frame* if it is valid in every model based on the frame (i.e., for every propositional valuation). Finally, a formula is valid in a set of frames if it is valid in every frame in the set.

[3] Here and elsewhere, $Rwv$ should be understood as shorthand for $v \in R(w)$.

We pause to note that the following three principles are valid already in this minimal setting:

$$\blacksquare(\psi \supset \chi) \supset ((\phi \mathrel{\Box\!\!\rightarrow} \psi) \supset (\phi \mathrel{\Box\!\!\rightarrow} \chi))$$
$$\blacksquare(\phi \equiv \psi) \supset ((\mathrel{\Box\!\!\rightarrow} \phi\chi) \equiv (\mathrel{\Box\!\!\rightarrow} \psi\chi))$$
$$\phi \mathrel{\Box\!\!\rightarrow} \top.$$

However, more standard principles of counterfactual logics are not valid, such as:

$$((\phi \mathrel{\Box\!\!\rightarrow} \psi) \wedge (\psi \mathrel{\Box\!\!\rightarrow} \phi)) \supset ((\phi \mathrel{\Box\!\!\rightarrow} \chi) \equiv (\phi \mathrel{\Box\!\!\rightarrow} \chi)).$$

We leave to the reader to find an appropriate counterexample. We could opt to make this principle valid by imposing standard restrictions on the selection function $S$. We do not pursue this and leave the demonstration of a completeness result for the minimal logic to the classically inclined logician. We focus instead on the interaction principles between the modalities that regulate vacuism.

**§3. Principles.** In this section, we lay out the schematic principles that sit at the core of our investigation. Each of these principles says something about the counterfactual conditional. We do not endorse all of these principles—far from it! Indeed, taken together these principles are inconsistent—and so, given our classical setting, trivial. Instead, we consider a range of connections and incompatibilities among them.

For each principle, we give a corresponding frame condition, in the sense of Blackburn, de Rijke, & Venema (2001, Definition 3.2): a frame meets this condition iff every instance of the principle is valid on the frame.[4] For example, the schema `id` is $\phi \mathrel{\Box\!\!\rightarrow} \phi$; its corresponding frame condition `ID` is that $S(w, X) \subseteq X$. (The frame conditions should be understood as implicitly universally quantified, so a frame meets this condition iff $S(w, X) \subseteq X$ for every world $w$ and proposition $X$.) This can be shown as follows. First, to show that `id` is valid on all frames meeting `ID`: we know that a world $w$ satisfies $\phi \mathrel{\Box\!\!\rightarrow} \phi$ iff $S(w, [\![\phi]\!]) \subseteq [\![\phi]\!]$. When a frame satisfies `ID` we have $S(w, X) \subseteq X$ for every proposition $X$, in particular we have this for $[\![\phi]\!]$, whatever proposition that turns out to be, in any model built on such a frame. Second, to show that when `id` is valid on a frame that frame must meet `ID`, we argue contrapositively: take any frame that does not meet `ID`. Then there must be some world $w$ and proposition $X$ in the frame such that $S(w, X) \not\subseteq X$. On that frame, we can consider a model such that $[\![p]\!] = X$. Now $p \mathrel{\Box\!\!\rightarrow} p$ fails at $w$ in such a model, so the schema $\phi \mathrel{\Box\!\!\rightarrow} \phi$ is not valid on the frame in question. In what follows, we claim correspondence facts without further proof; each needed proof is just like this one, mutatis mutandis.

Our principles come in five families of three plus four odds and ends. The first family includes the principle `id` plus two weakenings.

We can think of a counterfactual $\phi \mathrel{\Box\!\!\rightarrow} \psi$ as saying that $\psi$ holds in the worlds that result from entertaining $\phi$. With this interpretation in mind, let an entertaining of $\phi$ count as *successful* iff $\phi$ holds in the resulting worlds.[5] Then `id` tells us that every entertaining succeeds, no matter what is being entertained. Its weakenings only require

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[4] As Blackburn *et al.* (2001) defines correspondence, it applies to individual formulas rather than schemas. But since we are concerned only with what is valid on a frame, these amount to the same thing.

[5] We borrow the term 'successful' here from the literature on belief revision, which takes a revision by $\phi$ to be successful iff $\phi$ itself ends up in the resulting belief set.

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| ID family | | | |
|---|---|---|---|
| id | | | $\phi \mathrel{\Box\!\!\rightarrow} \phi$ |
| ID | | | $S(w, X) \subseteq X$ |
| cid | $\blacklozenge\phi$ | $\supset$ | $\phi \mathrel{\Box\!\!\rightarrow} \phi$ |
| CID | $X \neq \emptyset$ | $\Rightarrow$ | $S(w, X) \subseteq X$ |
| mid | $\Diamond\phi$ | $\supset$ | $\phi \mathrel{\Box\!\!\rightarrow} \phi$ |
| MID | $R(w) \cap X \neq \emptyset$ | $\Rightarrow$ | $S(w, X) \subseteq X$ |

certain entertainings to succeed: `cid` requires that entertaining $\phi$ always succeeds when $\phi$ is conceivable; and `mid` requires that entertaining $\phi$ always succeeds when $\phi$ is (metaphysically) possible.[6]

The pattern connecting the members of this family repeats for our other four families. We consider one unqualified principle about counterfactuals, together with the principles that result from qualifying it first with the claim that the counterfactual antecedent is conceivable, and second with the claim that it is possible.

