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THE MANAGEMENT OF RISKS IN BANKING

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ABSTRACT

This paper takes an overview of the various financial risks which need to be managed in banking. It then looks in detail at the specific areas of operational risk, market risk and pricing loans. A cash flow model is then developed, which takes explicit account of the various financial factors which should influence the interest rate charged. The model is applied to price loans with various features. The results of the model are shown, and weaknesses in the model and possible areas for further work indicated.

KEYWORDS

Financial Risk Management; Banking; Actuarial Techniques

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1. INTRODUCTION

1.1 Actuaries work in many financial services industries, managing financial risks using statistical and mathematical techniques. Yet, with a few exceptions (see Allan *et al.*, 1996), actuaries rarely work for banks. Where they do work for banks, it tends to be in the investment functions rather than in the commercial or retail operations. In the Institute of Actuaries' *Members' Handbook* (1996), banks do not appear as a distinct sector of employment. This situation in the United Kingdom differs from the situation in France and Australia (see Griffin, 1996), where there is more involvement by actuaries in the banking sector.

1.2 It is, perhaps, surprising that actuaries do not work for banks in large numbers. Like insurance companies, banks are intermediary financial institutions. Banks have assets and liabilities which need to be managed from the market risk and credit risk points of view. Banks produce products which need pricing. Banks suffer bad debts on their loan portfolios and need to forecast how large these bad debts might be. Most banks take part in investment activity, the risks from which need managing. This paper discusses various risks which need to be managed in a bank, and approaches which are used to manage them. In some cases, particularly in product pricing, the authors believe that techniques which have been used in traditional actuarial fields could have some value in banking.

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Specifically, we propose a product pricing model. In other fields, the techniques used in banking and those used in traditional actuarial fields differ only in the terminology which is used. Nevertheless, the language barrier can be a significant one, and there are other differences between insurance and banking. For example, the nature of the assets and liabilities are quite different, as are the rules by which capital requirements are determined.

1.3 There is a trend for life insurance companies to set up banks and for banks to take over life insurance companies. There is also a trend to greater competition in financial services. As the market for financial services becomes more competitive and more closely integrated, we would expect that actuaries would need to understand the techniques and language used by other financial professionals. Similarly, models developed by actuaries may assist banks in pricing products and in using capital in an efficient way. At a corporate level the actuarial and banking professions will need to understand how the approaches to risk management, pricing and capital allocation in insurance companies and banks relate to each other. There is a growing focus on profitability, and institutions providing a range of financial services need to find a consistent way of measuring profitability.

1.4 The paper is structured as follows. Section 2 looks at the various different risks faced by a bank and summarises some of the management techniques. Most of the issues discussed in Section 2 then have a separate section devoted to them later in the paper. However, the issue of interest rate risk, familiar to actuaries, and which is managed in a familiar way by banks, is not taken any further. Section 3 considers operational risk, which could be regarded as being at the apex of all the other risks. Section 4 analyses market risk and the value-at-risk approach to managing market risk. There is already a considerable amount of literature on this subject, and we do not add anything original to work which has already been published. Sections 5 to 8 look at product pricing and how risk and capital costs can be taken into account when setting interest margins on loans. Specifically: Section 5 examines risk factors in the mortgage market; Section 6 looks at statistical techniques which can be used for analysing risk; Section 7 develops a pricing model; and Section 8 compares mortgage lending with large corporate lending. Throughout the paper we draw analogies between banking risks and other financial risks which are more familiar to actuaries.

1.5 This paper takes an actuarial view of banking risks. It explains risks in a language which is familiar to actuaries. Actuaries who wish to know more about the subject should refer to books such as Cade (1997), which discusses banking, but using terminology which is more common in the banking industry.

2. Introduction to Risks in Banking

2.1 *Risks faced by Banks*

Risk to a banker means the perceived uncertainty connected with some event related to the banking business. For example: will borrowers default on loans; will new business fall; will the price of assets in an investment portfolio fall; will the bank suffer losses from a change in long-term interest rates; is lending profitable? These risks are not intrinsically different from other financial risks with which actuaries are used to dealing. Partly in reaction to their bad debt experience in the early 1990s, banks have, over recent years, focused increased attention on how banking risk can be measured and controlled. Appropriate analytical techniques are necessary to analyse banking risk, and some of these techniques are used by actuaries in other fields. So far, however, the techniques of measurement and control of risk in different financial services industries have grown up separately, and terminology is often quite different. Actuaries have not generally been involved with the management of risk in banking. It is, therefore, necessary to begin by looking at basic terminology and the classification of risks faced by banks.

2.2 *Credit Risk and Pricing*

2.2.1 Capital is required to protect a bank against insolvency in the event of the assets declining in value. The capital will be made up of equity and debt capital. A bank's assets consist mainly of corporate loans, personal loans and mortgages. Associated with these loans is the credit risk that some borrowers will experience financial difficulties and will not be able to repay their loans in full, so causing the bank to experience some bad debt losses. The approach to credit risk that banks follow is to identify two items:

— *the 'expected' loss* — the average loss which may be expected over an economic cycle; this may be regarded as a cost of doing lending business, and must be allowed for in pricing loans; and
— *the 'unexpected' loss* — the volatility of recorded losses around the expected loss ratio; the greater the volatility, the greater the need for capital.

2.2.2 It is of interest to note that the credit risks of banks, on the assets side, share a number of characteristics with the liability risks of non-life insurers. The potential losses are highly correlated with risk category, and tend to be cyclical. There is little interest rate risk, because assets and liabilities tend to be short term or floating rate. Whilst the statistical characteristics of this source of risk may be similar to those of general insurance liabilities, financially these risks are equivalent to the C1 risk experienced by insurance institutions, as classified in the United States literature: this is, essentially, the risk of a fall in asset values caused by default. This includes any decline in the market value of debt due to the perception of increased default risk and any actual decline caused by actual default. The former is only directly quantifiable for marketable securities debt. However, for non-securitised debt, the bank should try to estimate the level of 'bad' debts. If risk is not properly taken into account in pricing, there will also be a mis-pricing risk.

2.2.3 The two risk measures proposed in ¶2.2.1 are used in the statistic

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commonly known as RARORAC (risk-adjusted return on risk-adjusted capital). This ratio measures the 'return on economic capital', where the numerator is the profit less the expected bad debts associated with a type of loan and the denominator is the amount of capital deemed necessary by the volatility of bad debts for this kind of loan. To grant a loan at a particular price and allowing for the perceived credit risk, the bank will wish to achieve a return on economic capital higher than its hurdle rate.

2.2.4 A fundamental problem of estimating credit risk, and therefore the capital required, is that the past is not a very good guide to the future as far as loan default probabilities are concerned (see Section 5). The lender must try to assess the default risk of any borrower (e.g. by using some measure of financial strength and taking a view on economic conditions in the near future). The lender can then choose to decline a request for a loan, or, if the request is acceptable, must decide at what level to set two key parameters: the interest rate charged on the loan; and the amount of capital set aside to back the loan.

2.2.5 The capital allocation policy adopted by banks is based on two considerations. Firstly, there is a general preference amongst shareholders for a stable pattern of returns. Variable losses, which could lead to variable returns, would require greater amounts of economic capital and so would reduce the expected return on capital, unless it were possible to increase interest rates. Secondly, there is a regulatory requirement to hold at least a certain amount of capital in respect of lending; this is to protect the bank against becoming insolvent.

2.2.6 Banks are required, under the Basle Accord, to hold capital of at least 8% of their risk-weighted assets, including at least 4% equity capital; the remainder will be debt capital. Risk-weighted assets include 100% of commercial loans, 50% of mortgages, 20% of interbank loans and 0% of government debt. Certain off-balance-sheet items are also covered by the Basle Accord. In practice, in the U.K., banks have generally held capital of around 10% of their risk-weighted assets, of which 6% has been equity capital. (The amounts do vary with time — both quantities increased through the first half of the 1990s — and between banks, with some banks holding total capital of 14% and equity capital of 9%.) Within each category (e.g. commercial loans or mortgages) the regulatory capital requirement includes no allowance for differences in default risk. This means that loans to large corporations need as much capital backing (per £ of loan) as loans to individuals. This is in contrast to 'risk adjusted', or 'economic', capital, which does take risks into account.

2.2.7 It is a concern among banks that this encourages high-risk lending, as it is inefficient to hold large amounts of capital for low-risk loans. There is also no allowance for portfolios of loans where the risks across the portfolio are not strongly correlated.

2.2.8 In the pricing analysis, in Section 7, we accept the regulatory requirements as given, but it is an important issue, to which actuaries can contribute, to determine the correct level of economic capital for a portfolio of loans. Its determination could use the techniques of dynamic solvency testing and stochastic modelling. An alternative approach to changing the capital held is to accept the regulatory capital and change the mix of business, so that business where regulatory capital is greater than economic capital is balanced by business where economic capital is greater than regulatory capital. However, this might be economically or operationally inefficient. Actuaries can also contribute to estimating bad debts, a problem which must have some similarities with incurred, but not reported (IBNR) claims in non-life insurance. We make no further comment on this issue in this paper. This is not because of its lack of importance, but because it is an area where we have not pursued research.

2.2.9 For corporate loans, prices are set by banks after taking account of the credit quality and other characteristics of individual borrowers. In retail lending, there is less pricing according to individual risks: in general, a uniform rate is applied to all those who pass a credit test, although there may be some variation according to items such as the size of loans or, in the case of mortgages, loan-to-value ratios. Also, in general, new loans are not priced taking into account the expense and capital requirements for new loans, but an overall interest margin is set, which ensures, on a year-on-year basis, that expenses are covered and the required return on capital is made. The traditional approach can lead to cross subsidies, and the major contribution of this paper is to propose an actuarial cash flow approach to pricing in Section 7.

2.3 *Liquidity Risk*

2.3.1 Bankers are also concerned about the danger of not having sufficient cash and borrowing capacity to meet deposit withdrawals, loan demand, and so on. Faced with liquidity risk, a bank may be forced to borrow emergency funds at excessive cost to cover its immediate cash needs, reducing its earnings. Very few banks ever actually run out of cash, because of the ease with which liquid funds can be borrowed from other banks. Something more common is a shortage of liquidity due to unexpected heavy deposit withdrawals, which forces a bank to borrow funds at an elevated interest rate (higher than the interest rates other banks are paying for similar borrowings).

2.3.2 Standard remedies for reducing a bank's exposure to liquidity risk include increasing the proportion of bank funds committed to cash and readily marketable assets, such as government securities, or using longer-term liabilities to fund the bank's operations.

2.3.3 In general, asset/liability risk is monitored by an asset-liability committee of senior executives — the ALCO — which determines the appropriate structure of the balance sheet. However, shorter-term risks, such as liquidity risk, will generally be managed by a treasury committee (see Whitely, 1992). When comparing with traditional actuarial risk, it is important to distinguish between liquidity risk and mis-matching risk. Banks tend to match their assets and liabilities by term, as far as interest rate risk is concerned. In doing this they take account of both the contractual and behavioural terms of

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their assets and liabilities. There is, therefore, limited mis-matching risk. Also, assets and liabilities are both, generally, at floating interest rates (although banks often lend at fixed rates, this will either be matched by borrowing or swaps will be arranged to leave the net cash flow floating rate). Nevertheless, banks do not have an effective mechanism to prevent a reduction in deposits which match their assets, which tend to be loans granted on a medium-term basis. There is, therefore, a liquidity risk.

2.4 *Operational Risk*

2.4.1 Banks also face significant operational risk. Because of its nature, across a wide range of operational activities, it is very difficult to quantify the bank's exposure to this type of risk. There are no standard parameters, such as credit ratings, financial ratios, etc., to define, categorise or measure operational risk. Additionally, operational risk frequently does not receive the same level of management scrutiny as do other types of risk.

2.4.2 A loss due to the realisation of an operational risk factor can be concealed from senior management. Because many of these risks are subjective, there is a great deal of judgement required to balance the implicit cost of the risk to the explicit cost of the controls. The most important criteria necessary to control operational risk, as stated in *Risk* (June 1995), are:

— an investment in qualified personnel;
— integrative systems appropriate to the size, scope and complexity of a company's activities;
— market and credit risk management functions independently organised from trading functions;
— documented policies and procedures detailing activities, limits, credit controls and reporting requirements;
— internal audits of activities ensuring that the policies, procedures and limits established are being followed; and
— an overview of the company's business and operations being undertaken by knowledgeable and involved senior management.

2.4.3 These operational risks tend to exist in all businesses. However, if there is a failure on the operational front, this can lead to the build up of the other risks (for example, the Barings problems ultimately related to market risk, but it may have been possible to prevent the Barings failure through better operational control). Operational risk may be more acute in financial institutions than in non-financial businesses, and this could be one factor which leads to opportunities for actuaries, who are normally well regarded for their skills in managing financial risk, in the management of banks. Operational risk will be discussed further in Section 3.

2.5 *Interest Rate Risk*

2.5.1 There is no significant difference between interest rate risk, as faced by

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banks, and that faced by other institutions. There will, therefore, not be a separate section relating to interest rate risk later in the paper. Some aspects of interest rate risk will be discussed in Section 4.

2.5.2 Movements in market interest rates can have serious effects on a bank's profit if the structure of the institution's assets and liabilities is such that interest expenses on borrowed money increase more rapidly than interest revenues on loans and investments. The most important measure of bank interest rate risk exposure is the ratio of interest-sensitive assets to interest-sensitive liabilities. When interest-sensitive assets exceed interest-sensitive liabilities in a particular maturity range, a bank is vulnerable to losses from falling interest rates. In contrast, when interest-sensitive liabilities exceed interest-sensitive assets, losses are likely to be incurred if interest rates rise. Whilst this ratio may be widely used, (see Rose, 1993), it does not, of course, indicate the extent of the risk.

2.5.3 In fund management, as would be expected, duration is used to measure interest rate risk. In an actuarial context, Redington (1952) introduced immunisation as a means for life insurance companies to mitigate the effects of interest rate changes on their portfolios. The technique is now used generally to manage fund portfolio interest rate risk. The essence of Redington's strategy is to set the average duration of the assets and the liabilities equal. By matching durations on both sides of the balance sheet, he showed that assets and liabilities would be equally price sensitive to changes in the general level of interest rates. For any change in yield, both sides of the balance sheet should be equally affected, and, therefore, the relative values of assets and liabilities are not changed. One way to achieve immunisation is by increasing or decreasing the duration of a portfolio using interest rate futures. The duration of a portfolio can be matched to the duration of a known, or expected, liability stream, and some insulation from interest rate movements may be achieved. However, duration does not involve cash flow matching, and some interest rate risk will continue to exist, for example as a result of convexity and from non-parallel yield curve movements. This risk is quantified for specific examples in Stanghellini (1996).

2.5.4 Interest rate risk is frequently referred to in the actuarial literature as C3 risk, and comprises the *mismatch risk* and the *disintermediation risk*. The mismatch risk arises when there is a mismatch in timing between asset and liability cash flow streams. The liability cash flow is the net expected cash flow for benefits, expenses and premiums, and the asset cash flow is the net expected cash inflow for the assets supporting the business. For equity assets, for example, the cash flow stream may reflect both expected dividends while the asset is held and expected proceeds on future sale. The consequences of mismatch risk are either asset reinvestment, or disinvestment, transactions in an uncertain future interest rate environment. When asset cash flow exceeds liability cash flow, the net amount must be reinvested, and when asset cash flow is less than liability cash flow, there is a need to sell assets to meet current cash requirements.

2.5.5 The disintermediation risk arises when options are available, to either asset or liability clients, that expose the financial intermediary, such as a bank or a life insurance company, to financial anti-selection. In the case of a bank, when interest rates fall, borrowers may have the option and financial incentive to prepay their loans and to refinance them at more favourable rates. Similarly, liability clients may have the right to make additional voluntary deposits at a guaranteed interest rate. In an insurance company, when interest rates rise, policyholders will have a financial incentive to surrender policies or to borrow on their security if guaranteed terms exist. Asset and liability clients, therefore, can contribute to the financial disadvantage of the financial institution by the exercise of their options. In both cases this risk can be controlled by adjusting the terms of the contract (preventing early repayment without penalty, for example) or by holding extra reserves. We will not discuss interest rate risk further. Its management and control is familiar to actuaries. There are situations in the management of bank activities where techniques with which actuaries are familiar can be used in the management of interest rate risk in portfolios of fixed-rate mortgages. Two examples are: the option given to a borrower who 'books' a fixed-rate mortgage ahead of completion; the booking fee is charged to protect against the borrower taking a loan out with another lender before completion if interest rates fall. Secondly, there is a redemption penalty charged if a re-mortgage is taken out during the fixed-rate period.

2.6 *Market Risk*

2.6.1 *Introduction to market risk*

2.6.1.1 In recent years banks have become increasingly involved in the trading of securities and derivatives. This expansion has taken place either through internal growth or by acquisition. These trading activities have given rise, primarily, to position, or market, risk. That is the risk that a change in the prices of the securities or derivatives in which a bank has a position will cause a loss. Trading-book exposures are taken with a view to resale or short-term profit, rather than holding securities until maturity, and, therefore, the assets are treated as short term and valued on a mark-to-market basis, i.e. at the current price at which they could be sold in the market. Market risk is not different, in principle, from interest rate or mis-matching risk. However, it arises from a completely different part of the banks' business, and is worth considering separately.

2.6.1.2 Market risk reflects uncertainty as to the asset's price when it is sold before its maturity date. The uncertainty exists because the asset's sale price depends on four basic components of market exposure, described by Walmsley (1992):

(1) the level of interest rates on the investor's horizon date (horizon is the point at which an asset will be liquidated or turned into cash; even a speculator has a horizon, which may be as short as one day);
(2) the shape of the yield curve;
(3) volatility exposure, for options; and
(4) basis risk between different markets.

2.6.1.3 The Bank of England, in its risk assessment of London institutions, includes a separate category of market risk: forced-sale risk. This aims to assess the extra risk arising from being a forced seller when seeking to close a position, and will depend on the degree of market liquidity.

2.6.1.4 Banks face the risk of changes in interest rates. If interest rates increase the market value of its assets will fall, and the bank, faced with the need to sell some assets, will take losses. For debt instruments, the longer the issue's term to maturity relative to the horizon date (and the lower its coupon), the greater the interest rate volatility, and hence the greater the risk. For equity investments or foreign currency investments, there is a risk of a fall in the capital value of equities or a change in value of the relevant foreign currency. To some extent, the capital value of equities will be bond-yield sensitive.

2.6.1.5 The derivatives markets pose their own special set of risks. Some of these are traditional actuarial risks transferred to another context: for example, the leverage effect of futures. The exposure is not merely the margin, but the amount of underlying assets controlled through the futures market. Measurement of this risk is similar to that for the risk on the underlying instrument on which the futures contract is based. Similarly, the risks in the interest rate and currency swap markets are generally similar to those in the underlying interest rate and foreign exchange markets.

2.6.1.6 Risks in the options market are more complex. One risk is the difficulty of valuing the contracts properly. In general, this is fairly limited for exchange-traded options, since the current price in the market can be generally taken as a good guide. For over-the-counter options, it may be very difficult to obtain a reliable market price, and, even if the options are correctly valued, the complexity of these markets means that the number of institutions involved is small and the forced-sale risk is much greater. An additional problem with options is that they give rise to non-linear pay-offs. Standard risk management models which assume a normal distribution, therefore, cannot always be applied.

2.6.2 *Measurement of market risk*

2.6.2.1 Measurement of market risk varies from institution to institution. Traditionally banks have broken down risk into predefined categories and have allocated risk limits to each category. A well-established method of looking at market risks in the banking industry is the asset/liability approach, which consists of projecting future estimated earnings under assumed market scenarios. Earnings are defined as earnings reported in the financial statements under generally accepted accounting principles. Unfortunately, changes in values only manifest themselves very slowly in the financial statements of a bank. In recent years, banks have begun to use an alternative methodology known as the value at risk (VAR) approach; see Section 4 for a more detailed discussion.

2.6.2.2 The aim of the VAR model is to calculate, on a consistent basis, the likely loss that a bank might experience on its whole trading book, allowing for the hedges that exist between (as well as within) different markets. VAR models assess

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likely price changes of instruments within individual markets and the extent to which prices in one market vary with those in others. More specifically, they will calculate the loss which should occur with a particular pre-determined probability.

2.6.2.3 There are two main VAR approaches: variance/covariance; and simulation. Under the variance/covariance approach, a bank uses statistics on the magnitude of past price movements and correlations between price movements to estimate potential losses in its portfolio of trading positions. Under the simulation approach, a bank bases its expectation of future potential losses on calculations of the losses that would have been sustained on that book in the past. Banks can use either approach to allocate the capital between their various operations. They can also use them to see how particular exposures change their value at risk. The variance/covariance approach has some similarities with basic modern portfolio theory. The simulation approach is similar to stochastic investment modelling, commonly undertaken by actuaries.

2.6.2.4 VAR models are only part of the risk measurement process. The other main part consists of 'stress tests', used to look at the effects of extreme market movements on a trading book. Stress tests calculate the possible extent of losses under extreme assumptions (rather than the likely loss). For example, the bank could test for a shift both ways of 1% in interest rates along the yield curve in all bond markets and a 20% fall in equity markets. This whole approach is rather like resilience testing or dynamic solvency testing, described in Booth (1997a). Again, we can draw parallels with traditional actuarial fields. The regulation of market risk tends to take a more arbitrary approach.

2.6.3 *Regulation of market risk*

2.6.3.1 The European Union approved a Capital Adequacy Directive (CAD) (EEC 93/6) which came into force in January 1996. The directive requires firms to put up capital against the following risk exposures:

— position risk, known as market risk (i.e. risk arising from positions in equities, debt securities and derivatives);
— settlement/delivery risks;
— counterparty risk;
— foreign exchange risks on the institution's overall business; and
— large exposures in activities primarily subject to market and credit risks.

2.6.3.2 Although the Directive covers all these risks, the main aim is to supervise market risk which includes both the positions taken by a firm in financial instruments in which it deals and the exposures of the firm relating to the provision of financial services to its customers.

2.6.3.3 The CAD applies to investment firms and credit institutions. The CAD defines an investment firm as an entity whose regular business is the provision of investment services to third parties on a professional basis (including securities, traders, brokers, investment fund managers and other financial services). Investment services generally comprise the following services:

— reception, transmission and execution of orders;
— managing portfolios of investments; and
— giving investment advice concerning transferable securities (e.g. shares, bonds), money market instruments and derivative investments.

Credit institutions are defined as undertakings whose business is the taking of deposits from the public and the granting of credits for its own account (generally banks). The CAD has brought banks and investment firms within the same regulation, because banks are engaging much more in trading activities, and thus have exposure to risks, besides credit risk.

2.6.3.4 The capital requirements for the trading book broadly cover three categories of risk:

(1) market risk;
(2) settlement/delivery and counterparty risk; and
(3) large exposure risk.

The most important of these, in practice, and the most complicated to calculate, is market risk. The CAD provides separate rules for each of the following categories:

— equities;
— traded debt instruments;
— derivatives (such as futures contracts, swaps, options, warrants, etc.); and
— foreign exchange risk.

2.6.3.5 The market risk on these categories is divided into two components (the specific risk and general risk components). The capital required is often calculated from an approach called the building block approach. The specific risk attaching to an instrument refers to the risk of a price change due to factors relating to the circumstances particular to an issuer, for example deterioration in its financial position or significant management changes. The general risk, however, refers to factors that affect the market as a whole, for instance changes in economic or monetary policy. This is similar to the modern portfolio theory interpretation of the terms.

2.6.4 *Calculation of specific risk for equities and bonds*

2.6.4.1 Capital requirements for specific risk on equity positions are calculated by applying the following steps:

(1) Calculate the net position for each equity issue. The net position is the excess of the long position over the short position (or vice versa).
(2) The sum of the net positions in all equities is referred to as the 'overall gross position'.
(3) Multiply the 'overall gross position' by 4% to give the capital requirement against specific risk.

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This capital requirement of 4% may be reduced to 2% for diversified portfolios, at the discretion of the regulatory authorities in any Member State.

2.6.4.2 To calculate the specific risk component for traded debt instruments, the bank must firstly establish its net position in each instrument, and multiply the net position by a risk weighting to find the 'weighted position'. The sum of all the weighted positions gives the appropriate capital requirement. Central government debt items have a zero risk weighting. Other qualifying bonds with maturity up to 6 months have 0.25% risk weighting; if their maturity is between 6 and 24 months this risk weighting is 1%; if it is over 24 months the risk weighting is 1.6%. Other items have a risk weighting of 8%. Qualifying items have to fall within various categories laid out in the CAD; one of the qualifications is that they are listed on a regulated E.U. market. The intention of this aspect of the CAD is to allow different risk weightings for different default probabilities.

2.6.5 *Calculation of general risk for equities and bonds*

2.6.5.1 The capital requirement in respect of general risk on a bank's equity portfolio is calculated as follows:

(1) The bank's net position in each equity is calculated in broadly the same manner as for specific risk (although exchange traded futures are treated differently).
(2) The total of the bank's long position is offset against the total of its short position, in order to arrive at its 'overall net position'.
(3) The overall net position is multiplied by 8% to give the capital requirement.

The overall net position is taken (instead of the gross position, as in the calculation of specific risk), because general risk represents the response to purely market factors. Market movements will have equal, and opposite, effects on short positions, as compared with long positions. Specific risks will not 'cancel out' in this way.

2.6.5.2 The approach, however, could be regarded as too general, in that it does not consider the beta values of equities. The beta value indicates the degree of responsiveness of shares' values relative to market movements. By netting long and short positions without considering the different sensitivity of different shares to markets movements, capital requirements can potentially be underestimated or overestimated. This problem can only really be overcome by a 'value at risk' approach (see ¶2.6.8 and Section 4).

2.6.5.3 In order to calculate the capital requirements for general risk to the debt instruments positions, the bonds are classified according to their residual maturities and the size of the coupon. A risk weighting is applied to each position. The detailed operation of the general risk calculation for bonds is as follows:

(1) Securities are categorised according to their residual maturity into one of three zones. Zone 1 covers securities with a residual maturity up to one year, zone 2 covers securities of maturity between one and four years, and zone 3 covers maturities over four years.

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(2) Each zone is sub-divided into a number of maturity bands.
(3) Once the bank's positions in securities have been assigned to the appropriate maturity bands, the next step is to apply the appropriate risk weighting to each band.

2.6.5.4 The risk weighting is greater the longer the residual maturity of the security, and is based on an approximation of the duration of each bond class. There is an assumed interest rate change which represents assumed interest rate volatility for the different time horizons. Short-term interest rate volatilities are realistically assumed to be higher than long-term interest rate volatilities, but they are based on historical levels, and cannot allow for subjective assessments of future changes in volatility levels.

2.6.5.5 Basically, the CAD methodology for general risk for debt instruments consists of multiplying the estimate of duration for each bond by the assumed yield change and obtaining price volatility as the measure of market risk. This approach is, perhaps, somewhat less sophisticated than approaches which could be taken, given today's computing power.

2.6.5.6 The directive draws a distinction between bonds carrying a coupon of less than 3% and those with a coupon of 3% or more. The former tend to have slightly higher risk weightings, the difference being most pronounced for the longer-dated securities. For example, the maximum risk weighting for a bond with a coupon of 3% or more is 5%, and this weighting applies when the bond has a residual maturity of more than 20 years. By contrast, a bond of similar maturity with a coupon of less than 3% carries a risk weighting of 12.5%. This is consistent with the fact that bonds with lower coupon rates have greater duration.

2.6.5.7 From an actuarial perspective, there would seem to be some inadequacies in the directive. However, it should be borne in mind that regulation should not be relied upon to replace management controls, which should be more sophisticated. In some respects, the CAD follows the arbitrary approach of the solvency requirements for life offices (see Abbott, 1987). It uses the concept of duration to measure the extent of the risk, although in an imprecise way. Unlike resilience testing or dynamic solvency testing in insurance, it does not attempt to test the solvency position against particular interest rate changes, although it could broadly be said that the bank would be protected from the interest rate changes implicitly assumed by the directive. As a management tool, stochastic simulations or dynamic solvency testing could be carried out on the bond position to ascertain the extent of market position risk. The values used in these tests could be based on empirical evidence of potential yield curve shifts or pivots.

