Advancing the Physics of Cosmic Distances
Proceedings IAU Symposium No. 289, 2012
Richard de Grijs, ed.

doi:10.1017/S1743921312021382

The Baade–Becker–Wesselink technique and the fundamental astrophysical parameters of Cepheids

Alexey S. Rastorguev, Andrey K. Dambis, Marina V. Zabolotskikh, Leonid N. Berdnikov, and Natalia A. Gorynya

Lomonosov Moscow State University, Sternberg Astronomical Institute,
13 Universitetskii prospect, Moscow, 119992 Russia
email: rastor@sai.msu.ru

Abstract. The Baade–Becker–Wesselink (BBW) method remains one of most often used tools to derive a full set of Cepheid astrophysical parameters. The surface brightness version of the BBW technique was preferentially used during the past few decades to calculate Cepheid radii and to improve period–luminosity–colour relations. Its implementation requires *a priori* knowledge of Cepheid reddening values. We propose a new version of the BBW technique, which allows one to independently determine the colour excess and the intrinsic colour of a radially pulsating star, in addition to its radius, luminosity and distance. The new technique is a generalization of the Balona light curve-modelling approach. The method also allows calibration of the function $F(\mathrm{CI}_0) = \mathrm{BC}(\mathrm{CI}_0) + 10\log[T_{\mathrm{eff}}(\mathrm{CI}_0)]$ for the class of pulsating stars considered. We apply this technique to a number of classical Cepheids with very accurate light and radial-velocity curves. The new technique can also be applied to other pulsating variables, e.g., RR Lyrae stars. We also discuss the possible dependence of the projection factor on the pulsation phase.

Keywords. stars: fundamental parameters, Cepheids, distance scale

1. Introduction

Classical Cepheids are key standard candles. They are used to set the zero point of the extragalactic distance scale (Freedman *et al.* 2001) and also serve as important tracers of young populations (Binney & Merrifield 1998). They owe their popularity to their high luminosities and photometric variability (which make them easy to identify and observe even at large distances) and the fact that the luminosities, intrinsic colours and ages of these stars are closely related to such an easy-to-determine quantity as the period of variability.

It would be best to calibrate the Cepheid period–luminosity (PL), period–colour (PC) and period–luminosity–colour relations using distances based on trigonometric parallaxes. However, the most precisely measured parallaxes of even the nearest Cepeheids are insufficiently accurate and, more importantly, they may be fraught with thus far uncovered systematic errors. Here the Baade–Becker–Wesselink method (BBW; Baade 1926; Becker 1940; Wesselink 1946) comes in handy, because it allows the Cepheid distances (along with the physical parameters of these stars) to be inferred, thereby providing an independent check of results based on geometric methods (e.g., trigonometric and statistical parallaxes). The surface brightness technique was used most frequently and effectively during the past few decades. It is based on the relation between the so-called limb-darkened surface brightness parameter and the normal colours of Cepheids (Barnes & Evans 1976). Moreover, it critically depends on the adopted reddening value.

Cepheid reddening values can be estimated from medium- and broad-band photometric observations (including multicolour PL relations) and from spectroscopic data (Dean *et al.* 1978; Fernie 1987, 1990, 1994; Fernie *et al.* 1995; Berdnikov *et al.* 1996, 2000; Andrievsky *et al.* 2002a,b; Kovtyukh *et al.* 2008; Kim *et al.* 2011). All these methods use proper calibrations and relationships between key stellar parameters. However, there exist large (up to 0.2 mag) scatter in the estimates of the colour excess for individual Cepheids, and the structure of the Cepheid instability strip is still vague. In the review on the Hubble constant and the Cepheid distance scale, Madore & Freedman (1991) noted that "...any attempt to disentangle the effects of differential reddening and true color deviations within the instability strip must rely first on a precise and thoroughly independent determination of the intrinsic structure of the period-luminosity-color relation", and next, "...independent reddenings and distances to individual calibrator Cepheids must be available."

It should also be noted that reliable values of the colour excess and of the total-to-selective extinction ratio, say, $A_V/E(V-I)$, are extremely important when we use Wesenheit functions to derive Cepheid luminosities from precise trigonometric parallaxes (Groenewegen & Oudmaijer 2000; Sandage *et al.* 2006; van Leewen *et al.* 2007). To convert the Wesenheit index W_{VI} to an absolute magnitude, M_V, the intrinsic colour of the Cepheid, $(V-I)_0$, and the proper value of $A_V/E(V-I)$ are needed. We suppose that self-consistent and independent reddening estimates for individual Cepheids can reduce underestimated systematic errors induced by large variations of the absorption law (Fitzpatrick & Massa 2007) and can even result in a different A_λ law in the optical and near-infrared regimes. Therefore, the search for independent estimates of Cepheid reddening values is still actual, and this is our primary aim.