Are the worlds that result from entertaining $\phi$ always possible, for every $\phi$? If so, we have the principle `cp`. This forms the basis of our next family.

| CP family | | | |
|---|---|---|---|
| cp | | | $\phi \mathrel{\Box\!\!\rightarrow} \psi \supset \Diamond\psi$ |
| CP | | | $R(w) \cap S(w, X) \neq \emptyset$ |
| ccp | $\blacklozenge\phi$ | $\supset$ | $\phi \mathrel{\Box\!\!\rightarrow} \psi \supset \Diamond\psi$ |
| CCP | $X \neq \emptyset$ | $\Rightarrow$ | $R(w) \cap S(w, X) \neq \emptyset$ |
| mcp | $\Diamond\phi$ | $\supset$ | $\phi \mathrel{\Box\!\!\rightarrow} \psi \supset \Diamond\psi$ |
| MCP | $R(w) \cap X \neq \emptyset$ | $\Rightarrow$ | $R(w) \cap S(w, X) \neq \emptyset$ |

These three principles give us conditions under which the worlds that result from our entertainments must be possible. According to `cp`, they must always be; according to `ccp`, they must be whenever the proposition entertained is conceivable; and according to `mcp`, they must be whenever the proposition entertained is possible. (The principle `mcp` is also known as `POS`, and studied under that name in Williamson, 2020; Berto *et al.*, 2017; and Girard, 2020.)

Let a proposition $\phi$ be *coherent* iff $\neg(\phi \mathrel{\Box\!\!\rightarrow} \bot)$. That is, $\phi$ is coherent[7] iff entertaining it does not result in a contradiction.[8] Is every proposition coherent? If so, we have the principle `ee`. This forms the basis of our next family.

6 You might not like our talk of *conceivability*, but we do not apologise for it. We thought of using 'conceptual possibility' instead, but figured it would be just as controversial. Whatever word best suits you, the distinction we have in mind is clear: `id` holds no matter what, `cid` holds when the antecedent is true in some world, and `mid` holds when the antecedent is true in some possible world.

7 No apologies!

8 Remember, we are remaining fully classical throughout; any contradiction at all is absurd.

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| EE family | | | |
|---|---|---|---|
| ee | | | $\neg(\phi \mathrel{\Box\!\!\rightarrow} \bot)$ |
| EE | | | $S(w, X) \neq \emptyset$ |
| cee | $\blacklozenge\phi$ | $\supset$ | $\neg(\phi \mathrel{\Box\!\!\rightarrow} \bot)$ |
| CEE | $X \neq \emptyset$ | $\Rightarrow$ | $S(w, X) \neq \emptyset$ |
| mee | $\Diamond\phi$ | $\supset$ | $\neg(\phi \mathrel{\Box\!\!\rightarrow} \bot)$ |
| MEE | $R(w) \cap X \neq \emptyset$ | $\Rightarrow$ | $S(w, X) \neq \emptyset$ |

These three principles give us conditions under which propositions are coherent. According to ee, every proposition is; according to cee, at least the conceivable propositions are; and according to mee, at least the possible ones are. In what follows, we are sometimes interested in the relations between conceivability and coherence. In particular, we look at cases where these are and are not equivalent. It's worth noting, then, that cee gives us one half of this equivalence.

FACT 1. *The schemas id and ee are not consistent with each other.*[9]

*Proof.* $\bot \mathrel{\Box\!\!\rightarrow} \bot$ is an instance of id, and $\neg(\bot \mathrel{\Box\!\!\rightarrow} \bot)$ is an instance of ee. □

This records the idea that entertaining an absurdity cannot be both successful and coherent. After all, if it's successful, the resulting scenario is absurd, so it's not coherent! Despite this, every other combination of one principle from the id family with one from the ee family is consistent. We just can't hold to the strongest principles in both families at the same time.

FACT 2. *cp entails ee.*[10]

*Proof.* Note that $\Diamond\bot$ is equivalent to $\bot$, and apply classical logic. □

This entailment records the idea that if entertaining $\phi$ must result in a possible scenario, it cannot result in an absurd one. For essentially the same reasons, ccp entails cee and mcp entails mee.

Are the scenarios that result from our entertainings limited by necessities? If so, we have the principle nc. This forms the basis of our next family.

| NC family | | | |
|---|---|---|---|
| nc | | | $\Box\psi \supset \phi \mathrel{\Box\!\!\rightarrow} \psi$ |
| NC | | | $S(w, X) \subseteq R(w)$ |
| cnc | $\blacklozenge\phi$ | $\supset$ | $\Box\psi \supset \phi \mathrel{\Box\!\!\rightarrow} \psi$ |
| CNC | $X \neq \emptyset$ | $\Rightarrow$ | $S(w, X) \subseteq R(w)$ |
| mnc | $\Diamond\phi$ | $\supset$ | $\Box\psi \supset \phi \mathrel{\Box\!\!\rightarrow} \psi$ |
| MNC | $R(w) \cap X \neq \emptyset$ | $\Rightarrow$ | $S(w, X) \subseteq R(w)$ |

[9] That is, no world in any model satisfies all instances of both.

[10] That is, all instances of cp put together entail each instance of ee, with entailment understood locally. Local entailment: $\Sigma \models \phi$ iff there is no world in any model that satisfies everything in $\Sigma$ but does not satisfy $\phi$.

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These three principles give us conditions under which entertaining a proposition yields worlds in which all necessary propositions hold. According to `nc`, this always happens; according to `cnc`, this happens at least when the proposition entertained is conceivable; according to `mnc`, this happens at least when the proposition entertained is possible.

The frame condition `MNC` corresponding to `mnc` is of some interest. It says that whenever we entertain from a world $w$ some proposition $X$ that is possible at $w$, the resulting scenario cannot include *any* worlds that are impossible at $w$. This is, we think, the natural statement in the present setting of the condition sometimes called the 'strangeness of impossibility condition', or `SIC`.[11] As far as we know, this is the first time that an object-language schema corresponding to a `SIC`-like condition has been identified.

