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Effectiveness evaluation of fighter using fuzzy Bayes risk weighting method

M. Suo, S. Li, Y. Chen, Z. Zhang, B. Zhu and R. An
buaasuozi@hotmail.com, lishunli@hit.edu.cn
School of Astronautics
Harbin Institute of Technology
Harbin
China

ABSTRACT

Multiple Attribute Decision Analysis (MADA), known to be simple and convenient, is one of the most commonly used methods for Effectiveness Evaluation of Fighter (EEF), in which the attribute weight assignment plays a key role. Generally, there are two parts in the index system of MADA, i.e. performance index and decision index (or label), which denote the specific performance and the category of the object, respectively. In some index systems of EEF, the labels can be easily obtained, which are presented as the generations of fighters. However, the existing methods of attribute weight determination usually ignore or do not take full advantage of the supervisory function of labels. To make up for this deficiency, this paper develops an objective method based on fuzzy Bayes risk. In this method, a loss function model based on Gaussian kernel function is proposed to cope with the drawback that the loss function in Bayes risk is usually determined by experts. In order to evaluate the credibility of assigned weights, a longitudinal deviation and transverse residual correlation coefficient model is designed. Finally, a number of experiments, including the comparison experiments on University of California Irvine (UCI) data and EEF, are carried out to illustrate the superiority and applicability of the proposed method.

Keywords: Effectiveness evaluation; Bayes risk; Weight assignment; Fuzzy; Gaussian kernel; Correlation coefficient; Fighter

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NOMENCLATURE

BP	Basic Performance
CA	Classification Accuracy
CCSD	Correlation Coefficient and Standard Deviation
CE	Conditional Entropy
CRITIC	CRiteria Importance Through Inter-criteria Correlation
CS	Cosine Similarity
DM	Decision Maker
DS	Decision System
DSOM	Decision System Objective Method
EEF	Effectiveness Evaluation of Fighter
FBRW	Fuzzy Bayes Risk Weight assignment method
GJC	Generalized Jaccarb Coefficient
GRA	Grey Relation Analysis
IS	Information System
ISOM	Information System Objective Method
JC	Jaccarb Coefficient
LTCC	Longitudinal deviation and Transverse residual Correlation Coefficient
MADA	Multiple Attribute Decision Analysis
MP	Manoeuvre Performance
NRS	Neighbourhood Rough Set
PCC	Pearson Correlation Coefficient
SD	Standard Deviation
SMC	Simple Matching Coefficient
WDBC	Wisconsin Diagnostic Breast Cancer
a	an attribute
A	attribute set
c	a conditional attribute
C	conditional attribute set
d	a class of decision
D	decision attribute
I	information function
LCC	Longitudinal Correlation Coefficient
N	neighbourhood
P	conditional probability
r	region
R	Bayes risk
TCC	Transverse Correlation Coefficient
U	universe
V	value
w	weight
W	weight vector
x	sample

Greek Symbol

$\triangle$	distance metric
δ	neighbourhood threshold
γ	loss function

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1.0 INTRODUCTION

The Effectiveness Evaluation of Fighter (EEF) is one of the most common approaches to measure the capabilities of fighter to accomplish some specific tasks, which could be applied to many aspects such as fighter design, combat simulation and military might comparison[(1)]. There are several categories of methods for EEF such as Analytic Hierarchy Process (AHP)[(2)], Availability-Dependability-Capability (ADC)[(3)], synthesised index method[(4)], Fuzzy Evaluation (FE)[(1)] and Multiple Attributes Decision Analysis (MADA)[(5)], etc. Therein, MADA, known to be simple and practical, can only rely on the characteristics of the data in the index system to obtain the evaluation results. In the MADA method, the decision result of the object is obtained by the weighted sum so as to evaluate the comprehensive performance of the object.

Attribute weight assignment plays a significant role in MADA, which can be generally divided into three categories of methods, i.e. subjective methods[(6,7)], objective methods[(9,13,14)] and hybrid methods[(15-18)], according to the extent of dependence on the preferences or subjective judgements of Decision Makers (DMs)[(8)]. In practical applications, some ideal weights are usually hard to be obtained by the subjective or hybrid methods when there is a lack of related field experts or no unanimous conclusion reached by DMs[(19,20)]. Fortunately, the objective weight methods can effectively solve the above problem, because attribute weights are generated by using data rather than the DMs' reference.

In terms of the data systems used in MADA, they can be broadly divided into two categories, namely Decision System (DS) and Information System (IS), of which DS is a set of data consisting of conditional attributes and decision attributes, and IS does not include decision attributes, i.e. labels. With respect to DSs, although they contain decision attributes, there are still many issues in MADA, such as effectiveness evaluation[(21)], classification[(22)] and fault diagnosis[(23)]. According to the applied data systems, there are two parts regarding the objective methods, including Information System Objective Method (ISOM) and Decision System Objective Method (DSOM). Based on these two categories, further introduction of objective methods are given in detail.