Both BBW techniques—surface brightness (radius-variation modelling) and maximum likelihood (light-curve modelling)—are based on the same astrophysical background but make use of somewhat different calibrations (limb-darkened surface brightness parameter, bolometric correction–effective-temperature pair) of the normal colours. Here we propose a generalization of the Balona (1977) light-curve modelling technique, which allows one to independently determine not only the star's distance and physical parameters, but also the amount of interstellar reddening, and even calibrate the dependence of a linear combination of the bolometric correction and effective temperature on intrinsic colour (Rastorguev & Dambis 2011).

2. Theoretical background

We now briefly outline the method. First, the bolometric luminosity of a star at any time is given by the following relation, which immediately follows from the Stefan–Boltzmann law:

$$L/\mathrm{L}_\odot = (R/\mathrm{R}_\odot)^2 (T/\mathrm{T}_\odot)^4. \tag{2.1}$$

Here L, R and T are the star's current bolometric luminosity, radius and effective temperature, respectively, and the '$\odot$' subscript denotes the corresponding solar values. Given that the bolometric absolute magnitude, M_{bol}, is related to bolometric luminosity as

$$M_{\mathrm{bol}} = M_{\mathrm{bol}\odot} - 2.5 \log(L/\mathrm{L}_\odot),$$

we can simply derive from Eq. (2.1),

$$M_{\mathrm{bol}} - M_{\mathrm{bol}\odot} = -5 \log(R/\mathrm{R}_\odot) - 10 \log(T/\mathrm{T}_\odot). \tag{2.2}$$

https://doi.org/10.1017/S1743921312021382 Published online by Cambridge University Press

Now, $M_{\rm bol}$ can be written in terms of the absolute magnitude M in some photometric band and the corresponding bolometric correction, BC, i.e.

$$M_{\rm bol} = M + {\rm BC},$$

and the absolute magnitude M can be written as

$$M = m - A - 5\log(d/10\ {\rm pc}).$$

Here m, A and d are the star's apparent magnitude and interstellar extinction in the corresponding photometric band, respectively, and d is the heliocentric distance of the star in pc. We can therefore rewrite Eq. (2.2) to

$$m = A + 5\log(d/10\ {\rm pc}) + M_{{\rm bol}\odot} + 10\log({\rm T}_\odot)$$

$$-5\log(R/{\rm R}_\odot) - {\rm BC} - 10\log(T). \tag{2.3}$$

Let us now introduce the function $F({\rm CI}_0) = {\rm BC} + 10\log(T)$, the apparent distance modulus, $(m - M)_{\rm app} = A + 5\log(d/10\ {\rm pc})$, and rewrite Eq. (2.3) as the light-curve model,

$$m = Y - 5\log(R/{\rm R}_\odot) - F, \tag{2.4}$$

where constant

$$Y = (m - M)_{\rm app} + M_{{\rm bol}\odot} + 10\log({\rm T}_\odot).$$

We now recall that interstellar extinction, A, can be determined from the colour excess (CE) as $A = R_\lambda$ CE, where R_λ is the total-to-selective extinction ratio for the passband–colour pair considered, whereas $M_{{\rm bol}\odot}$, ${\rm R}_\odot$ and ${\rm T}_\odot$ are rather precisely known quantities. The quantity $F({\rm CI}_0) = {\rm BC} + 10\log(T)$ is a function of intrinsic colour index ${\rm CI}_0$ = CI − CE. Balona (1977) used a very crude approximation for the effective temperature and bolometric correction, reducing the right-hand side of the light-curve model, Eq. (2.4), to a linear function of the observed colour, with the coefficients containing the colour excess in latent form.