2.6.6 *Derivative instruments*

Derivative instruments, such as interest rate and exchange rate contracts, bought and sold over-the-counter equity options and covered warrants, give rise to market risk exposures, and are, therefore, subject to capital requirements. To

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incorporate these instruments into the 'building block' approach, they are broken down into long and short positions in the respective underlying debt or equity instruments. Hence, for example, an interest rate swap, where the institution receives at fixed interest rate and pays floating, would be split into a long position in a fixed rate debt instrument and a short position in a floating rate instrument, maturing at the next interest rate reset date. These long and short positions are then treated as notional holdings in bonds, and attract a nil specific risk weighting. For the general risk requirement, these positions are to be slotted into their respective maturity or duration time zones. Thus, derivatives are treated in a fashion consistent with the underlying instruments. Kemp (1997) covers the treatment of derivatives in more detail.

2.6.7 *Foreign exchange risk*

2.6.7.1 Fluctuations in exchange rates have an impact on the bank's financial position. The CAD requires that, once the bank's overall net foreign exchange position exceeds 2% of its own funds, capital must be provided to cover the excess. The bank's overall net foreign exchange position is calculated as the higher of: (a) the total net short positions; and (b) the total net long positions, in all currencies other than the bank's reporting currency, converted at the spot rate into the reporting currency.

2.6.7.2 Considerable discretion is given to each Member State's regulators to set alternative procedures to those specified above. Lower capital requirements may be set in respect of positions in currencies which are 'closely correlated' — namely currencies in respect of which historical exchange rate data indicate a very low likelihood of significant rate variation. The capital requirements are:

- for matched positions in closely correlated currencies, 4% of their value; and
- for unmatched positions in closely correlated currencies and for all positions in other currencies, 8% of the higher of: (a) the total net short positions; and (b) the total net long positions, in all currencies.

2.6.8 *The Basle proposals on market risk*

2.6.8.1 In April 1993, the Basle Committee on Banking Supervision published a consultative document with proposals for the measurement of banks' market risk exposures. Like the CAD, the general approach proposed for traded securities is based on the 'building block methodology', whereby the capital requirement calculated for each position is the sum of two components: a specific risk requirement; and a general risk requirement.

2.6.8.2 The Basle Committee is now considering allowing banks to use their in-house VAR models. The main advantage of such an approach would be that it would not generate excessive capital requirements for a widely diversified book in the way that the 'building block approach' does. One problem, however, is that, even where banks' VAR models are built along similar lines, they use different parameters: some may cover price changes over monthly periods, others daily; some may include a 95% confidence interval and others 99%. To reduce the differences between models, the Basle Committee is proposing to fix a number of the parameters to govern the way in which models are specified, for example:

— the use of price changes over a two-week period as the basis for the price volatility calculations;
— a minimum sample period of one year for the past data;
— a 99% one-tailed confidence interval; and
— a requirement to take into account, in some way, the non-linear behaviour of option prices.

2.6.8.3 Fixing these parameters, however, would not address the problem that the historical correlations used in VAR models may not hold in extreme periods (see Section 4), although it allows a more empirical approach, albeit a more subjective one. Within a risk category, the Basle Committee is proposing to allow banks to use the correlations within and between markets that they consider appropriate; but no hedging or diversification allowances will be permitted between different risk groups (for example, between equity risk against bond risk). This would be compatible with a view that, in times of extreme market turbulence, markets can all 'collapse' together, and historical correlation estimates break down. The outcomes of the VAR model for each risk group will be added together. The Committee is considering requiring banks to multiply the outcome of the VAR model by a factor of three, to reach an appropriate capital requirement for stress periods. This may overstate the amount of capital justifiable on economic grounds, and may reduce the apparent attractions of a VAR model relative to the CAD 'building block' approach.

2.6.8.4 As a further safeguard, the Committee is proposing that banks applying the VAR approach must also use a rigorous stress testing programme covering a range of possibilities which could create extraordinary losses or gains. The stress tests would cover extreme price changes, such as those at the time of the 1987 equity market crash. It also requires banks to report information on the largest losses experienced during the reporting period, which could be compared with the capital requirement produced by the VAR model.

2.6.8.5 Banks and regulators have to bear in mind that the models themselves, however accurately they measure risk, are almost useless without effective systems and controls (see Section 3).

2.6.9 *Comparison of the building-block and the VAR approaches*

2.6.9.1 The building block approach to capital requirements, as embodied in the CAD, is based on an essentially arbitrary approach. However, some weight is put on concepts such as duration and empirical movements in market levels. Broadly, it views the risk in each part of the trading book separately, rather than looking at the extent of the overall risks. It assumes that risks in different geographical markets should be simply added together, since banks could face adverse movements in each market simultaneously. This assumption introduces a

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more conservative approach to market risk, since it could give rise to higher than economic levels of capital requirements for some institutions, depending on their risk profiles. Some allowance for diversification is given.

2.6.9.2 In contrast, the VAR methodology, which will be discussed in detail in Section 4, allows us to consider past correlations between movements in different markets, and uses these to estimate the extent of the overall risks faced. Some VAR models also take account of correlations, not only between geographical markets in the same risk class, but also between risk classes (bonds, equities and foreign exchange). It is argued that this approach permits a more efficient use of capital, as it prevents an overstatement of the risk to which banks are actually exposed.

2.6.9.3 Another difference between the VAR model and the building block approach is that there are not separate requirements to reflect markets' differing volatilities in the CAD. VAR models take the price data, and, therefore automatically, take the different volatilities of individual markets into account. Similarly, they are likely to measure spread and basis risk in particular markets more accurately than the building block approach. The VAR approach may be difficult to apply in a regulatory framework, due to its subjective nature. However, it should be used as one of a range of management tools, and its appraisal, in that context, is the subject of Section 4.

2.7 *Summary*

Thus, in this section, we have considered a number of different types of risk in the banking sector. These include credit risk, liquidity risk, operational risk, interest rate risk and market risk. The details of these risks and their methods of control are different from the risks controlled, and the methods used by actuaries in non-bank fields. There is also a significant difference in terminology. However, there should not be any insurmountable barriers between actuaries and the banking world. In Sections 3, 4 and 5, respectively, we look in more detail at operational risk, market risk and credit risk and pricing.

3. Operational Risk

3.1 Operational risk in banks is not easily defined. It covers a wide range of risks that do not fall under the headings: credit risk, market risk and liquidity risk. In their *Generally Accepted Risk Principles*, Coopers & Lybrand (1996) set out a useful categorisation under two distinct headings: operational risk and business/event risk. This categorisation is shown in Table 3.1.

3.2 Operational risk can be very substantial. The most dramatic examples in recent years have arisen through unauthorised trading — not only at Barings, but also at NatWest, Sumitomo Bank and Daiwa Bank. As a result of this, banks have focused on the needs to control limits and to separate front and back offices. Another type of problem, which is not known to have occurred, but which could

Table 3.1. Operational risk and business/event risk

Operational risk	Transaction risk	Execution error Product complexity Booking error Settlement error Commodity deliver risk Documentation/contract risk
	Operational control risk	Exceeding limits Rogue trading Fraud Money laundering Security risk Key personnel risk Processing risk
	Systems risk	Programming error Model/methodology error Market-to-market (MTM) error Management information IT systems failure Telecommunications failure Contingency planning
Business event risk	Currency convertibility risk Shift in credit rating Reputation risk Taxation risk Legal risk	
	Disaster risk	Natural disasters War Collapse/suspension of markets
	Regulatory risk	Breaching capital requirements Regulatory changes

Source: Coopers & Lybrand (1996)

be equally serious, could follow prolonged computer failure. Large numbers of uncompleted payments would accumulate, and it would become very difficult to clear the backlog. For this reason banks have access to alternative computer facilities — either of their own, or at some contingency site. Even where individual losses arising from processing errors are small, the large scale of bank operations can lead to significant losses, compounded by the costs of rectifying errors. Substantial operational risk can exist, even in areas where there is no credit or market risk, such as in the administration of fund management activities.

3.3 Operational risk can be hard to measure. There is, at present, no generally accepted 'best practice'. There are many different types of operational risk. It is relatively straightforward to measure the frequency and size of small errors in processing activities. Operational risk is particularly hard to measure when the

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probability is very small, but the extent of potential loss is very large. Banks may feel that the most extreme risks associated with rogue trading or prolonged computer failure are so unlikely that they are not measurable. However, it may be that insurance experience in measuring other extreme risks, such as earthquakes, could be of value in the analysis of this category of operational risk. In practice, while banks may use very complex models to quantify credit and market risk, detailed models for operational risk are used by only a small percentage of banks. A pragmatic approach, followed by some banks, has been to estimate operational risk as a percentage of costs — perhaps 25% to 50%. As banks focus on the extent of operational risk, however low its probability, they are collecting data for more detailed models.

3.4 Operational risk may be hard to insure. There appears to have been an increasing number of incidents of operational risk. Products, computer systems and international payments are increasingly complex. Exclusion clauses, however, limit insurance cover. Given the difficulty in quantifying operational risks and the difficulty of obtaining insurance, it is likely to be cheaper for banks to self-insure than to insure externally. Even if insurance is not feasible, consideration of its costs can act as a powerful discipline on management, making them more aware of the extent of operational risks.

3.5 Given the potential scale of operational risk, banks should maintain adequate capital to cover the risk. Quantification of the appropriate amount of capital is difficult. However, in an environment where loan losses are limited by both strong internal controls and benign economic developments, the regulatory capital which banks are required to maintain for credit and market risks is likely to be greater than their computed economic capital requirements. In practice, banks are likely to measure this balance, and consider whether it is sufficient to cover operational risks. The balance may also be compared with the chosen percentage of costs which is thought to represent the long-term expected cost of operational risk. Banks recognise the limitations of the current approach. Many banks are attempting to build up databases of past frequencies and losses, and to use these data to compute the amount of capital required to give protection against operational risk — rather than judge the adequacy of the balance between regulatory and economic capital. In the past, many banks had an additional cushion in that they maintained capital ratios comfortably in excess of the regulatory minima. Now there is shareholder pressure on banks to maximise their return on equity, and, if they cannot make profitable acquisitions, to return excess capital through increased dividends or share repurchase schemes. Given that the regulatory capital of banks is greater than their economic capital, this means that they tend to hold less capital than used to be the case. Their cushion against operational risk is smaller.

3.6 In the case of Barings, substantial losses from unauthorised trading caused the bank to become insolvent and to be sold to ING; but even where the amounts of capital are more than adequate to cover these kind of losses, they can damage the reputation of a bank and can even threaten its future. For example, trading losses at NatWest have been one factor leading to speculation as to whether the bank will survive as an independent organisation or merge with another financial group.

3.7 Banks are acutely aware of operational risk, and have the following safeguards to limit its potential impact:

(a) A fundamental control system which comes through the appointment of strong management, who understand the risks and how to limit them.
(b) In general banks have a risk-averse culture, which may be quick to recognise practical difficulties associated with business opportunities.
(c) Banks have invested in detailed plans to ensure business continuity and disaster recovery, in the event of a wide range of difficulties.
(d) Banks have strong group risk and internal audit functions which report directly to the centre and monitor various issues, including operational risk, and require improved procedures and controls, if appropriate.
(e) The internal controls are supported by the need to report to regulators on operational as well as business issues.
(f) An important additional safeguard may come from experienced staff, who recognise problems, or potential problems, and bring them to the attention of their supervisors. There is a danger that, with substantial change in banking and pressure for greater efficiency, some of the traditional values may be breaking down with less reliance on the experience of staff.

3.8 New entrants to the market may have more difficulty controlling operational risk. For example, professional fraudsters may systematically target new entrants in the market, on the appearance of new advertisements for unsecured lending, to see if they can find loopholes in the underwriting process.

4. Market Risk

4.1 *Introduction*

4.1.1 The trading of securities, derivatives and other instruments by banks has exposed them to market risk; i.e. the risk caused by changes in the market value of the portfolio. It is essential that this risk is measured and monitored, and, in recent years, there have been developments in methodology and also in regulation. The E.U. Capital Adequacy Directive, which came into force in 1996, was outlined in Section 2. In 1994, J.P. Morgan launched a tool for measuring market risk, called RiskMetrics, which is now jointly produced and distributed through the World Wide Web by J.P. Morgan and Reuters.

4.1.2 This section reviews the measurement of market risk, and attempts to set out some of the issues involved. Vinas (1997) and the RiskMetrics Technical Document (1996), along with other publications, have been used as the basis for this discussion. The reader could miss out the mathematical aspects in Sections 4.2 and 4.3 without loss of continuity.

4.2 *Value at Risk*

4.2.1 The value at risk (VAR) is a single figure which presents the maximum loss that is likely (with a certain probability) over a given time period. Let $V(t)$ be the value of the portfolio at time t. Then the value at risk at time 0 using horizon T, with probability α, is $VAR(\alpha, T)$, where:

$$P(V(T) < V(0) - VAR(\alpha,T)) = \alpha. \tag{4.1}$$

If we denote the return on the portfolio over $(0, T)$ by $r(T)$ and the value at risk by:

$$VAR(\alpha,T) = V(0)(1 - e^{\hat{r}(\alpha,T)}) \tag{4.2}$$

then equation (4.1) can be rewritten as:

$$P(V(0)e^{r(T)} < V(0)\, e^{\hat{r}(\alpha,T)}) = \alpha \tag{4.3}$$

that is:

$$P(r(T) < \hat{r}(\alpha,T)) = \alpha. \tag{4.4}$$

Thus, the value at risk can be calculated using equation (4.2) and the lower bound on the return derived from equation (4.4), as an alternative to using equation (4.1). This gives the following equation for the value at risk:

$$VAR(\alpha,T) = V(0)(1 - e^{F^{-1}_{r(T)}(\alpha)}) \tag{4.5}$$

where $F_{r(T)}(\cdot)$ is the distribution function of the return $r(T)$. From this, it can be seen that the calculation of the value at risk requires the specification of α and T, the specification and estimation of $F_{r(T)}(\cdot)$, and the calculation of $V(0)$. A usual value of α is 0.05, and the value at risk is calculated on a daily, monthly or other period basis. Section 4.3 discusses the important issue of the choice of distribution for $r(T)$, and Section 4.4 describes how $V(0)$ may be measured.

4.2.2 The calculation of *VAR* provides the bank with an estimate of the loss to which the bank could be exposed with a given probability (normally 5% or 1%). It can then compare that loss with its capital. The *VAR* might be an appropriate figure to calculate if the bank was worried about losing a given amount of capital, but not worried about the extent of losses beyond that point. It may be an appropriate measure in determining capital requirements. However,

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it is not always a good measure of risk for general management purposes. A bank would also need to know about the extent of losses below the VAR point. This could involve using risk measures calculated as lower partial moments (see Booth, 1997b; Clarkson, 1989; and Markowitz, 1952). These look at the probability distribution of returns and calculate a measure such as:

$$\int_{-\infty}^{L} (L-l)^p f(l)dl$$

where l is the level of loss arising, $f(l)$ is its probability density function and L is a 'benchmark' level of loss. The exponent p characterises the particular form of risk which is being measured, for example $p = 2$ produces 'semi-variance'. Downside risk measures are not necessarily helpful where return distributions are symmetric, because they produce results similar to *VAR*, in terms of ordering of portfolios by risk, but may be very appropriate when considering the risk implied by asset portfolios which include derivatives, such as options, with non-linear pay-offs.

4.3 *Distribution of Returns*

4.3.1 *Normal distribution*

4.3.1.1 The obvious choice for the distribution function $F_{r(T)}(\cdot)$, and for the distribution of all returns is the normal distribution. In this case:

$$F^{-1}_{r(T)}(\alpha) = \mu_{r(T)} - z_\alpha \sigma_{r(T)}$$

where z_α is the 100αth percentile of the standard normal distribution. For example, $z_{0.05} = 1.645$ is a common choice. Then the value at risk is given by:

$$VAR(\alpha, T) = V(0)(1 - e^{\mu_{r(T)} - z_\alpha \sigma_{r(T)}})$$

and $(\mu_{r(T)} - z_\alpha \sigma_{r(T)})$ is a 100αth percent lower bound on the overall return on the portfolio over the time interval $(0, T)$.

4.3.1.2 The use of a normal distribution has the advantage that the value at risk depends only on the first two moments of the distribution of $r(T)$. Consider a portfolio, consisting of n assets, for a certain value of T, the length of the time interval considered. For ease of exposition, we will drop the reference to T in the following equations. The expected return, μ_r, can be calculated from:

$$\mu_r = \sum_{i=1}^{n} \pi_i \mu_{r_i}$$

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where μ_{r_i} is the expected return on asset i and π_i is the amount invested in asset i. The variance of the return can be calculated from:

$$\sigma_r^2 = \sum_{i=1}^{n} \pi_i^2 \sigma_i^2 + 2\sum_{i<}\sum_{j} \pi_i \pi_j \rho_{ij} \sigma_i \sigma_j$$

where σ_i^2 is the variance of return on asset i and ρ_{ij} is the correlation between returns on assets i and j.

4.3.1.3 The normal distribution is defined by the first and second moments, but there is considerable flexibility for the way in which these can be modelled. The simplest choice is a constant mean and variance for the return on each asset and constant correlations between the returns on different assets. Other than this, stationary time series models could be applied, along the lines of the stochastic investment models familiar to actuaries (see, for example, Wilkie, 1995), conditionally heteroscedastic models could be applied, or other similar approaches investigated. It should, of course, be noted that, in general, the time period considered in the measurement of market risk is likely to be far shorter than that usually considered in stochastic asset modelling by actuaries, and that this will influence the choice of modelling approach.

4.3.1.4 In RiskMetrics, it is assumed, when using the normal model outlined above, that the mean returns are zero. The justification given for this is that when considering the variance of the return for daily data, $\sigma_i^2 = E[r_i^2] - (E[r_i])^2$, the first component $E[r_i^2]$ typically dominates the second $(E[r_i])^2$ by a factor of about 700 to 1. In other words, when considering such risk over a short time period, the variance of returns is a much more important influence on price movements than the mean return.

4.3.1.5 RiskMetrics also assumes that returns are not autocorrelated. However, it is assumed that the variances of returns are autocorrelated. It is straightforward to construct forecasts over time periods of any length for the model used by RiskMetrics. For example, the forecast variance over 25 days (one month) is 25 times the forecast variance over one day. Obviously estimation using daily returns and monthly returns may not give identical results, but it is possible to use just daily returns to carry out the estimation necessary for the calculation of *VAR*. Hence, we can now simplify the notation by considering just daily returns, and defining $r(t)$ to be the return at time t. Considering the set of n securities, as in ¶4.3.1.2, the model used by RiskMetrics for the return on asset i is:

$$r_i(t) = \sigma_i(t)\varepsilon_i(t)$$

where $\varepsilon_i(t)$ is the ith component of a multivariate normal random variable whose mean is zero, and whose covariance matrix is the matrix of correlations between the securities; i.e. the covariance matrix is:

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$$\begin{bmatrix} 1 & \rho_{12}(t) & \mathrm{L} & \rho_{1n}(t) \\ \rho_{12}(t) & \mathrm{O} & & \mathrm{M} \\ \mathrm{M} & & \mathrm{O} & \rho_{n-1,n}(t) \\ \rho_{1n}(t) & \mathrm{L} & \rho_{n-1,n}(t) & 1 \end{bmatrix}.$$

The variances and correlations are estimated using exponential smoothing of the daily estimates. This assumes that these quantities are fairly stable over time, so that forecasts can be made using historical data.

4.3.2 *Other distributions and models*

4.3.2.1 It is well known that the empirical evidence does not support the use of the normal distribution in modelling returns. In particular, the distribution of returns is usually found to be skewed and leptokurtotic. This has been noted elsewhere in the actuarial literature — for example, see Geoghegan *et al.* (1992). Various suggestions have been made for alternative distributions to replace the normal distribution: the Student-t distribution, the stable Paretian distribution and other stable distributions. One recent paper, which also studies ruin probabilities and uses stable distributions, is Finkelstein (1997). Further study of the effect of using the normal distribution in the context of assessing market risk would be worthwhile.

4.3.2.2 The actuarial literature contains many examples in which stochastic investment models, which postulate correlations between successive returns, are used. In short-term applications, for example when estimating the value at risk over 1 day, it may be reasonable to assume that returns are not autocorrelated. However, the longer the time period, the more significant the effect of autocorrelated returns becomes. The extension of the model to allow for autocorrelated returns when considering the value at risk over time periods of one month or more would be worthwhile.

4.3.2.3 In RiskMetrics a relatively unsophisticated method is generally used to smooth the estimates of the variance and correlation of returns. However, other methods, such as multivariate GARCH models, have been used to model volatilities, and are suggested by RiskMetrics as alternatives. It may be useful to investigate their use in this context further..

4.3.3 *Bootstrapping, Monte Carlo simulation and the Gibbs sampler*

4.3.3.1 Simulation methods are popular in actuarial applications, and a useful reference on their application is the second part of Daykin, Pentikäinen & Pesonen (1994). Straightforward simulation can be used, in conjunction with the models outlined above, to investigate scenarios, or properties of portfolios which are difficult to analyse otherwise. RiskMetrics allows for these possibilities, but it should be emphasised that the results of the simulation will still depend on the distributional assumptions made.

4.3.3.2 Bootstrapping, which is also a simulation technique, uses the empirical distribution rather than a fitted parametric distribution to simulate from. It can, therefore, overcome some of the problems faced by incorrect distributional assumptions. An investigation which made a comparison between estimates of the value at risk from bootstrap samples and parametric models would be useful.

4.3.3.3 In some ways related to bootstrapping is the Gibbs sampler. This is a technique which is under intense development in the statistical literature; a useful reference is Smith & Roberts (1993) and the related discussion. The Gibbs sampler is a method of simulating from the Bayesian posterior distribution when it is not feasible to use any other methods, analytical or otherwise. This approach allows much greater flexibility in the model, and has already found a number of actuarial applications (see, for example, Makov, Smith & Liu, 1996; Carlin, 1992; and Boskov & Verrall, 1994).

4.3.4 *Discussion*

4.3.4.1 Modelling investment returns is a much-studied area, and there are many issues involved. For a discussion of some of the philosophical background, see Huber & Verrall (1997). It would not be appropriate here to cover all the issues involved, but there are a number of points which should be emphasised. The first is that RiskMetrics and any similar methods use past data (usually recent) to assess value at risk over various future time periods. The implication of this is that the future will be like the past, and that adverse movements in prices are likely to behave in a similar way to previous movements. Clearly, this will depend on the period chosen over which to estimate the parameters of the model, as was demonstrated by Huber (1997). Therefore, care should be taken in interpreting the VAR, and explicit allowance should be made for regime changes, crashes, periods of instability and other effects not included in the data used in the estimation process. VAR techniques are tools which do not eliminate the need for judgement. It should be appreciated that the nature of events, such as regime changes, is such that they can be difficult to predict using statistical techniques. Such events may also be more important than regular market conditions in causing significant losses.

4.3.4.2 These features of VAR models can lead to two difficulties: firstly, the models can be so complex that all but the most sophisticated users do not understand their weaknesses, or know how to interpret the results; secondly, they are expensive to apply. An alternative risk management tool which lies between arbitrary, regulatory capital requirements and value at risk modelling is 'stress testing' or 'scenario testing'. This is not strictly a statistical method, but it does provide information on those deterministic scenarios which are chosen by the modeller. Management can then consider how likely are those scenarios which lead to an unacceptable loss of capital. Such an approach is relatively cheap, and the results are easy to interpret. They may be appropriate for medium-sized banks for whom the investment in full VAR models is not economic.

4.4 *Estimation of the Value of the Portfolio*

4.4.1 *Introduction*

The calculation of the current value of a portfolio $V(0)$ is usually done on a mark-to-market basis. We then need to determine an estimate for the variance of the portfolio returns. To simplify the estimation, variances and correlations are analysed in RiskMetrics for a representative set of securities (known as vertices), rather than for each individual cash flow. Thus, for the estimation of the variance of the return on the portfolio, it is necessary to reduce the set of securities to the vertices. The way in which this is done is outlined in the following sections.

4.4.2 *Fixed income securities*

These are valued by relating them, in cash flow terms, to the standard grid of maturities on the vertices. Thus, for example, a cash flow in $6\frac{1}{4}$ years time is represented by a weighted combination of cash flows at 5 and 7 years. The variances and correlations of returns are estimated on the vertices. These values are then used to produce variances and correlations for the fixed income securities in the portfolio.

4.4.3 *Equities*

Equities are mapped onto the appropriate index, using the principles of the capital asset pricing model. In other words, the value at risk at time t for a stock VAR_i, which is related to a market index denoted by the suffix m, is given by:

$$VAR_i = V(t)\beta(t)z_\alpha\sigma_m(t)$$

where:

— $V(t)$ is the market value of the investment at time t;
— $\beta(t)$ is the beta value for the stock in relation to the market index at time t; and
— $\sigma_m(t)$ is the standard deviation of the market index at time t.

Thus, variances and correlations are calculated for the market indices and foreign exchange rates, which are then used to calculate the value at risk for the stocks in the portfolio.

4.4.4 *Other instruments*

Other instruments, such as options, can present greater difficulties because of the non-linear relationship between the value of the position and the market rates. In this case, either approximations based on Taylor series expansions or Monte Carlo simulation methods can be used. These are standard methods, which are described in some detail in the RiskMetrics technical document.

4.5 *Discussion*

4.5.1 Estimation of market risk depends on the approach to investment modelling. The stochastic model used in this process will make certain assumptions about the likely behaviour of the securities in the portfolio. Stochastic investment models have become increasingly widely used in actuarial work, and the issues surrounding them have been discussed in the actuarial literature.

4.5.2 Some of the points raised by Huber (1997) are of particular relevance to market risk. In particular, Huber showed that the data used in the estimation of the parameters of the investment model described by Wilkie (1986) should be regarded as consisting of a number of distinct groups. Within each group separate stationary models could be applied, but it was hard to justify using a stationary model over the whole range of the data. Haberman (1995), in the discussion of Wilkie (1995), concluded that, over the long term, although a model may not accurately reflect particular effects such as sudden underlying changes, it may still provide a reasonable representation of stochastic variability. The issues in the measurement of market risk are of a different nature, being shorter term, and such an argument may not be valid in this context. Hence, further investigation, perhaps using a model of the type described by Smith (1996), would be worthwhile.

4.5.3 It is worthwhile drawing analogies between the measurement of market risk in banks and techniques which actuaries are familiar with in the management of solvency. The regulatory approach to calculating capital requirements is similar to arbitrary solvency margins. The value at risk approach would be similar to the calculation of risk-based capital using stochastic models (although the interpretation is different). Stress testing would be like dynamic solvency testing or resilience testing. However, the applications are different; in a bank, there is more concern with short-term fluctuations, whereas, particularly in life insurance, actuaries are more interested in the evolution of the net capital position over a longer period. The statistical methods used, therefore, need different properties. Investment portfolios are also likely to be more complex in banking.

5. Risk Factors in the Mortgage Market

5.1 *Outline*

5.1.1 In this section we outline the nature of the information available concerning loan failures and the associated default risk factors in one particular lending market (mortgages). The mortgage market is used because the pricing model derived in Section 7 is applied to mortgages. However, the points raised here apply more generally to personal lending. Large corporate lending is discussed in Section 8. There are other risk factors in the mortgage market, such as early repayment, which are considered in Section 7.

5.1.2 The publicly available information is inadequate for the purpose of analysing risk factors.

5.1.3 The process of mortgage default is as follows. If a borrower falls behind on payments, an attempt is made to come to an agreed rescheduling of payments. If this fails (through either no agreement or a broken agreement), the borrower may be taken to a County Court. The court will often try to arrange a new repayment plan. This state is 'suspended' possession. If this agreement fails, the possession becomes final. Judges have varying attitudes as to how long the borrower should have to pay off the arrears; it may be as long as the remainder of the mortgage.

5.2 *The Importance of Risk Factors*

5.2.1 A cash flow model for a particular class of loan requires values for the expected default rates and the costs in the event of default. In order to set prices according to risk, it is necessary to identify the risk factors and to determine how the default rates and costs depend on these factors.

5.2.2 While it may not be feasible to use information on some possible risk factors, such as family size or occupation, to set different mortgage rates, it would still be useful to know what risk factors exist. It would also be beneficial in analysing trends, as such information would help to break data down into homogeneous classes.