Before we look at our last family, we have some individual principles to introduce. The first is `ecp`. We mentioned above that we would be interested in the conditions under which coherence and conceivability turn out to be equivalent. `cee` gives us one half of the equivalence; `ecp` is the other half.

| ECP | |
|---|---|
| ecp | $\neg(\phi \mathrel{\Box\!\!\rightarrow} \bot) \supset \blacklozenge\phi$ |
| ECP | $S(w, \emptyset) = \emptyset$ |

FACT 3. *`id` entails `ecp`.*

*Proof.* If a world $w$ satisfies $\neg(\phi \mathrel{\Box\!\!\rightarrow} \bot)$, then $S(w, [\![\phi]\!])$ must be nonempty, and if $w$ obeys `id`, then we must have $S(w, [\![\phi]\!]) \subseteq [\![\phi]\!]$. It follows that $[\![\phi]\!]$ is nonempty, but this is all it takes to make $\blacklozenge\phi$ hold at $w$. □

The second individual principle, `mp`, is a standard principle of conditional logics. It tells us that if entertaining $\phi$ results in worlds where $\psi$ holds, and if $\phi$ is true, then $\psi$ too must be true. Its corresponding frame condition (weak centering) is the same here as elsewhere. For more discussion of `mp`, see Chellas (1975).

| MP | |
|---|---|
| mp | $\phi \mathrel{\Box\!\!\rightarrow} \psi \supset (\phi \supset \psi)$ |
| MP | $w \in X \supset w \in S(w, X)$ |

It's now time to turn to the principles we aim to avoid. One of them is our third individual principle.

| collapse |
|---|
| $\blacklozenge\phi \supset \lozenge\phi$ |
| $R(w) = W$ |

It follows from our basic setup that everything possible is conceivable; `collapse` gives the other direction. It is part of our basic approach to counterpossibles that there are

[11] For further discussion of `SIC`, which has some slightly different statements and relatives in the literature, see Nolan (1997), Mares (1997), and Berto *et al.* (2017).

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impossible worlds. Worlds impossible from $w$ can still play a role in the satisfaction conditions for counterfactuals at $w$. To collapse possibility with conceivability would be to remove this texture. So we aim, in what follows, to steer clear of `collapse`.

Before we consider our fourth individual principle, we present our final family. Can we coherently entertain something that is impossible? This is the core issue that divides *vacuists* from *nonvacuists*. As we have flagged from the outset, our aim is to develop workable nonvacuist theories, so we want to insist that some impossibilities are coherent. If none are, we have the principle `vac`, which forms the basis of this family.

| `VAC` family | | | |
|---|---|---|---|
| `vac` | | | $\neg\Diamond\phi \supset \phi \,\Box\!\!\rightarrow \psi$ |
| `VAC` | | | $R(w) \cap X = \emptyset \Rightarrow S(w, X) = \emptyset$ |
| `cvac` | $\blacklozenge\phi$ | $\supset$ | $\neg\Diamond\phi \supset \phi \,\Box\!\!\rightarrow \psi$ |
| `CVAC` | $X \neq \emptyset$ | $\Rightarrow$ | $R(w) \cap X = \emptyset \Rightarrow S(w, X) = \emptyset$ |
| `mvac` | $\Diamond\phi$ | $\supset$ | $\neg\Diamond\phi \supset \phi \,\Box\!\!\rightarrow \psi$ |
| `MVAC` | $R(w) \cap X \neq \emptyset$ | $\Rightarrow$ | $R(w) \cap X = \emptyset \Rightarrow S(w, X) = \emptyset$ |

These three principles give us conditions under which we *cannot* coherently entertain something impossible. According to `vac`, this happens always. According to `cvac`, this happens at least when the impossible thing is conceivable. And according to `mvac`, this happens when the metaphysically impossible thing is possible. We continue to hold firmly to classical logic, so `mvac` is a (vacuous) tautology; we do not discuss it further.

`cvac`, however, continues to play a role in what follows. In particular, it (and, perhaps surprisingly, not `vac`) is our starring villain. We find it very hard to understand a worldview that could endorse `cvac` without endorsing `vac`. Although such a worldview would be consistent, it would bizarrely hold that the only exceptions to `vac` are inconceivable; we could coherently entertain contradictions, but not (for example) that water is distinct from H2O. We see no appeal to such a view, and so think it's reasonable to aim to avoid `cvac`.

This is convenient, because `cvac` is entailed by `collapse` as well as by `vac`. (`collapse` renders `cvac` equivalent to `mvac`, which is a tautology.) So if we make sure to steer clear of `cvac`, we can in one swoop make sure we've avoided both `collapse` and `vac`. This is what we aim to do in what follows.

We close this section by mentioning the principle `nec`, discussed under the name 'NEC' in Williamson (2020) and Berto *et al.* (2017).

| `NEC` | |
|---|---|
| `nec` | $\Box(\phi \supset \psi) \supset \phi \,\Box\!\!\rightarrow \psi$ |
| `NEC` | $S(w, X) \subseteq R(w) \cap X$ |

We don't directly consider `nec` in any detail in this paper. It is equivalent to the conjunction of `id` and `nc`, so does not require separate consideration. In addition, `nec` entails `vac`, because $\neg\Diamond\phi$ entails $\Box(\phi \supset \psi)$. So our desire to avoid vacuism leads us away from `nec` as well.