The key point of our approach is that the values of function F are computed from the already available calibrations of the bolometric correction ${\rm BC}({\rm CI}_0)$ and effective temperature $\log T({\rm CI}_0)$ (Flower 1996; Bessell *et al.* 1998; Alonso *et al.* 1999; Sekiguchi & Fukugita 2000; Ramirez & Melendez 2005; Biazzo *et al.* 2007; Gonzalez Hernandez & Bonifacio 2009). These calibrations are expressed as high-order power series of the intrinsic colour:

$$F({\rm CI}_0) = a_0 + \sum_{k=1}^{N} a_k {\rm CI}_0^k, \tag{2.5}$$

with known $\{a_k\}$ and $N = 7$; in some cases, the decomposition also includes the metallicity, [Fe/H], and/or gravity ($\log\ g$) terms.

As for the stellar radius, R, its current value can be determined by integrating the star's radial-velocity curve over time, using ${\rm d}t = (P/2\pi){\rm d}\varphi$:

$$R(t) - R_0 = -{\rm PF}\int_{\varphi_0}^{\varphi} [v_{\rm r}(t) - v_\gamma](P/2\pi){\rm d}\varphi,$$

where R_0 is the radius at phase φ_0 [we use the mean radius, $\langle R\rangle = (R_{\rm min} + R_{\rm max})/2$], v_γ the systemic radial velocity, φ the current phase of the radial-velocity curve, P the star's pulsation period and PF the projection factor, which accounts for the difference between the pulsation and radial velocities. Given the observables (light curve and apparent magnitudes, m, colour curve and apparent colour indices, CI, and radial-velocity curve and v_r) and known quantities for the Sun, we end up with the following unknowns: distance, d, mean radius, $\langle R \rangle$, and colour excess, CE, which can be found simply using a maximum-likelihood technique (nonlinear optimization).

For Cepheids with large amplitudes of their light and colour curves (ΔCI $\geqslant 0.4$ mag), it is also possible to apply a more general technique by setting the expansion coefficients $\{a_k\}$ in Eq. (2.5) free and treating them as unknowns. We expanded the function $F = \mathrm{BC} + 10\log(T)$ in Eq. (2.4) into a power series of the intrinsic colour index $\mathrm{CI}_0^{\mathrm{st}}$ of a well-studied 'standard' star (e.g., α Per or some other bright star) with accurately known T^{st},

$$F = \mathrm{BC}^{\mathrm{st}} + 10\log(T^{\mathrm{st}}) + \sum_{k=1}^{N} a_k(\mathrm{CI} - \mathrm{CE} - \mathrm{CI}_0^{\mathrm{st}})^k. \tag{2.6}$$

The best fit to the light curve is provided by the optimal expansion order $N \simeq 5$–9. We use this modification to calculate the physical parameters and reddening CE of the Cepheids, as well as the calibration $F(\mathrm{CI}_0) = \mathrm{BC}(\mathrm{CI}_0) + 10\log[T_{\mathrm{eff}}(\mathrm{CI}_0)]$ for a star of given metallicity [Fe/H] and log g (Rastorguev & Dambis 2011).

3. Observational data, constants and best calibrations

Cepheid photometric data were obtained by L. Berdnikov; his extensive multicolour photo-electric and CCD photometry of classical Cepheids is described in Berdnikov (1995). Very accurate radial-velocity measurements of 165 northern Cepheids were measured with the Moscow CORAVEL spectrometer (Tokovinin 1987) during the period 1987–2011 (Gorynya *et al.* 1992, 1996, 1998). The total number of individual measurements is approximately 11,000 (the latest data are currently being prepared for publication). The photometric and spectral data sets are nearly synchronous, to prevent any systematic errors in the computed radii (up to 30%) and other parameters owing to evolutionary period changes resulting in phase shifts between light, colour and radial-velocity variations. We adopt $\mathrm{T}_\odot = 5777$ K and $M_{\mathrm{bol}\odot} = +4.76$ mag (Gray 2005). We proceeded from $(V, B-V)$ data and found as the best solutions for the V-band light curve and $(B-V)$ colour curve those computed using the $F[(B-V)_0]$ function based on two calibrations (Flower 1996; Bessell *et al.* 1998) of similar slope; the poorer results obtained using the other cited calibrations can be explained by the fact that the latter involved an insufficient number of supergiant stars.

4. The projection factor

There is as yet no consensus as to which projection factor (PF) should be used for Cepheid variables (Nardetto *et al.* 2004, 2007, 2009; Groenewegen 2007). Different approaches (constant or period-dependent PF values) lead to systematic differences in the inferred Cepheid parameters, first and foremost in the radii.