5.2.3 A study of risk factors would be helpful in understanding the risks in a portfolio of loans. It would be useful to know whether diversification, for example by region, house price or initial loan-to-house-value ratio, could lead to less volatility in the default rates.

5.3 *The Principal Risk Factors*

5.3.1 *Factors affecting the probability of default*

The main cause of mortgage default is loss of income to the borrower, and the prime cause of this is unemployment. Hence, the key risk factor in respect of a particular borrower is employability (which has to be represented by proxies such as occupation and location). In addition to the relative risk of individual borrowers, the overall level of defaults is linked to the state of the economy and, in particular, the employment market.

5.3.2 *Factors affecting the cost of default*

As mortgages are secured on the property being bought, a default will only lead to losses for the lender if the value of the property is lower than the amount outstanding on the mortgage plus the amount of costs incurred in pursuing the arrears, and the subsequent possession and sale. (There may well be a discount to market value on sale if a property has been possessed.) Therefore, the initial loan to value ratio of the mortgage is a risk factor. House price inflation also plays an important role in determining the cost of a default.

5.4 *National Statistics on Mortgage Lending*

5.4.1 *What statistics are available?*

5.4.1.1 Data on arrears and possessions have been collected since 1969 by the Building Societies Association (for example, Building Societies Association,

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1985) and later the Council of Mortgage Lenders (for example, Council of Mortgage Lenders, 1996). The number of properties taken into possession during a year (or half year since 1981) is available as a percentage of the number of mortgages outstanding, as is the percentage of mortgages at year end where the arrears amount to between 6 and 12 monthly payments (data for shorter and longer arrears have been available since 1993 and 1982 respectively).

5.4.1.2 There is also information on house prices, mortgage interest rates, loans made to first time buyers and loan-to-house-value ratio, amongst other categories. However, there is no breakdown of number of defaults or loans in arrears according to any of these categories.

5.4.1.3 There is evidence of cyclical behaviour in the 1970s in the proportions of mortgages ending in possession. There were peaks in 1970 and 1975, and troughs in 1973 and 1979. The rate rose throughout the first half of the 1980s, peaking in the first half of 1987, with an annualised rate of 0.33% of mortgaged properties taken into possession, more than three times as much as in 1975, which was the worst year in the 1970s. There followed a sharp fall for a couple of years, but to a level above that of the 1970s. However, between the first half of 1989 and the second half of 1991 the annualised rate rose from 0.17% of properties being taken into possession to 0.8%. There has been a substantial fall since then, but the level is still well above the 1980s peak.

5.4.2 *Problems with using these statistics*

5.4.2.1 An example of a question which might be of interest is: "How does the general level of inflation in the economy influence mortgage defaults and their costs?" There are, however, a number of fundamental problems in using national statistics to address any questions related to risk factors.

5.4.2.2 We will mention three problems:

— the absence of economic costs from any of the statistics;
— structural changes in the mortgage market; and
— changes in the economy.

5.4.2.3 Most published statistics on mortgage problems relate to the number of borrowers who are behind on their payments and the number of properties taken into possession by the lender. These are not, however, the same as the economic costs to the lender of mortgage failures, which are better measured by provisions in the accounts, write-offs of bad debt and recoveries. In their published accounts some lenders automatically make a provision for all possessions, while others consider the necessary charge for each possession. The accounts do not identify what loans have been written off or recovered (e.g. by indicating in which year the corresponding provisions were first made).

5.4.2.4 A further complication is that connection between possessions and economic costs is likely to be changeable, with lenders being less inclined to take hold of a property in a depressed housing market. Court attitudes may also complicate the interpretation of possession data. Court decisions affect the conversion of arrears to possessions; this has been examined by Muellbauer & Cameron (1997). See also Ford *et al.* (1995) for a discussion of the role of County Courts in the settling of mortgage problems.

5.4.2.5 The mortgage market has been subject to profound structural change over the past fifteen years. These changes include the break up of an interest rate 'cartel' which existed until the early 1980s, a substantial opening of the lending market, with banks and centralised lenders becoming more involved following de-regulation (the Financial Services Act 1985 and the Building Societies Act 1986), and a large increase in the number of mortgages taken out as the Government encouraged home ownership. There has also been a number of new entrants into the mortgage market, including 'direct' providers.

5.4.2.6 Low or negative house price inflation makes mortgage lending much more risky. In times when house prices are rising, a borrower who has a drop in income is able to use the growing equity in the house to cover future mortgage payments. This route is unavailable if the value of the house does not rise; but, even in an economic downturn, if general inflation rates are high (as in the 1970s), a fall in real house prices may not be too catastrophic for borrowers if nominal house prices are rising. Having said this, our experience of high inflation was in an era when nominal interest rates stayed low because of the cartel. It is likely that high inflation today would lead to high nominal interest rates, the cost of which could offset the benefits of increases in equity.

5.4.2.7 Unless personal savings increase greatly, it is plausible that future economic downturns will prompt a large number of defaults if general inflation stays low. Also, as it can take around ten years for a mortgage repayer to gain a substantial portion of equity in a property when there are no house price rises, long-term low inflation could mean that borrowers are exposed to two or more periods of recession before they have a cushion against reduced income.

5.4.2.8 Greater job insecurity also makes the risks in the mortgage market different from those faced in the past.

5.4.2.9 Because of the changes in the economy and the structural changes in the mortgage market, it is difficult to rely on any model of defaults which is derived from historic data.

5.5 *Analysis of Arrears and Possessions*

5.5.1 *Questionnaire evidence*

5.5.1.1 This section is based on a housing research report to the Department of the Environment by Ford *et al.* (1995). The research was carried out in 1994, using interviews and questionnaires. The authors examined several risk factors related to mortgage lending.

5.5.1.2 The basic conclusion in the report was that borrowers got into trouble through loss of income much more than through increases in interest rates. The losses came about through unemployment, changes to lower income employment and, to a significant extent, failure (complete or partial) of the businesses of the self-employed. The rise in the number of possessions was exacerbated: by the

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widening of home ownership to a greater proportion of the population; by the unusually high unemployment among (home owning) managers and professionals; and by the increasing trend towards self-employment.

5.5.1.3 While it is a loss of income that tends to cause arrears, the effect of the arrears and the decision to possess depend on the amount of equity which the borrower has in the property. A fall in house prices meant that it became impossible to re-arrange mortgages by using the borrower's equity in the property as security.

5.5.1.4 Other points mentioned in the report include the following:

— The main reasons given for arrears were (in order): job loss; loss of earnings; failed self-employment; relationship breakdown.

— Compared to a 1989 survey, the four main reasons given above have grown in importance (small business failure was not a feature in 1989); over-commitment, unexpected bills and high interest rates have become less important.

— High interest rates were only a minor problem for the borrowers (although they were more important for possessions). (However, high interest rates would be a factor affecting employment, if employment levels were affected by monetary policy.)

5.5.2 *Survey evidence*

5.5.2.1 Burrows & Ford (1997) have analysed the results of a survey carried out in 1994-95, which provided data on characteristics of around 8,300 households with mortgages. Of these households, about 160 were in arrears of three months or more. (This small number of 'in arrears' households, from what is the largest published survey, illustrates the difficulty of obtaining good quality statistical data for analysing mortgage risks.)

5.5.2.2 Fifteen demographic and socio-economic factors were considered, all of which might, *a priori*, have been related to the risk of being in arrears. A statistical analysis showed that several of the factors which appeared to show an association with the probability of being in arrears were, in fact, due to interrelations with other factors. This category includes the age of the head of household, the social class of the head of household, the council tax band of the property (a proxy for price), and region.

5.5.2.3 The following list of parameters is from Table 2 of Burrows & Ford (1997). It shows which risk factors are important predictors of mortgage arrears. Along with the risk factors, the various subdivisions are given in order, with the lowest risk subdivision first. (Some of the orderings may be unreliable due to small sample sizes.) Also given are the relative odds of being in arrears for members of the highest risk category, compared with the lowest risk category:

— 100% mortgage: no, yes — ratio 1.88;

— household structure: couples with no dependent children; couples with dependent children; large adult household; single female; single male; lone parents — ratio 4.57;

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— economic status of head of household: retired; employed full time; employed part time; unable to work; unemployed — ratio 10.20;
— self-employed or employee status: employee in public sector; employee in private sector; self-employed with employees; self-employed sole trader — ratio 6.77;
— type of accommodation: detached house or bungalow; flat; terraced house; semi-detached house — ratio 2.49;
— when property was purchased: post-1989; pre-1987; 1987–1989 — ratio 2.90; and
— anyone in household previously subject to mortgage possession: no, yes — ratio 6.22.

5.6 *Modelling Mortgage Defaults*

5.6.1 There have been several attempts to model default risk for mortgages. These include an econometric model of the mortgage market in the U.K. by Breedon & Joyce (1993). This paper derives six equations linking house prices, arrears and possessions to each other and to a range of demographic indices, such as unemployment rate and disposable income.

5.6.2 The possessions and arrears data each consist of a series of about 30 numbers (from the sources mentioned in ¶5.4.1.1). The demographic indices were broad brush; for example, average loan-to-value ratio for new borrowers (rather than, say, the proportion of loans where this ratio was over 95%) and a national house price index (rather than regional indices, which would have been important, given that housing market problems varied from region to region).

5.6.3 However, the derived equations do make sense, with, for example, a high rate of arrears being associated with high unemployment and high borrowing, while a low rate is associated with high real disposable income and unwithdrawn equity.

5.6.4 One of the derived relations suggests that the number of possessions is very closely linked to the number of mortgages in arrears. More recent data than those used by Breedon & Joyce (1993) indicate that this link has been changed. Other econometric models include those by Brookes, Dicks & Pradhan (1994), which is very similar to the Breedon & Joyce (1993) model, and Allen & Milne (1994).

5.6.5 Another approach, taken by Kau *et al.* (1992), is to treat mortgage defaults in much the same way as a put option which may be exercised by borrowers with negative equity. (So, for example, in a volatile housing market the option becomes more valuable and the borrower is less likely to choose to use the option to default.)

5.7 *The Role of Insurance*

5.7.1 The Government provides some income support for mortgage interest. However, since October 1995 this has only been available after a wait of nine months. In the interim (and also for people not claiming income support)

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mortgage payment protection insurance is available. Currently the take up rate of this insurance is around 20% (Burrows & Ford, 1997).

5.7.2 Mortgage protection plans have not been very beneficial according to Ford *et al.* (1995). The problems are two-fold: insurers often exclude those who appear to be risky (part-time workers, self-employed, borrowers aware of impending redundancy, and so on); and the cover is of limited duration.

5.7.3 This issue, which is not related to the thrust of this paper, could be an area of future collaboration between the actuarial profession and bankers. Particularly important is the designing and pricing of suitable income protection products for mortgages.

5.8 *Discussion*

5.8.1 Publicly available data on problem mortgages are not adequate for the purposes of analysing risk factors.

5.8.2 Ideally, data should be collected for individual loans over a period of years. A large number of loans should be monitored, because the proportion of mortgages which fail is low and because it is necessary to have data on many failed loans if several risk factors are to be considered.

5.8.3 Lenders do keep such information. However, as the losses from mortgage lending were very low until the end of the 1980s, it is likely that most lenders do not have extensive data on mortgages starting before that time. Lenders use the information to build scorecards, which are used to assess the level of risk associated with a new loan applicant (see Section 6). The scorecards are primarily useful as indicators of relative risk, not the overall level of risk, which is driven by the state of the economy and the housing market.

5.8.4 Useful information about a loan at the time it is made includes: date; size of the loan; value of the property (hence the loan to value ratio); and location. Other potentially relevant information about the borrower includes: general credit information (relating, for example, to the borrower's accounts at the bank, or evidence of prior credit repayment difficulties); income cover (the ratio of income to mortgage costs); time in employment; occupation, whether employed or self-employed; age; marital status and number of dependants.

5.8.5 The progress of each loan should be recorded: what is the current status of the loan; whether there have been any repayment problems so far; and any changes in the borrower's circumstances, such as the level of income cover.

5.8.6 For all failed loans, records should be kept of the ultimate cost to the lender, and also of when the lender first became aware of the difficulties.

5.8.7 If all these data are kept, the analytical pricing methods we develop in Section 7 could be used with reliable estimates of default parameters. Methods for estimating the default parameters from the data are discussed in Section 6.

6. STATISTICAL TECHNIQUES FOR ANALYSING RISK FACTORS

6.1 *Introduction*

6.1.1 *Credit scoring*

6.1.1.1 When deciding whether to grant an applicant a loan, a lender will try to ascertain the likelihood that this applicant will be able to repay the loan and the interest charged on it. The likelihood will be calculated by comparing various characteristics of this potential borrower with those known, from analysis of historical data, to be indicative of a good or bad risk.

6.1.1.2 The central task of a statistical analysis of risk factors is to find the links between the characteristics of historical data and the level of risk. Some actuaries have been involved in this type of work; see, for example, the contribution by Wilkie in the discussion of Brockman & Wright (1992).

6.1.1.3 In general, an analysis will begin with historical data of previously made loans of a similar type (although this is not necessary for corporate lending). The information will include the values of a set of characteristics, or risk factors, for these loans, at the time that the loans were made. Also, each loan will be labelled 'good' or 'bad' according to whether the loan was repaid successfully or not. This set of data will enable the lender to identify the typical qualities of good borrowers, and any potential new borrower can be compared with these qualities.

6.1.1.4 Some, but not all, of the statistical methods produce a 'scorecard', which enables a score to be calculated according to the characteristics of a loan applicant. This score will be directly related to the probability of default. Whether the loan will be issued, or not, depends on this probability and on the costs to the lender of making a mistake (i.e. missing the profit by rejecting a good customer or acquiring debts by accepting a bad risk).

6.1.1.5 In addition to this 'lend or do not lend' question, the bank could derive an interest rate appropriate to the risk. (This may be done using cash flow models to link net present values, interest rates and default probabilities; see Section 7.) This varying of interest rates for risk may be done between lending categories (for example unsecured overdrafts will pay higher interest than that applicable to mortgages), and is done for large corporate borrowers, but is not widely applied within categories such as personal lending or mortgages; see Section 8.4.

6.1.1.6 In contrast to general insurance, where each insurer will charge a variety of premiums for, say, car insurance, lenders tend to set up several distribution channels, each channel targeting a particular group of potential borrowers, and each charging different interest rates. This has been happening for several years with credit cards, and has, more recently, been introduced for mortgages and personal loans. However, the interest rate differential within the mortgage category is not that great.

6.1.1.7 There are two quantities of particular interest: the probability of the borrower defaulting; and the cost to the lender in the event of a default (these are directly analogous to claim frequency and claim size). Almost all of the literature on statistical analysis of credit risk is concerned with the former quantity, the

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default probability. For a review of classification and credit scoring, see Hand & Henley (1997).

6.1.2 *Practical considerations*

6.1.2.1 Credit risk is strongly linked to the state of the economy. This means that it is best to analyse loans issued close together in order to isolate risk factors pertaining to the borrower (rather than caused by the economy). The presence of this 'background' risk, due to the economy, causes problems in predicting the absolute level of risk for any future borrowers (although the relative level of risk may be known more accurately).

6.1.2.2 Information about the factors used to assess risk will be strongly influenced by commercial considerations, including the following. People wanting to borrow money generally prefer simple application forms, thus limiting the amount of data which can be collected for future analyses. There are some concerns about privacy — what data should the lender be able to collect from other sources concerning the past credit worthiness of loan applicants? There will be different sorts of information for different applicants (e.g. those who have a current account with the lender and those who do not). This might lead to the production of several scorecards, which should be made broadly consistent. Also, it is generally regarded as unacceptable to use some characteristics of the applicant (e.g. sex) to determine whether a loan is made.

6.1.2.3 The default rates on most classes of lending are low (often less than 1% failing in any year); but to produce a reliable scorecard (or other form of risk measure), it is necessary to have data on many bad loans. This need is increased if many risk factors are to be considered. Typically, a few thousand bad loans (and a similar number of good loans) are needed for reliable results. This can lead to a weak definition of bad, such as any loan which was in arrears for three months (even if it was eventually paid off successfully).

6.1.2.4 The use of past loan data to establish future credit risk can cause bias. These past loans were made because the lender thought the borrowers were not high risks. This means that the population of borrowers who form the historical data set is different from the population of people who will apply for loans in the future. It is possible that a scorecard derived from the former population will not be applicable to the wider population.

6.1.2.5 For corporate lending this problem does not apply, as the probability that a company fails to repay a loan is similar to the probability that the company becomes bankrupt. Bankruptcy predictions can be based on information from a wide set of companies, and is not restricted to companies that have previously been provided with a loan.

6.2 *Regression*

6.2.1 *Overview*

6.2.1.1 This section and Sections 6.3 to 6.6 describe, in outline, a variety of

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statistical techniques which have been used in assessing credit risks. References are given to fuller treatments of these methods.

6.2.1.2 The statistical techniques used in risk analysis in finance are quite varied, and include some methods which are substantially different from the techniques most often used in general insurance. However, one method which is common to bank lending and insurance is regression. Regression methods are used in general insurance for modelling claim frequency (Brockman & Wright, 1992) and in other areas of actuarial work, such as graduation (Renshaw, 1995). They can also be used in analysing default frequency in bank lending (Altman et al., 1981).

6.2.1.3 There are various approaches which come under the heading of regression, the one outlined here is a multifactor generalised linear model.

6.2.2 *Method*

6.2.2.1 Suppose mortgages are to be analysed in terms of four factors: age of borrower; loan to value ratio; income cover; and time that the borrower has been with the bank. Each factor will be divided into several levels; for example, age may be split into: under 25; 25 to 30; 30 to 40; 40 to 60; and over 60. This provides $L_1 \times L_2 \times L_3 \times L_4$ cells, where L_1 is the number of levels for the first factor (five in this case), and so on. Historical data can be analysed to find, for each cell, the number of loans made and the number of these which 'failed'.

6.2.2.2 Note that the levels should be 'ordered', in the sense that there is a natural progression (in terms of risk) between them. Hence, it is not always simple to decide how to arrange the levels for some factors, such as location or occupation.

6.2.2.3 Any cell or loan can, in the above mortgage example, be represented by a vector of four indices (each index identifying the level in one of the four categories). The following notation may be used:

- $\mathbf{X} = \{x_1, x_2, x_3, x_4\}$, the identification of a cell;
- $n(\mathbf{X})$, an exposure measure; either the number of loans made with the characteristics of cell $\mathbf{X}$ or the number of loan years;
- $d(\mathbf{X})$, the number of these loans which defaulted (or met whatever definition of 'failure' is being used); and
- $r(\mathbf{X}) = d(\mathbf{X})/n(\mathbf{X})$, the ratio of defaults to the number of loans made.

6.2.2.4 Given data summarised in this form, a model is required for $r(\mathbf{X})$. One method is to assume that the loan defaults are the result of a Poisson process in which the default probability per loan (per year) is $f(\mathbf{X})$ and the loans are considered to be independent. Thus, the expected number of defaults (and the variance) are given by:

$$E[d(\mathbf{X})] = n(\mathbf{X}) \cdot f(\mathbf{X})$$

and

$$\mathrm{Var}[d(\mathbf{X})] = n(\mathbf{X}) \cdot f(\mathbf{X}).$$

The expected proportion of loans defaulting and the variance in this quantity are:

$$\mathrm{E}[r(\mathbf{X})] = f(\mathbf{X})$$

and

$$\mathrm{Var}[r(\mathbf{X})] = f(\mathbf{X}) / n(\mathbf{X}).$$

6.2.2.5 A generalised linear model has $f(\mathbf{X})$ being some function (called an inverse link function) of a linear combination of the indices $\{x_1, x_2, x_3, x_4\}$, for example:

$$f(\mathbf{X}) = \exp(\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4).$$

The choice of the inverse link function (in this case exponential) depends on what transformation is needed to enable a linear combination of the factors to be a good model of the data. Many transformations are possible (see Renshaw, 1995; and Altman *et al.*, 1981, for discussions of useful transformations).

6.2.2.6 The parameters of the model (in this case α_1, α_2, etc.) can be estimated according to a maximum likelihood procedure. Also, confidence intervals may be obtained.

6.2.2.7 For a loan applicant who is characterised by **X**, a score may be calculated according to:

$$\text{Score} = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4$$

and this is related to a default probability by:

$$\text{Probability of default} = \exp(\text{score}).$$

The exponential function is relevant because it was chosen as the inverse link function.

6.2.2.8 It is possible to avoid the requirement that a linear form is used. Generalised additive models allow a non-parametric transformation of the levels of each factor to be performed as part of the fitting process (see Hastie & Tibshirani, 1990).

6.3 *Discriminant Analysis*

6.3.1 *Overview*

6.3.1.1 Discriminant analysis is used frequently in assessing the risks of

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bankruptcy in corporations (Altman *et al.*, 1981), and, by extension, it is useful in predicting the probability of a corporate loan defaulting. The technique is much more naturally suited to continuous variables, such as profit or the ratio of working capital to total assets, than categorical variables (such as occupation or marital status).

6.3.1.2 The idea behind discriminant analysis is that there are separate populations of 'companies which will fail' and 'companies which will not fail'. These populations have characteristic values for particular variables (usually balance sheet ratios). Past data can be used to determine these values. When estimating the future behaviour of a particular company, it must be decided whether this company's characteristics make it more 'similar' to failed companies or to those which did not fail.

6.3.2 *Method*

6.3.2.1 A standard form of discriminant analysis treats the two populations ('good' and 'bad', or 'g' and 'b') as being described by multivariate normal distributions. Thus, if there are m variables, each good company will have properties $\mathbf{X}$ drawn from the appropriate normal distribution, i.e.:

$$\mathbf{X} \sim \text{Normal}(\mu_g, \Sigma_g)$$

with μ_g a vector of length m and Σ_g the $m \times m$ covariance matrix for this population. The bad population is similarly described, with vector of means μ_b and covariances Σ_b. The means and covariances will have to be estimated from the data. Denoting the probability density functions of these two populations by $f_g(\mathbf{X})$ and $f_b(\mathbf{X})$ and the relative sizes of the two populations by π_g and π_b, the probability that a company with characteristics $\mathbf{X}$ belongs to the good population is:

$$p(\text{good} \mid \mathbf{X}) = \frac{\pi_g \cdot f_g(\mathbf{X})}{\pi_g \cdot f_g(\mathbf{X}) + \pi_b \cdot f_b(\mathbf{X})}.$$

6.3.2.2 If the two populations have the same covariances, this leads to $\mathbf{X}$ being classified as good if:

$$\mathbf{X}^T \cdot \gamma \geq \alpha + \log[(c(g \mid b) \cdot \pi_b)/(c(b \mid g) \cdot \pi_g)]$$

where $c(g \mid b)$ is the cost (e.g. to the lender) of misclassifying a bad company as good, and vice versa for $c(b \mid g)$:

$$\gamma = \Sigma^{-1} \cdot (\mu_g - \mu_b)$$

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and

$$\alpha = \frac{1}{2}(\mu_g + \mu_b)^T \cdot \gamma.$$

6.3.2.3 The above equation produces a score for a given **X** which is linear in the individual characteristics. If the covariances of the populations are not equal, a quadratic equation may be derived (which includes cross terms such as $x_1 x_2$).

6.3.2.4 Note that this model requires all characteristics to be distributed as normal variables within a given population.

6.3.2.5 With regression, it is possible to test whether a variable is significant or not in determining if a borrower is risky or not (e.g. by using t-statistic tests on the coefficients of the regression model). There is no directly equivalent procedure which can be used with discriminant analysis although many suggestions have been made (see Altman *et al.*, 1981, pp135-150). Hence, the optimal choice of variables is not straightforward when using discriminant analysis.

6.4 *Cluster Analysis*

6.4.1 *Overview*

6.4.1.1 It is possible to introduce more flexibility into the analysis of loan data by using non-parametric techniques. These avoid the necessity for making assumptions about formulae describing populations of good and bad risk (as is needed in discriminant analysis) or requiring a linear combination of risk factors to be a good measure of risk (as is necessary with generalised linear models).

6.4.1.2 One non-parametric method is cluster analysis. The purpose of cluster analysis is to identify patterns in a set of data.

6.4.1.3 The particular method described here was used by Henley & Hand (1996) for analysing credit risk. It is just one of many forms of cluster analysis (see Everitt, 1993, for a review of other varieties).

6.4.1.4 Note that, even though this is called a 'non-parametric' method, there are two parameters associated with it (k and λ, defined later in this section) and three 'structures' are imposed on the data (to obtain D, $\tilde{\mathbf{X}}$ and **W**). It does not impose a formula for the probability of default or for the description of a population.

6.4.2 *Method*

6.4.2.1 At the heart of the technique is a distance measure, or metric, which indicates how close a new loan applicant is to each loan in a historical data set. Once the distance has been calculated, the probability that this loan will fail is determined according to the proportion of the nearest loans which fail. (The number of nearest neighbours used k, is determined empirically from the data set; it will be chosen to maximise the ability of the technique to separate the good and bad loans.)

6.4.2.2 There are many possible metrics. The one used by Henley & Hand (1996) is:

$$D(\mathbf{X},\mathbf{Y}) = [(\tilde{\mathbf{X}} - \tilde{\mathbf{Y}})^T(\mathbf{I} + \lambda \mathbf{W}\mathbf{W}^T)(\tilde{\mathbf{X}} - \tilde{\mathbf{Y}})]^{\frac{1}{2}}.$$

In this equation:

— D is the distance between two loans described by **X** and **Y**.
— **X** and **Y** are length m vectors if there are m characteristics for each loan.
— $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ are transformed representations of the same loans. For example, the first element of $\tilde{\mathbf{X}}$ is related to the first element of **X** according to:

$$\tilde{x}_1 = \log[p(\text{good} \mid x_1) / p(\text{bad} \mid x_1)].$$

The probabilities are determined from the data set of past loans. This transformation provides an order to the levels of a factor, and also makes it possible to compare the differences in one factor with the differences in another factor, by putting them on the same scale.
— **I** is the $m \times m$ identity matrix.
— **W** is a length m vector, which points in the direction along which the probability ratio (of good loans to bad loans) is steepest. This can be estimated for the whole data set (i.e. a global value, independent of **X**), or it could be calculated local to each **X**.
— λ is a parameter which increases the flexibility of the metric, and the value is determined in the same manner as k, i.e. it is chosen so that the ability to discriminate between good and bad loans is maximised.

6.4.2.3 This method does not produce a scorecard as such, but it is able to give a probability that a borrower with given characteristics **X** will default.

6.4.2.4 It is similar to discriminant analysis in that the aim is to identify populations of good and bad loans and then assign a new borrower to one of these classes. However, the variables used do not have to be normally distributed, nor do they need to be continuous.

6.5 *Classification Trees*

6.5.1 *Overview*

6.5.1.1 Trees are used in another non-parametric technique for classification.

6.5.1.2 In classification, a tree is a type of data structure. The mathematical ideas are discussed by Breiman *et al.* (1984). One particular technique, called recursive partitioning, has been discussed in finance literature by Marais *et al.* (1984), Frydman *et al.* (1985) and Boyle *et al.* (1992). This is the method described in Section 6.5.2.

6.5.1.3 A tree can be used to produce groups (e.g. of loans) with similar characteristics, where similarity is measured in terms of risk factors. By

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combining a set of simple rules, the overall method of identifying similar groups can be very flexible.

6.5.2 *Method*

6.5.2.1 A tree can be constructed in the following way from a set of loans which were either 'good' or 'bad', and each was characterised by *m* risk factors:

— Assign all loans in a data set into a single 'root' node.

— Find what single discriminator is best able to split the data in this node into two groups. 'Best' is measured in terms of the homogeneity of the two groups; they should be as close as possible to 'all good' or 'all bad'. Each of the *m* risk factors will have one value which produces a best split; it is the best of these *m* discriminators which is used. Each of the two groups is assigned to a new node.

— The process continues, each node being split according to its own best discriminator, until either there is no discriminator which works well or until there are too few loans left in the node. Note that the two nodes linked to the root node will have their own discriminators: different from each other and different from the root. A node which is not split is called a 'terminal node' or a 'leaf'.

6.5.2.2 In practice, it may be best to set the criterion for there being 'too few' loans in a node as a small number when deciding whether a node should be terminal or not. This produces a complex tree, which may have to be 'trimmed' (this process is discussed by Marais *et al.*, 1984).