**§4. Troubles.** Let a theory be *acceptable* iff it does not entail cvac. Our main goal in this paper is to present a range of acceptable theories determined by selections of the above principles. Since acceptable theories do not entail cvac, they also do not entail either vac or collapse, and they are consistent. In this section, we catalog a few combinations of principles that entail cvac. As such, any theory containing these combinations cannot be acceptable. We use these results in later sections to set the boundaries of our inquiry.

FACT 4. *ee is inconsistent with ecp.*

*Proof.* Consider the instances of each with $\phi = \bot$. Applying modus ponens to these instances gives $\blacklozenge\bot$, which is a contradiction. □

FACT 5. *cid and ccp together entail collapse.*

*Proof.* Supposing $\blacklozenge\phi$ holds at a world $w$, by cid we can conclude $\phi \boxright \phi$. Drawing on both these formulas, by ccp we can conclude $\Diamond\phi$. So with cid and ccp both in place, we can conclude $\blacklozenge\phi \supset \Diamond\phi$; this is collapse. □

FACT 6. *cid and cnc together entail cvac.*

*Proof.* Suppose every instance of cid and cnc both hold at a world $w$, and consider any $\phi$ with $\blacklozenge\phi$ and $\neg\Diamond\phi$ holding at $w$ as well, aiming to show that $\phi \boxright \psi$ holds at $w$, for any $\psi$.

By cnc, $\Box\neg\phi \supset (\phi \boxright \neg\phi)$. Since $\Box\neg\phi$, this gives $\phi \boxright \neg\phi$. That is, $S(w, [\![\phi]\!]) \subseteq [\![\neg\phi]\!]$. By cid, $\phi \boxright \phi$ holds as well. That is, $S(w, [\![\phi]\!]) \subseteq [\![\phi]\!]$. So $S(w, [\![\phi]\!]) \subseteq [\![\neg\phi]\!] \cap [\![\phi]\!]$. But all our worlds are classical, so $[\![\neg\phi]\!] \cap [\![\phi]\!] = \emptyset$. Thus, $S(w, [\![\phi]\!]) \subseteq \emptyset \subseteq [\![\psi]\!]$, for any $\psi$, which ensures that $\phi \boxright \psi$ holds at $w$. □

Those three problems alone are enough to structure the remainder of our discussion. In what follows, we outline four sets of principles and show that they are the maximally acceptable sets: the sets such that they are acceptable, and adding any further principle from our catalog to one of these sets would result in a set that is unacceptable.

**§5. Maximally acceptable theories.** In this section, we locate the four *maximally acceptable theories*. These are maximally acceptable in the following sense: they are acceptable (that is, they do not entail cvac), and no other principles from our initial list can be added to any of them without violating acceptability.

Consider the theories determined by the following four collections of principles:[12]

| Maximally acceptable theories | | | | | | |
|---|---|---|---|---|---|---|
| Successful-Anti-Vacuism | cid | mnc | ee | mcp | mp | |
| Necessary-Anti-Vacuism | mid | nc | (ee) | cp | mp | |
| Necessary-Quasi-Vacuism | mid | nc | (cee) | ccp | mp | ecp |
| Successful-Quasi-Vacuism | id | mnc | cee | mcp | mp | (ecp) |

[12] That is, each of these theories should be understood as the set of all sentences $\phi$ such that $\phi$ is valid on all models $M$ such that the principles listed in the collection in question are all valid on $M$.

(The principles listed in parentheses do not need to be listed separately to specify these collections, since they follow from the other given principles. We list them anyhow for clarity.) In this section, we show that these four are *maximal acceptable sets* of principles, and that they are the only such. As an exercise in logical cartography, this provides the maximal logical landscape for classical metaphysical enquiry. That is, every theory in between the minimal logic and one of those four theories is an acceptable theory.

It has been suggested to us that a theory that only validates mee (and not cee or ee) might be more plausible upon metaphysical consideration. We concur. Our mission is not to identify the *best* theories in this landscape, but rather to indicate the *limits* of acceptability. Choosing one or another acceptable theory would have to be done on metaphysical, not logical grounds. That's not something we will attempt here. However, in the next section, we do discuss the kind of metaphysical interpretations the maximal theories might sustain.

THEOREM 1. *Successful-Anti-Vacuism, Necessary-Anti-Vacuism, Necessary-Quasi-Vacuism, and Successful-Quasi-Vacuism are maximal consistent acceptable sets of principles.*

*Proof.* To show this, we construct models that jointly satisfy all principles of a collection without also satisfying cvac (witnessing acceptability). We then argue that none of the other principles from our list could be added to this collection on pain of unacceptability.

In each case, the models share a universe $W = \{x, y\}$ and metaphysical selection function $R$, according to which $R(w) = \{w\}$ for each $w \in W$. They differ from each other in their counterfactual selection functions.

- First, Successful-Anti-Vacuism. Consider the frame $F_1 = \langle W, R, S_1\rangle$, where $S_1(w, X) = \begin{cases} \{w\} & \text{If } w \in X \\ W \setminus \{w\} & \text{otherwise} \end{cases}$
  $F_1$ validates cid because it meets the condition CID: whenever $X \neq \emptyset$, we have $S_1(w, X) \subseteq X$ for all $w$. $F_1$ validates mnc because it meets the condition MNC: whenever $X \cap R(w) \neq \emptyset$, $X = \{w\}$ or $X = W$, and in either case we have $S_1(w, X) \subseteq R(w)$. Furthermore, in these cases $R(w) \cap S_1(w, X) = \{w\} \neq \emptyset$, so $F_1$ meets the condition MCP, and so validates mcp. $S_1(w, X)$ is never empty, so $F_1$ meets the condition EE and thus validates ee. Finally, $F_1$ meets MP: whenever $w \in X$, then $w \in S_1(w, X)$; so it validates mp. Therefore, $F_1$ is a Successful-Anti-Vacuism frame; every instance of every principle in Successful-Anti-Vacuism is valid on $F_1$.