Bearing in mind the specific features of CORAVEL measurements, Rastorguev (2010) introduced a phase-dependent PF. Its value is calculated from flux integration across the stellar limb and depends on the limb-darkening coefficient, ϵ (linear darkening law: $D(\varphi) = 1 - \epsilon + \epsilon\ \cos\varphi$; φ is the angle between the line-of-sight direction and the surface-element normal vector), and the photospheric velocity, $\mathrm{d}r/\mathrm{d}t$. The true line profile will be broadened by any spectral instrument, and usually we approximate it by a Gaussian curve to measure the radial velocity as the coordinate of the maximum (see Fig. 1). In this case, the measured radial velocity will additionally depend on the instrument's

https://doi.org/10.1017/S1743921312021382 Published online by Cambridge University Press

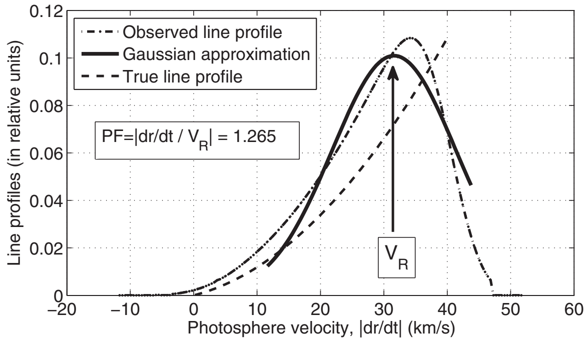

Figure 1. Example: true line profile ($\epsilon = 0.75, \mathrm{d}r/\mathrm{d}t = 40$ km s^{-1}), observed line profile and its Gaussian approximation (with a spectrograph instrumental linewidth of $S_0 = 4$ km s^{-1}). The measured v_r value is indicated by the arrow; the PF value is shown.

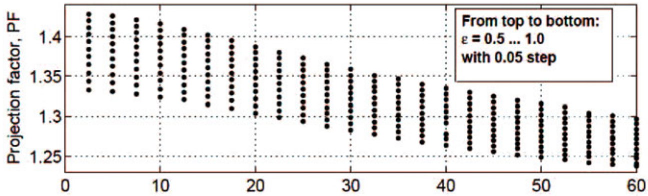

Figure 2. Projection factor, PF, as a function of $|\mathrm{d}r/\mathrm{d}t|$ for an instrumental width of $S_0 = 6$ km s^{-1} and for different values of the limb-darkening coefficient $0.5 < \epsilon < 1$.

spectral linewidth, S_0 (so the PF value should be 'adjusted' to the spectrograph used for radial-velocity measurements), and on the photospheric velocity, $|\mathrm{d}r/\mathrm{d}t|$.

PF values were estimated for $2.5 < |\mathrm{d}r/\mathrm{d}t| < 60$ km s^{-1}, $0.5 < \epsilon < 1$ and $4 < S_0 < 8$ km s^{-1}. We see that the maximum of the normal approximation is shifted relative to the tip of the line profile. This shift depends on $|\mathrm{d}r/\mathrm{d}t|$ and S_0. Fig. 2 shows that the calculated PF values may differ considerably from the 'standard' and widely used value PF = 1.31. Variations in PF values have also been reported by, e.g., Nardetto *et al.* (2004). We provide a useful analytical approximation for the projection factor as a three-parameter exponential expression, based on 10,000 numerical experiments (Rastorguev 2010),

$$\mathrm{PF} \approx a_1 \exp[-(\mathrm{d}r/\mathrm{d}t)^2/(2a_2^2)] + a_3,$$

where a_1, a_2 and a_3 are functions of ϵ and S_0,

$$a_1 \approx -0.068\epsilon - 0.0078S_0 + 0.217;$$
$$a_2 \approx +1.69\epsilon + 2.477S_0 + 9.833;$$
$$a_3 \approx -0.121\epsilon + 0.009S_0 + 1.297.$$

Overall, the rms residual for this analytical expression is approximately 0.003. In practice, the PF value for a given radial velocity should be determined by iteration for known values of S_0. To compare our results for Cepheids with other calculations, we finally adopted a moderate dependence of PF on the period advocated by Nardetto *et al.* (2007), although we repeated all calculations with other variants of the PF's dependence on the period and pulsation phase to ensure the stability of the calculated colour excess.