6.5.2.3 Once a tree has been constructed, a potential new borrower is classified by comparing the characteristics of this borrower with the discriminator in the root node, and thus assigning the borrower to one of the two nodes branching from the root. Then the borrower is compared with the discriminator in this node and assigned to one of the two nodes branching from it. This continues until the borrower has been assigned to a terminal node. The default probability of the loan is then estimated as the proportion of loans in this node which were bad (this proportion should be close to 0 or 1 if the discriminators were effective).

6.6 *Artificial Intelligence Methods*

6.6.1 *Overview*

6.6.1.1 Many techniques have been developed in the field of artificial intelligence (AI) to address classification problems. We briefly describe some of them in this section.

6.6.1.2 Some AI techniques automate existing procedures, by systematising the judgements currently used by people who are expert at making decisions about whether to lend; others use statistical techniques to make the required judgements.

6.6.1.3 As has been pointed out in other sections, the problem of classifying credit risks has much in common with that of insurance underwriting. Lecot

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(1993), Wright & Rowe (1993) and Rowe & Wright (1993) describe some developments in the latter area. Chorafas & Steinmann (1991) describe some applications of AI techniques in the field of banking.

6.6.2 *Expert systems*

6.6.2.1 The most widely known success story in this area is the 'Authorizer's Assistant' system used by American Express. Authorizer's Assistant expresses the knowledge used by human experts as a series of rules. The system is used when a request is made for authorisation of a purchase to be charged to a credit card. The AI system is used by customer service representatives who must give or refuse authorisation over the telephone, and has both decreased the response time and increased the accuracy of the decisions (Holsapple *et al.*, 1988; Leonard-Barton & Sviokla, 1988).

6.6.2.2 Many rule-based expert systems rely on extensive interviews with experts. However, it is sometimes possible to derive a set of rules using statistical techniques. Trees, such as those described in Section 6.5, can be converted into rules.

6.6.2.3 Expert systems are widely used to perform tasks that, when performed by people, are thought to rely on judgement or intuition. These are often tasks for which all information is not available, or for which the information that is available is imprecise or uncertain. Several techniques have been developed for handling these problems.

6.6.2.4 The use of 'fuzzy logic' is claimed to be a powerful tool in the design and construction of decision support systems (Cox, 1995). Fuzzy logic is based on fuzzy sets (Zadeh, 1965). A fuzzy set is a collection of objects that might belong to the set with a degree of certainty which can vary from 0 (does not belong at all) to 1 (belongs fully). In AI, fuzzy logic is used to express the vagueness that often surrounds reasoning from assumptions, and that leads to uncertainty in judgements.

6.6.2.5 An alternative way of representing and reasoning about uncertainty, which may come more naturally to actuaries, is to use probabilities. Bayesian networks, based on work by Pearl (1988), use Bayesian probability theory to manage uncertainty by explicitly representing conditional dependencies between different facts, assumptions and hypotheses. Their use provides an intuitive graphical visualisation of the knowledge, including the interactions among the various sources of uncertainty. Efficient algorithms have been developed for them, and they are becoming widely used in a variety of fields, especially in applications that involve some kind of analysis of problems. An obvious use for them in banking would be to estimate the probability of default on a loan.

6.6.3 *Neural networks*

6.6.3.1 Artificial neural networks (ANNs) are computer systems that can be used to 'learn' how to classify inputs (Lowe, 1997). An ANN consists of a number of simple processing units, called nodes, which take a number of inputs

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and produce a single output. The nodes can be tuned to respond to inputs in different ways by assigning a different weight to each input. A node's output is then a weighted function of all its inputs. The nodes are connected together in a network, so that the inputs of one node are the outputs of others.

6.6.3.2 There are a number of different algorithms for tuning, or 'training' ANNs. The basic idea is to present a number of training examples to the ANN, and adjust the weights on the individual nodes until the correct outputs are produced.

6.6.3.3 ANN techniques are becoming increasingly widely used for classification in the credit and banking area. For comparisons with other classification methods and a range of opinions on about the success, or otherwise, of neural networks in credit and banking, see, for example, Jost (1993), Brennan (1993a, 1993b) and Desai *et al.* (1996)

6.6.3.4 Overall, it seems likely that neural networks may prove to be useful, especially because of their ability to handle nonlinearity and to classify inputs into any number of classes. However, it is clear that more investigations should be made in order to confirm or deny this hypothesis.

6.7 *Summary*

6.7.1 There is a range of techniques which can be used to analyse loan default risks. The following general points can be made.

6.7.2 For most categories of lending, there is less analysis of risk undertaken than would be desirable. Actuaries could use techniques which are used in other actuarial fields to analyse default risk. It would be worthwhile to test whether the various methods produce significantly different results.

6.7.3 A scoring process is often undertaken, but this is normally used to accept or reject a loan. The score is rarely used to provide default parameters, and thence a risk-based rate of interest. To price for risk, in this way, for lending categories such as personal lines or mortgages would be a major step forward in loan pricing.

6.7.4 To perform a detailed analysis, more data must be collected. Risk parameters can then be estimated, and used in models such as those developed in Section 7. We commented, in ¶¶5.8.4 to 5.8.6, on what information would be useful for assessing risk for mortgages. Similarly for other personal lending, both credit history and demographic factors will be important.

7. PRICING OF RISK USING CASH FLOW MODELLING

7.1 *Current Approach*

7.1.1 *Method*

7.1.1.1 The profitability of lending can be measured on a global basis. The net interest income generated by lending minus the expenses and bad debts costs associated with the loans can be compared with the amount of equity capital held to back the loans. This calculation leads to a figure for the return on equity

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capital. If the lender has a target return on equity, the margin on the lending business, and hence the interest rates being charged on the loans, can be changed to produce this desired return.

7.1.1.2 Separate calculations are done for different lines of business, for example mortgages, personal loans and credit cards. The lines have different expense levels and margins.

7.1.1.3 Risk enters the price through bad debts; an increase in the level of bad debts requires an increased margin to produce the same return.

7.1.1.4 Extra terms can be added to the model, so that the effect on profitability caused by varying these terms may be examined. Possible items include: default rates, default costs, debt capital costs; and fees paid by borrowers (e.g. for changing the terms of a loan).

7.1.1.5 This method is most appropriate to a portfolio of existing loans, which has a (roughly) level population of loans outstanding throughout a year and also a (roughly) constant amount of capital to back the portfolio. At any time, such a portfolio would include newly issued loans as well as loans near to completion.

7.1.2 *Problems with the current approach*

7.1.2.1 There are a number of limitations to the global approach when it comes to pricing loans. One problem is the lack of focus on various features which do have an important impact on profitability. For example, the size and duration of a loan are both very significant, but are not represented in the broad brush approach. It is necessary to look at each loan, and consider the income and expenses which relate to it.

7.1.2.2 Another aspect which is missing from this method is any consideration of timing. The net income generated by the lending business is compared with the amount of capital currently being held. A more accurate method would take into account the delay between the outlay of capital for a loan and the income generated by the loan.

7.1.2.3 Timing is also important for expenses, but initial expenses and general on-going expenses are included jointly in these loan profitability calculations.

7.1.2.4 The failure to distinguish between initial cash flows and more general cash flows which occur throughout the loan, can lead to cross-subsidies between different cohorts of loans.

7.1.2.5 Some numerical examples illustrating the importance of an accurate treatment of timing are included in Section 7.9.4.

7.2 *Reasons for using Cash Flow Modelling*

7.2.1 We propose a new approach, based on cash flow modelling techniques familiar to actuaries. This new approach considers more accurately the economic costs and benefits which can be attributed to bank or building society lending. We can look at the specific outgoings (cost of borrowing to finance the mortgage,

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cost of capital to back the mortgage, expenses and losses due to default) and the specific items of income (interest charged and any fees). We can consider the timing of these items and make an explicit assessment of profitability.

7.2.2 Cash flow models can be used to examine a cohort of loans, giving explicit attention to the timing of income and outgo. They would be essential for new entrants into the market. The capital allocated to a loan at the outset, the initial expenses and the running expenses can all be included. Moreover, parameters, such as loan size and duration, may readily be included in a cash flow model, so that their effect on profitability can be calculated. In the current environment, loans are often marketed with compulsory purchase of insurance, etc. In a competitive market such a 'full-line pricing' approach should not be possible. From a management perspective, it is important to know the element of cross subsidy arising from marketing joint products. A cash flow model can determine that.

7.2.3 Cash flow models may also be used for more complex and general work, such as examining the interactions between interest rates and early repayment fees (see Section 7.5).

7.2.4 In the remainder of Section 7 we develop a cash flow model. The model's possible use in examining the sensitivity of loans to a large range of parameters is also illustrated (Section 7.8). We also explore how the model might be applied to the tasks of designing fee structures relating to loans which are repaid early (Section 7.5) and costing the provision of cash back payments to borrowers (Section 7.10).

7.3 *Model Development*

7.3.1 *Basic methodology*

7.3.1.1 Cash flow models of bank lending can be constructed for a cohort of similar loans issued at the same time, for an existing portfolio of loans, or for the whole business of lending (including the costs of setting up computer systems, training staff, and so on, as well as a model of the growth rate of the business, and all of the cash flows arising directly from the lending). We will consider only the first of these three situations. In this paper 'cohort' and 'tranche' are both used when describing loans issued at the same time.

7.3.1.2 There are many possible outputs from cash flow models. We are most interested in the net present value (NPV), internal rate of return (IRR) and break-even loan rate (i.e. the rate of interest which must be charged in order that the loan is sufficiently profitable).

7.3.1.3 The model developed here is quite general (for example it could be used with either secured loans, such as mortgages, or unsecured loans). However, in order to illustrate the behaviour of the model and the sensitivity of the parameters included, we concentrate on one particular type of loan: the retail mortgage. The model could be applied to both banks and building societies in pricing mortgage products. Walsh & Booth (1997) look at more general cases.

7.3.1.4 When making a loan, the lending section of the bank will have to obtain the same amount of money from the bank's treasury, which, in turn, will acquire the money from retail deposits or short-term borrowing in the wholesale markets. There are also two types of capital backing the loan (equity and debt). There are, therefore, several types of interest rate in the model. The customer pays interest on the loan at one rate and the bank borrows from the treasury at another rate. No capital is involved up to this point. However, capital is needed to back the loan (for example, to provide protection against bad debts). The net cash flows to the bank need to provide a return on that capital (with possibly a different rate being provided on debt and equity capital). In order to assess the profitability of a loan, it is essential to separate those elements of a cash flow which may be regarded as profit or return on equity capital and those which may not. Because the loan is much larger than the capital backing it, most cash flows do not relate to the provision of equity capital.

7.3.1.5 The separation of a cash flow into several streams is familiar to actuaries, for example in the context of unit-linked life policies (e.g. Squires, 1986) where premium income is split between a unit fund ('belonging' to the policyholder) and a sterling fund ('belonging' to the office). A closer analogy to the two sources of funds required in bank lending is where a negative sterling fund is used in a life office (e.g. Hare & McCutcheon, 1991). In such a situation, the initial strain caused by setting up a policy is partly backed by capital, which requires one rate of interest, and partly by internal funds, which require a lower rate of interest.

7.3.1.6 The cash flow equations become complicated (see Walsh & Booth, 1997), but the ideas are not different from those used in other actuarial cash flow models: there are terms for the amount of income and outgo; the timing of these; the probability that they occur; and a discount factor. The complexity arises because there are many parties to consider (the shareholders, the borrower, the bank's treasury and the providers of debt capital) and because two ways of cutting short the loan are considered (default and early repayment), both of which give rise to income (including default recoveries and 'surrender fees').

7.3.2 *The repayment pattern*

7.3.2.1 In general, a personal loan or mortgage will be repaid over time, with each instalment including both an interest component and some capital repayment, rather than requiring a series of interest only payments and a final return of capital; although there is still a substantial number of loans which pay interest only, and repay the capital at the end of the term using an endowment, personal equity plan or pension policy. Nevertheless, we will consider the former type of loan. The cash flow model is flexible, and can be adapted to any type of loan.

7.3.2.2 Denoting the outstanding loan at the end of month t by L_t, and assuming that the loan is repaid in arrears over n months, the total repayment at the end of month t will be $L_{t-1}/a_{\overline{n+1-t}|}$, where $a_{\overline{m}|}$ is the present value of an annuity of one per month paid in arrears for m months. The annuity is evaluated at the monthly interest rate charged to the borrower i_L. The monthly repayment

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includes an interest payment of $i_L \cdot L_{t-1}$, with the remainder reducing the principal outstanding by $L_{t-1} - L_t$. This concept is familiar to actuaries. See, for example, McCutcheon & Scott (1986).

7.3.2.3 In this situation there is also a release of capital each month, as the capital requirement is likely to be proportional to the amount of the loan outstanding. If the capital held is equal to the regulatory capital, this is directly proportional to the size of the loan (see ¶¶2.2.5 to 2.2.7). This procedure should not be followed if analysis suggests that the loan is becoming more risky; the capital backing the lending should be kept at a level sufficient to cover future losses. In practice, loans probably tend to become less risky over time, but the capital is not likely to be reduced as a proportion of the loan outstanding. We will assume, throughout, that capital follows regulatory requirements (or exceeds the requirements by a fixed proportion). However, theoretically, the amount of capital that should be held should, itself, be a decision variable. This is a fruitful area for further research.

7.3.2.4 We will assume that the lending section of the bank borrows from the treasury at the start of each month an amount equal to the loan outstanding. This will be repaid to the treasury with some interest at the end of the month. In this approach, it is implicitly assumed that it will be possible, throughout the term of a long loan, for the treasury to be able to borrow the amount of money which has already been lent by the bank. It is also assumed that borrowing is always arranged for just one month. Given the liquidity and low costs in the cash market and the ability of banks to borrow from retail and wholesale sources, this is a reasonable assumption.

7.3.3 *Expenses*

7.3.3.1 There are several ways of dealing with expenses, particularly initial expenses, and these lead to different values for the profitability of a loan and also to the sensitivity of the return on equity capital to changes in parameters such as default rate.

7.3.3.2 Expenses will have to be paid relating to setting up the loan, maintaining it and closing it. The cash flow treatment for the set-up costs is best considered separately.

7.3.3.3 The initial expenses included in the loan pricing calculations refer only to the costs directly attributable to selling and setting up new loans. They do not include 'overhead' costs or the costs of setting up a line of business.

7.3.3.4 Initial expenses might, theoretically, be met by borrowing money (from the treasury), by using equity capital, or from the net income generated by existing loans. These three methods are discussed below.

7.3.3.5 If the initial expenses are borrowed from the treasury, they would have to be repaid, with interest, at some later time, using the repayments received on the loan. We will assume that, if initial expenses are borrowed, they will be repaid to the treasury in equal instalments over the typical duration of a loan (although this will change if there are defaults or early repayments).

7.3.3.6 This is not the only possible treatment of the initial costs. They could be met from capital, on the grounds that there is a risk that they will not be recovered because the borrower fails to make sufficient payments. As there is a (cumulatively) greater chance of the later payments failing than the early payments, the likelihood of being unable to recover these initial expenses is minimised if the first few instalments paid by the borrower are used for the purpose of meeting the expenses rather than contributing to profit. (In life insurance profit testing, the initial expenses are generally charged to capital; see Squires, 1986.)

7.3.3.7 Because initial expenses can be quite large, it would require a substantial increase in the capital outlay for a loan if the expenses had to be met in this way. Moreover, this capital would be consumed immediately, and, therefore, would be unable to earn any interest. Hence this would be a costly approach. Note that there is clearly a risk that loans will not be repaid in full, but the existence of such a risk does not require the bank to use 100% capital to support the loan. Instead, around 10% (5% for mortgages) is generally regarded as sufficient, of which 6% might be equity capital. So, the above suggestion that initial expenses are fully met by capital is a very cautious approach. Any capital used for this purpose would be 'economic' rather than 'regulatory' capital, because regulatory capital only relates to the amount of the loan.

7.3.3.8 It is current practice to use a method which has an effect similar to that of borrowing from the treasury. If expenses were charged to capital, it would reduce the variability of returns to capital (because similar variability in cash flows due to defaults, etc. would be related to a greater amount of capital provided). Therefore, it would be reasonable to reduce the required internal rate of return on capital (or reduce the risk discount rate) if this approach were used. In practice, it may not make much difference whether expenses are charged to capital and a low IRR (or risk discount rate) is used or expenses are borrowed from the treasury and a higher IRR (or risk discount rate) is used. We will generally use the latter approach, which is closer to current practice. We will illustrate the effect of these two ways of paying for initial expenses in Section 7.7. The NPV and IRR are both reduced by the requirement that extra capital be used for paying these expenses, but the variability of the returns is also reduced.

7.3.3.9 In practice, the expenses for each loan are not treated separately from those of other loans. The initial costs of one loan are paid from positive cash flows generated by other loans and, similarly, later positive cash flows produced by this loan will be reduced to pay for a portion of the initial expenses of subsequent loans. Thus, there is neither any input of extra capital nor any borrowing of funds from the treasury. Therefore, when considering a particular loan, the initial costs could be regarded as having simply been deferred until there had been sufficient positive cash flows. No extra charges (such as interest) need to be paid, except, possibly, due to expense inflation. This grouping of loans, rather than handling each cohort separately, gives rise to cross-subsidies, and our cash flow modelling approach is designed to avoid this.

7.3.3.10 Small running cost expenses can be met directly each month from the borrower's repayments. ('Small' here means less than the net monthly interest margin from the loan.) These can be included in a straightforward manner by a deduction from each month's net income.

7.3.3.11 If the finalising costs are larger than the net monthly income, they will have to be met in advance using earlier months' income.

7.3.3.12 Expenses are a very significant part of the cash flow calculation, as they may be more than a half of the cumulative net interest margin. They are also difficult to quantify. For example, how should the staff costs of a branch network be attributed to a particular type of loan? The correct treatment of expenses is vital. It may be the case that a different treatment will lead to different conclusions about charging structures (policy fees, margins, etc.). Just as with insurance, it is important to carry out an accurate expense attribution analysis.

7.3.3.13 The expenses will not, in general, vary directly with the size of the loan. This implies that NPV will not vary directly in proportion to loan size. Small loans may be unprofitable, except at very high interest rates.

7.3.4 *Contributions towards other expenses*

7.3.4.1 There will be expenses related to lending which are not directly attributable to any particular loan, advertising costs being an example. It is possible to include these costs in cash flow models by introducing an extra levy. This is the approach we take, with a monthly charge proportional to the size of the loan.

7.3.4.2 Other possible approaches include increasing the hurdle rate or requiring the expected net present value of the loan to exceed some chosen level.

7.4 *Base Model*

7.4.1 *Type of loan*

Because there are many variables which are unfamiliar to actuaries, it is useful to build the model step by step. The starting point for illustrating cash flow models of lending is a 25-year mortgage. It is assumed that all borrowers repay their mortgage early, after exactly seven years (this is a typical average duration in practice), and that there is no early repayment fee. In later sections we consider the effect of repayments at other times, repayment fees and defaults. Payments are made monthly in arrears.

7.4.2 *Parameter values*

7.4.2.1 The initial term of the loan and the date when all loans are repaid are: n = 25 years, and n_r = 7 years. Initial expenses are amortised over n_E = 7 years (see ¶7.3.3.5).

7.4.2.2 The initial size of the mortgage is: L_0 = £100,000.

7.4.2.3 Equity capital of 3% of the outstanding loan is held at all times, and debt capital of 2% is held. The minimum equity capital percentage is 2, but most lenders will retain more than this.

7.4.2.4 In their accounts, banks do not provide a breakdown of costs which separates initial expenses, running expenses and other types of expense. A rough figure for the total costs associated with mortgage lending, per year, is 1% of the amount of loans outstanding. In our base model, initial expenses related to the loan are E_0 = £2,500, and monthly running expenses are E_t = £10. This combination means that the average expenses over the seven year existence of the mortgage is just under 0.5% of the average loan outstanding. These initial expenses are far larger than the total running expenses. In the base model, we assume that the initial expenses are borrowed from the treasury (see Section 7.3.3 for a discussion of this choice, and Section 7.7 for an analysis of its importance).

7.4.2.5 As mentioned, the assumptions for initial expenses and maintenance costs in this model add up to roughly half of the typical level of expenses in mortgage lending. The remainder of the expenses are in the form of a monthly levy (Section 7.3.4). The model, therefore, includes a levy at an annualised rate of 0.5% of the amount of loan outstanding. This approximately equal split between expenses related to loans actually made and expenses unrelated to them is an assumption.

7.4.2.6 Five interest rates are included in the model:

— the risk free, or 'cash', interest rate, r_C = 8%, which is earned on set-aside equity and debt capital;
— the interest rate paid to the treasury (the 'cost of funds'), r_F = 8.5%;
— the interest rate paid to the providers of debt capital, r_{T2} = 9.5%;
— the interest rate charged on the loan, r_L = 10.3%; and
— the hurdle rate used to discount future income, r_H = 20%.

7.4.2.7 The margin between the cost of funds and the loan rate is 1.8%, which is close to a typical value of about 2% or higher. The loan interest rate is treated here as a 'market rate' applicable to mortgages. The cash flow model shows how profitable a loan is expected to be if this market rate is charged. It can also derive the minimum interest rate which is necessary to make the loan sufficiently profitable. As with insurance premium setting, the model can determine whether the bank can enter the market at the market rate and the minimum level of interest which is acceptable.

7.4.2.8 The hurdle rate is a measure of the return required on the equity capital backing the loan. It is set according to the riskiness of the loan, and, therefore, the risk of losing the capital backing the loan. It will be higher than the rate of interest charged by the treasury, because the treasury is not taking on any risk. The treasury will have a prior claim on any income, and if there is any shortfall (e.g. because of a loan default) capital will have to be used to make up the difference. A value for the hurdle rate of approximately 20% (pre-tax) is used, in practice, for bank lending, generally derived from the capital asset pricing model. It is not the subject of this paper to criticise that assumption, although the authors recognise that this could be a useful area for further research.

7.4.2.9 As this paper is not focusing on risks relating to changes in base rates, the same interest rates are used throughout the term of the loan.

7.4.2.10 In Section 7.8 we look at the effect of changing the parameters from those used in this base model.

7.4.3 *Base model outputs*

7.4.3.1 This loan has the following properties:

— net present value at the 20% hurdle rate, NPV = £768.37;
— internal rate of return (annualised value), IRR = 28.29%; and
— break even loan rate at the 20% hurdle rate, 10.08%.

7.4.3.2 The monthly cash flows are shown in Table 7.1. Columns B to E show the various amounts outstanding at the start of each month. The money received from the borrower (the sum of columns F and G) is constant at £897.69 per month, except in the final month, when all of the remaining principal is returned. The interest paid to the treasury (column H) is less than that received from the borrower, while the return of principal is the same (column I). Four items relating to expenses are in columns J to M, covering initial expenses with interest, the levy for expenses not caused by this loan, and monthly running expenses. Flows related to equity and debt capital are in columns N, O and P.

7.4.3.3 The monthly net cash flow in column Q is calculated as F + G – H – I – J – K – L – M + N + O – P. This decreases slowly until the last month, when there is a large positive cash flow because of the return of equity capital. Columns R and S show the effect of discounting. (The final four columns are concerned with surrender fees, and are discussed in Section 7.5.)

7.4.3.4 There are two rows after the final month for columns F to Q. The first of these gives sums for each column (total interest paid over the loan, and so on). The other row shows the present value of the payments in each column.

7.4.3.5 The net present value of the loan is calculated by summing column S (or equivalently by combining the values in the final row) and subtracting the initial capital outlay.

7.5 *Fees for Early Repayments*

7.5.1 The terms of a loan will often allow the borrower to repay the loan early; this is the case in the base model used above, where all borrowers repay early. Because of the capital and expense outlay at the beginning of a loan, borrowers who repay early incur an extra cost on the bank. There is also a loss of future profits. This situation is familiar to actuaries who are used to calculating 'actuarially fair' surrender values. In bank lending, similar approaches could be taken. For example, arrangement fees could be charged which reduce initial net outlay, or early repayment fees could be introduced (similarly, interest rates could be reduced on a 'loyalty basis' for loans which went beyond the average term).

7.5.2 Early repayments can be a very important feature of long-term loans

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such as mortgages, where many borrowers move house or switch between lenders in search of the lowest interest rates. Hence, the lender cannot rely on the receipt of a full number of interest payments to provide the required profits. The problem is amplified by the fact that mortgage loans often include a reduced interest rate in the first year or so, and also by the concentration of expenses and default risks near the beginning of the loan period. The lender will, therefore, be relying on later interest payments to make the lending worthwhile.

7.5.3 We will consider two ways for the lender to recoup the lost profit caused by early repayments. The first method is to calculate surrender fees equal to the forgone profit.

7.5.4 A surrender fee can be calculated as:

Fee = Discounted value, at the time of repayment, of expected future profit forgone
+ Discounted value, at the time of repayment, of expected future return of equity capital
+ Expenses
− Equity capital returned early

where the missed profits are between the actual surrender date and the typical surrender date (seven years in the base model). The expenses are those directly related to the surrender, and also any other expenses which are still owed to the treasury. We will ignore the former, but the base model does include some expenses which had been met by borrowing from the treasury.

7.5.5 There are two rates of interest which might reasonably be used in the discounting of future profit to calculate surrender fees. One is the hurdle rate of interest which is used to calculate the NPV for the loan. Using this rate ensures that the NPV of the loan is not affected by early repayments. The other rate which might be used is the internal rate of return achieved on loans repaid at the 'intended' time (7 years in the base model). Using the IRR to discount future profit ensures that the IRR of the loan is unaffected by early repayments. The choice between the two rates depends on whether NPV or return on equity capital is regarded as the more important quantity.

7.5.6 The surrender fee can be calculated retrospectively (the same fee is obtained). In the case where the IRR is used to determine the correct fee, the retrospective calculation is: (minus) the accumulated value, at the internal rate of return, of all the net cash flows which happen up to, and including, early repayment, and including the initial equity capital outlay.

7.5.7 Usually the IRR will be greater than the hurdle rate (otherwise the loan would not have been issued), hence the surrender fee is smaller when the IRR is used to discount future profit. The difference between the surrender fee calculated on the IRR basis and the surrender fee on the hurdle rate basis changes according to the time of surrender; it starts at the NPV of the whole loan and reduces to zero as the repayment date approaches the seven year average loan length.