Most objective weight assignment methods aim at IS. Among them, Entropy method[(8,9)] is the most popular one in ISOMs, based on which a number of approaches are developed to obtain more satisfactory results of weight assignment[(8,10,11)]. To mention a few, Valkenhoef and Tervonen[(10)] discussed the entropy-optimal weight constraint elicitation problem with additive multi-attribute utility models. He et al[(11)] proposed a linguistic entropy weight method to determine the attribute weights in the linguistic MADA. Yang et al[(8)] designed a three-stage hybrid weight assignment approach based on entropy theory. In this category, another widely used method is Principal Components Analysis (PCA)[(12)]. Moreover, Diakoulaki et al[(14)] raised a weight determination method based on the quantification of two fundamental nations of MADA: the contrast intensity and the conflicting character of the evaluation criteria, which is named CRiteria Importance Through Inter-criteria Correlation (CRITIC). Deng et al[(9)] employed Standard Deviation (SD) to obtain the weights of attributes. In order to consider the relationships of attributes, Wang and Luo[(13)] proposed a Correlation Coefficient and Standard Deviation (CCSD) integrated method for determining attribute weights. To the best of the authors' knowledge, all of the above-mentioned methods do not take account of the contributions of decision attributes to the determination of conditional attribute weights, when applying to DS. With respect to DS, there is usually a single decision attribute[1] that can

[1] In fact, systems with multiple decision attributes can also be transformed into ones with a single decision attribute.

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be regarded as a generalisation of the overall system and an abstract of all the conditional attributes. Each conditional attribute provides a particular contribution to its system and an individual support degree to the abstract of the decision attribute, which could be depicted by the weight of the conditional attribute. Therefore, the weight determination of the conditional attribute cannot ignore the role of the decision attribute in DS. The approaches, such as the Conditional Entropy (CE)[24] approach, Grey Relation Analysis (GRA)[25] approach and Rough Set (RS)[26] approach, could be considered as the alternatives for the weight assignment of the conditional attribute in DS because of taking into account the coupling relationships between the conditional attributes and the decision attribute.

In fact, with regard to MADA, the final decision produced by any decision-making unit will be accompanied by some risks. These risks are usually derived from the difference in the distribution of data between the conditional attributes and the decision attribute. Consequently, each of the conditional attributes will generate a unique risk for the final decision, which could employ the weight of attribute as a metric. In addition, the fuzzy membership of a sample induced by conditional attributes to a decision attribute can be considered as a kind of main relationship between the two types of attributes. However, the aforementioned objective methods, whether ISOMs or DSOMs, have not taken the decision risk as a main factor to determine the weights of attributes, and not taken the above fuzzy membership into account as well. Furthermore, with respect to the weight assignment in a multi-layer attribute set, it usually needs the help of experts (DMs) or is achieved through some complex combination methods[15], which have greatly limited the application of a weight assignment in a multi-layer index system. These inadequacies of the present research motivate this work.

To handle the aforementioned issues and overcome the deficiencies of the existing methods, we propose a simple and effective objective attribute weight assignment method based on a Fuzzy Bayes Risk (named FBRW), which is not only suitable for ISs and DSs, but can be also applied to single-layer and multi-layer index systems. Bayes risk utilises the calculation of probability to estimate the risk of event, which takes full account of the causality and dependency between each event, i.e. the relationship of the conditional attributes and the decision attribute in DS. Therefore, Bayes risk is very suitable for estimating the weights of conditional attributes. The loss function in Bayes risk, however, is usually determined by experts or through a large number of statistical tests[35,36], which greatly limits the practical application and extension of Bayes risk theory. Based on Gaussian kernel, a loss function model is proposed to cope with this drawback, in which the loss values of samples are obtained by the distribution characteristics of data. Furthermore, considering the fuzziness of the data system, the fuzzy similarity of each sample and the fuzzy membership between conditional attributes and the decision attribute are employed in our method. On the other hand, as a significant part of the weight assignment, the weight evaluation has not been paid enough attention in literature. Hence, we propose a correlation coefficient named Longitudinal deviation and Transverse residual Correlation Coefficient (LTCC) that considers two directions, i.e. the longitudinal direction and the transverse direction, to measure the similarity between the assigned weights and the reference ones. Subsequently, a number of comparison experiments are carried out to illustrate the superiority of the proposed method. Finally, we demonstrate and verify the applicability of the proposed method through the effectiveness evaluation of fighter. Therefore, the main contributions of this work lie in that

(1) A simple and effective objective attribute weight assignment method based on fuzzy Bayes risk is proposed.

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(2) A Gaussian kernel loss function model is raised, which could promote the application and extension of Bayes risk theory.

(3) A longitudinal deviation and transverse residual correlation coefficient model is raised for weight evaluation.