5. Results and discussion

To test the new method, we used the maximum-likelihood technique to solve Eq. (2.4) for the V-band light curve and $(B-V)$ colour curve for several classical Cepheids residing

https://doi.org/10.1017/S1743921312021382 Published online by Cambridge University Press

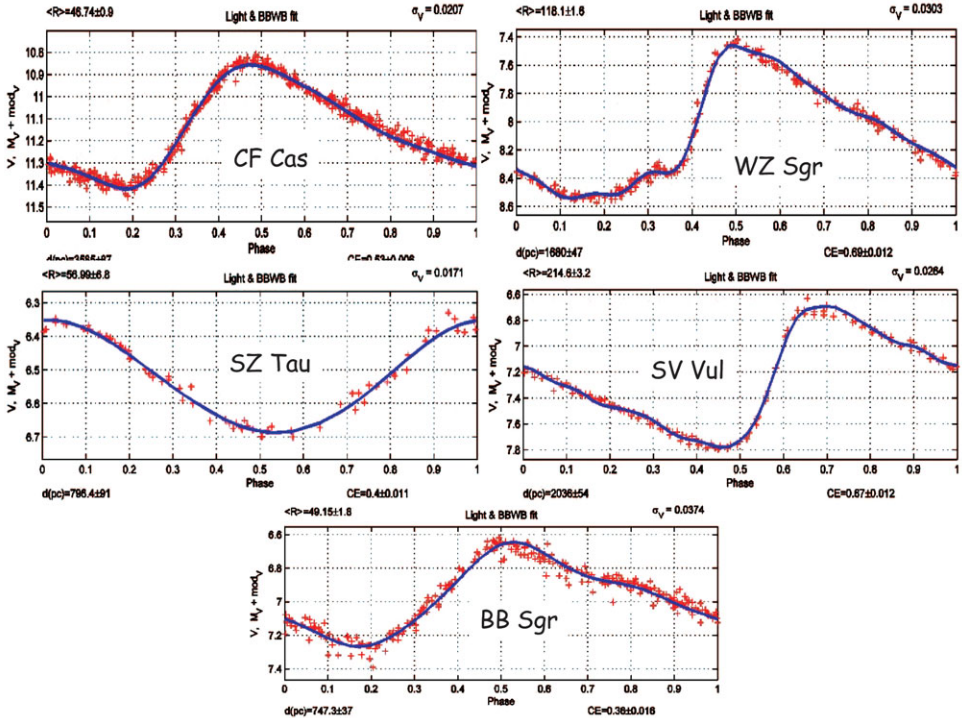

Figure 3. Examples of the light-curve modelling. Crosses: observations; solid line: model. Fit quality: rms ~ 0.02–0.04 mag.

in young open clusters, including SZ Tau, CF Cas, U Sgr, DL Cas and GY Sge, and found good agreement for the calculated reddening values with those determined for the host clusters (Rastorguev & Dambis 2011). A weak sensitivity of the calculated reddening, $E(B-V)$, on the adopted PF value (constant or period/phase-dependent) is explained by the very strong dependence of the light curve's amplitude on effective temperature, $\sim 10\log(T)$, and—as a consequence—on the dereddened colour. Although the internal errors of the reddening $E(B-V)$ seem very small, the values determined using the two best calibrations (Flower 1996; Bessell *et al.* 1998), may differ by as much as 0.03–0.05 mag because of the systematic shift between these two calibrations. Fig. 3 shows the final fit to the V-band light curves for some of the Cepheids in our sample.

The PL relation and the instability strip for our Cepheid sample are shown in Fig. 4. Note that the inferred radii and luminosities of a large fraction of the Cepheids with $P < 4$ days are too large for fundamental-tone pulsation; in most cases, this may be indirectly evidenced by their low colour amplitudes.

To refine the calibration of $F(\mathrm{CI}_0)$, Eq. (2.6), we tried to use α Per as the 'standard' star, with $T^{\mathrm{st}} \approx (6240 \pm 20)$ K, [Fe/H] $\approx -0.28 \pm 0.06$ dex (Lee *et al.* 2006), $(B-V)^{\mathrm{st}} \approx$ 0.48 mag and $E(B-V) \approx 0.09$ mag (WEBDA, for the α Per cluster). To take into account the effects of metallicity on the zero point of $F(\mathrm{CI}_0)^{\mathrm{st}}$, we estimated the gradient $\mathrm{d}F(\mathrm{CI}_0)^{\mathrm{st}}/\mathrm{d}[\mathrm{Fe/H}] \approx +0.24$ from the calibrations of Alonso et al. (1999), Sekiguchi & Fukugita (2000) and Gonzalez Hernandez & Bonifacio (2009). In some cases (particularly for large-amplitude colour variations) the 'free' calibration, i.e. Eq. (2.6), can markedly improve the model fit to the observed light curve of the Cepheid variable. Fig. 5 shows an example of the calibrations of the F functions derived from nine Cepheids with different

https://doi.org/10.1017/S1743921312021382 Published online by Cambridge University Press