  But cvac is not valid on $F_1$. To see this, consider a model $M_1$ based on $F_1$ with $V(p) = \{y\}$, and $V(q) = \emptyset$. Then $y \in [\![p]\!]$, so $x \in [\![\blacklozenge p]\!]$. Since $y \notin R(x)$, $x \notin [\![\Diamond p]\!]$. However, $S_1(x, \{p\}) = \{y\} \not\subseteq [\![q]\!]$, so $x \notin [\![p \mathrel{\Box\!\!\rightarrow} q]\!] = \emptyset$. Therefore, $F_1$ doesn't validate cvac. Since it does validate all the principles of Successful-Anti-Vacuism, those principles don't entail cvac, and thus Successful-Anti-Vacuism is acceptable.

  All that remains to show is that Successful-Anti-Vacuism is maximal: that we cannot add any more principles from our list without violating acceptability. The missing principles are cnc, ccp, and ecp.[13] We can't add cnc, since by

[13] nc and cp are also missing, but as we can't add the weaker cnc or ccp they're addressed.

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fact 6 `cid` with `cnc` entails `cvac`. We can't add `ccp`, since by fact 5 `cid` with `ccp` entails `collapse`. And we can't add `ecp`, since by fact 4 `ee` with `ecp` is inconsistent.

- Second, Necessary-Anti-Vacuism. Consider the frame $F_2 = \langle W, R, S_2 \rangle$, where $S_2(w, X) = \{w\}$ for each $w \in W$. $F_2$ validates `mid` because it meets the condition `MID`: whenever $R(w) \cap X \neq \emptyset$ this means that $w \in X$ and thus, as $S_2(w, X) = \{w\}$,
that $S_2(w, X) \subseteq X$. $F_2$ validates `nc` because it meets the condition `NC`: $\{w\} = S_2(w, X) \subseteq R(w) = \{w\}$. $F_2$ meets condition `CP`, as $R(w) \cap S_2(w, X) = \{w\} \neq \emptyset$, so it validates `cp`. Finally, $F_2$ is also a frame for `mp`, since always $w \in S(w, X)$, so the condition `MP` is always met. Therefore, $F_2$ is a Necessary-Anti-Vacuism frame; every instance of every principle in Necessary-Anti-Vacuism is valid on $F_2$.

  But `cvac` is not valid on $F_2$. To see this, consider a model $M_2$ based on $F_2$ with $V(p) = \{y\}$ and $V(q) = \emptyset$. This is a model in which $x \in [\![\blacklozenge p]\!]$ and $x \notin [\![\Diamond p]\!]$ (and thus $x \in [\![\neg\Diamond p]\!]$), while $x \notin [\![q]\!]$, and thus as $S_2(x, [\![p]\!]) = \{x\} \not\subseteq [\![q]\!] = \emptyset$, we have $x \notin [\![p \mathrel{\Box\!\!\rightarrow} q]\!]$, so `cvac` is not valid.

  To confirm that Necessary-Anti-Vacuism is maximal, we need to check the principles `cid` and `ecp`. We can't add `cid`, since by fact 5 `cid` with `ccp` entails `collapse`; but we already have `cp` here, which suffices for `ccp`. And we can't add `ecp`, since by fact 4 `ee` with `ecp` is inconsistent.

- Third, Necessary-Quasi-Vacuism. Consider the frame $F_3 = \langle W, R, S_3 \rangle$, where
$$S_3(w, X) = \begin{cases} \emptyset & \text{If } X = \emptyset \\ \{w\} & \text{otherwise} \end{cases}$$
$F_3$ validates `mid` because it meets the condition `MID`: whenever $R(w) \cap X \neq \emptyset$ this means that $w \in X$, and thus that $S_3(w, X) = \{w\} \subseteq X$. $F_3$ validates `nc` because it meets the condition `NC`: $S_3(w, X) \subseteq \{w\} = R(w)$ for each $w$. $F_3$ validates `ccp` because it meets the condition `CCP`: if $X \neq \emptyset$ then $S_3(w, X) = \{w\} = R(w)$, and thus $S_3(w, X) \cap R(w) = \{w\} \neq \emptyset$. $F_3$ validates `mp` because it meets the condition `MP`, since $w \in S_3(w, X)$ whenever $w \in X$. Finally, $F_3$ validates `ecp` because it meets the condition `ECP`: $S_3(w, \emptyset) = \emptyset$. Therefore, $F_3$ is a Necessary-Quasi-Vacuism frame; every instance of every principle in Necessary-Quasi-Vacuism is valid on $F_3$.

  But `cvac` is not valid on $F_3$. To see this, consider a model $M_3$ based on $F_3$ with $V(p) = \{y\}$ and $V(q) = \emptyset$. This is a model in which $x \in [\![\blacklozenge p]\!]$ and $x \notin [\![\Diamond p]\!]$ (and thus $x \in [\![\neg\Diamond p]\!]$), while as $S_3(x, [\![p]\!]) = \{x\} \not\subseteq [\![q]\!] = \emptyset$, we have $x \notin [\![p \mathrel{\Box\!\!\rightarrow} q]\!]$, so `cvac` is not valid.