7.5.8 Surrender values for the base model are shown in the last three columns

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Table 7.1. Monthly cash flows from a loan

A Month	B Loan at start of month	C Initial expenses owed to treasury at start of month	D Equity capital at start of month	E Debt capital at start of month	F Interest paid by borrower	G Return of principal by borrower	H Interest paid to treasury on loan	I Return to treasury of principal for loan	J Interest paid to treasury on expenses	K Return to treasury of principal for expenses	L Levy for other expenses
1	100000.00	2500.00	3000.00	2000.00	820.29	77.40	682.15	77.40	17.05	22.14	41.57
2	99922.60	2477.86	2997.68	1998.45	819.66	78.03	681.62	78.03	16.90	22.29	41.54
3	99844.57	2455.56	2995.34	1996.89	819.02	78.67	681.09	78.67	16.75	22.45	41.51
4	99765.89	2433.11	2992.98	1995.32	818.37	79.32	680.55	79.32	16.60	22.60	41.47
5	99686.57	2410.52	2990.60	1993.73	817.72	79.97	680.01	79.97	16.44	22.75	41.44
6	99606.60	2387.76	2988.20	1992.13	817.07	80.63	679.47	80.63	16.29	22.91	41.41
7	99525.97	2364.85	2985.78	1990.52	816.41	81.29	678.92	81.29	16.13	23.07	41.37
8	99444.68	2341.79	2983.34	1988.89	815.74	81.96	678.36	81.96	15.97	23.22	41.34
9	99362.73	2318.56	2980.88	1987.25	815.07	82.63	677.80	82.63	15.82	23.38	41.31
10	99280.10	2295.18	2978.40	1985.60	814.39	83.31	677.24	83.31	15.66	23.54	41.27
11	99196.80	2271.64	2975.90	1983.94	813.71	83.99	676.67	83.99	15.50	23.70	41.24
12	99112.81	2247.94	2973.38	1982.26	813.02	84.68	676.10	84.68	15.33	23.86	41.20
13	99028.13	2224.08	2970.84	1980.56	812.32	85.37	675.52	85.37	15.17	24.03	41.17
14	98942.76	2200.05	2968.28	1978.86	811.62	86.07	674.94	86.07	15.01	24.19	41.13
15	98856.69	2175.86	2965.70	1977.13	810.92	86.78	674.35	86.78	14.84	24.35	41.10
16	98769.91	2151.51	2963.10	1975.40	810.20	87.49	673.76	87.49	14.68	24.52	41.06
17	98682.42	2126.99	2960.47	1973.65	809.49	88.21	673.16	88.21	14.51	24.69	41.02
18	98594.21	2102.30	2957.83	1971.88	808.76	88.93	672.56	88.93	14.34	24.86	40.99
19	98505.28	2077.44	2955.16	1970.11	808.03	89.66	671.95	89.66	14.17	25.03	40.95
20	98415.62	2052.42	2952.47	1968.31	807.30	90.40	671.34	90.40	14.00	25.20	40.91
21	98325.22	2027.22	2949.76	1966.50	806.56	91.14	670.72	91.14	13.83	25.37	40.88
22	98234.08	2001.85	2947.02	1964.68	805.81	91.89	670.10	91.89	13.66	25.54	40.84
23	98142.20	1976.31	2944.27	1962.84	805.05	92.64	669.48	92.64	13.48	25.72	40.80
24	98049.56	1950.59	2941.49	1960.99	804.29	93.40	668.84	93.40	13.31	25.89	40.76

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	M Monthly expenses	N Interest earned on equity capital	O Return of equity capital	P Net interest paid to debt capital	Q Net Cash flow at end of month	R Discount factor	S Present value of net cash flow	T Cash flows accumulated at IRR	U Early repayment fee based on IRR	V Fee as % of loan at end of month	W Fee/ monthly interest payment
1	10.00	19.30	2.32	2.32	66.68	0.98	65.68	66.68	2476.43	2.48	3.02
2	10.00	19.29	2.34	2.31	66.62	0.97	64.62	134.70	2452.71	2.46	2.99
3	10.00	19.27	2.36	2.31	66.55	0.96	63.58	204.07	2428.86	2.43	2.97
4	10.00	19.26	2.38	2.31	66.48	0.94	62.56	274.83	2404.86	2.41	2.94
5	10.00	19.24	2.40	2.31	66.41	0.93	61.55	347.00	2380.72	2.39	2.91
6	10.00	19.23	2.42	2.31	66.33	0.91	60.56	420.62	2356.43	2.37	2.89
7	10.00	19.21	2.44	2.30	66.26	0.90	59.58	495.70	2332.00	2.35	2.86
8	10.00	19.19	2.46	2.30	66.19	0.89	58.62	572.29	2307.43	2.32	2.83
9	10.00	19.18	2.48	2.30	66.12	0.87	57.67	650.42	2282.71	2.30	2.80
10	10.00	19.16	2.50	2.30	66.04	0.86	56.74	730.11	2257.84	2.28	2.77
11	10.00	19.15	2.52	2.30	65.97	0.85	55.82	811.39	2232.83	2.25	2.75
12	10.00	19.13	2.54	2.29	65.90	0.83	54.91	894.31	2207.67	2.23	2.72
13	10.00	19.11	2.56	2.29	65.82	0.82	54.02	978.89	2182.36	2.21	2.69
14	10.00	19.10	2.58	2.29	65.74	0.81	53.15	1065.17	2156.91	2.18	2.66
15	10.00	19.08	2.60	2.29	65.67	0.80	52.28	1153.18	2131.31	2.16	2.63
16	10.00	19.06	2.62	2.29	65.59	0.78	51.44	1242.97	2105.56	2.13	2.60
17	10.00	19.05	2.65	2.28	65.51	0.77	50.60	1334.55	2079.66	2.11	2.57
18	10.00	19.03	2.67	2.28	65.43	0.76	49.78	1427.99	2053.61	2.08	2.54
19	10.00	19.01	2.69	2.28	65.36	0.75	48.97	1523.30	2027.41	2.06	2.51
20	10.00	19.00	2.71	2.28	65.28	0.74	48.17	1620.53	2001.06	2.04	2.48
21	10.00	18.98	2.73	2.28	65.20	0.73	47.39	1719.72	1974.57	2.01	2.45
22	10.00	18.96	2.76	2.27	65.11	0.72	46.61	1820.91	1947.92	1.98	2.42
23	10.00	18.94	2.78	2.27	65.03	0.71	45.85	1924.14	1921.12	1.96	2.39
24	10.00	18.93	2.80	2.27	64.95	0.69	45.10	2029.46	1894.17	1.93	2.36

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	B Loan at start of month	C Initial expenses owed to treasury at start of month	D Equity capital at start of month	E Debt capital at start of month	F Interest paid by borrower	G Return of principal by borrower	H Interest paid to treasury on loan	I Return to treasury of principal for loan	J Interest paid to treasury on expenses	K Return to treasury of principal for expenses	L Levy for other expenses
25	97956.16	1924.70	2938.68	1959.12	803.53	94.17	668.21	94.17	13.13	26.07	40.72
26	97861.99	1898.63	2935.86	1957.24	802.76	94.94	667.56	94.94	12.95	26.25	40.68
27	97767.05	1872.39	2933.01	1955.34	801.98	95.72	666.92	95.72	12.77	26.42	40.64
28	97671.34	1845.96	2930.14	1953.43	801.19	96.50	666.26	96.50	12.59	26.61	40.60
29	97574.84	1819.36	2927.25	1951.50	800.40	97.29	665.61	97.29	12.41	26.79	40.56
30	97477.54	1792.57	2924.33	1949.55	799.60	98.09	664.94	98.09	12.23	26.97	40.52
31	97379.45	1765.60	2921.38	1947.59	798.80	98.90	664.27	98.90	12.04	27.15	40.48
32	97280.55	1738.45	2918.42	1945.61	797.99	99.71	663.60	99.71	11.86	27.34	40.44
33	97180.85	1711.11	2915.43	1943.62	797.17	100.53	662.92	100.53	11.67	27.53	40.40
34	97080.32	1683.58	2912.41	1941.61	796.34	101.35	662.23	101.35	11.48	27.71	40.36
35	96978.97	1655.87	2909.37	1939.58	795.51	102.18	661.54	102.18	11.30	27.90	40.32
36	96876.79	1627.97	2906.30	1937.54	794.67	103.02	660.84	103.02	11.11	28.09	40.27
37	96773.77	1599.88	2903.21	1935.48	793.83	103.86	660.14	103.86	10.91	28.28	40.23
38	96669.91	1571.59	2900.10	1933.40	792.98	104.72	659.43	104.72	10.72	28.48	40.19
39	96565.19	1543.12	2896.96	1931.30	792.12	105.58	658.72	105.58	10.53	28.67	40.14
40	96459.62	1514.45	2893.79	1929.19	791.25	106.44	658.00	106.44	10.33	28.87	40.10
41	96353.17	1485.58	2890.60	1927.06	790.38	107.31	657.27	107.31	10.13	29.06	40.06
42	96245.86	1456.52	2887.38	1924.92	789.50	108.19	656.54	108.19	9.94	29.26	40.01
43	96137.66	1427.25	2884.13	1922.75	788.61	109.08	655.80	109.08	9.74	29.46	39.97
44	96028.58	1397.79	2880.86	1920.57	787.72	109.98	655.06	109.98	9.54	29.66	39.92
45	95918.60	1368.13	2877.56	1918.37	786.81	110.88	654.31	110.88	9.33	29.86	39.87
46	95807.73	1338.27	2874.23	1916.15	785.90	111.79	653.55	111.79	9.13	30.07	39.83
47	95695.94	1308.20	2870.88	1913.92	784.99	112.71	652.79	112.71	8.92	30.27	39.78
48	95583.23	1277.92	2867.50	1911.66	784.06	113.63	652.02	113.63	8.72	30.48	39.74

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	M Monthly expenses	N Interest earned on equity capital	O Return of equity capital	P Net interest paid to debt capital	Q Net Cash flow at end of month	R Discount factor	S Present value of net cash flow	T Cash flows accumulated at IRR	U Early repayment fee based on IRR	V Fee as % of loan at end of month	W Fee/ monthly interest payment
25	10.00	18.91	2.82	2.27	64.87	0.68	44.37	2136.90	1867.07	1.91	2.33
26	10.00	18.89	2.85	2.27	64.78	0.67	43.64	2246.51	1839.81	1.88	2.29
27	10.00	18.87	2.87	2.26	64.70	0.66	42.93	2358.34	1812.41	1.86	2.26
28	10.00	18.85	2.90	2.26	64.61	0.65	42.22	2472.42	1784.85	1.83	2.23
29	10.00	18.83	2.92	2.26	64.53	0.64	41.53	2588.82	1757.14	1.80	2.20
30	10.00	18.82	2.94	2.26	64.44	0.63	40.85	2707.57	1729.27	1.78	2.16
31	10.00	18.80	2.97	2.25	64.35	0.62	40.18	2828.72	1701.26	1.75	2.13
32	10.00	18.78	2.99	2.25	64.27	0.61	39.52	2952.33	1673.09	1.72	2.10
33	10.00	18.76	3.02	2.25	64.18	0.61	38.87	3078.44	1644.76	1.69	2.07
34	10.00	18.74	3.04	2.25	64.09	0.60	38.23	3207.11	1616.29	1.67	2.03
35	10.00	18.72	3.07	2.25	64.00	0.59	37.60	3338.39	1587.66	1.64	2.00
36	10.00	18.70	3.09	2.24	63.91	0.58	36.98	3472.33	1558.87	1.61	1.96
37	10.00	18.68	3.12	2.24	63.82	0.57	36.37	3608.99	1529.93	1.58	1.93
38	10.00	18.66	3.14	2.24	63.72	0.56	35.77	3748.42	1500.84	1.55	1.89
39	10.00	18.64	3.17	2.24	63.63	0.55	35.18	3890.68	1471.59	1.53	1.86
40	10.00	18.62	3.19	2.23	63.54	0.54	34.60	4035.84	1442.19	1.50	1.82
41	10.00	18.60	3.22	2.23	63.44	0.54	34.03	4183.95	1412.64	1.47	1.79
42	10.00	18.58	3.25	2.23	63.35	0.53	33.46	4335.06	1382.93	1.44	1.75
43	10.00	18.56	3.27	2.23	63.25	0.52	32.91	4489.25	1353.06	1.41	1.72
44	10.00	18.54	3.30	2.22	63.15	0.51	32.36	4646.58	1323.04	1.38	1.68
45	10.00	18.51	3.33	2.22	63.05	0.50	31.83	4807.11	1292.87	1.35	1.65
46	10.00	18.49	3.35	2.22	62.96	0.50	31.30	4970.91	1262.54	1.32	1.61
47	10.00	18.47	3.38	2.22	62.86	0.49	30.78	5138.05	1232.06	1.29	1.57
48	10.00	18.45	3.41	2.21	62.76	0.48	30.26	5308.60	1201.43	1.26	1.53

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	B Loan at start of month	C Initial expenses owed to treasury at start of month	D Equity capital at start of month	E Debt capital at start of month	F Interest paid by borrower	G Return of principal by borrower	H Interest paid to treasury on loan	I Return to treasury of principal for loan	J Interest paid to treasury on expenses	K Return to treasury of principal for expenses	L Levy for other expenses
49	95469.60	1247.44	2864.09	1909.39	783.13	114.56	651.25	114.56	8.51	30.69	39.69
50	95355.04	1216.76	2860.65	1907.10	782.19	115.50	650.46	115.50	8.30	30.90	39.64
51	95239.54	1185.86	2857.19	1904.79	781.24	116.45	649.68	116.45	8.09	31.11	39.59
52	95123.09	1154.75	2853.69	1902.46	780.29	117.40	648.88	117.40	7.88	31.32	39.54
53	95005.68	1123.43	2850.17	1900.11	779.33	118.37	648.08	118.37	7.66	31.53	39.50
54	94887.31	1091.90	2846.62	1897.75	778.35	119.34	647.27	119.34	7.45	31.75	39.45
55	94767.97	1060.15	2843.04	1895.36	777.38	120.32	646.46	120.32	7.23	31.97	39.40
56	94647.66	1028.18	2839.43	1892.95	776.39	121.30	645.64	121.30	7.01	32.18	39.35
57	94526.35	996.00	2835.79	1890.53	775.39	122.30	644.81	122.30	6.79	32.40	39.30
58	94404.05	963.59	2832.12	1888.08	774.39	123.30	643.98	123.30	6.57	32.62	39.25
59	94280.75	930.97	2828.42	1885.61	773.38	124.31	643.14	124.31	6.35	32.85	39.19
60	94156.43	898.12	2824.69	1883.13	772.36	125.33	642.29	125.33	6.13	33.07	39.14
61	94031.10	865.05	2820.93	1880.62	771.33	126.36	641.43	126.36	5.90	33.30	39.09
62	93904.74	831.76	2817.14	1878.09	770.29	127.40	640.57	127.40	5.67	33.52	39.04
63	93777.34	798.23	2813.32	1875.55	769.25	128.44	639.70	128.44	5.45	33.75	38.98
64	93648.89	764.48	2809.47	1872.98	768.20	129.50	638.83	129.50	5.21	33.98	38.93
65	93519.40	730.50	2805.58	1870.39	767.13	130.56	637.94	130.56	4.98	34.21	38.88
66	93388.84	696.28	2801.67	1867.78	766.06	131.63	637.05	131.63	4.75	34.45	38.82
67	93257.21	661.84	2797.72	1865.14	764.98	132.71	636.15	132.71	4.51	34.68	38.77
68	93124.49	627.15	2793.73	1862.49	763.89	133.80	635.25	133.80	4.28	34.92	38.71
69	92990.70	592.23	2789.72	1859.81	762.80	134.90	634.34	134.90	4.04	35.16	38.66
70	92855.80	557.08	2785.67	1857.12	761.69	136.00	633.42	136.00	3.80	35.40	38.60
71	92719.80	521.68	2781.59	1854.40	760.57	137.12	632.49	137.12	3.56	35.64	38.54
72	92582.68	486.04	2777.48	1851.65	759.45	138.24	631.55	138.24	3.32	35.88	38.49

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	M Monthly expenses	N Interest earned on equity capital	O Return of equity capital	P Net interest paid to debt capital	Q Net Cash flow at end of month	R Discount factor	S Present value of net cash flow	T Cash flows accumulated at IRR	U Early repayment fee based on IRR	V Fee as % of loan at end of month	W Fee/ monthly interest payment
49	10.00	18.43	3.44	2.21	62.66	0.47	29.76	5482.62	1170.64	1.23	1.50
50	10.00	18.41	3.47	2.21	62.55	0.47	29.26	5660.18	1139.70	1.20	1.46
51	10.00	18.38	3.49	2.20	62.45	0.46	28.77	5841.38	1108.61	1.17	1.42
52	10.00	18.36	3.52	2.20	62.35	0.45	28.29	6026.26	1077.36	1.13	1.38
53	10.00	18.34	3.55	2.20	62.24	0.45	27.82	6214.93	1045.96	1.10	1.34
54	10.00	18.32	3.58	2.20	62.14	0.44	27.35	6407.44	1014.41	1.07	1.30
55	10.00	18.29	3.61	2.19	62.03	0.43	26.90	6603.89	982.70	1.04	1.27
56	10.00	18.27	3.64	2.19	61.92	0.43	26.45	6804.35	950.85	1.01	1.23
57	10.00	18.25	3.67	2.19	61.82	0.42	26.00	7008.91	918.84	0.97	1.19
58	10.00	18.22	3.70	2.19	61.71	0.41	25.56	7217.65	886.68	0.94	1.15
59	10.00	18.20	3.73	2.18	61.60	0.41	25.13	7430.66	854.37	0.91	1.11
60	10.00	18.17	3.76	2.18	61.49	0.40	24.71	7648.03	821.91	0.87	1.07
61	10.00	18.15	3.79	2.18	61.38	0.40	24.29	7869.85	789.30	0.84	1.02
62	10.00	18.13	3.82	2.17	61.26	0.39	23.88	8096.21	756.55	0.81	0.98
63	10.00	18.10	3.85	2.17	61.15	0.38	23.48	8327.20	723.64	0.77	0.94
64	10.00	18.08	3.88	2.17	61.04	0.38	23.08	8562.93	690.59	0.74	0.90
65	10.00	18.05	3.92	2.16	60.92	0.37	22.69	8803.48	657.39	0.70	0.86
66	10.00	18.03	3.95	2.16	60.80	0.37	22.31	9048.97	624.05	0.67	0.82
67	10.00	18.00	3.98	2.16	60.69	0.36	21.93	9299.49	590.56	0.63	0.77
68	10.00	17.97	4.01	2.16	60.57	0.36	21.56	9555.14	556.92	0.60	0.73
69	10.00	17.95	4.05	2.15	60.45	0.35	21.19	9816.04	523.15	0.56	0.69
70	10.00	17.92	4.08	2.15	60.33	0.35	20.83	10082.30	489.23	0.53	0.64
71	10.00	17.90	4.11	2.15	60.21	0.34	20.47	10354.02	455.16	0.49	0.60
72	10.00	17.87	4.15	2.14	60.09	0.33	20.12	10631.31	420.96	0.46	0.56

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	B Loan at start of month	C Initial expenses owed to treasury at start of month	D Equity capital at start of month	E Debt capital at start of month	F Interest paid by borrower	G Return of principal by borrower	H Interest paid to treasury on loan	I Return to treasury of principal for loan	J Interest paid to treasury on expenses	K Return to treasury of principal for expenses	L Levy for other expenses
73	92444.43	450.16	2773.33	1848.89	758.32	139.38	630.61	139.38	3.07	36.13	38.43
74	92305.05	414.03	2769.15	1846.10	757.17	140.52	629.66	140.52	2.82	36.37	38.37
75	92164.53	377.66	2764.94	1843.29	756.02	141.67	628.70	141.67	2.58	36.62	38.31
76	92022.86	341.04	2760.69	1840.46	754.86	142.84	627.73	142.84	2.33	36.87	38.26
77	91880.02	304.17	2756.40	1837.60	753.69	144.01	626.76	144.01	2.07	37.12	38.20
78	91736.02	267.05	2752.08	1834.72	752.51	145.19	625.78	145.19	1.82	37.38	38.14
79	91590.83	229.67	2747.72	1831.82	751.31	146.38	624.79	146.38	1.57	37.63	38.08
80	91444.45	192.04	2743.33	1828.89	750.11	147.58	623.79	147.58	1.31	37.89	38.01
81	91296.87	154.15	2738.91	1825.94	748.90	148.79	622.78	148.79	1.05	38.15	37.95
82	91148.08	116.01	2734.44	1822.96	747.68	150.01	621.77	150.01	0.79	38.41	37.89
83	90998.06	77.60	2729.94	1819.96	746.45	151.24	620.74	151.24	0.53	38.67	37.83
84	90846.82	38.93	2725.40	1816.94	745.21	90846.82	619.71	90846.82	0.27	38.93	37.77
Total payments					66101	100000	54969	100000	793	2500	3350
Present value of total					37418	30165	31117	30165	526	1320	1896

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Table 7.1 (continued). Monthly cash flows from a loan

A Month	M Monthly expenses	N Interest earned on equity capital	O Return of equity capital	P Net interest paid to debt capital	Q Net Cash flow at end of month	R Discount factor	S Present value of net cash flow	T Cash flows accumulated at IRR	U Early repayment fee based on IRR	V Fee as % of loan at end of month	W Fee/ monthly interest payment
73	10.00	17.84	4.18	2.14	59.96	0.33	19.78	10914.30	386.62	0.42	0.51
74	10.00	17.82	4.22	2.14	59.84	0.32	19.44	11203.11	352.14	0.38	0.47
75	10.00	17.79	4.25	2.13	59.72	0.32	19.11	11497.84	317.52	0.35	0.42
76	10.00	17.76	4.29	2.13	59.59	0.32	18.78	11798.64	282.77	0.31	0.38
77	10.00	17.73	4.32	2.13	59.46	0.31	18.46	12105.61	247.88	0.27	0.33
78	10.00	17.71	4.36	2.12	59.33	0.31	18.14	12418.90	212.85	0.23	0.28
79	10.00	17.68	4.39	2.12	59.20	0.30	17.83	12738.63	177.70	0.19	0.24
80	10.00	17.65	4.43	2.12	59.07	0.30	17.52	13064.94	142.42	0.16	0.19
81	10.00	17.62	4.46	2.11	58.94	0.29	17.22	13397.96	107.00	0.12	0.14
82	10.00	17.59	4.50	2.11	58.81	0.29	16.92	13737.84	71.46	0.08	0.10
83	10.00	17.56	4.54	2.11	58.68	0.28	16.63	14084.71	35.79	0.04	0.05
84	10.00	17.54	2725.40	2.10	2779.37	0.28	775.67	17159.56	0.00		
Totals	840	1555	3000	187	8018						
Present values	471	880	905	106	3768						

Sum of present value of monthly cash flows	3768.37
Initial outlay of equity capital	3000.00
Net present value of loan	768.37

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of Table 7.1. These values have been calculated so that the IRR achieved by the lender on equity capital is not affected by the early repayments. The values are given in absolute terms, as a fraction of the loan outstanding at the time, and in terms of the next monthly interest payment (this last figure is a multiple of the interest payment, not a percentage). The values are very large at the start, as they have to cover the cost of the initial expenses, and they do not decrease quickly. The value of accumulated cash flows (column T) is calculated by accumulating the previous month's value at the internal rate of return and adding this month's net cash flow. The early repayment fee (column U) is calculated from: accumulated value of initial capital outlay – column T + column C – column D. (The surrender fee based on keeping the NPV fixed is £3,239.55 at the end of the first month, £2,909.03 at the end of the first year and £1,633.38 at the end of the fourth year.)

7.5.9 Imposing such fees would be commercially implausible. (Although large surrender fees are charged in the market if the borrower has chosen a mortgage with low introductory interest rates, which cause the lender to make a loss on the loan over this introductory period.) An alternative would be to predict the rate of early repayments, charge a nominal fee to those who do repay early, and charge a higher interest rate to all borrowers to compensate for the expected reduction in number of interest payments received. This is the second method we model.

7.5.10 As an example, we consider a situation starting with the base model, but where 1% of the loans remaining are repaid each month after the end of the first year (in which no early repayments are allowed). This leads to 48.5% of loans surviving to the end of the seventh year. A fee of £50 is charged for each early repayment before 5 years are complete, with no fee charged to those who repay after this point.

7.5.11 As with the base model, we ignore the gains made by loans extending beyond 7 years, and assume all loans which reach the end of the seventh year are repaid then.

7.5.12 A portfolio of loans which has this repayment pattern would produce the following results, (where the NPV is per loan of £100,000):

— net present value at the 20% hurdle rate, NPV = £237.78;
— internal rate of return (annualised value), IRR = 23.09%; and
— break-even loan rate at the 20% hurdle rate, 10.22%.

7.5.13 This portfolio is still profitable, as the market rate of interest, taken as 10.3%, is higher than the break-even loan rate, but the profits are lower than for the loans of the base model, because the repayment fees are lower than the 'neutral' fees shown in Table 7.1.

7.5.14 There are problems with this approach of relying on small fees and a sufficiently high rate of interest on the loan to cover the losses due to early repayment of mortgages. If there are more repayments than expected, especially at the earliest times, the fees will not be sufficient. These problems do not arise with the 'neutral' surrender fees, because each loan produces the same internal rate of return (or NPV, see ¶7.5.5) no matter when it is repaid.

7.5.15 Calculating correct charging structures is an important part of the use of cash flow models. Many banks and building societies offer good initial discounts as a marketing tool, but few offer loyalty bonuses. This may work against long-term profitability and create cross-subsidies, as it may increase surrender rates.

7.5.16 If the repayment rate were to increase to 2% per month (double the expected value in this model), there would be a loss (NPV = – £127.09 per £100,000). The loan interest rate would have had to be at least 10.35% to produce a positive NPV.

7.5.17 This vulnerability to borrowers switching to other lenders will exist unless fees are charged which are sufficient to pay for initial expenses (or the loan contract prohibits early termination of the mortgage). Moreover, if fees are not charged, higher interest rates will have to be charged to all borrowers, and this will encourage borrowers to look for other lenders who have not increased the margin on their rates. Borrowers who stay with a lender will be subsidising those who leave.

7.5.18 The difficulties with early repayment arrangements (very high fees or high interest rates and the risk of losses) indicate the importance of the repayment clause in the mortgage contract. They also underline the need for a pricing model which takes into account timings of events and recognises the existence of initial costs distinct from general expenses.

7.5.19 If the difference between initial expenses and other expenses were ignored, the repayment fees would be enormously reduced. For example, if we start with the base model, but replace the initial expenses with an increased levy of 0.873% per month (see ¶7.8.2.2 for the motivation for this choice), the repayment fees are actually slightly negative when calculated to make the IRR independent of early repayment. (The alternative surrender fee, based on maintaining NPV, is £1,197 at the end of the first month.) This substantial decrease in fees occurs because no account is taken of the levy income which is forgone when this loan stops early; thus the model implicitly assumes new, equally profitable loans will replace the ones ending early.

7.5.20 In ¶7.5.1 the possibility of some form of bonus was mentioned for those who remain with the lender beyond the average term. We will not examine potential schemes in detail here, but we note that, if the base model loan were to continue beyond seven years, the sum of the monthly profits (in excess of an annualised 20% charge for equity capital) earned in the 8th year amounts to £1,057.11. This gives an indication of the size of bonus which might be made available. (This value applies if no expense levy is charged once a loan has passed the seventh year. It would be £609.01 if the levy was maintained.) The corresponding values for the 10th, 15th, 20th and 25th years are: £1,001.44, £803.66, £480.77 and – £46.38 (a loss; this is due to the inclusion of a constant £10 per month running cost in the loan, whereas all other items have been

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substantially reduced by the last year). Furthermore, as there is less risk associated with loans once the amount of principal outstanding has been reduced (see Section 7.6), a lower hurdle rate would be suitable for this stage of the mortgage, and the excess profits could be regarded as even larger.

7.5.21 A second quantity which illustrates the value to the lender of borrowers who stay for a long time is the internal rate of return. In Table 7.2 the IRR is given as a function of the time the loan is repaid. No early repayment fees or loyalty bonuses were included in the calculations for Table 7.2. Note that the internal rate of return does not increase substantially for loans lasting beyond about 10 years. This is partly because the size of the loan has decreased by this time, hence, although the returns over the later period are high, the amounts involved are small in comparison with the initial capital outlay. Another factor is that a given positive cash flow has less impact on the IRR where the IRR is already large and the cash flow has to be discounted over a long time.

7.5.22 Without some kind of loyalty bonus structure, borrowers do not have the incentive to take decisions which are economically beneficial to the bank, i.e. to remain long-term borrowers. Banks and building societies rely on loyalty and inertia, which, as marketing methods improve, may be eroded.

Table 7.2. Internal rate of return for various repayment times

Time loan is repaid (years)	Internal rate of return on equity capital (%)
2.5	2.37
3.0	10.37
5.0	24.39
7.0	28.29
10.0	31.61
15.0	32.86
20.0	33.05
25.0	33.06

7.6 *Loan Defaults*

7.6.1 Some borrowers will default on the repayment of their loans, and the lender will not be able to recover the full amount of the outstanding principal.

7.6.2 The frequency of default and the costs of default must both be included in any model. These are essentially the risk parameters in the cash flow model, analogous to, for example, disability rates in an actuarial premium setting model. They will have to be estimated in advance, perhaps from historical data relating to similar loans (see Section 6).

7.6.3 Defaults may be included in cash flow models by introducing 'survival probabilities', and multiplying terms by these probabilities. As mortgages are secured on property, an extra term must be added to take account of the money recovered by selling the properties which were being purchased by the mortgages which have now defaulted.

7.6.4 In Section 7.8 we examine the impact of default rates and costs on the loan profitability; here we describe how the default rate and costs are parameterised.