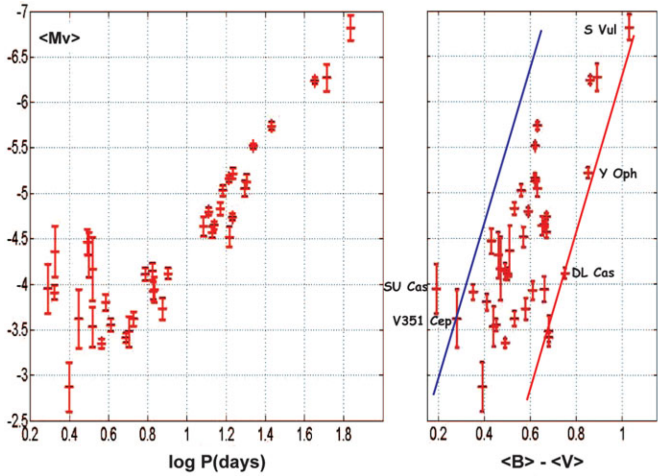

Figure 4. PL relation (left) and the instability strip (right) for approximately 40 Galactic Cepheids. Absolute magnitudes corresponding to the mean intensities as well as colours are shown. Most short-period Cepheids seem to be overtone pulsators. Cepheids residing near the edges of the instability strip and labelled by their names all have very small amplitudes of their light and colour curves.

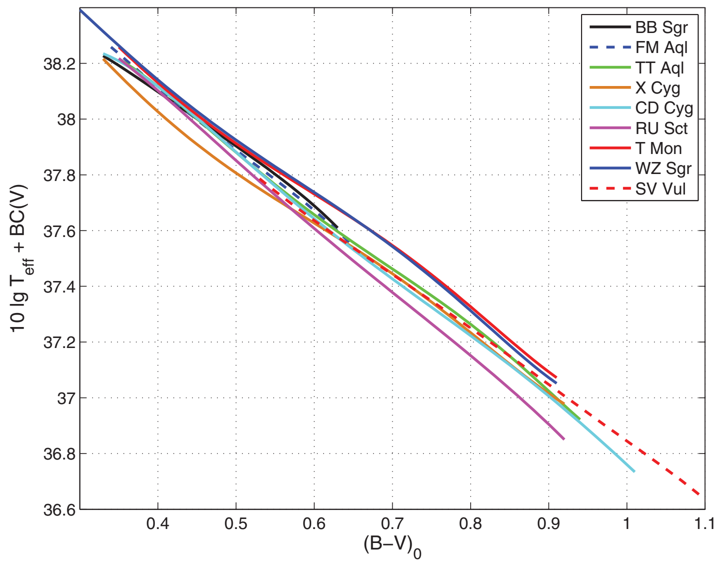

Figure 5. Inferred calibrations of the $F((B-V)_0)$ function, Eq. (2.6), for nine Cepheids with large amplitudes of their colour variation.

[Fe/H] and log g values. The temperature scatter at $T_{\rm eff} \sim 5100$– 6600 K amounts to 3–5%.

When applied to an extensive sample of Cepheid variables with homogeneous photometric data and detailed radial-velocity curves, the new method is expected to lead to a completely independent scale of Cepheid reddening values and to refine the PL relation.

https://doi.org/10.1017/S1743921312021382 Published online by Cambridge University Press