  To confirm that Necessary-Quasi-Vacuism is maximal, we need to check the principles `cid`, `ee`, and `cp`. We can't add `cid`, since by fact 5 `cid` with `ccp` entails `collapse`. We can't add `ee`, since by fact 4 `ee` with `ecp` is inconsistent. And we can't add `cp`, since by fact 2 `cp` entails `ee`, and we've just seen we can't add `ee`.

- Fourth, Successful-Quasi-Vacuism. Consider the frame $F_4 = \langle W, R, S_4 \rangle$, where
$$S_4(w, X) = \begin{cases} \{w\} & \text{If } X = W \\ X & \text{otherwise} \end{cases}$$
$F_4$ validates `id` because it meets the condition `ID`: in all cases $S_4(w, X) \subseteq X$. $F_4$ validates `mnc` because it meets the condition `MNC`: in any case where we have $R(w) \cap X \neq \emptyset$ we have $w \subseteq X$ and thus $S_4(w, X) = \{w\} \subseteq R(w)$. $F_4$ validates `cee` because it meets the condition `CEE`: if $X \neq \emptyset$ then $S_4(w, X) \neq \emptyset$ for each

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$w \in W$. $F_4$ validates mcp because it meets the condition MCP: when $R(w) \cap X \neq \emptyset$, which is to say $w \in X$, we have $R(w) \cap S_4(w, X) = \{w\} \neq \emptyset$. Finally, $F_4$ validates mp because it meets the condition MP: the only way to have $w \notin S(w, X)$ is by having $w \notin X$. Therefore, $F_4$ is a Successful-Quasi-Vacuism frame; every instance of every principle in Successful-Quasi-Vacuism is valid on $F_4$.

But cvac is not valid on $F_4$. To see this, consider a model $M_4$ built on $F_4$ with $V(p) = \{y\}$ and $V(q) = \emptyset$. This is a model in which $x \in [\![\blacklozenge p]\!]$ and $x \notin [\![\Diamond p]\!]$ (and thus $x \in [\![\neg\Diamond p]\!]$), while as $S_4(x, [\![p]\!]) = \{y\} \not\subseteq [\![q]\!] = \emptyset$, we have $x \notin [\![p \mathrel{\Box\!\!\rightarrow} q]\!]$, so cvac is not valid.

To confirm that Successful-Quasi-Vacuism is maximal, we need to check the principles cnc, ee, and ccp. We can't add cnc, since by fact 6 cid with cnc entails cvac; but we already have id here, which suffices for cid. We can't add ee, since by fact 1 id with ee is inconsistent. And we can't add ccp, since by fact 5 cid with ccp entails collapse; but we already have id here, which suffices for cid. □

At this point we have demonstrated that at least the above four selections are maximal acceptable selections of principles from our set. In fact, they are the only four such selections.

THEOREM 2. *Every acceptable theory given by a selection of our principles is contained in at least one of Successful-Anti-Vacuism, Necessary-Anti-Vacuism, Necessary-Quasi-Vacuism, or Successful-Quasi-Vacuism.*

*Proof.* Suppose otherwise: that there is an acceptable theory given by a selection of our principles contained in none of these theories. Call this new acceptable theory $X$.

Since $X$ is not contained in any of our four sets, for each of these sets $X$ must include at least one principle not in that set.[14] Since it is not contained in Necessary-Anti-Vacuism, for example, it must contain either cid or ecp. And since it is not contained in Necessary-Quasi-Vacuism, it must contain at least one of cid, ee, or cp.

Now, suppose towards a contradiction that $X$ does not contain cid. Then, by the above disjunctions, $X$ must contain ecp, and it must contain either ee or cp. Since cp entails ee (fact 2), $X$ must contain both ecp and ee. But then it is inconsistent (fact 4). So $X$ must contain cid.

Since $X$ is not contained in Successful-Anti-Vacuism, it must contain at least one of id, cnc, ccp, or ecp. But since it contains cid, it cannot contain ccp (fact 5) or cnc (fact 6). Thus, $X$ contains either id or ecp; but as id entails ecp (fact 3), $X$ contains ecp.

Finally, since $X$ is not contained in Successful-Quasi-Vacuism, it must contain at least one of cnc, ee, or ccp. We have already seen that, due to its containing cid, $X$ cannot contain either cnc or ccp (facts 5 and 6). So it must contain ee. But then $X$ is contradictory, since it also contains ecp (fact 4). So we have a contradiction: there is no such $X$. □

**§6. Worldviews.** Though acceptable in our technical sense, the four theories of the previous section depict different worldviews that might not all be equally plausible from a metaphysical point of view. What they all do, however, is avoid both vacuism

[14] And these cannot be principles like vac or collapse, since then $X$ would not be acceptable.

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and collapse. As nonvacuist theories of classical counterpossibles, they each deserve to be discussed in their own right. In this section, we give an impression of the kind of worldview each theory yields.

In what follows, we consider a range of examples. For these purposes, we take "The liar sentence is both true and not true," an explicit contradiction, as our example of a inconceivable sentence, and take both "13 has 2 and 6 as factors" and "Water is an element" as our examples of conceivable but metaphysically impossible sentences. Our point, though, is not about the modal status of these particular sentences; we're just using them to illustrate the commitments of these various views. If you disagree with us about these examples, feel free to substitute your own.

A useful grouping of the views is with respect to the ee family of principles. What can be coherently entertained? If a view contains ee, it holds that anything can be coherently entertained; call such a view anti-vacuist. We have two anti-vacuist views to consider: Successful-Anti-Vacuism and Necessary-Anti-Vacuism. Our other two views, Necessary-Quasi-Vacuism and Successful-Quasi-Vacuism, allow that some things cannot be coherently entertained, but still insist that everything conceivable can be coherently entertained. Call these views quasi-vacuist.

