DENSITY OF GLACIER ICE

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Abstract. The density of glacier ice containing a given amount of air can be computed if it is assumed that both ice and air are subjected to a pressure due to the weight of overlying material. In this way it is possible to deduce the form that a curve of density versus depth should have for ice of a constant air content. Reasons for the divergence of observed depth-density profiles from those predicted by this theory are discussed, in particular the effect of the plastic-viscous behaviour of ice, which results in an air pressure differing from the hydrostatic pressure of the ice above. The empirical power-law relation between depth and density is discussed in the light of this theory, and is also used to derive relations for useful parameters characterizing the densification process such as rate of subsidence and rate of densification in terms of the depth, accumulation, and the constants entering the power law.

Résumé. On a calculé la densité de la glace des glaciers comprenant une proportion donnée d'air, en présumant que la glace et l'air sont soumis tous les deux à la pression des matières qui les recouvrent. De cette façon il est possible de déduire le tracé de la courbe de la densité en fonction de la profondeur pour une glace dont le contenu d'air est constant. Les profils profondeur/densité observés ne correspondent pas à ceux prévus sur la base de cette théorie; on discute les raisons de cette divergence: en particulier l'effet du caractère plastique de la glace, qui fait que la pression atmosphérique ne se trouve pas identique à la pression hydrostatique de la glace dans les couches supérieures. Les relations empiriques entre la profondeur, z, et la densité, ρ, s'expriment d'habitude par la formule $z = k\rho^n + c$, k, c et n étant constants. La loi théorique, quoiqu'elle ne soit pas de cette forme, a pourtant avec la loi empirique un rapport approximatif quand il s'agit d'une variation de profondeurs s'échelonnant en-dessous d'une certaine limite; cette loi empirique, plus simple, est donc utilisée pour la série de profondeurs considérables, et sert à calculer des rapports de paramètres utiles caractéristiques du procédé de densification tels que la vitesse de l'affaissement et la vitesse de la densification; on peut ainsi exprimer ces vitesses comme fonctions de la profondeur, de l'alimentation et des constants k, c et n.

Zusammenfassung. Die Dichte von Gletschereis von bekanntem Luftgehalt lässt sich berechnen, wenn angenommen wird, dass sowohl Eis als auch Luft einem Druck des darüber lastenden Materials ausgesetzt werden. Auf dieser Weise ist es möglich, die Form abzuleiten, die eine Dichtenkurve in Abhängigkeit von Tiefe für Eis bei konstantem Luftgehalt haben sollte. Die Gründe für die Divergenz der beobachteten Tiefen-Dichtenprofile von den nach dieser Theorie vorhergesagten werden besprochen, insbesonders die Auswirkung des plastisch-viskosen Verhaltens des Eises, was einen Druck ergibt, der sich von dem hydrostatischen Druck des darüber liegenden Eises unterscheidet. Die empirische Beziehung $z = k\rho^n + c$, zwischen Tiefe z und Dichte ρ, wobei k, n und c Konstanten sind, wird im Hinblick auf diese Theorie besprochen und wird ferner benutzt, um verwendbare Parameter abzuleiten, die den Verdichtungsvorgang charakterisieren, wie z.B. die Senkungs- und Verdichtungsgeschwindigkeit ausgedrückt als Funktion von Tiefe, Akkumulation und den Konstanten k, n und c.

Introduction

Glaciers are moving accumulations of ice of atmospheric origin containing a considerable amount of air (up to 11 per cent of the total volume). As fresh layers are precipitated in the accumulation area, the ice steadily subsides—sometimes up to 3 or 4 km.—and subsequently melts in the ablation area of the glacier. While subsiding, the ice is compressed and increases in density under the pressure of the overlying layers, and when it returns to the surface it partially expands again and becomes less dense. The pressure within a glacier depends both on the depth below the surface and on the ice density, the latter also changing under the influence of the different temperatures in the various parts of the glacier. For a given rate of accumulation or ablation of ice at the surface, the rate of subsidence or elevation of the ice depends on its densification or expansion. Accordingly the age of ice at a given depth may differ.