7.6.5 For mortgages the risk to the lender is mainly concentrated in the first few years of the loan, because, unless house prices fall, if a borrower has repayment problems later on the mortgage will be covered by the house value — so that the bank could either try to rearrange the mortgage or take possession of the property. Two factors cushion the financial impact of default in the later years of the mortgage: a likely upward drift in house prices; and the fact that capital is paid off during the loan. Additionally, the likely increase in income of the mortgagee over time reduces the default probability.

7.6.6 There are many possible ways to parameterise the cost of default. For illustrating the effect of defaults, we assume that the default rate is constant for some period, and there are no defaults after this period. As a starting point, we take the period to be 3 years. The cost of default is related to the loan-to-house-price ratio at the time of default. The expression we use for the ratio of the cost of default to the amount of loan outstanding is:

$$\frac{\text{Cost of default}}{\text{Loan outstanding}} \equiv f_t = \begin{cases} (L_t - L_{36})/L_t + 0.05 & t \leq 36 \\ 0 & t > 36. \end{cases}$$

The time t is measured in months, and L is the size of the loan. (More precisely L_t is the intended size of the loan at the start of month t according to the repayment schedule. This may differ from the actual amount outstanding if the borrower is in arrears with payments.) The motivation for this formula is as follows: the cost of default is equal to the amount of loan outstanding, including arrears, *minus* the sale price of the property. This may be rewritten as: scheduled loan outstanding plus accumulated debt *minus* sale price of the property. The formula includes assumptions for the size of the debt and for the house price. The accumulated debt is taken to be 5% of L_t; this is roughly half a year's missed payments. The assumption for the house price is that it has fallen below the original amount of the loan. For convenience, we assume that the level the house price has fallen to is the same as the amount outstanding on the mortgage after 3 years. (This means that the house price has fallen to about £96,900, compared with an initial loan of £100,000.) With the parameters of the base model, the maximum value of f_t is 8.12% (in the first month) and falls to 5% after three years. There are assumed to be no later defaults. Both the default parameters and costs of default should be estimated from empirical data. Techniques familiar to actuaries can be used. We are simply using these assumptions to illustrate the cash flow technique in the absence of realistic data.

7.6.7 Starting with the base model of Section 7.4, and assuming the default rate is 0.2% per month throughout the first three years, the loan has the following properties:

— net present value at the 20% hurdle rate, NPV = £227.95;
— internal rate of return (annualised value), IRR = 22.36%; and
— break-even loan rate at the 20% hurdle rate, 10.23%.

7.6.8 This shows that the lending is still profitable at an interest rate of 10.3%, although the profitability has been reduced by the defaults, and the profits would be below the required level if the interest rate had to be reduced by 0.1%. The default rate of 0.2% per month for three years, as used in the model, is quite high for mortgages. Taken together with the seven year typical duration of a loan, it gives an annual default rate of close to 1%, which is similar to the number of possessions which took place at the height of the housing market problems at the start of the 1990s.

7.6.9 In theory, interest rates could be varied according to the level of risk. With mortgages this would mean that they fall as the loan is paid off. (In practice, the opposite is more likely, as banks offer low introductory rates to attract customers.) Walsh & Booth (1997) show that, if a lender charges a high rate of interest at the outset of a mortgage and reduces it as the mortgage progresses, the average interest rate charged over the duration of the loan can be lower than the flat interest rate which does not respond to risk. However, the reduction was small (less than 0.1%). The issues here are similar to those relating to surrender penalties. Current marketing trends lead banks to charge rates which are 'actuarially too low' at early durations and 'actuarially too high' at late durations. Banks and building societies could be vulnerable to competition taking remortgage business of mature loans if customer inertia decreases. There is evidence that lenders are becoming more sensitive to re-mortgaging, and are offering special terms to customers who they are aware may be about to re-mortgage.

7.7 *Analysis of Expense Assumptions*

7.7.1 We noted, in Section 7.3.3, that initial expenses might be met by capital, on the grounds that there is some risk attached to their recovery due to defaults and early repayments. Here we examine the returns achieved under two methods of paying for initial expenses (borrowing or capital). This is done for the situation where the default rate varies, and where it is assumed that all repayments are at seven years. The starting point for the calculations is the base model.

7.7.2 The internal rate of return has been calculated for three default rates, where the costs of default are as in Section 7.6, and there are no defaults beyond three years. The results are shown in Table 7.3.

7.7.3 Using capital to pay for initial expenses is very expensive, causing a substantial drop in the returns generated by the loan. Paying for the initial expenses should reduce the risks of subsequent losses (as there is less owed to the treasury), so the bank may be able to operate with less equity capital. However, even if the equity capital backing the loan is reduced to the minimum allowed (2% of the loan), the returns are still worse than if initial expenses are borrowed.

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Table 7.3. Internal rates of return

Method of financing initial expenses	Initial equity capital backing loan	Monthly default rates throughout the first three years		
		0.0%	0.2%	0.4%
		Internal rate of return		
Borrowing	£3,000	28.29%	22.36%	17.18%
Capital	£3,000	20.74%	17.40%	14.26%
Capital	£2,000	24.28%	19.88%	15.83%

7.7.4 However, the volatility of returns (caused by varying default rates) is reduced by using capital to pay for initial expenses, hence a lower hurdle rate might be acceptable where there is this extra capital outlay.

7.7.5 The gap between the IRR in the borrowing scenario compared with the IRR in the expenses from capital scenario decreases as the default rate increases. If the default rates get very high (over 0.8% per month), then the IRR is higher in the case where capital is used for initial expenses; but this is because the IRR has fallen below the cost of funds (i.e. borrowing is more expensive than using capital).

7.7.6 We offer no further guidance as to the treatment of expenses in the model. Whilst it is a non-trivial problem, the particular approach may not matter too much, as long as the relationship between the treatment of expenses and the appropriate hurdle rate is understood.

7.8 *Parameter Dependence*

7.8.1 *Introduction*

7.8.1.1 Table 7.4 shows how the net present value (at a 20% hurdle rate), the internal rate of return and the break-even loan rate (also at a 20% hurdle rate) change as the various inputs of the cash flow model are altered one at a time. These three values are given for the case where initial expenses are met by borrowing. (One line in the table uses a 25% hurdle rate rather than 20%. For this entry, both the NPV and break-even loan rate are calculated at 25%.)

7.8.1.2 The base model has the following parameters: length of loan $n = 25$ years; time loans are repaid $n_r = 7$ years; period over which initial expenses are amortised $n_E = 7$ years; size of loan $L_0 = £100{,}000$; initial equity capital $K_0 = 3\%L_0$; initial debt capital ${}^{T_2}K_0 = 2\%L_0$; initial expenses $E_0 = £2{,}500$; monthly running expenses $E_t = £10$; levy for other expenses $= 0.5\%L_t$; interest earned on set aside capital $r_C = 8\%$; cost of funds $r_F = 8.5\%$; interest charged on debt capital $r_{T_2} = 9.5\%$; rate charged on the loan $r_L = 10.3\%$; and hurdle rate $r_H = 20\%$.

7.8.1.3 All other entries in the table differ only in one value, except in lines E and F where n_r and n_E are changed together.

7.8.1.4 The effect of varying parameters is discussed below. The base model has no defaults; and has all loans repaid at the same time with no repayment fee. The effect of varying the default rate and costs and of varying repayment fees

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Table 7.4. The dependence of profitability on the parameters of the loan

Parameter changed	New value	NPV £	IRR %	Break-even loan rate %	Line
None		768.37	28.29	10.08	A
L_0	£50,000	-774.16	4.59	10.74	B
L_0	£150,000	2,310.89	37.09	9.86	C
n	15 years	552.77	26.54	10.13	D
n_r & n_E	10 years	1319.82	32.68	9.97	E
n_r & n_E	5 years	202.98	22.55	10.23	F
K_0/L_0	2%	1,173.22	39.85	9.96	G
$^{T2}K_0/L_0$	3%	715.75	27.71	10.09	H
E_0	£2,750	583.79	26.26	10.13	I
E_t	£11	721.28	27.77	10.09	J
levy/L_t	0.55%	579.21	26.20	10.13	K
r_H	25%	268.32	28.29	10.21	L
r_L	11.3%	4,255.40	72.50	10.08	M
r_F	7.5%	4,365.11	74.40	9.05	N
r_{T2}	10.5%	698.70	27.52	10.10	O
r_C	8.5%	856.74	29.28	10.05	P

and the distribution of repayment times are shown in Sections 7.8.7 and 7.8.6 respectively. In a practical application of the model, defaults would, of course, need to be incorporated at every stage. However, in this illustration, they would simply obscure the parameter sensitivity analysis.

7.8.2 *Expenses: see lines I, J and K*

7.8.2.1 Table 7.4 shows the effect of increasing the three components of expenses, independently, by 10%. As the (borrowed) initial expenses and the levy give rise to much larger costs than the running expenses, they have a larger impact. However, a change of just £1 per month in respect of this £100,000 loan alters the annual return on equity by 0.5%. Since expenses are unlikely to be known to such precision, there will inevitably be considerable uncertainty in the expected return.

7.8.2.2 In practice, lenders do not treat initial expenses differently from other expenses. We now consider removing initial expenses from the cash flow models and adjusting the expense levy. In the base model the total amount raised towards other expenses by the levy is £3,349.90, compared with £2,500 for initial expenses and £792.58 of interest paid on the money borrowed to pay for the initial expenses. If all initial expenses are ignored, and, consequently, the levy is increased to make up the difference, the effect is as follows:

— Increasing the levy (to 0.991%) to match the initial expenses and associated interest the NPV becomes £760.39; the IRR is 28.14% and the break-even loan rate is 10.08%.

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— Increasing the levy (to 0.873%) to match the initial expenses only the net present value becomes £1,205.14, the internal rate of return becomes 33.13% and the break-even loan rate becomes 9.95%.

7.8.2.3 The first of these two adjustments makes little difference to the profitability of the loan, while the second alternative makes the loan look much more attractive to the lender. However, it is this second method which more accurately reflects the way business is done and the total level of expenses. The reason that loans look substantially better (from the lender's viewpoint) is that each generation of loans is receiving a subsidy from previous generations. The cash flow model we propose avoids this problem.

7.8.2.4 Note that, if the levy is used to meet all expenses, and if the levy is proportional to the size of the loan, all monetary values will scale exactly with the loan size. The internal rate of return and the break-even loan rate will be independent of L_0. This is another feature of current methodology which leads to inaccurate pricing of loans.

7.8.2.5 If everything is as in the base model, except that initial expenses are met from capital rather than from borrowing, the loan has a NPV of £114.16, an IRR of 20.74% and a break-even loan rate of 10.27%.

7.8.3 *Interest rates: see lines L, M, N, O and P*

7.8.3.1 Changing the hurdle rate has much less impact on profitability than changing the margin on the loan. A 1% increase in the margin improves the IRR by over 40%. (A 0.1% increase in margin improves the IRR by nearly 4%.)

7.8.3.2 The interest earned on set-aside capital and the interest paid to the holders of debt capital do not (on their own) have much impact on the rate of interest which should be charged on the loan. Nevertheless, the return on equity does vary, emphasising just how sensitive this quantity is to almost every facet of the loan.

7.8.4 *Size and length of loan: see lines B, C and D*

7.8.4.1 Large loans are much more profitable than small loans, both in absolute terms and per unit of capital deployed. This suggests that interest rates which vary with loan size would be an appropriate charging policy, as has been noted in ¶7.8.2.4. This effect is masked by the pricing methodology which does not allow specifically for 'set up' expenses and fixed repayment expenses.

7.8.4.2 Short loans produce less profit than long loans. This, again, is something masked by pricing according to an interest margin, i.e. pricing without using cash flow projections.

7.8.5 *Capital: see lines G and H*

The amount of equity capital is much more significant than the amount of debt capital. If the total capital is fixed at 5% of the loan, but the proportions of the equity and debt capital are reversed compared with the base model (so equity capital is 2% and debt capital 3% of the loan) the NPV would be £1,120.42, the IRR 38.90% and the break-even loan rate 9.98%. It is clear that the efficient deployment of capital is important in any banking operation.

7.8.6 *Repayments: see lines E and F*

7.8.6.1 Table 7.4 shows that, if the typical duration of a loan changes by two or three years, then the IRR changes by around 5% and the break-even loan rate by 0.1%. (As the typical duration is altered in lines E and F, we also change the period over which initial expenses are amortised. The period is chosen as the typical duration in each case; see ¶7.3.3.5.) The control of early repayments and the encouragement to borrowers to keep loans on the book should be an important aspect of pricing and marketing policy. If an appropriate charging and incentive structure does not exist, profitability on new tranches of loans could be reduced significantly if customer inertia reduces.

7.8.6.2 It was noted that the repayment fees can be very large, and an alternative fee structure (with lower fees and consequently higher interest rates) was compared in ¶¶7.5.10 to 7.5.16. Table 7.5 illustrates the effect of changing the repayment rate and fee payments on the loan profitability. Table 7.5 is similar to Table 7.4, in that it starts with a particular model, and then shows the effect of varying one parameter at a time. The starting point is the base model together with the fee structure of ¶7.5.10, i.e. a fee of £50 is charged for repayments before the end of the fifth year, there are no early repayments in the first year and repayments happen at a rate of 1% per month thereafter.

Table 7.5. The dependence of profitability on parameters relating to early repayment

Parameter changed	New value	NPV £	IRR %	Break-even loan rate %	Line
None		237.78	23.09	10.22	A
Fraction repaying each month	2%	−127.09	18.07	10.35	B
Fee	100	249.22	23.24	10.21	C
Fees stop	end of fourth year	236.01	23.07	10.22	D
Repayments start	first month	−15.35	19.79	10.31	E

7.8.6.3 Because the fees are only token amounts, doubling the fee (line C) or stopping the fees one year earlier (line D) make little difference. Allowing repayments in the first year (line E), without imposing large fees, leads to a reduction in IRR of 3.3%, or makes it necessary to raise interest rates for all borrowers by at least 0.09% to maintain the return on equity. If repayments are more frequent than planned for (line B), the internal rate of return can be substantially reduced, again illustrating the advantage of charging surrender fees which actually reflect the cost to the lender.

7.8.6.4 We have, so far, considered how parameters relating to repayments

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and repayment fees influence loan performance. Another important aspect is how the other loan parameters affect repayment fees. Table 7.6 gives actuarially neutral surrender fees at the end of the first, third and fifth years, as a monetary amount and in terms of the borrower's monthly interest payments. These are calculated using the equation in ¶7.5.4, so that the internal rate of return is unaffected by early repayments. Each loan differs from the base model by one parameter, except for line E where n_r and n_E are changed together.

Table 7.6. Early repayment fees

Parameter changed	New value	End of first year		End of third year		End of fifth year		Line
		Fee £	Fee / interest	Fee £	Fee / interest	Fee £	Fee / interest	
None		2,208	2.72	1,559	1.96	822	1.07	A
L_0	£150,000	2,209	1.81	1,561	1.31	822	0.71	B
E_0	£2,750	2,428	2.99	1,715	2.16	905	1.17	C
Levy/L_t	0.55%	2,207	2.72	1,558	1.96	822	1.07	D
n_r & n_E	5 years	2,063	2.54	1,097	1.38	0	NA	E
r_L	10.5%	2,209	2.67	1,562	1.93	823	1.05	F

7.8.6.5 The repayment fee, based on ensuring that the same internal rate of return is achieved whether the loan is repaid early or not, depends only on the initial expenses (line C) and the average duration of the loan (line E); it does not depend on the size of the loan (B), the interest rate charged (F) or the levy for other expenses (D). This means that the proportionate repayment fee should be larger for small loans.

7.8.7 *Defaults*

7.8.7.1 In this section we examine the sensitivity of the loan's performance to the parameters in the default model, introduced in Section 7.6. The model included a default cost which combined a fixed proportion of the loan (5%, which represents payments in arrears) together with an extra amount which decreases as the loan is paid off (this represents the difference between the outstanding loan and the sale price of the house). Defaults were assumed to cease after 3 years, and before this the default rate was 0.2% per month.

7.8.7.2 The parameter sensitivity is shown in Table 7.7, which is similar to both Tables 7.4 and 7.5. In this case the starting point is the base model together with the default parameters of Section 7.6. (In the fifth line, where the defaults continue to the end of the fifth year, the loan loss fraction is given by an amended form of the expression given in ¶7.6.6. Instead of the house price falling to the same level as the loan remaining after 3 years, it is taken to have fallen to the same level as the loan remaining after 5 years. Hence L_{36} is replaced by L_{60}.)

7.8.7.3 Changing the number and costs of default does have a significant

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Table 7.7. The dependence of profitability on parameters relating to loan default

Parameter changed	New value	NPV	IRR	Break-even loan rate
		£	%	%
None		227.95	22.36	10.23
Monthly defaults	0.4%	−282.65	17.18	10.39
Fixed part of cost	10%	−29.92	19.70	10.31
Fixed part of cost	0%	485.83	25.25	10.15
Defaults stop	End of fifth year	−102.60	18.94	10.33
Include time-dependent cost?	No	323.82	23.42	10.20

effect on the returns achieved by the lender. However, the impact is not as large as for some of the other parameters, namely: the size of the loan; the average duration of the loan; and the interest margin. Nevertheless, there is clearly a case for differential pricing based on default risk, even if this only takes into account factors such as loan to value ratio.

7.9 *Other Issues*

7.9.1 *Synergies*

7.9.1.1 We noted, in Section 7.3.3, that cross-subsidies are implicit in any calculation which pools the income and expenses for a portfolio of loans rather than focusing on a particular cohort of loans. This comparison of the individual loan with the portfolio approach leads us to the question of synergies between different loan portfolios. A number of different forms of synergy might exist. For example, there may be expense synergies, tax synergies or risk synergies. The first two can be dealt with by taking a marginal expense and tax rate approach to pricing (actuaries will be familiar with these concepts from the tax and expense synergies which arise with the 'interest minus expenses' tax basis discussed, for example, in Hylands & Gray, 1989).

7.9.1.2 Risk synergies are more difficult to deal with. They arise from the aggregation of less than perfectly correlated risks, which can reduce the variability of default rates, and, therefore, reduce economic (although not necessarily regulatory) capital. We do not believe that there would be a significant reduction in risk from the marginal addition of loans to a mortgage portfolio (but there may be a reduction caused by geographical diversification of the portfolio, as house price inflation does vary markedly between regions). Risk synergies would, however, be important for large project lending and, possibly, for lending to large corporations. We believe that approaches suggested in Lewin *et al.* (1995) may well be appropriate (see Section 8).

7.9.2 *Whole project cash flows*

7.9.2.1 When estimating the profitability of a new line of business, say personal loans sold by telephone, working out the value of each loan is not sufficient. There will be substantial start-up costs, and marketing expenses may be much higher per loan actually arranged in the first year or so compared with loans made later. These extra costs must be spread across all loans of this class made over a period of several years. A full cash flow analysis will, therefore, have to include these initial costs, estimates of the growth in volume of lending (e.g. quarterly estimates for the first five years), together with the income and outgo pertaining to each loan.

7.9.2.2 A new line of business would be worthwhile if the combined net present value of all future loans generated by it exceeds the net present value of the extra outlays, including the required return on capital. However, once a line of business is up and running, it should be continued so long as the on-going net income exceeds the cost of capital. An individual loan should be issued if the expected net present value is positive.

7.9.2.3 Again, these issues are not significantly different from those handled in actuarial cash flow analysis.

7.9.3 *Uncertainty*

7.9.3.1 Although the expected NPV is a useful quantity to calculate, it is not the only important value, and the requirement that NPV > 0 is not the only way to decide whether the terms of a loan provide adequate returns for the bank's shareholders.

7.9.3.2 The distribution of NPV or accumulated cash flows is important. In particular, if the NPV is highly uncertain for a particular line of business, then having the expected NPV as positive at the hurdle rate used on other lines of business may not be enough. Possible quantitative measures of uncertainty are the standard deviation of the NPV or of the internal rate of return; the latter may be particularly relevant to shareholders who regard the variance of return on possible investments as a good measure of risk. (The uncertainty could be caused by difficulties in predicting default rates, early repayments and so on.)

7.9.3.3 In practice, the more uncertain the profit of a loan product is, the higher the rate of return that it will be required to produce. This will be reflected in a higher hurdle rate used in the calculation of NPV. Then, so long as the expected NPV at this rate is positive, the loan will be deemed worth providing. It is, therefore, important to adjust the hurdle rate to allow for the risk of the tranche of business if a given amount of capital is held against the business. Therefore, to analyse the profitability of business, it is not only necessary to differentiate between lines where expected values of parameters are different, but also between lines where the degree of certainty attached to parameters varies.

7.9.4 *Comparison of cohort-based pricing with portfolio-based pricing*

7.9.4.1 Loans can be priced without going into as much detail as we have done earlier in this section.

7.9.4.2 Suppose that the lender continually issues loans, each with the same properties as the base model loan, subject to the default risks and costs as described in Section 7.6. Each year, for every £1 million of outstanding loans,

supported by £30,000 of equity capital, the income and expenditure will comprise the following:
— loan interest income of £98,435;
— other interest (a net outgo) of £81,010;
— expenses of £10,170;
— loan default costs of £791; and hence
— net cash flow of £6,464.

7.9.4.3 A return on equity can be calculated from net cash flow ÷ capital, giving 21.55%. The corresponding value from the full cash flow model is 22.36% (first line in Table 7.6). Similarly, since the lender 'breaks even' when the net cash flow equals the cost of equity capital, a break-even loan rate can be calculated as:

$$10.30\% - (\text{net cash flow} - \text{hurdle rate} \times \text{equity capital})/£1 \text{ million}$$

which gives 10.25%, whereas the value from Table 7.6 is 10.23%. These whole portfolio calculations could also be done on a monthly or continuous basis. Using the monthly flows leads to an annualised return on equity of 23.81% and break-even loan rate of 10.20%.

7.9.4.4 These differences between the detailed cash flow model and the annual accounting-based model are not very large in comparison to some of the uncertainties in the cash flow model and the sensitivity to some parameters. However, problems arise when considering the effect of changing some value related to the loans and calculating the impact of the change. As an example, what happens if the default rate doubles (to 0.4% per month)? From Table 7.7, the internal rate of return and break-even loan rate change to 17.18% and 10.39% respectively.

7.9.4.5 The portfolio-based returns can be found most simply by doubling the default costs per year. This leads to an IRR of 18.91% and a break-even loan rate of 10.33%. Note that the full cash flow model has a decrease in IRR of 5.18%, while the portfolio method gives just 2.64%. In some situations the portfolio approach overestimates returns (e.g. in this higher default rate calculation), while in others it leads to underestimates (e.g. in the original calculation).

7.9.4.6 The portfolio approach will provide a good approximate method of pricing in stable business conditions when parameters do not differ much between loan types. However, the results of a cash flow approach are generally more appropriate, as they take into account the full economic costs of the cohort of loans.

7.10 *Pricing of Cash Backs*

7.10.1 *The pricing of features*

7.10.1.1 As well as showing whether a loan is likely to provide a profit when a particular interest rate is charged, cash flow models can also be used to examine possible ways of charging for features of loans. Such features include

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cash backs (which we will concentrate on), or low introductory interest rates (which are similar), fixed-interest rate guarantees, and agreements by the borrower to insure against missing loan payments.

7.10.1.2 Cash flow models are adaptable, which makes them particularly useful tools in an area such as bank lending, where the loans often come with a variety of features other than simply their term and their interest rate.

7.10.2 *Cash backs*

7.10.2.1 Many loans are provided which either give the customer some extra cash at the outset of the loan or offer some discount on the loan rate charged for the first year. These features are designed to attract customers to the bank, but an alternative would be to offer a constant rate of interest throughout the loan, which would be lower than the rate charged in the cash back scheme and lower than the rate charged beyond the first year in the discount scheme.

7.10.2.2 Any cash back would be met from equity capital. This makes it a very expensive feature. Starting with the base model (in which defaults and repayments before the seventh year are ignored), a cash back of 1% of the loan requires an increased interest rate of 10.37% to be charged to the borrower for the bank to break-even. This is 0.29% higher than the break-even rate without any cash back. This is quite a substantial difference in a competitive loan market. The extra interest rate increases nearly linearly with the amount of cash back.

7.10.2.3 If the portfolio-based annual cash flow method (Section 7.9.4) is used instead of the full cash flow model, the severity of the cost of cash backs appears to be greater, with the break-even loan rate increasing by 0.36%. (Cash backs enter the equation for the interest rate in two ways: a cash outflow spread over seven years; and an increase in the amount of capital on which a 20% return is needed.)

7.10.2.4 The impact of cash back on the break-even loan rate is greater for loans of shorter duration than the base model, and also for loans where the hurdle rate is larger. The size of the loan, the amount of equity capital, the level of expenses and the method of financing the initial expenses are all unimportant as far as the effect cash back has on the break-even loan rate.

7.10.2.5 Because cash backs are costly to the lender, surrender penalties based on future net interest payments which are no longer paid become very high for those who repay their loan early. The actuarially neutral surrender fees calculated for a loan with a 1% cash back, but where the other assumptions are as the base model, are £3,116 at the end of the first year and £2,244 after three years (the corresponding values without cash backs are given in the first line of Table 7.6 as £2,208 and £1,559). These surrender fees were calculated so that the internal rate of return was maintained on early repayment.

7.10.2.6 It is possible to separate the surrender fee into a return of cash backs and an additional amount. As an illustration, consider the repayment arrangements of ¶7.5.10, where a fee of £50 was payable on any loans terminating before the end of the fifth year, no repayments were allowed in the

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first year, and 1% of remaining loans were repaid each subsequent month. If £1,000 of cash back is provided to the borrower, with no adjustment to the surrender terms, the break-even loan rate becomes 10.57% (it was 10.22% without the cash back). If those who repay the mortgage within the first three years are required to return all of the cash back as well as the £50 fee, the break-even rate is 10.52%.

7.10.2.7 This shows that introducing this 'return of cash back' penalty enables the loan interest rate to be reduced by (only) 0.05%. The reduction would be greater if the expected early repayment rate were higher (the rate in the model is 1% per month after the end of the first year), or if the cash back were greater. However, this is not the end of the story. A cash back feature without a surrender penalty provides an option to the borrower. Without the surrender penalty, one would expect the option to be exercised by more customers, thus raising the early repayments and the break-even loan rate. Once again, we find familiar actuarial problems coming up in the banking area.

7.11 *Personal Loans*

Cash flow modelling can be used for personal loans as well as for mortgages. Some key parameters will change. The size of the loan will usually be much smaller and the duration shorter, the unrecovered portion of defaulted loans will be higher because such loans are usually unsecured, the regulatory capital requirements are twice as high and early repayments will be less prevalent. Walsh & Booth (1997) generalise the model in a way which makes it suitable for personal lending. There are no new issues in principle. Where the model can be applied to accounts which can be in credit or debit (for example current accounts), explicit account should be taken of the cost of account transactions. Currently there are cross subsidies on personal current accounts (interest is normally paid, but transactions are free). These could be monitored and costed using a cash flow approach.

7.12 *Customer Loyalty*

In this section, there has been much discussion of the problem of cross subsidies. We do not believe that all cross subsidies are necessarily inappropriate. Cross subsidies can be regarded as an essential marketing tool to generate customer loyalty, or could be regarded as fundamental to the philosophy of a mutual business such as a building society. However, we would make two points. Firstly, the cost of cross subsidies should be ascertainable. Secondly, some cross subsidies may be more appropriate than others. A small cross subsidy between large and small loans may be appropriate to build customer loyalty. Charging those who default an early repayment fee might be inappropriate. On the other hand, there is no clear reason why loyal customers should subsidise short-term customers. Indeed, as competition grows and inertia in the mortgage market decreases, that particular cross subsidy may cause serious difficulties to banks and building societies. We also see no clear reason why loyal customers should cross subsidise cash backs and short-term discounts on new mortgages.

8. COMPARISON OF MORTGAGE LENDING WITH CORPORATE LENDING

8.1 *Risk Factors and Risk Assessment*

8.1.1 In assessing lending risk, a lender can group large borrowers according to expected default risk, perhaps using about ten risk categories. The allocation of a borrower to a risk group will generally be based on accounting ratios. Statistical techniques (such as regression or multiple discriminant analysis, see Section 6) are applied to historical data on bankruptcies or loan defaults, together with these accounting ratios, to produce a set of weights for the ratios.

8.1.2 Any potential borrower's accounts can be studied to provide a 'score', which is meant to be a predictor of default risk.