### *6.1. Anti-vacuist views.*

| Successful-Anti-Vacuism | | Necessary-Anti-Vacuism | |
|---|---|---|---|
| cid | $\blacklozenge\phi \supset \phi \mathrel{\Box\!\!\to} \phi$ | mid | $\Diamond\phi \supset \phi \mathrel{\Box\!\!\to} \phi$ |
| mnc | $\Diamond\phi \supset \Box\psi \supset \phi \mathrel{\Box\!\!\to} \psi$ | nc | $\Box\psi \supset \phi \mathrel{\Box\!\!\to} \psi$ |
| ee | $\neg(\phi \mathrel{\Box\!\!\to} \bot)$ | ee | $\neg(\phi \mathrel{\Box\!\!\to} \bot)$ |
| mcp | $\Diamond\phi \supset \phi \mathrel{\Box\!\!\to} \psi \supset \Diamond\psi$ | cp | $\phi \mathrel{\Box\!\!\to} \psi \supset \Diamond\psi$ |
| mp | $\phi \mathrel{\Box\!\!\to} \psi \supset (\phi \supset \psi)$ | mp | $\phi \mathrel{\Box\!\!\to} \psi \supset (\phi \supset \psi)$ |

A decision between the Successful-Anti-Vacuism and Necessary-Anti-Vacuism is one that trades between commitments between the id and nc families, i.e., between successful entertainments and entertainment limited by necessities. On Successful-Anti-Vacuism, everything conceivable can be successfully entertained, and the worlds resulting from entertainments are not limited by what is necessary. On Necessary-Anti-Vacuism, by contrast, all worlds selected from entertainments are bound to be not just coherent, but even possible. As a result, fewer things can be successfully entertained. Because everything can be coherently entertained, $\blacklozenge\phi$ is not definable as $\neg(\phi \mathrel{\Box\!\!\to} \bot)$.

What do anti-vacuist views have to say about counterpossibles? Consider the sentence "If the liar were both true and not true, then some sentence would be both true and not true." On either anti-vacuist view, this counterfactual must be false: because of ee, the scenario resulting from entertaining the antecedent cannot be absurd, but the consequent is absurd. When a counterfactual has an inconceivable consequent it is false, on either anti-vacuist view, no matter its antecedent.

One view, Necessary-Anti-Vacuism, is even more restrictive: it engages in this same phenomenon with sentences that are even just impossible, owing to the combination of ee with nc. On Necessary-Anti-Vacuism, then, the sentence "If 13 had 2 and 6 as factors, then 13 would have 2 and 6 as factors" must fail. By ee, the worlds that result from entertaining 13 having 2 and 6 as factors must not be absurd, but by nc it must be worlds in which all necessary truths hold. In particular, then, it must be one in which 13 does *not* have 2 and 6 as factors. (And since it is not absurd, it cannot also be one in which 13 *does* have 2 and 6 as factors, so the entertainment cannot be successful.) So on Necessary-Anti-Vacuism, when a counterfactual has an impossible consequent it is false, no matter its antecedent.

The other view, Successful-Anti-Vacuism, allows for true counterfactuals with impossible consequents, so long as those consequents are conceivable, and so long as the antecedent is impossible. So the sentence "If some compounds were elements, water would be one of them" can be either true or false, as far as the principles of Successful-Anti-Vacuism are concerned, despite its necessarily false consequent. According to Successful-Anti-Vacuism, we can successfully entertain anything conceivable; it does not need to be in addition possible.

Because of mcp and mnc the Successful-Anti-Vacuism worldview reasons cautiously with possible entertainments. Because of mcp possible worlds result whenever an entertained proposition is possible. The strangeness of impossibility condition is secured by mnc so any possible entertainment yields exclusively possible worlds. When guarded by possible antecedents, couterfactual reasoning proceeds in the good old ways.

Both anti-vacuist views, then, end up in the position of calling a good number of counterpossibles false; they are at no risk of falling into vacuism. Indeed, it is even possible to add collapse to each of these and still avoid vacuism.[15] The challenge for an advocate of an anti-vacuist view is to make sure enough counterfactuals can still be counted true.

### 6.2. *Quasi-vacuist views.*

| Necessary-Quasi-Vacuism | | Successful-Quasi-Vacuism | |
|---|---|---|---|
| mid | $\Diamond\phi \supset \phi \boxright \phi$ | id | $\phi \boxright \phi$ |
| nc | $\Box\psi \supset \phi \boxright \psi$ | mnc | $\Diamond\phi \supset \Box\psi \supset \phi \boxright \psi$ |
| cee | $\blacklozenge\phi \supset \neg(\phi \boxright \bot)$ | cee | $\blacklozenge\phi \supset \neg(\phi \boxright \bot)$ |
| ccp | $\blacklozenge\phi \supset (\phi \boxright \psi \supset \Diamond\psi)$ | mcp | $\Diamond\phi \supset \phi \boxright \psi \supset \Diamond\psi$ |
| mp | $\phi \boxright \psi \supset (\phi \supset \psi)$ | mp | $\phi \boxright \psi \supset (\phi \supset \psi)$ |
| ecp | $\neg(\phi \boxright \bot) \supset \blacklozenge\phi$ | ecp | $\neg(\phi \boxright \bot) \supset \blacklozenge\phi$ |

Quasi-vacuist views both include ecp. Contraposed, ecp tells us that when something is not conceivable, the scenario that results from entertaining it is absurd. They both also include cee, which strengthens this to a biconditional: the only way an absurd scenario can result from an entertainment is when the thing entertained is not conceivable. This means that, for the quasi-vacuist views, conceivability is the same thing as coherence, and we can if we like define $\blacklozenge$ from $\boxright$.