References

Alonso, A., Arribas, S., & Martínez-Roger, C. 1999, *A&AS*, 140, 261
Andrievsky, S. M., Kovtyukh, V. V., Luck, R. E., *et al.* 2002 *A&A*, 381, 32
Andrievsky, S. M., Kovtyukh, V. V., Luck, R. E., *et al.* 2002 *A&A*, 392, 491
Baade, W. 1926, *Astron. Nachr.*, 228, 359
Balona, L. A. 1976, *MNRAS*, 178, 231
Barnes, T. G. & Evans, D. S. 1976, *MNRAS*, 174, 489
Becker, W. 1940, *Zeitschr. Astrophys.*, 19, 289
Berdnikov, L. N. 1995, in: *Astrophysical applications of stellar pulsation*, Proc. IAU Colloq. 155 (Stobie, R.S., & Whitelock, P.A., eds.), *Astron. Soc. Pac. Conf. Ser.*, 83, 349
Berdnikov, L. N., Vozyakova, O. V., & Dambis, A. K. 1996, *Ast. Lett.*, 22, 839
Berdnikov, L. N., Dambis, A. K., & Vozyakova, O. V. 2000, *A&AS*, 143, 211
Bessell, M. S., Castelli, F., & Plez, B. 1998, *A&A*, 333, 231
Biazzo, K., Frasca, A., Catalano, S., *et al.* 2007, *Astron. Nachr.*, 328, 938
Binney, J. & Merrifield, M. 1998, *Galactic astronomy*, Princeton, NJ: Princeton Univ. Press
Dean, J. F., Warren, P. R., & Cousins, A. W. 1978, *MNRAS*, 183, 569
Fernie, J. D. 1987, *AJ*, 94, 1003
Fernie, J. D. 1990, *ApJ*, 354, 295
Fernie, J. D. 1994, *ApJ*, 429, 844
Fernie, J. D., Evans, N. R., Beattie, B., & Seager, S. 1995, *IBVS*, 4148, 1
Fitzpatrick, E. L. & Massa, D. 2007, *ApJ*, 663, 320
Flower, P. J. 1996, *ApJ*, 469, 355
Freedman, W. L., Madore, B. F., Gibson, B. K., *et al.* 2001, *ApJ*, 553, 47
Gonzalez Hernandez, J. I. & Bonifacio, P. 2009, *A&A*, 497, 497
Gorynya, N. A., Irsmambetova, T. R., Rastorguev, A. S., & Samus', N. N. 1992, *Sov. Ast. Lett.*, 18, 316
Gorynya, N. A., Samus', N. N., Rastorguev, A. S., & Sachkov, M. E. 1996, *Ast. Lett.*, 22, 175
Gorynya, N. A., Samus', N. N., Sachkov, M. E., Rastorguev, A. S., Glushkova, E. V., & Antipin, S. 1998, *Ast. Lett.*, 24, 815
Gray, C. D. F.. 2005, *The Observation and Analysis of Stellar Photospheres*, Cambridge: Cambridge Univ. Press
Groenewegen, M. A. T., & Oudmaijer, R. 2000, *A&A*, 356, 849
Groenewegen, M. A. T. 2007, *A&A*, 474, 975
Kim, C., Moon, B.-K., & Yushchenko, A. V. 2011, *J. Kor. Astron. Soc.*, 43, 153
Kovtyukh, V. V., Soubiran, C., Luck, R. E., *et al.* 2008, *MNRAS*, 389, 1336
Lee, B.-C., Galazutdinov, G. A., Han, I., *et al.* 2006, *PASP*, 118, 636
Madore, B. F. & Freedman, W. L. 1991, *PASP*, 103, 933
Nardetto, N., Fokin, A., Mourard, D., *et al.* 2004, *A&A*, 428, 131
Nardetto, N., Mourard, D., Mathias, P., *et al.* 2007, *A&A*, 471, 661
Nardetto, N., Gieren, W., Kervella, P., *et al.* 2009, *A&A*, 502,951
Ramirez, I. & Melendez, J. 2005, *ApJ*, 626, 465
Rastorguev, A. S. 2010, in: *Variable Stars, the Galactic halo and Galaxy Formation* (Sterken, C., Samus', N., & Szabados, L., eds.), p. 225; see also Rastorguev, A. S. 2010, arXiv:1001.1648v2
Rastorguev, A. S. & Dambis, A. K. 2011, *Astron. Bull.*, 66, 47
Sandage, A., Tammann, G. A., Saha, A., *et al.* 2006, *ApJ*, 653, 843
Sekiguchi, M. & Fukugita, M. 2000, *AJ*, 120, 1072
Tokovinin, A. A. 1987, *Sov. Astron.*, 31, 98
van Leeuwen, F., Feast, M. W., Whitelock, P. A., *et al.* 2007, *MNRAS*, 379, 723
Wesselink, A. J. 1946, *Bull. Astron. Inst. Neth.*, 10, 91

https://doi.org/10.1017/S1743921312021382 Published online by Cambridge University Press