Theory of the Depth-Density Relation

Ice-I, of which the glaciers are composed, has a volumetric compressibility [1] χ of $1\cdot2\times10^{-5}$ bar^{-1}, a coefficient of linear expansion (averaged over the different crystallographic directions)[2] α_i of $5\cdot1\times10^{-5}$ °C.$^{-1}$ and a density at a pressure p_n of one atmosphere and a temperature θ of 0° C. given [3] by $\rho_i = 0\cdot91670$ g. cm.$^{-3}$, while the air included in the ice compresses

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according to Boyle's Law, having a density ρ_a of $1 \cdot 294 \times 10^{-3}$ g. cm.$^{-3}$ and a coefficient of volume expansion β_a of $3 \cdot 66 \times 10^{-3}$ ° C.$^{-1}$.

The analytical solution of the problem of the density of glacier ice at a given depth using these data is possible only under the assumption that the processes of ice densification and expansion are purely elastic and instantaneous, so that the ice density at any instant depends solely on the external pressure and temperature.

If the initial ice density is ρ_o and the initial pressure (equal to the atmospheric pressure at the surface) is p_o, then we can consider a thin upper layer of ice weighing $\Delta p = 1$ per unit area and having a thickness $h_o = 1/\rho_o g$ where g is the acceleration due to gravity. If the temperature of the ice is 0° C. and if we neglect the weight of the included air, the part l_o of the layer thickness consisting of ice is

$$l_o = \frac{1-\chi(p_o-p_n)}{\rho_i g}, \tag{1}$$

and the part q_o consisting of air is

$$q_o = h_o - l_o = \frac{1}{\rho_o g} - \frac{1-\chi(p_o-p_n)}{\rho_i g}. \tag{2}$$

Correspondingly at a temperature θ_o,

$$l_o = \frac{1-\chi(p_o-p_n)}{\rho_i g}(1+\alpha_i\theta_o) \tag{3}$$

and

$$q_o = \left\{\frac{1}{\rho_o g} - \frac{1-\chi(p_o-p_n)}{\rho_i g}\right\}(1+\beta_a\theta_o). \tag{4}$$

Let us now suppose that our layer has subsided to a depth z and is subjected to a pressure p. Its density ρ_p is given by

$$\rho_p = 1/h_p g = 1/(l_p+q_p)g \tag{5}$$

where

$$l_p = l_o\{1-\chi(p-p_o)\} \tag{6}$$

and

$$q_p = q_o p_o/p. \tag{7}$$

Hence, neglecting the small term $\chi^2(p-p_o)(p_o-p_n)$,

$$\rho_p = 1 \Big/ \left[\frac{1-\chi(p-p_n)}{\rho_i}(1+\alpha_i\theta_o) + \left\{\frac{1}{\rho_o} - \frac{1-\chi(p-p_n)}{\rho_i}\right\}\frac{p_o}{p}(1+\beta_a\theta_o)\right]. \tag{8}$$

If moreover the temperature of the ice changes from θ_o to θ, then, neglecting the small terms $\alpha^2_i\theta_o(\theta-\theta_o)$ and $\beta^2_a\theta_o(\theta-\theta_o)$, we have

$$\rho_{p,\theta} = 1 \Big/ \left[\frac{1-\chi(p-p_n)}{\rho_i}(1+\alpha_i\theta) + \left\{\frac{1}{\rho_o} - \frac{1-\chi(p-p_n)}{\rho_i}\right\}\frac{p_o}{p}(1+\beta_a\theta)\right]. \tag{9}$$

The depth z_p corresponding to a pressure p may be determined by summing the thicknesses h_i of infinitely thin ice layers, each under the pressure of the overlying ice layers and of the atmosphere:

$$z_p = \lim_{\substack{h\to 0\\ n\to\infty}} \sum_{i=0}^{n-1} h_i = \int_{p_o}^{p} h\,dp = \int_{p_o}^{p} (l+q)\,dp$$

$$= \frac{p-p_o}{\rho_i g}\left\{1-\chi\left(\frac{p+p_o}{2}-p_n\right)\right\}(1+\alpha_i\theta) + \left\{\frac{1}{\rho_o g} - \frac{1-\chi(p_o-p_n)}{\rho_i g}\right\}p_o(\ln p - \ln p_o)(1+\beta_a\theta). \tag{10}$$