On these views, then, sentences like "If the liar sentence were both true and not true, then the moon would be made of green cheese" come out vacuously true; the antecedent here is not even conceivable, and so the scenario that results from entertaining it is absurd. In some sense, then, these quasi-vacuist views are 'vacuist-like': counterfactuals with inconceivable antecedents come out vacuously true. For this reason, collapse cannot be added to either quasi-vacuist view without entailing vac. These views, unlike the anti-vacuist views, depend on the failure of collapse for their nonvacuism.

15 Although of course not cvac.

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Neither view, however, is committed to vacuism in the sense we've given here (or, as we've seen, even to cvac); we can still coherently entertain metaphysical impossibilities. These views, then, can still allow for counterfactuals like “If 13 had 2 and 6 as factors, then cicadas would still have 13-year life cycles” or “If water were an element, it wouldn't be on the periodic table” to be false.[16]

Where they differ from each other is in what results from entertaining something that is conceivable but impossible. On Necessary-Quasi-Vacuism, the resulting worlds must contain all necessities (by nc) and be possible (by ccp). Although these impossible antecedents can be coherently entertained on Necessary-Quasi-Vacuism, the resulting worlds are always bound by possibility and necessity. As such, these entertainments are never successful.

On Successful-Quasi-Vacuism, by contrast, we have (for the first time!) full id: *every* entertainment is successful, no matter what it is an entertainment of. As such, entertaining an impossible antecedent results in a scenario not bound by possibility: it must be one in which that impossible antecedent obtains. However, the presence of mnc and mcp ensures that entertaining a *possible* antecedent still does not go beyond the possible.[17]

**§7. Conclusion.** So we have four classical theories of counterpossibles, two of which forbid vacuously true counterfactuals (Anti-Vacuism), the other two accepting vacuous truth for inconceivable things (Quasi-Vacuism). The various theories trade between assumptions on what can be entertained, what can be entertained successfully and how much metaphysical modal import obtains. The four theories are acceptable because they do not entail cvac, and they are maximal, because extending them with any other principles entails cvac, making them unacceptable. We proved further that they are the only four theories that are maximally acceptable in this sense. We have offered an impression of the worldviews that each theory yields.

All theories are developed in entirely classical logic, with every world closed under logical consequence, and no appeal to a separate class of impossible worlds. Yet we can formalise reasoning about counterfactuals that go beyond the realm of possibility, such as countermathematicals. We hope this might help reconcile those with stronger classical inclinations to the possibility of nonvacuous reasoning with counterpossibles.

We haven't considered how the various principles interact with typical restrictions on modalities and counterfactuals, such as reflexivity and transitivity of the accessibility relation R, or various monotonicity principles for S. For example, Necessary-Anti-Vacuism entails the D axiom $\Box\phi \supset \Diamond\phi$, because it contains both nc and cp. We leave this kind of technical excursion, along with various axiomatisations and completeness results, for the future.

To keep the paper focused on nonvacuist logical theories, we have not engaged in much metaphysical investigation here. We take ourselves to be acting as logicians for

[16] For discussion of similar sentences to these, see Baron, Colyvan, & Ripley (2017) and Mares (1997).

[17] This view is similar to the nonvacuist view put forward in Berto *et al.* (2017), which, after imposing the frame condition MNC (under the name SIC), claims it to be ‘easily checked’ that this condition suffices for (an analogue of) mcp to be validated. It does not. The needed extra assumption, not explicitly provided for there, is MEE. (In the notation of that paper, that $x \Vdash A$ for some $x \in P$ implies that for all $w$ there is some $w'$ with $wR_Aw'$.)

hire, and we offer this piece as a service paper for logically inclined metaphysicians whom we hope will be motivated to take up that investigation. Perhaps we'll be lucky enough to have the opportunity to work with you on such a project.

A natural follow-up to this paper would add predicates and identity. The idea would be to allow for the nonvacuous analysis of *counteridenticals*, such as "If Hesperus wasn't Phosphorus, the Greeks would have been right about them" or "If Hesperus wasn't Phosphorus, one of the planets wouldn't be Venus." The idea is in principle simple: preserve the metaphysical necessity of identicals, but allow conceptual variation, just as we saw with mathematics in general. The devil is in the details, however, and we leave this project for future research.

Another natural follow-up, one that drives a lot of interest in the study of counterpossibles, is that of counterlogicals. For instance, "if paraconsistent logic were correct the rule of explosion wouldn't be valid" is a true counterlogical. However, "if paraconsistent logic were correct, some contradiction would be true" is a false counterlogical, because not all paraconsistent logics are dialetheist. One way to think of counterlogicals is at the top level: redo the exercise of this paper, but with your favourite paraconsistent (or any nonclassical) logic instead of classical logic. You still wouldn't get counterlogicals in your object language, but at least, from a meta-theoretical stance, you could see the difference between a classical and a paraconsistent universe of worlds. A more concrete way to approach counterlogicals is to adapt our models with multiverses as their domain, and add a counterfactual operator $\circ\!\!\rightarrow$ that shifts between universes to our language. This multiverse counterfactual (or counterlogical, as we would call it) would allow to select worlds from different universes. In the Lewisian parlance, a counterlogical operator picks out the *most similar worlds* from different universes (those with a different logic) that make the antecedent true. For example, "if the rule of explosion is invalid, then there are true contradictions" could be seen to be false by having a paraconsistent universes with nondialetheic worlds. The thought is that worlds without contradictions from a paraconsistent universe are more similar to the actual classical world. We think that this vague but basic idea is a promising logical and philosophical research project.

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