perspective: excellent examples are Brian Hall's *Lie Groups, Lie Algebras and Representations* and John Stillwell's *Naïve Lie Theory*. Both books do quite a bit with matrix groups without having to mention general manifolds; a similar discussion here would have been more elementary and linear-algebraic in nature. Of course, all this is a matter of personal taste: there may well be instructors out there who think a graduate course in linear algebra is an appropriate venue for discussions of differentiable manifolds or Lie groups, and for such people these two chapters should prove very useful.

The presence of chapters 4 and 5 do, however, make this book a useful reference source for people wanting a quick overview of these areas of mathematics without tackling more sophisticated texts.

Given the subject matter covered in this volume, it is not surprising that *LA II* is a somewhat more demanding text than is *LA I*. Nevertheless, it retains a number of useful pedagogical features of the first volume, and, in fact, adds a new one. The retained features include generally clear exposition, a decent number of examples, and a good number of exercises (including nicely chosen and useful True-False problems). As in the first volume, some exercises are embedded in the text itself, and others appear collected at the end of each chapter. Also as in volume I, no solutions are provided. The newly added feature is the presence of an overview for each chapter, generally several pages long, which helps set the stage for the more detailed discussion that is to follow. These overviews seemed to me to be informative and valuable.

In one respect, however, this second volume is problematic: Volume I had an Index, but this book does not. This, I think, is an inexcusable omission for any mathematics text (or, for that matter, *any* text). On the flip side, Volume I lacked a bibliography; this book has one, but it's very small (6 items), and is limited to books on the subject matter of the last two chapters, namely differential geometry and Lie theory. A list of some of the many books covering linear algebra at a reasonably sophisticated level would have been appreciated. The books listed are also fairly old (the most recent one was published in 2002).

So, to summarise and conclude: the eclectic selection of topics in *LA II* might pose problems with using this book as a stand-alone text for a traditional graduate course in linear algebra; it could more readily be used in conjunction with *LA I*, but that would require the students to purchase two texts. However, over and above the use of this book as a text, it would make an interesting reference source, especially for people who wish to see a relatively quick overview of differential geometry or Lie theory.

Reference

1. Frederick P. Greenleaf and Sophie Marques, *Linear Algebra I*, American Mathematical Society (2019).

10.1017/mag.2022.47

MARK HUNACEK
Iowa State University,
Ames, IA 50011, USA
e-mail: *mhunacek@iastate.edu*

Advanced calculus explored by Hamza Alsamraee, pp. 448, $26.99 (paper), ISBN 978-0-578-61682-7, Curious Math Publications (2019)

One of the most surprising things about this book is the age of its author. At the time of its publication Alsamraee was in his final year of secondary school. As incredible as I found this, in a writer who is so young come both advantages and drawbacks, and I shall come to both shortly. Despite the word ‘calculus’ in its title, *Advanced calculus explored* is really a book on advanced techniques for evaluating definite integrals. G. H. Hardy famously said he could never resist the challenge of a definite integral and I would have to tend to agree with him. The type of integrals found here reminded me of those considered in books such as those by Paul Nahin [1] and Cornel Valean [2], or those that appear from time to time on MATHEMATICS STACK EXCHANGE, an online question-and-answer site—most cannot be found by elementary means and are not of the type one usually encounters on first meeting integration. The techniques considered include the use of infinite series, and 'Feynman's trick', differentiating under the integral sign after the introduction of a parameter [3, pp. 86-87]. Other techniques include the use of integrals I and J and finding $I + J$ and $I - J$ first, or using the symmetry of the integrand. The author also gets great mileage out of what I refer to as a 'symmetric border flip' [4, pp. 214–215]. Here, for a continuous function f on the interval $[a, b]$ with $a < b$, one has $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$, and by recognising when it applies, many a challenging integral can be laid low. A good variety of interesting and difficult integrals are given throughout, as either examples or end-of-chapter exercises. If worked through and understood these are guaranteed to give the reader a solid grounding in the evaluation of such integrals. Some well-known classical but tricky integrals such as Euler's log-sine integral are considered (see Example 2 starting on page 67). I was also pleased to see an evaluation of the double integral (Example 7 on page 119)

$$\int_0^\infty \int_0^\infty \frac{\sin x \sin y \sin(x + y)}{xy(x + y)} dx\, dy$$

by first exploiting symmetry in the integrand before using Feynman's trick in an interesting way. This integral first appeared as a problem in *The American Mathematical Monthly* [5, 6]. Raabe's log-gamma integral from 1840 of

$$\int_0^1 \log\left(\Gamma(x)\right) dx$$

also makes an appearance (Example 4 on page 71 or Example 2 on page 272 for a generalisation) and more incredibly when the integrand is squared (Exercise 6 on page 276), an integral only recently the subject of investigation [7].