The first term in equation (10) is the total thickness of pure ice and the second is that of the included air. Here θ should represent the mean temperature of the whole ice sheet. (The influence of thermal expansion may be calculated more accurately if the law of temperature change with depth is known.) An analogous solution is applicable to any multi-phase system with varying compressibility and thermal expansion of the phases.

Figure 1 shows the change of ice density with depth computed from these formulae using $\theta = 0°$ C. and $p_0 = 1$ atmosphere, and with initial densities $\rho_0 = 0{\cdot}9167$ g. cm.$^{-3}$ (i.e. pure ice devoid of air) and $\rho_0 = 0{\cdot}815$ g. cm.$^{-3}$ (ice very rich in air inclusions). As subsidence proceeds, the ice increases rapidly in density in the upper layers of a glacier owing to the compression of included air, but at greater depths the density of even the most air-rich ice hardly differs from the density of pure ice, and accordingly the rate of change of density with depth is infinitesimal.

In fact, however, the compaction process proceeds in exactly such a manner only in a single-phase system, i.e. in a single crystal free of any impurities. On the other hand in poly-crystalline ice containing air inclusions the process is not purely elastic and fully reversible, since it partly results from a plastic-viscous flow which requires time to occur; thus for its completion, this process requires theoretically an infinitely long time. Therefore in a glacier, where the load is constantly changing (rising in the accumulation and falling in the ablation area) the ice density does not correspond completely to the external pressure at any given moment. The greatest lag of densification behind the increase of load is observed in the upper layers, where in order to reach equilibrium with the external pressure the rate of plastic viscous flow would have to be at its maximum. At greater depths where, for the same rate of pressure change the density alters very little and mainly by the elastic compression of ice, the lag of compaction behind load increase is practically imperceptible.

In ice containing air the lag of ice densification or expansion behind the load change is connected with the difference which arises between the external pressure and the pressure of the included air. This difference is very clearly manifested in the existence in the upper parts of a glacier accumulation area of a layer of snow and firn, which, in cold regions, may reach a thickness of a hundred or more metres. In intercommunicating pores in the snow and firn, the pressure is equal to that of the atmosphere, despite the fact that the pressure of the overlying layers in the lower part of a thick firn sheet may exceed 6 to 8 bars. To reach equilibrium with the external pressure, even the uppermost layers of a glacier would have to be composed of ice with its air inclusions subjected to a pressure exceeding that of the atmosphere by the weight of ice overlying the given level. Such ice has a density of no less than $0{\cdot}815$ to $0{\cdot}820$ g. cm.$^{-3}$, whereas instead there lies snow of density $0{\cdot}1$ to $0{\cdot}5$ g. cm.$^{-3}$. When the deep ice melts out to the surface, it may, conversely, preserve the higher pressure of the included air, which in cold regions reaches 10 to 14 bars. In this way a hysteresis loop arises (Fig. 2).

Thus at greater depths, and also in the upper layers when the ice accumulation or ablation rate is very low, the density distribution in a glacier practically corresponds to the ultimate state of equilibrium with the external pressure, described by equations (9) and (10). However, when the ice accumulation or ablation proceeds more rapidly, the compaction or expansion of the ice in the upper layers of a glacier lags considerably behind the rise or fall of external pressure. In these cases the density distribution can only be determined empirically, since the processes involved have not been sufficiently studied.