8.1.3 The procedure has been in widespread use since the 1960s, and some of the formulae used have been published. It is often found, however, that weighting factors determined from one data set are not the same as those from another set: e.g. U.S. derived weights are not applicable in the U.K. and 1970s values are not useful now.

8.1.4 A review of many analyses of corporate default has been presented by Altman (1983). For a paper specifically relating to risk factors in the U.K., see Taffler (1982). References to some more recent papers may be found in Altman (1996); this paper also gives a formula for a score which is a predictor of default risk:

$$\text{Score} = c_0 + c_1X_1 + c_2X_2 + c_3X_3 + c_4X_4$$

with:
— X_1 = working capital / total assets;
— X_2 = retained earnings / total assets;
— X_3 = (earnings before interest and taxes) / total assets; and
— X_4 = equity (book value) / total liabilities.

The c_1, etc. are coefficients, all of which are positive. A high score is indicative of a low probability of default. The same author derived a formula 25 years earlier (given in Altman, 1983) which is quite similar, except that the coefficients are different, the original formula for X_4 used market value of equity, and there was a fifth ratio included (sales / total assets).

8.1.5 One practical difference between corporate lending and mortgage lending is the availability of accounting ratios for assessing the risk in large corporate lending. Another difference was mentioned in ¶6.1.2.5; default risks in corporate lending can be derived from analysis of a large set of companies which is not restricted to companies which have been clients of the lender. With

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mortgages, the bank will have more past information about its borrowers than it will about the population in general, and it is likely to base its statistical analysis on this limited group.

8.2 *Diversification of Risk*

8.2.1 Concentration of risk can be an important problem for corporate lending. One reason is that the number of large loans is small (relative to the number of mortgage loans, for example). Another reason is that the lender may have developed expertise in a few particular areas, such as loans relating to Latin America or the construction industry, and hence have a lack of diversification.

8.2.2 The same problems could occur in mortgage lending, in particular there is a risk for niche lenders which target one region or a specific risk group (such as borrowers wanting mortgages for the full value of the property); but for a national lender with a given amount of lending there should be less difficulty in diversifying a mortgage portfolio than a corporate loans portfolio.

8.3 *Required Returns*

8.3.1 The required return on a corporate loan may depend, not only on the risks associated with the borrower, but also on how the loan fits in with the lender's portfolio. A similar situation applies in investment, where the capital asset pricing model links the expected return on a security with the covariance of the return on this security with the return on the whole portfolio. Lewin *et al.* (1995) linked this to the required return on capital projects, the same arguments apply to corporate lending.

8.3.2 The connection between returns and the covariance with respect to the portfolio applies most naturally to cases where the distribution of returns is well represented by two parameters (mean and variance). This is not the case with loans, because these have a highly skewed distribution of returns with a well-defined maximum, which has a high probability of being achieved (this happens when the loan is repaid successfully), and a wide range of possible lower returns which depend on when the loan defaults and how much capital is repaid. It is possible to overcome this difficulty by treating each loan as a member of some risk group, and then comparing the behaviour of this group against the behaviour of the portfolio (Altman, 1996). 'Behaviour' might be more likely to refer to default rates rather than returns, because there is more information available on corporate defaults than on returns on particular categories of loan. (The returns of a homogenous group of loans will tend towards having a normal distribution, due to the central limit theorem.)

8.3.3 The return required on mortgage lending could, theoretically, be calculated in the same manner, with correlations found between default rates on a class of mortgages (e.g. borrowers in the South East) and default rates on the whole mortgage portfolio. In practice, a more likely procedure would be to set a required return for mortgage lending in general by comparing the returns on the mortgage business with other returns available to shareholders.

8.3.4 This required return might then be adjusted in an *ad hoc* way for loans which are thought to be more risky; but this 'risk' could be related to the type of loan rather than to any correlations between this type of mortgage and other mortgages. Although correlations between types of mortgages may not be directly allowed for, the maximum degree of diversification should still be sought.

8.4 *Interest Rates*

8.4.1 In corporate lending and sometimes with personal overdrafts, it is established practice to charge different interest rates to different borrowers according to the perceived risks. This is not so with mortgages or other mass-marketed personal lending, where it is common for anyone regarded as creditworthy to be charged the same interest rate for a particular type of loan. There are several possible explanations for this difference, including:

— *Efficiency*. The larger the loan, the more likely it is to be cost effective to analyse information about a borrower in detail, and thus derive an interest rate specific to the borrower. Marketing costs may also be lower when the same mortgage conditions apply to a large number of borrowers. However, it should not be difficult to analyse mortgages according to standardised risk factors, just as is done for general insurance policies. The appropriate methodology has been discussed in Section 6.

— *Competition*. It is unlikely that a low risk corporate borrower will accept being charged the same interest rate as 'average' corporate borrowers. Competition will lead to lenders charging low interest rates to low risk businesses. With mortgage lending, the price of the loan has not been the primary focus of competition (convenience and 'special offers' have been important). This may change in a more competitive mortgage market.

— *Fairness*. As it has not been standard practice to offer different interest rates to each mortgage borrower, it is possible that any lender which tries to introduce risk-related interest rates will be charged with being unfair. Again, with more demutualisation and more competition, this may change.

— *Quality of information*. For mortgage loan applicants, there is no equivalent to accounting data, although we have suggested, in Section 5, the type of information which would be useful.

— *Security*. The degree of security for mortgage lending is typically very high, because the loan is secured on the property being bought. Corporate loans are often unsecured, relying, instead, on the ability of the borrower to generate income. Nevertheless, as the late 1980s and early 1990s showed, there is risk attached to mortgages, and we would contend that this risk should be priced using methods similar to those proposed in Section 7.

8.4.2 Some risk pricing does exist in mortgage lending. It is not unusual for a lender to charge a higher interest rate to borrowers with high initial loan to

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value ratios (e.g. value of loan 95% or more of the property's value) than to those whose property provides more security.

9. Conclusion

9.1 We note that there is no history, in the U.K., of significant actuarial involvement in the affairs of commercial banks. However, we have found significant areas of overlap between the techniques necessary for the managing and pricing of risks in commercial banks and those used in institutions in which actuaries have traditionally been involved. This should not be surprising. Banks, like insurance companies and pension funds, are financial intermediary institutions with financial assets and liabilities.

9.2 We can divide the overlap into four groups. Firstly, there are those areas, such as the management of interest rate risk, where the techniques used in banking are very little different from those used by actuaries in other fields. Secondly, there are areas, such as loan pricing, where different techniques are used by the banking professionals from those which would be used by actuaries. We believe there is room for the development of new techniques in loan pricing which show more explicitly the effect of changes in financial variables on interest rates. Thirdly, there are areas where analogies can be found with traditional actuarial areas of work and where it may be possible for actuaries and banking professionals to learn from each other. There are three examples covered in this paper. Reserving for operational risk may be similar to reserving for rare events, for example in earthquake insurance. The statistical analysis of default rates may be similar to the analysis of certain general insurance risks. The analysis of market risk is not different, in principle, from the analysis of capital requirements for insurers. Another example might be in reserving for cyclical events (such as bad debts), which, again, may be similar to reserving for cyclical general insurance risks. The final overlap arises because of the integration of financial services. Higher management positions in banking involve understanding how a number of different types of risk interact at a conceptual level. This is similar to the role that actuaries play in higher management positions in insurance companies. Indeed, as bancassurance develops, the risks across insurance company-led groups may well become very similar to the risks across bank-led groups.

9.3 Overall, we believe that the risks managed within banking groups have a considerable amount in common with those traditionally managed by U.K. actuaries in insurance companies. We have discussed the nature of banking risks at a level which should be accessible to actuaries. In addition, we have proposed a pricing model which, we believe, can make a contribution to the development of more analytical techniques for loan pricing in commercial banks. It is an example of one of the types of contribution that the actuarial profession could make in the banking field.

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ABSTRACT OF THE DISCUSSION

Mr P. M. Booth, F.I.A. (introducing the paper): In Section 7 we introduce a pricing model, and it is worth making further points about the model.

First, it is a pity that we have had to illustrate the effect of defaults merely using some sample default rates, rather than being able to use actual default rates estimated from empirical data. There is a chicken and egg situation here. Without proper data, analytical models for pricing will not be developed. On the other hand, without models, people may not develop and collate the appropriate databases. We have either hatched the chicken or laid the egg, and we now hope that the cycle will continue so that data sources will develop. We need data by risk category in order to price differentially for risk.

Then, there is no stochastic element to the model that we have produced. There are several reasons for this; and the first relates to the fact that it is desirable to walk before you can run. We have published a pricing model, and it can be developed later. The second reason is the difficulty, in practice, of incorporating a stochastic element. The financial risk of a given mortgage can be represented by a compound distribution, combining probability of default with probability of losses of different sizes. This is rather like the frequency and size of claim in general insurance. Data on default are sparse enough; data on size of loss which is related to mortgage size relative to forced sale value are even more sparse. For differential risk pricing we need data on both aspects, sub-divided by risk category, such as the occupation of the borrower, loan size, loan to value ratio, etc.

There is a range of other problems; both probability of default and loss on default are related to background economic parameters, such as unemployment, interest rates and inflation. Interest rates are already a parameter in the model, and all three of these variables are related to each other. A further problem is that estimates of default rates and losses on default are structurally unstable, and no strong relationships can be found between the relevant variables.

Despite the differences in terminology and practical differences in the problems encountered, I am sure that actuaries can contribute to the management of risk in banking. We will need to develop our educational syllabuses, although not necessarily in the direction or in the particular detail that the current review is taking us.

Mr J. M. Pemberton, F.I.A. (opening the discussion): Many banking institutions have entered long-term insurance. Life assurers are now increasingly entering banking. There is an increasing need for understanding risks across both businesses. Moreover, there is much to be learnt from comparing the treatment of risk by bankers with the treatment of risk by actuaries. This comparison can considerably deepen our understanding.

In Sections 2 to 4 the authors have provided a comprehensive and helpful account of banking risks. In the area of credit and pricing risk, the main contribution of the paper is the development of a cash flow model, set out in the second half of the paper. Lending to less developed countries, in the 1970s, saw the price of risk bid down in the market to a level which, in retrospect, was clearly too low. This is a good example of the common problem of pricing credit risk in a competitive market. This is a problem which banks widely recognise. Problems arise in this area largely because past experience is not, as suggested by the paper, generally a good guide to the future. If a risk has not occurred recently, then there is pressure in the market for it not to be priced adequately. Actuaries recognise the need to do two things: to choose carefully the relevant historic data; and to introduce adjustments to past experience when looking forward. The disciplined use of a cash flow model, as proposed by the authors, offers a potentially helpful step forward in this area.

As the authors note, operational risk is not often subject to elegant mathematical treatment. Experience has shown that this has sometimes meant that it has received less management attention than it deserves. The difficulty of quantifying the risk does not mean that it is small or can be ignored. As the authors note, in ¶3.7, one of the fundamental ways to control operational risks is to appoint strong management who understand the risks. It is noticeable that, where mistakes have occurred, we generally find black box approaches to risk control which are not understood, on an intuitive level, by the senior management. Allowing such situations to arise or to persist is fundamentally unsatisfactory. Actuaries have long grappled with the problems of communicating risk. Such techniques as the use of scenario models can be helpful in this regard. I agree with the authors that the area of operational risk is one where actuaries might add value.

The discussion of interest rate and market risk, in Sections 2.5 and 2.6, highlights the dichotomy between interest rate and equity risks. There is much to be learnt from deeper investigation. The nature of the products written by the United States insurance industry is often closer to that of banking products than their United Kingdom counterparts are. The U.S. experience may act as a bridge between banking and U.K. actuarial approaches. The French experience — where much of the banking balance sheet is held in insurance subsidiaries for tax reasons — is also likely to be relevant.

The paper contains a helpful discussion of the regulation of market risk within banks; in particular, the comparison of the 'building block' and 'value-at-risk' (VAR) approaches. As the authors explain, the building block approach involves classifying the portfolio, e.g. into equities and into short / medium / long-term bonds, and then applying various fixed percentages to each to derive a capital requirement. The VAR approach, on the other hand, takes explicit account of historic experience and the interaction of risk across the book. The paper explains the RiskMetrics model — the widely used variance/covariance approach to VAR.

The authors correctly note the difficulty of estimating the correlation of price movements in downside risk situations — experience suggests that the correlations which hold in normal daily trading may break down in such extremes. As noted in Section 4.3.4, it is important to make allowance for regime changes.

The building block approach starts with powerful simplifying assumptions which are applied universally, and this, as noted in the paper, can sometimes appear to be somewhat arbitrary.

The VAR method is much closer to an actuarial approach — especially where simulation is used. An expert interpretation of the facts is used to develop a model for the specific situation. From the regulatory point of view, the VAR approach is more problematic because it is less uniform and thus more difficult to control, but, as the authors suggest, the greater degree of realism within this approach is a great step forward. Bankers are to be encouraged in their moves towards the more sophisticated VAR approach.

The recent Basle Committee proposals also embrace the concept of resilience testing. This is a useful addition to risk control techniques, even for larger banks, and can assist in the communication of risk analysis to executives or boards. These moves shift the regulatory framework closer to an actuarial approach, and actuarial science has much to offer in helping to take this forward. Especially where longer-term and inter-temporal issues are to the fore, the profession's practical experience of stochastic models and asset/liability management has particular relevance.

In Section 5 the authors are right to identify that a loss only occurs on default if the house price is below the mortgage outstanding, with the addition of accrued expenses. Until 1988 many models of loss adopted an econometric approach, which failed to use house prices as a variable. Even following the 1988-93 house price falls, the view that house price changes are a major driver of mortgage default costs has remained surprisingly controversial amongst many traditional practitioners. It is good to see the paper adopting a sound common sense approach on this point.

I am sure that the authors are right to conclude that publicly available information is insufficient to analyse risk factors. To the list of problems cited by the paper, I would add the difficulty of finding a reliable measure of house prices. Leading indices can differ by over 10% in a single year, and it is by no means clear that they provide a reliable guide to the pattern of house price changes. It would be helpful to get some suitable proprietary data to pursue the exploration further, flagged in Section 6, which suggests that actuarial techniques would lead to different assessments to those derived from banking methods.

The development of the cash flow model, in Section 7, is an important addition to the literature. It provides a practical tool for exploring the application of actuarial methods to banking. Perhaps the best starting point for this exploration is a set of discount rates derived from the structure of market prices. The model allows the implications, for banking business, of a wide range of parameters to be explored. These include expenses, timing effects, guarantees and capital requirements. It also provides a basis for exploring the relationship between average and marginal profit — an exploration whose importance is becoming increasingly required. The model is a step towards the development of a common unit of risk across the banking and insurance industries.

The authors have, sensibly, included early repayment fees as a feature of their model. The introduction and extension of lock-in periods on mortgages has been a major development over the past four to five years. This, together with front-end discounts and cash backs, has helped to drive a radical change in the profit profile of mortgages — the first year loss followed by subsequent year profits has brought the profit signature of mortgages much closer to that which actuaries would recognise from standard life products. While acknowledging the difficulties of moving towards a stochastic model, I nevertheless think that, in time, that would be a helpful step forward. It certainly would be fascinating to see an exploration of the possible cyclical behaviour of house prices.

Section 8 compares mortgage and corporate lending. I would note that the skill base, information base and geographic presence required to manage the operating risks associated with writing commercial business are very considerable. They represent a different order of proposition from writing a diversified portfolio of residential mortgages.

I agree with the authors that the actuarial approach has the potential to enhance the treatment of risk by bankers, and, conversely, that there is much for actuaries to learn from the bankers' approach to risk.

Mr Y. Guérard (a visitor): I make my comments wearing three different hats. First, as Secretary-General of The International Actuarial Association (IAA), I recognise this paper as an extremely valuable contribution to an area of our professional expertise that is in fast development. The IAA created the AFIR section (Actuarial Approach to Financial Risk) in Helsinki in 1988, and, at the time, it looked like being in advance of the evolution of the market. The growth in the interest of actuaries into financial risk has been tremendous since.

My second hat is that of a Canadian actuary. We could take this paper, put it in a word processor, and, by doing a few careful substitutes, make it a background paper for our own solvency margin system: minimum capital and continuing surplus requirement (MCCSR) for an insurance company, where you have more or less the same structure of C1, C2, C3 and C4 risk. In Canada we have the privilege of having a single statement which is accepted both as the statutory statement and the GAAP statement. One of the keys to having this unique statement acceptable for both ends is this careful definition of value at risk, and therefore the solvency margins that it requires.

My third hat is that of an actuary who spends a lot of time in Indonesia. If this paper had been published earlier, it could have been very useful in avoiding the debacle that occurred in their banking system, which ground to a halt for about 48 hours, which was quite dramatic, because it meant that all trade and commerce was stopped. One large factor was that they were not managing the risk in banking, particularly the C3 risk. The mis-matching of duration was a minor part of the problem, but the mis-matching of currency was a major problem. If you explain that they should not have liabilities in U.S. dollars and assets in their domestic currency, the rupiah, they wonder what you are talking about. Now they are wiser.

Professor R. S. Clarkson, F.F.A.: My comments are restricted to three areas of a specific nature. The first relates to the Basle Committee proposal to have a minimum sample period of only one year for VAR volatility calculations. I suspect that a one-year period will often give a misleadingly low estimate of the underlying volatility, and perhaps two years, or even three years, might be a more appropriate minimum period.

Next, I agree with the authors' observations that a VAR model does not always give a good measure of risk for general management purposes, and that a downside integral measure, along the lines suggested in Clarkson (1989), may offer a better conceptual framework.

Then, as the authors state in Section 4.3.2, the empirical evidence does not support the use of the normal distribution in modelling financial returns. I had already addressed this problem in the context of option pricing, and, in my paper 'An Actuarial Theory of Option Pricing' (*B.A.J.* **3**, 321-380), I suggested, as an alternative to the normal distribution, a compound distribution where the logarithm of the return is the combination of a normal distribution and harmonic motion. This appears to encapsulate the higher central peak and fatter tails phenomena that have now been comprehensively documented by many researchers, such as Bouchaud & Sornette and Géman & Ané over the past few years. Ignoring these fatter tails in the context of risk investigations, as many of today's proprietary models appear to do, could be dangerously unsound.

Mr J. P. Ryan, F.I.A.: Modelling corporate capital requirements is fundamental to the capital management of a financial institution. The approach, while complex, is, in many ways, relatively unsophisticated to the actuary. We can all think of improvements to the Basle model, and this is undoubtedly an area where the actuarial profession could contribute to the banking industry. Conversely, it should also be recognised that risk-based capital concepts were introduced earlier into banking institutions than into insurance companies, via the Basle Committee. We, as a profession, can learn from this, and apply some of those lessons that the bankers have learned to insurance and reinsurance operations.

The authors point out the impact of the covariance or diversification credits in Section 8.3 and elsewhere in the paper. These credits are of fundamental importance, and have a major impact on the overall strategies adopted by a bank and the banking sector as a whole. Thus, writing a small amount of non-correlated business will only marginally increase the overall capital requirements of the bank, and, therefore, the returns required by that additional line can be very low. It is this concept that is driving the desire of banks to move into other areas of financial services.

In Section 7 the authors go into much detail on cash flow modelling. Clearly this is something that the actuarial profession has to offer, but it is extremely important to recognise that applying it in different areas means that we cannot take the short cuts that work in the life assurance field, or we will get the wrong answer — and I can give some examples of this. In particular, the concept of capital being the 'cash tied up in a product' only works if regulatory capital is much greater than the risk-based capital set by the financial institution for itself. This condition is clearly met by the life assurance industry, but, in general, it is not for many banking institutions or, indeed, many non-life reinsurance operations.

Another difference in the banking industry, as opposed to the life assurance area, is the need to consider how all the variables may move together and model on a multi-variate basis, rather than just moving each variate one at a time. Again, this is often not generally an issue in life assurance or pension fund modelling, although it is in the general insurance field, and therefore the more simplistic techniques that are generally used in life assurance, for instance cash flow modelling, need care when translated into other arenas, such as banking or non-life insurance. The authors are undoubtedly aware of this problem, and refer to it a number of times in the paper. However, it is worth exploring further, because, in some cases, the authors' examples give conclusions which are likely to be erroneous, because they have not undertaken the full analysis, no doubt for illustrative purposes. For example, in Section 7.8.4 they state that large loans appear to be more profitable than small loans. They demonstrate that by doing the cash flows in a deterministic way, and the conclusion appears obvious. However, almost certainly default rates do vary by loan size. Indeed, the authors, themselves, hint as much in their analysis of corporate loans. Consequently, it is quite possible that large loans are less attractive than small loans, i.e. the opposite conclusion to that shown in the example. The default rate analysis, difficult though it might be, can often be more important than the cash flow analysis. It is important, therefore, that the profession, as a whole, does not make the same mistake if it is moving into an area with which it is not completely familiar.

Similarly, in ¶7.10.2.2 the authors conclude that cash back mortgages are an expensive option. However, it is by no means certain that this is the case. The first aspect is that it is probably unreasonable to assume that the cash back is financed out of capital, as, for most banking institutions, a risk-based capital formula will be greater than the cash invested in the product. This means that the cost of capital, as determined by the authors in their example, is too high. Secondly, it is almost certain that the cash back content will have some impact on default rates. If the cash back element were to reduce the default rate materially, then it could be a very attractive proposition, and different again from the conclusion drawn by the authors. Clearly this is an area for further research, but it also illustrates the importance of multivariate modelling, and how actuaries might get it wrong!

An area that the authors did not touch upon, but where the multivariate approach is important, is econometric modelling. Actuaries should be reasonably familiar and comfortable with the econometric approaches, even if they do not have the full economic background. Econometric models can have a fundamental impact on the cash flow models, because of the way that the returns and other variables inter-relate.

I used to have responsibility for the investment analysis of clearing bank shares at a major stockbroking firm. This was at the time when banks had only recently started producing and publishing 'proper profit' figures, and so there was much learning as to the drivers of the profitability, not only amongst the analysts, but also amongst the banks themselves. One of the issues that I found was that deposit mixes, bad debt provisions and interest rates were all correlated, often significantly. Consequently, moving variables independently far overstated results. Indeed, they could give a totally erroneous progression. For example, a drop in interest rates was often accompanied by an increase in the proportion of current account balances. Thus, the so-called endowment effect was very much less than a cash flow model, as outlined by the authors, would indicate, without adjusting for the multivariant consequences. Also, the various interest margins and balance sheet effects need to be taken into account when undertaking cash flow modelling.

The authors refer to the capital required to earn a 20% hurdle rate. This is derived from the capital asset pricing model (CAPM). While I have no reason to doubt the source of the authors' information on this, it is likely to prove to be too high a rate to achieve in the U.K. in the current competitive market position. Since the market demands returns for that level of risk, the only way forward is for the bank concerned to reduce its capital requirements; which means diversification of risk; which means moving into different lines. It is this diversification aspect that provides a major driver of banks moving into other financial products. Indeed, if they fail to do so they will find themselves earning inadequate returns on their capital, because the market will not allow them to derive the returns that they need to service their capital in isolation. Since banks are going to diversify, it will likely have a similar impact on returns in the insurance industry, and actuaries are going to have to learn more about banking, whether they like it or not.

Mr P. J. Akers, F.I.A.: As some of my remarks are critical of some of the authors' conclusions, I feel that it is necessary to explain my experience in this field, both to establish the legitimacy of my remarks and to dispel the suggestion in the paper that actuaries have not yet been involved in risk management in banks.

I was employed by a U.K. retail bank from 1988 to 1993. My responsibilities included aspects of risk management, and specifically included the insurance of residential mortgage lending risk. Since leaving that bank, I have, for the last four years, been a consultant, and a large part of my practice has centred on residential mortgage risk management. As a result of this, I have seen the unfolding experience of over two-thirds of U.K. residential lending made since 1990. It is this experience which gives me cause to argue with some of the authors' conclusions, particularly those related to residential lending in Section 5.

My experience leads me to make two general conclusions: that data are absolutely key; and that all lenders are different, both in the availability and the quality of relevant data, and in their loan default experience.

The authors assert, in ¶5.8.1, that the "publicly available data on problem mortgages are not adequate for the purposes of analysing risk factors", and I would agree with this. However, there is a potential implication that the required data do not exist anywhere, and therefore risk analysis is not currently possible. Indeed, the authors go on to state, in ¶6.7.2, that "there is less analysis of risk undertaken than would be desirable", and, in ¶6.7.4, that "to perform a detailed analysis, more data must be collected". I would agree that the more risk analysis there is the better, but, in fact, such data do exist, within individual lenders, and risk factor analyses and pricing for risk exercises can be done and are done. The point is that, yes, data are key, and yes, data availability and quality frequently do constrain the work that can be performed, but data are crucially scarce only in the public domain. In

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general, lenders do have sufficient data to support such work, but they do not make these data available publicly. In my view, this is a perfectly justifiable position to take, given the competitive value that such data represent.

In Section 5.4.2 the authors indicate some of the problems associated with decision making based on statistical analysis, and arrive at the conclusion, in ¶5.4.2.9, that "it is difficult to rely on any model of defaults which is derived from historic data". Again I agree, in part. I agree that relying solely on such exercises would be unwise, but the implication that such exercises have no value is incorrect. Firstly, to assert that exercises based on historic data have no value is the same as asserting that the past is absolutely no guide to the future. The point is that understanding the past can be extremely helpful in deciding how to manage the future. That is fundamental to the concept of the control cycle. Secondly, general insurers must justify their reserving on the basis of past statistics, so analyses based on historic data are, not only helpful, but required in some circumstances.

The authors also reach a conclusion which I would describe as actuarially logical, but commercially inappropriate. This is in the way that they describe the impact of pricing for risk. In ¶5.2.1 they state that it is necessary to identify risk factors "in order to set prices according to risk", and in ¶6.7.3 they state "to price for risk... would be a major step forward". I certainly agree that risk factor analysis is important and beneficial, especially when an organisation comes to determine its loan pricing, but it is simplistic to suggest that the availability of risk factor analysis should necessarily lead to pricing for risk. Rather, a lender will benefit if it knows the risk factor impacts when it comes to determine its prices, because such knowledge will certainly help it to estimate future profitability, but it is not necessary that the lender actually charges a price which varies by risk factor. The extent to which the knowledge that comes from risk factor analysis actually changes the price charged will depend on many considerations, not least what the current market pricing is, what customer price sensitivity is, and what pricing discrimination is permissible.

I feel that, as a profession, we need to be very careful when suggesting that we have skills useful in new fields, to make sure that we are not making inaccurate assumptions about the current state of the art. I fear that the authors have come dangerously close to this in their comments about the current state of risk management in U.K. banking, especially in their comments in Sections 5, 6 and 7, suggesting that data are insufficient and methods inappropriate. If the rest of the paper is as far away from reality as my practical experience shows it to be in the areas I have mentioned, then we would do well to proceed with some humility, or else the U.K. retail banking industry will simply resent our intrusion.

Mr S. Chandaria, F.I.A.: My comments are directed on the operational risk areas. The authors state that the more significant risks are found in operational areas. These risks are large enough to ruin a bank or to cause severe reputational damage. Unfortunately, this is the area which is least extensively covered in the available literature. Operational risks are very wide ranging, and there is no consistent definition in the banking industry. Definitions range from narrow ones, covering transaction processing only, to wider ones such as 'any risks other than market or credit risks'. This lack of definition makes the consistent measurement of operational risks difficult, and it is compounded by two further factors:

(a) the inability to quantify some operational risks, for example those resulting in reputational damage; and
(b) the inability to find a causality link between an operational risk and an actual loss.

Not surprisingly, measurement techniques for operational risks are less well advanced than for market and credit risks, and, indeed, are often very broad brush, comprising only a loading on expenses. This lack of accepted measurement technique makes operational risks an area where actuaries are most likely to be able to add significantly to existing knowledge in the banking industry. Actuarial methods could be used as part of an overall measurement framework, perhaps dovetailing into the existing risk-adjusted return on risk-adjusted capital method used for market and credit risks. There are a number of methods used by general insurance actuaries which could assist (perhaps after suitable modification).