Each of these different techniques appears in a chapter of its own. A bare minimum of theory is introduced; eight to ten examples are then given showing how the technique can be used and applied in practice, and each chapter concludes with a similar number of exercises. There are also several chapters on supporting material. Chapter 1 provides a quick overview of the concepts of the limit and the derivative, and chapter 4 of simple series where the arithmetic, geometric, arithmetic-geometric, and telescoping series are introduced. Chapter 5 defines absolutely and conditionally convergent series and gives standard tests for the convergence or divergence of an infinite series. Interestingly the chapter also provides criteria under which an infinite summation and integration can be interchanged. The author opts for the Lebesgue dominated convergence theorem; though measure theory is well beyond the scope of the book the author does a good job of presenting a definition that is able to deal with the types of examples encountered in his book. The evaluation of some series using either integration or by first converting the series into a double summation is considered in chapter 6. The harmonic numbers H_n defined by $\sum_{k=1}^{n} \frac{1}{k}$ are introduced here and some series containing these numbers, which now go by the name of Euler sums, are considered. There are separate chapters on four important special functions that arise repeatedly: the gamma function (chapter 9), the polygamma functions (chapter 10), the beta function (chapter 11), and the Riemann zeta function (chapter 12). Four chapters at the end of the text provide examples of applications.

The youthful exuberance with which the author launches into the material is on clear display at many places throughout the book. This is in some ways an advantage as the writing style of the author encourages you and buoys you along but it can also be a disadvantage, particularly when the author is overtaken by his desire to impress his readers with his knowledge. An example can be found in the first chapter. When considering limits, the fifth example to be given is one involving nothing less than the Riemann zeta function! It is a nice example and the urge to present something different and a little unusual is hard to resist, but a proper preparation of the requisite background material is important if the author is not to lose many of his readers very early on. For readers for whom the Riemann zeta function or the gamma function is completely new, these two functions crop up a little too often before they are considered in the text. For such readers this will no doubt cause a very disjointed reading experience as one is forced to flip forward and back again.

A few small quibbles. The layout of the text is not justified but is instead ragged right. This made, at least for me, a somewhat uncomfortable read. The text has also been only lightly edited and suffers throughout from large areas of whitespace, reflecting no doubt its self-published nature. The order of the chapters is also less than ideal. I would have liked to see the chapters on the gamma and beta functions introduced much earlier.

Evaluating integrals has always had its coterie of dedicated admirers. This book is likely to appeal to such an audience. For the more experienced integrator, if one overlooks the more serious shortcomings one is left with a useful collection of interesting and more difficult examples and problems. For the neophyte I expect the going will be much tougher. Mastery of elementary integration techniques of the type considered in [4] is expected and a course in real analysis would prove useful, particularly in handling infinite series. With some effort and persistence it should be possible for beginning students to navigate their way through the text and grasp most of the material. While this book is an admirable attempt at lifting the lid on the attraction and fascination many find in evaluating difficult integrals, I believe the definitive book on such methods is yet to be written.

References

1. P. J. Nahin, *Inside interesting integrals*, Springer (2015).
2. C. I. Valean, *(Almost) impossible integrals, sums, and series*, Springer (2019).
3. R. P. Feynman, *Surely you're joking, Mr. Feynman*!, Vintage Books (1992).
4. S. M. Stewart, *How to integrate it*, Cambridge University Press (2018).
5. C. I. Valean, Problem 11956, *Amer. Math. Monthly* **124** (January 2017) p. 83.
6. R. Chapman, Using Plancherel to evaluate a double integral. Solution to Problem 11956, *Amer. Math. Monthly* **125** (November 2018) pp. 854-856.
7. T. Amdeberhan, M. W. Coffey, O. Espinosa, C. Koutschan, D. V. Manna and V. H. Moll, Integrals of powers of loggamma, *Proc. Amer. Math. Soc.* **139** (2011) pp. 535-545.

10.1017/mag.2022.48 © The Authors, 2022
Published by Cambridge University Press on behalf of The Mathematical Association

SEAN M. STEWART
9 Tanang Street, Bomaderry
NSW 2541, Australia
e-mail: *sean.stewart@physics.org*