The function $\rho = f(z)$ may take various forms and, as Bader [4] has noted, it depends on how much labour one is willing to expend to obtain a close fit—i.e. it depends on the degree of accuracy. Experiments on the artificial compaction of snow, firn and ice have shown, however, that there is a power-law relationship between the relative compression or densification and the magnitude of the load. Hence, if the occasional fluctuations due to variations of the initial density of the layers deposited under different conditions are removed by averaging, the distribution curve for ice density ρ in a glacier with depth z will generally be represented by part of an nth-order parabola

$$z = k\rho^n + c \qquad (11)$$

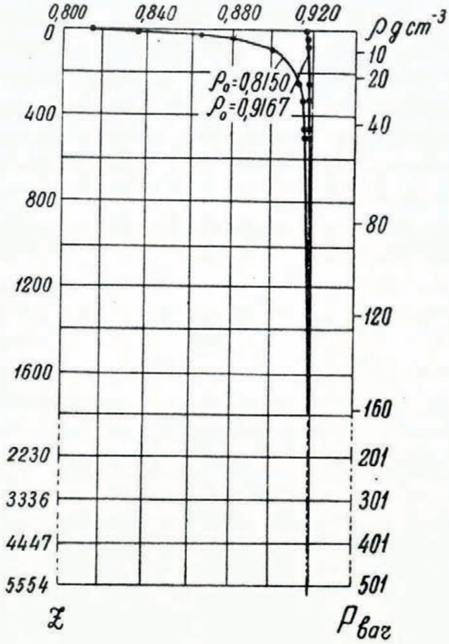

Fig. 1. The variation of ice density with depth when the included air pressure is equal to the pressure of the overlying layers

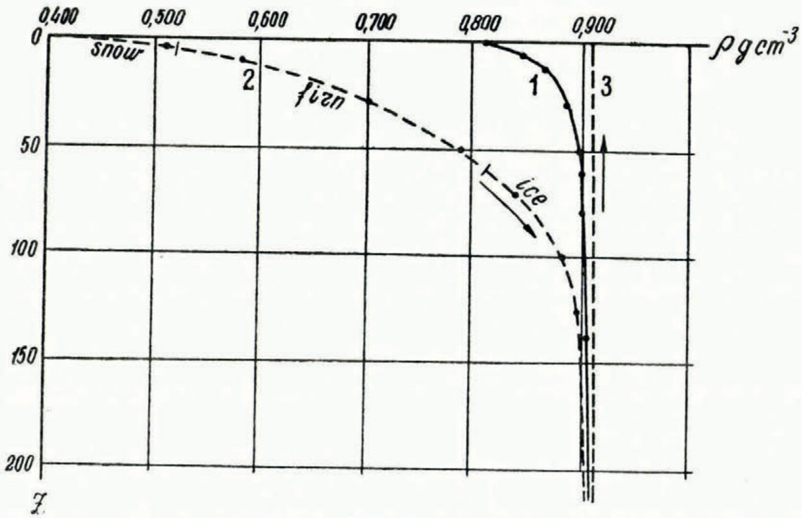

Fig. 2. The variation of ice density with depth:

1. *for ice with $\rho_0 = 0 \cdot 815$ g. cm.$^{-3}$ at $\theta = 0°$ C., $p_0 = 1$ bar and included air pressure equal to the pressure of the overlying layers (ultimate equilibrium state)*
2. *for snow, firn and ice in the accumulation area of a polar glacier (during densification)*
3. *for ice with $\rho_0 = 0 \cdot 815$ g. cm.$^{-3}$ at $\theta = 0°$ C., $p_0 = 1$ bar in the ablation area of a glacier (during) expansion*

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The parameters k, n and c depend on the initial density, structure, temperature, rate of load increase, and peculiarities of the densification mechanism. For this reason they vary for regions of different glacier regime. These parameters also vary in the vertical section of a glacier, and the density curve is divided by more or less pronounced inflexions into at least three parts, corresponding to the snow, firn and ice states. In each of these parts densification proceeds more slowly than in the one above. Owing to the breaking of layers loosened by sublimation processes, snow compacts faster than firn, while ice compacts more slowly than firn because its pores are closed off, and included air can no longer be freely pressed out of the subsiding ice but is compressed elastically in the enclosed bubbles so that it begins to resist compaction.