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Whilst actuarial methods may not be implemented immediately because of the extensive data which may need to be gathered, it would seem that immediate opportunities for actuaries to assist in the measurement of banking operational risks would include the following areas:

(a) an assessment of existing available data and their suitability for actuarial methods;
(b) specification of data to be gathered, with the aim of implementing more sophisticated actuarial methods in the future; and
(c) implementation of new measurement methods in the short term, moving towards more sophisticated actuarial methods in the longer term as more data are gathered.

Even where existing data are available, their completeness and accuracy may be questionable, given that they were probably not compiled for actuarial purposes. Further, whilst some suitable data may be available now, invariably these are likely to be confined to those areas of operational risk which are not the most significant. For example, very little data (if any) are available on the type of low frequency, high severity operational breakdowns which resulted in the market losses leading to the collapse of Barings. Nevertheless, as part of the overall process, actuaries will be able to help management construct an appropriate measurement framework for risks where there are currently insufficient data or where data will always be sparse, such as the low frequency, high severity risks. Any methods implemented will need to be refined over time as more data become available.

The data collection process has already been started in some banks and, in at least one case, an actuarial approach has been implemented. Such data will enable these banks to adopt a more scientific approach to the allowance for operational risks in both pricing and capital requirements, perhaps giving them an edge over their competitors.

Mr S. J. Perry, F.I.A.: The retail financial services industry is changing very rapidly. Banks have been providing insurance products for some time, and, more recently, several insurance companies have set up their own banks. New entrants to the industry are providing both insurance and banking products. It is now very difficult to know where the boundary between the life industry and the rest of the financial services industry actually lies.

Many of the techniques discussed in the paper will be familiar to those who have had an insurance upbringing. The cash flow modelling approach to pricing mortgages, as discussed in Section 7, is of particular interest. The benefits of using cash flow modelling techniques in the pricing of life insurance products are well known by actuaries. Cash flow modelling and value-added techniques are not just used for pricing, but are also now used extensively to value life companies and in the general day-to-day management of them. Only by projecting future amounts of distributable profits and then valuing those profits to measure the value being added to the company by the business being sold, can one fully understand how the business works, and only by understanding how the business works, can sound business decisions be made. However, as discussed in the paper, cash flow projections are not yet being used as extensively by banks as they are by insurance companies.

In addition to being long term in nature, there are many ways in which mortgages are very similar to long-term insurance products. There are costs involved with selling the products and setting them up. There are running costs in administering the products. Instead of mortality rates and lapse rates, think of default rates and rates of early redemption. Instead of surrender penalties, there are early redemption penalties; instead of annual management charges, there are interest rate margins. When one prices a unit-linked insurance contract, one can vary the different elements of the charging structure. Exactly the same can be done to price a mortgage. If one looks at the profit signature for a mortgage product, one sees a very similar shape to that of an insurance contract. There is generally a loss in the first year, because of the costs of selling and setting up the contracts and also of setting aside regulatory capital. This is followed by profits in later years. Therefore, a method of pricing products which allows for both the likelihood of making profits and the time value of those profits seems to be particularly appropriate.

The market for residential mortgages has now become extremely competitive, with most providers offering substantial interest rate discounts and cash backs at the start of the contract. They can afford to do this by charging higher interest rates later on in the contract. They are, therefore, relying on tie-

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in periods and redemption penalties to ensure that they will not make a loss should the mortgage holder decide to redeem his mortgage early. It is clear to see how cash flow modelling could help in setting the various features of such a mortgage.

There are one or two areas where refinements could be made to the model described in the paper. For example, the model assumes that everyone repays their mortgage after a fixed time, and then goes on to show how sensitive the results are to the date of repayment. This does not give the full picture. Early repayment rates depend, among other things, on the penalties charged for repaying early, and, therefore, cannot be assumed to be the same at all periods during the contract. For example, one would expect a large number of repayments to take place at the end of the tie-in period. Therefore, I would propose using a profile of early repayment rates which depends on the length of time for which the mortgage has been held. The model also assumes that funding from the Treasury can be used to cover the costs of the initial expenses. An insurer might use reinsurance to achieve a similar effect. It is not clear, however, in this model, who is taking the risk of the expenses not being repaid, should the mortgage holder default. Nevertheless, cash flow models, such as this one, could be used to set appropriate levels of reserves to cover this sort of risk.

One final benefit of cash flow modelling, as discussed in the paper, is the ability to test and communicate the sensitivity of the bank's profits to various factors, including early redemptions and expense levels. Mortgages have become very complex products. With features such as cash backs, discounts and flexible repayment schedules, they are now becoming as complicated as insurance products. Communication within the bank of the effects of future uncertainty on the profitability of complex products is of the utmost importance, and should be an area where actuaries are well qualified to assist.

It should be pointed out that there are a number of important differences between banks and insurance companies. As the paper mentions, the nature of the assets and liabilities differs, as do the rules for determining capital requirements. Banks also account for their business quite differently to insurers. Additionally, the language used by each can be quite different. One of the main barriers to actuaries moving into this wider field is a lack of familiarity with the jargon and the techniques currently used. This paper goes a long way in addressing this problem.

Dr P. W. Poon (a visitor): I come to this paper with a background in the banking sector, and I start by saying that I whole-heartedly agree with the aims and conclusions in this paper. I started my career as a research mathematician and engineer, so, when I entered the world of banking, my natural inclination was to persuade my colleagues of the benefits of really getting underneath the numbers and what drives them. My techniques were met with just polite interest. This paper goes a long way towards illustrating the benefits of deeper analysis.

Banking is now becoming a much more competitive business than before, with insurers, overseas financial services providers and even retailers entering the market. Margins will be squeezed, and the banks with the best understanding of where they make money will be the ones that emerge as the winners. There is some progress being made. In the last few years the analysis that supports credit rating, pricing and investment decisions — to name but a few — has become more sophisticated. Unfortunately, it is difficult to find out exactly how much more sophisticated, because these techniques are jealously guarded. Nevertheless, in recent years we have seen a number of changes, for example in the way that banks price their products. Witness, for example, the introduction of lock-in periods and early redemption penalties on mortgages in order to immunise the bank against pre-payment risk.

The biggest barrier to progress is not techniques and not their dissemination, but the need to change the culture in the banks, within the retail banking sector especially. Banks are full of people who have worked in banking for years. These people have an intuitive feel for what they can charge and the level of risk that they are undertaking. This intuition, call it management judgement if you like, has served them well for many years. However, now the culture is changing, and changing to one in which managers need to justify their business decisions using the sort of analysis with which many of you would be familiar. Perhaps the greatest influence bringing about this change is the MBA graduate. Increasingly, MBA graduates turn up in banks as managers rather than strategy consultants,

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and they bring their discounted cash flow analyses with them. If actuaries are to bring their skills and techniques into the banking sector, it is these MBA graduates who will believe that they are already doing what you are suggesting that actuaries should do. The next phase in banks' development may well be the rise of the analyst as a key influencer, but I would suggest that the winners will be those banks who manage to merge the rigor of the analyst with the intuition of the experienced banker.

Banks are making some progress, although, of course, there is plenty of scope for more. The example that the authors give of cross-subsidisation is a pertinent one, and one which many banks have yet to address, or, in some cases, recognise. Customer loyalty is currently a much discussed, but little understood, topic, which could benefit from some more rigorous analysis. Another topical issue, which has been touched on in this discussion already, is reserving for operational risk, particularly 'rare' events, such as rogue traders, terrorist bombs or computer failure, where the methods developed by general insurance actuaries could be applied. These are just a few of the topics in the bank sector to which actuarial discipline could be brought to bear with great effect.

Mr J. C. T. Leigh, F.I.A.: The authors write, in ¶2.5.5, that a redemption penalty is "charged if a re-mortgage is taken out during the fixed-rate period". This is true, but I think that it is worth drawing attention to the fact that this is not generally linked to the loss that the bank may suffer. It is generally related to the interest that is being paid at the time, and, if interest rates have gone up very much, the bank may actually make something of a profit if the loan is repaid early. If interest rates have fallen, there is no extra penalty paid to take account of the fact that the bank cannot re-lend the same money at the same rate.

Paragraph 5.5.2.3 lists the various risk factors that have been found relevant to default risk. Other speakers have mentioned the fact that the past may not necessarily be a good guide to the future. Although the principle here may be similar to life underwriting, the practice, I think, is extremely different, given that we can be sure, or reasonably sure, that the factors that make a person a poor risk for a life policy are going to be the same from year to year. We cannot be so sure that the factors that will make a person a poor credit risk will be the same in a year's time. I think that we can be reasonably certain that the person who is greatly stretched in paying his mortgage will always be among the poorer risks, but if we look at the list of risk factors in ¶5.5.2.3, I cannot see any obvious reason why the type of accommodation which may have been a reasonable predictor in the last series of poor experience in this area will necessarily be so in the next.

Another point was made in ¶5.5.1.2. about defaults. The conclusion was that borrowers got into trouble through loss of income much more than through increases in interest rates. This seems, on the face of it, fairly logical, and, indeed, obvious in that an increase in interest rates on a mortgagor's monthly payment squeezes his income, whereas unemployment can completely destroy it. However, if interest rates go up and the general economic situation is squeezed, then that, itself, is a major cause of unemployment, and therefore of defaults. Secondly, when interest rates rose, it became more expensive to buy a house, and that, itself, had a depressing effect on the market. Thirdly, many defaults came at a time when high interest rates pushed up the amount that was owed on a mortgage and made the consequences of a default for the lender that much more serious. So, while the immediate cause of mortgage defaults may be seen to be unemployment rather than interest rate rises, those rises are a cause of mortgage defaults of profound importance.

Looking at the model itself, I was not sure that the authors actually realise how serious the effects of defaults could be on the internal rate of return. At the time of the stringent prices in the market, we had 15% interest rates. It takes a couple of years of non-payment of interest, in most cases, to get a property repossessed and to get it re-sold. A couple of years at a 15% interest rate can take a mortgage that is 75% of the value of a home up to 100%. At the same time, if the price of the property falls 20%, and there can be £5,000 or £10,000 of expenses involved in a claim, you will find that a mortgage which started at 75% has become much higher than the value of the security, and will be a major source of loss to the bank. It is significant to note that 75% has, for some lenders, been considered, traditionally, as the risk-free level of loan-to-value ratio, the point at which lenders no longer thought that they needed to buy mortgage indemnity insurance. I re-ran some of the authors' cash flows, assuming that losses on mortgages were something like 25% of the value of the loan on a default, and I found that it reduced the internal rate of return to low single figures.

The authors also state, as has been mentioned several times already, that companies, at the moment, do not price risk sensitively. There are, of course, some very good reasons for this. If mortgage lenders were, brutally speaking, seen to be charging more to poor people than to rich people, then they would incur a degree of odium that they would probably not wish to incur. There may be other ways in which it can be done, a sort of flight to quality might be imagined, whereby high interest rates were charged by some lenders and lower rates by others, and the risk profile then sorted itself out in the risks that they were prepared to accept.

The authors also comment on the existence of cash backs and early discounts. I think that the reason for these is that they give a very strong marketing advantage. People take out mortgages at a time of being highly strapped for cash, and they need to spend money on other things. Some research has been done, I believe, on the effective implicit discount rates that people employ in their own financial dealings, and come up with very high levels indeed. For example, certainly in selling ordinary consumer items like house insulation and new boilers, that may well save on fuel bills, it seems that it is reasonably easy to get a sale to a consumer only when the money can be recovered within a year. This suggests that they have a much higher implicit discount rate in their own thinking than a bank does, and that there is actually a sort of arbitrage difference that a mortgage lender can reasonably exploit and that they have exploited. On the other hand, it does suggest that significant early repayment penalties of the nature that the authors suggest may not be quite so difficult to impose as the authors seem to think.

In a paper that talks about the management of risks in banking, I was a little disappointed to read nothing about insurance, both of the credit risk through mortgage indemnity guarantee and risk management through captives. Mortgage indemnity guarantee has been much studied by members of this Institute in the last few years, but mainly from the point of view of insurance companies. It would have been interesting to see it explored from the point of view of banks.

Mr C. G. Lewin, F.I.A.: I comment on some important aspects of a bank's risk management strategy.

Where a corporate loan is large, it may be worth analysing the risks facing the borrowing company. An individual approach specific to the borrowing company is required, not a rule of thumb. This may mean analysing the risks inherent in the borrowing company's major capital projects, including both new ones and ongoing ones, particularly the disaster risks which could affect or imperil the borrower's ability to repay its loans. A useful new tool for this process may be RAMP, which stands for risk analysis and management for projects. It provides a strategic framework within which risks can be analysed and controlled. It is a disciplined process for complex activities, which has been developed by a joint working party of the actuarial and civil engineering professions, with help from some economists. It has the advantage of making sure that risks are not overlooked, and that they are properly evaluated and mitigated. It pays special attention to disaster risks, even where the probabilities of occurrence are thought to be small. It ensures that risks identified at the analysis stage are properly controlled thereafter. RAMP will be published in July 1998.

For the larger corporate loans made by a bank, RAMP could be applied by the bank or by a consulting actuary on the bank's behalf, to look at the borrowing companies and the major risks which they face. This would help banks to assess their client's creditworthiness in the longer term, and assess whether extra mitigation of some of the risks is needed.

Loans for new projects can be illustrated by the Government's Private Finance Initiative, where a bank's client is wanting to undertake the financing of a project in order to provide a service to the public sector. If there were an independent risk assessment for such projects by the RAMP method, which highlighted the residual risks remaining after mitigation action had been taken, this would assist, not only banks, but also equity investors, to assess those risks and the ability to control them, and hence whether lending or investment for those projects can be justified.

A bank could apply RAMP to itself as an on-going business. I have recently been associated with applications of RAMP to on-going pension schemes and to a trade association. The techniques used in applying RAMP can often be elementary. You do not necessarily need to use a computer. So, one of the needs which RAMP supplies is to provide a strategic framework for looking at risks comprehensively, and placing a financial value on them. It ensures that you do not spend too much time looking at certain risks because they are capable of mathematical analysis, while ignoring bigger risks that are less capable of a rigorous treatment. RAMP forces you to look at everything. It will accommodate mathematical or stochastic techniques where appropriate, but does not insist on them.

Professor A. D. Wilkie, C.B.E., F.F.A., F.I.A.: Rather than using a linear model in credit scoring, as in Section 6.2.2, so that factors like age are split into cells and then the levels are ordered, treating age as a number, I would much rather use binary type variables. So, for five cells you have four variables. For under 25 they are 0000; for 25 to 30 they are 1000; and for 30 to 40 they are 0100, and so on. That gives you many more variables, and is a much neater structure than assuming that you can actually order them.

Mr D. S. Parmee, F.I.A.: A paper such as this is, in many respects, simply a product — albeit a very good one — to put on the shelves of actuarial wares. All of us here who work in retail financial services will know that having good products on the shelf is a necessary condition for success, but it is by no means a sufficient condition. Effective distribution of the products by the means appropriate to the target market place is required to turn the good products into profitable business for the beneficial penetration of a new market sector. I would be interested to know from those who are responsible for the development of the profession what plans there are to capitalise further on this paper through professional contact with the leaders of the banking sector. I would also be interested to know whether or not the plans allow for integrated efforts between the profession and those organisations which might benefit most from expansion of the profession into this relatively new area. In ventures such as these, it is surely important to find the right blend between the professional and the commercial considerations.

Suppose that contact between the profession and the banking sector was to persuade senior management in the sector of the need for greater mathematical and statistical analysis of the risks of their business — or, perhaps, it would be fair to say to *confirm* the need in the minds of senior management in that sector. Why should the senior management choose actuaries to carry out the work? During 1997 I was under contract to a leading investment bank and members of its research team, the in-house 'rocket scientists'. The researchers were collaborating with leading academics to investigate the application of advanced mathematical and statistical techniques, neural networks and artificial intelligence to many aspects of the bank's business. The initial impetus for this research had stemmed from the quantitative asset management and structured product areas. I suspect that those researchers, none of whom is an actuary, have many counterparts in other investment banks and in the larger banking groups. We need to be clear, in any presentation to bankers, certainly the ones to which I was contracted, why actuaries have an advantage over their own rocket scientists.

By way of contrast, during the first half of the 1990s I advised a leading retail bank, not on banking issues, but in doing so I met the members of its Assets and Liabilities Committee (ALCO) and its Board of Directors. The members of ALCO and the Board struck me as highly pragmatic people, often highly experienced in their own field, with an instinct for the business built over many years, who took their responsibilities very seriously, and who each needed to understand, personally, the rationale behind the decisions being made. The implication here, if my perceptions were correct, is that we need to work very hard on how we communicate the nature and the value of the potentially complex mathematical and statistical bases of our work.

Concerning risk management generally and the generally accepted risk principles that are mentioned in the paper, an overview is given in Sections 2, 3 and 4. The authors then concentrate on the probability-based methodologies for risk measurement, reporting and control, with particular emphasis on the value at risk. However, there are other issues for which professional judgement and experience need to be integrated with rigorous methodologies to form a comprehensive solution. It is, perhaps, here where our particular professional judgemental skills, derived from what I believe to be our unique training and post-qualification experience, will differentiate the actuary from others who work in the area.

Particular topics in risk measurement reporting and control, to which we should be able to make a valuable contribution are:

— the analysis and decomposition of transactions into their component risks, to ensure that each risk is identified, accurately quantified and managed;
— the design of an appropriate risk measurement framework, which is both mathematically and statistically rigorous and comprehensible to senior management;
— the derivation of a series of risk sensitivity measures and dynamic processes for stress testing;
— the comparison of estimated risk exposure to the actual behaviours which emerge, both at the product level and at the portfolio level;
— the calculation of capital at risk for the company of each of its business units by aggregating measures of market credit and other quantifiable risks, and properly reflecting the correlation between these risk factors;
— the derivation of risk limits and capital-at-risk limits for the company as a whole and for each of its business units, perhaps down to the level of the individual trading desk for some risk factors; and
— the design and methodologies which those of us in life insurance would be well used to for the recognition of revenue, for the allocation of costs and for transfer pricing.

Mr R. E. Brimblecombe, C.B.E., F.I.A. (closing the discussion): I believe that this paper, demonstrating the role that actuaries can play in banking, will be seen as a seminal one in terms of actuaries working in the wider field in the same way that the paper Lewin (1995) was seen when it appeared. This paper and the discussion have demonstrated that, over a wide range of issues in banking, actuaries do have a role to play in assisting, in particular in assessing and quantifying risk.

The paper states that the increasing convergence of the financial services industry, particularly between banks and insurance companies, may act as a catalyst — if only because of the cross-fertilisation of senior management. I believe that the main message from the paper is that the banking world may have much to learn from insurance companies, and from actuaries in particular.

It has been said in the paper and by various speakers that, over the years, insurance companies and banks have been dealing with the same problems, but with different languages and probably from a different end of a telescope. It is interesting to note that, in ¶2.2.5, it is suggested that shareholders of banks have a general preference for a stable pattern of returns, which, perhaps, is at variance with shareholders in insurance companies, who, it is hoped, have come to learn the effects of underwriting cycles on share prices.

The earlier parts of the paper refer to various business and financial risks to which banks are exposed, and this has been reflected several times in the discussion. Managing interest rate risks, for example, is similar to that faced by many financial institutions, including insurance companies, and, of course, operational risks apply to many industries, and not just to those in financial services. However, the paper has done a service, I believe, in identifying and categorising such risks and discussing their characteristics. Actuaries and bankers are actually often assessing the same risk, but the difference is in communication and in the language used. That is something that, perhaps, we need to address.

To me, the most interesting part of the paper and, indeed, of the discussion is the area of credit risk, and particularly the question of mortgages, on which I want to concentrate my comments. The assessment of risk and the pricing thereof, through the collection and analysis of data, are food and drink to actuaries. However, as has been stated, in the mortgage market, traditionally, it has been a question of pricing the product first and considering the risk afterwards. Also, over many years, at least until the mid to late 1980s, the flat pricing of mortgages was standard, an approach which seems unnatural to actuaries. As well, the interest structure was, in the main, on a short-term basis, so that a substantial increase in short-term interest rates — usually correlated with a downturn in the economy — did have a material effect on the individual's ability to pay. I believe that this correlation

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(although at least one speaker stated that this was not particularly relevant) is relevant, and certainly led to some of the problems in the late 1980s.

Section 2.2.2 states that the prime task of insurance companies is managing liabilities, whereas mortgage lending is the management of assets, and this is a prime example of where there has been, perhaps, a difference in language, although the problems are similar. I hope that the main sections of the paper, dealing with the pricing and management of personal domestic lending, will be used increasingly by mortgage lenders. The experience of the late 1980s, which led to the problems of negative equity, bad debts and arrears of repayments, if nothing else, was, and still is, a social problem to which, perhaps, price differentiation could have had some alleviating effect, if only because some borrowers would not have over-stretched themselves.

As has been said by several speakers, these problems are very similar to those of general insurance companies, particularly those writing mortgage guarantee business, but I suspect that the problems of lending institutions were exacerbated by the pricing structure. It has also been suggested, as a side issue perhaps, that low-start mortgages, issued with alacrity at the behest of marketing departments in the heydays of the mid-1980s, which were starting to reach the end of their low-start period, also exacerbated the problems. It seems to me that both insurance companies and lenders should have made some allowance in their projections for a downturn of the economy coupled with an increase in unemployment at the same time as a collapsing housing market. It has often been said that, perhaps, we could not have foreseen all three events occurring at once, but a sensitivity analysis would, at least, have allowed us to understand the major downside.

Several speakers have commented on Sections 5, 6 and 7, which develop a model for use in private domestic lending, and it can be used, as has been said, to develop an actuarial approach to pricing personal lending.

I was rather surprised that Mr Akers stated that, whilst the question of differential pricing in the assessment of the various factors in mortgage lending could be used by lenders to assess the risk that they were undertaking and, perhaps, the level of their capital, he seemed to be less keen on the ability to use those various aspects in differential pricing. I believe that the mortgage market is now so sophisticated that differential pricing and differential products are what the public is seeking.

I am sorry that Section 8 is relatively short on corporate lending. Although Mr Lewin has expanded on this in relation to large loans, I would have liked to have seen a little more on the problems arising from lending to small corporate businesses. It has been stated that the recession that started in the late 1980s was exacerbated by banks withdrawing support from small businesses. The perception, rightly or wrongly, was that this, in some way, seemed almost indiscriminate in its effect, without a proper assessment of the risks involved. This, coupled with an appropriate pricing structure, at least for new loans, rather than the lenders going completely out of the market, in my view would have prevented this blanket approach, and I would like to suggest that, if actuaries had been involved more in banking at that time, perhaps some of the problems could have been averted, at least to some extent, thus reducing the effect of the recession. Similarly, if actuaries had been involved earlier in the area of personal mortgage lending, at least some of the social pain at the time would have been avoided. I do not agree with the paper where it states that the only real risk is the capital loss to the lender. The general problems arising from bad debts and arrears of mortgage repayments and so on are, I believe, a major issue from the individual borrower's point of view.

On the question of the future involvement of actuaries in banking, we will probably have to train actuaries in a more detailed way to assist institutions such as banks. Perhaps the time has come for the profession to consider a specialist finance subject at Fellowship level, which would go alongside investment and the other core subjects that we have at present. I believe that our Australian colleagues have gone down that route, and I would like to see it introduced here.

Mr Parmee wondered whether the paper would find its way onto the shelf gathering dust. I hope and trust that it will not do so. This is one of the areas in which, I hope, actuaries will continue to be involved, and I am sure that the profession and, in particular, the Wider Fields Board will ensure that we do press ahead with trying to talk with members of other professions, particularly in the banking field, to see exactly how we can help each other. It is not a question of actuaries telling bankers what to do, or vice versa; it is a question of working together, and I hope that some of the comments in this discussion will help in that area.

I believe that this discussion will be seen as a landmark for increasing actuarial involvement in banking, and I hope that, by the paper and by the discussion, the actuarial profession has more than demonstrated that its involvement in this area could assist both banks and, just as importantly, in the public interest, their personal and corporate clients.

Mr J. N. Allan, F.F.A. (replying)**:** The opener pointed out two natural areas where actuaries could enthusiastically get involved in banking. One was our understanding of the value-at-risk approach, and the other was our recognition of profit signatures of mortgages with cash backs and other features.

Mr Ryan raised the interesting strategic point that the low covariance between returns from different forms of business is one of the reasons why commercial banks are moving into other areas. As he commented, that is leading to lower hurdle rates of return from equity for banks. One bank has said, publicly, that its after-tax hurdle rate of return from equity is now about 10%, around 15% before tax. These hurdle rates are coming down as banks become more diversified.

Mr Akers raised the very valuable point that, as we actuaries attempt to help bankers, we must not offend them. We must be sensitive to their background and their cultures. He also raised the point about the availability of data and the use of that data. The point that we are trying to make is that the market in some retail financial products is not using whatever data individual companies do have. In the mortgage market there are very attractive prices for new mortgages, and it is, perhaps, only a degree of apathy that is preventing re-mortgage companies cherry picking away some of the more mature borrowers.

As the closer said, my belief is that differential pricing is what the market needs.

In this paper we have identified a number of areas where actuarial skills in risk analysis could be applied in banking. We have concentrated on the application of actuarial techniques for the development of models for pricing banking products, but, with similar considerations, actuarial techniques could be used to estimate the amount of capital required to support banking businesses, allowing for credit risk, market risk and, of course, the more difficult operational risk.

However, the broader purpose of the paper is to stimulate discussion about the threats and opportunities for actuaries arising from the very substantial changes which are taking place in financial services in the U.K. and, as we approach the common currency, across Europe. A major theme of these changes is the convergence of banking and insurance at a corporate level and at a product level. At a corporate level, many banks are now selling insurance products and some insurance companies are moving into banking. Some banks have bought insurance companies. The consolidation of financial services is expected to continue and to lead to the emergence of large, strong bancassurance groups. At a product level, Individual Savings Accounts are just one example of the need for products crossing the traditional boundaries of banking, insurance and fund management.

So, what about the threats and opportunities for actuaries? The threats are that, in these broader financial groups, actuaries might be restricted to their insurance divisions or to the role of financial engineer. The opportunities are for actuaries to participate in wider aspects of finance and in the management, as well as in the operation, of these large groups. To take advantage of these opportunities, we may have to broaden our horizons to banking as well as to insurance, and to continental Europe as well as to the U.K.

This suggests two points: first, we should encourage the Wider Fields Board to lead the profession in this area; and second, we should consider the inclusion of some wider financial topics in the examination syllabus.

The President (Mr D. G. R. Ferguson, F.I.A.): It just remains for me to express my own thanks and the thanks of all of us here tonight to all four of the authors for their paper, which has provoked a stimulating discussion. I hope they will be pleased with it.

WRITTEN CONTRIBUTION

The authors subsequently wrote: We agree with Mr Ryan's comments regarding the importance of multivariate modelling. Our preferred approach would be to estimate default parameters, based on available data, for different classes of borrower. The classifications may include the size of loan, as suggested by Mr Ryan, although, in practice, loan-to-value ratio (which will be correlated with loan size) may be more important. The variation of default parameters with loan size will have more effect on the break-even interest rates for unsecured lending than for mortgages, because the loss is limited to the accumulated amount of the loan plus expenses less the value of the property for mortgages. Our illustrative results, using the cash flow model, should be regarded as *ceteris paribus* results, and multivariate modelling is needed.

We agree with Mr Akers that the major difficulty with regard to data does relate to publicly available data, and that data do exist internally. We make this clear in the paper, and did not mean to imply that data did not exist in any form. However, we would contend that internally held data are not always held in the appropriate form, and the discipline of developing a cash flow pricing model is useful in focusing the minds of those who work in pricing to ensure that data of the appropriate form are kept and used. In suggesting that actuaries should not *rely* on a model of defaults which is derived from historic data, we did not mean to imply that such a model has *no* value. Indeed, there would have been no point in the authors developing a cash flow model if it were not possible to develop estimates of default rates. However, it is clear from an analysis of the studies of historical data that there is little structural stability in the estimates of default rates. Historical data should be one of many inputs in the analysis of default rates, but need not be the only input. We do not agree with Mr Akers that we suggest that an analysis of risk factors should necessarily lead to pricing for risk. Indeed, we specifically say, in Section 7.12, that all cross subsidies are not necessarily inappropriate and could be regarded as an essential marketing tool. Nevertheless, cross subsidies should be quantifiable, and competitive pressures in the future may ensure that pricing for risk is essential if profitability is to be maintained.