In some cases the curve of density distribution in a vertical direction also shows inflexions towards faster compaction downwards, as, for instance, between a winter snow layer and the underlying regelated firn that has been compacted during the thawing period, or in regions of high wind where the well packed snow at the surface settles more slowly under low pressures than the deeper layers loosened by sublimation processes.

Beginning at a certain depth in the ice, the empirically determined power law for the depth-density relationship gives a result close to the ultimate theoretical curve corresponding to equilibrium between internal and external pressure, which can also be approximated to by an equation of the form of (11). For instance the ultimate theoretical curve for the initial density $\rho_0 = 0{\cdot}91670$ g. cm.$^{-3}$ is satisfactorily described by equation (11) with the following parameters (in c.g.s. units):

$$k = 8{\cdot}9208\times 10^7 \text{ cm.}^{3n+1}\text{ g.}^{-1},\; n = 43{\cdot}6 \text{ and } c = -2{\cdot}0111\times 10^6 \text{ cm.}$$

Derivation of Parameters Characterizing the Densification Process

When the relationships we have discussed above are known, it is easy to determine all the parameters characterizing the densification or expansion processes for glacier ice, snow or firn. Thus the pressure p_z at depth z will be

$$p_z = p_0 + \int_0^z \rho_z g\, dz = p_0 + \frac{(z-c)^{(n+1)/n} + c^{(n+1)/n}}{k^{1/n}(n+1)/n} g. \tag{12}$$

The parameter c and the quantity $p_c = \dfrac{ng}{k^{1/n}(n+1)}\, c^{(n+1)/n}$ are interesting indicators of the snow and firn densification processes, characterizing the glaciological conditions in a given area as a function of climatic conditions. $-c$ corresponds to the depth, and $-p_c$ to the weight,* of the snow or firn layer that would have to lie over the existing surface for densification to occur under the given processes from zero to the initial density observed at the surface.

The gradient of ice densification with depth $d\rho/dz$ at depth z is given by

$$\frac{d\rho}{dz} = \frac{(z-c)^{(1-n)/n}}{nk^{1/n}}. \tag{13}$$

The densification or compression δ_z relative to the surface density is given as a function of z by

$$\delta_z = \frac{\rho_z - \rho_0}{\rho_0} = 1 + \left(\frac{c}{z-c}\right)^{1/n}, \tag{14}$$

and the rate of deceleration of relative compression with depth $d\delta/dz$ at depth z is equal to

$$\frac{d\delta}{dz} = -\frac{c^{1/n}}{n(z-c)^{(n+1)/n}}. \tag{15}$$

* c is a negative quantity, and $c^{(n+1)/n}$ and similar quantities are also assumed to be negative here and in subsequent workings.

If in addition the rate of accumulation or ablation $A = dm/dt$ of mass m at the glacier surface is also known, then assuming A to be constant (i.e. a steady regime) it is also possible to determine for any depth z under the surface a number of other characteristics:

Age of ice
$$t_z = (p_z - p_0)/Ag. \tag{16}$$

Rate of ice subsidence
$$v_z = \frac{dz}{dt} = \frac{A}{\rho_z} = A\left(\frac{k}{z-c}\right)^{1/n}. \tag{17}$$

Rate of ice densification
$$\frac{d\rho}{dt} = \frac{A}{n(z-c)}. \tag{18}$$

Rate of relative compression of ice
$$\frac{d\delta}{dt} = -\frac{Ak^{1/n}c^{1/n}}{n(z-c)^{(n+2)/n}}. \tag{19}$$

Variation of velocity of ice subsidence with depth
$$\frac{dv}{dz} = -\frac{Ak^{1/n}}{n(z-c)^{(n+1)/n}}. \tag{20}$$

It should, however, be noted that the formulae given above presuppose the absence of any ice movement in the horizontal direction, and that this is not true for glaciers. When considering the density and other characteristics of the ice at considerable depths, it is necessary to take into account changes in the initial density and the accumulation or ablation rate at a place up-glacier by the distance which the ice has moved horizontally since its formation.

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