(4) The demonstration and analyses of the fighter effectiveness evaluation have guiding significance for other similar engineering applications.

The remainder of this paper is organised as follows. Section 2 introduces the preliminaries for this work. The basic theories and analyses of the proposed method are presented in Section 3. The weight evaluation model is depicted in Section 4. The results and analyses of numerical experiments are given in Section 5, and the effectiveness evaluation of fighter is demonstrated in Section 6. Then, some discussions are brought in Section 7. Finally, conclusions and future work are described in Section 8.

2.0 PRELIMINARIES

This work takes the decision system as the research object, and employs the Bayes risk and neighbourhood relationship to realise the assignment of attribute weight. Thus, in this section, the aforementioned theories and concepts are introduced, which will pave the way for the further development of the following sections.

Definition 1. *(Decision system)*[28] *A decision system is a 4-tuple* $DS = (U, \{A/A = C \cup D\}, \{V_a/a \in A\}, \{I_a/a \in A\})$, *where* U *is a finite set of objects called universe and* $U = \{x_1, x_2, \cdots, x_m\}$, A *is the attribute set,* C *is the set of conditional attributes,* D *is the decision attribute,* $C \cap D = \emptyset$, $D \neq \emptyset$, V_a *is a set of values of each* $a \in A$, *and* I_a *is an information function for each* $a \in A$.

On account of such a form of the four tuples (i.e. U, A, V, I) and the three basic elements (i.e. U, C, D) of the decision system in Definition 1, a decision system is often denoted as $DS = (U, A, V, I)$ or $DS = (U, C \cup D)$ for short. It is noted that the above definition of the decision system only considers one decision attribute, and such a definition has been widely used in practical applications. In fact, systems with multiple decision attributes can be also transformed into the ones with single decision attribute. Specifically, a decision system is also called an information system $IS = (U, C)$ if its decision attribute forms an empty set[29].

Definition 2. *(Neighbourhood)*[27] *Given an arbitrary sample* $x_i \in U$ *and a conditional attribute subset* $B \subseteq C$, *a neighbourhood* $N_B(x_i)$ *of* x_i *in* B *is defined as:*

$$N_B(x_i) = \{x_j | x_j \in U, \triangle_B(x_i, x_j) \leq \delta\}, \quad \ldots (1)$$

where $\triangle$ *is a metric,* δ *is a threshold.*

In order to consider the fuzziness between each sample, a fuzzy similarity relation[33] is introduced as follows:

$$\triangle_a(x_i, x_j) = \begin{cases} 1 - |x_i - x_j|, & |x_i - x_j| \leq \delta, \\ 0, & |x_i - x_j| > \delta, \end{cases} \quad \ldots (2)$$

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where $a \in C$, δ is a neighbourhood threshold and $0 < \delta \leqslant 1$. Obviously, the following properties hold:

(1) $\triangle_a(x_i, x_j) = \triangle_a(x_j, x_i)$;

(2) $\triangle_a(x_i, x_i) = 1$;

(3) $0 \leqslant \triangle_a(x_i, x_j) < 1$.

The samples with respect to x_i satisfied the first condition in Equation (2), are denoted by $N_a(x_i)$, and named the fuzzy neighbourhood of x_i.

For a subset $B \subseteq C$, $a \in B$, the fuzzy neighbourhood relation $\triangle_B(x_i, x_j)$ between x_i and x_j is defined as follows:

$$\triangle_B(x_i, x_j) = \cap_{a \in B} \triangle_a(x_i, x_j), \qquad \ldots (3)$$

where $\cap$ is the fuzzy conjunction.

Definition 3. *(Bayes risk)*[30] *Given a domain of objects* X ($X = \{x_1, x_2, \cdots, x_m\}$) *and a set of classes* Y ($Y = \{y_1, y_2, \cdots, y_n\}$)*. For a classification function* $C : X \rightarrow Y$ *that maps each object to one class, the risk of classifying* x_i ($x_i \in X$) *into* y_p ($y_p \in Y$) *is defined as follows:*

$$R(y_p(x_i)) = \sum_{q=1}^{n} \lambda_q^p \cdot P(y_q|x_i), \qquad \ldots (4)$$

where λ_q^p *is a loss function that measures the error of classifying object* x_i *into class* y_p *knowing that the possible class is* y_q*, and* $P(y_q/x_i)$ *is the probability of object* x_i *belonging to class* y_q.

It is worth noting that different loss functions will yield different decision risks. In practical applications, the loss function is usually difficult to be provided, which has been the key bottleneck of the application and extension of Bayes risk theory.

3.0 WEIGHT ASSIGNMENT BASED ON FUZZY BAYES RISK

Through the previous analyses in Sections 1 and 2, we can know that the decision risk can be considered as an important factor in the attribute weight determination in DS. Therefore, in this section, the single-layer dataset is taken as an example to analyse the presented form of the above-mentioned risk in DS. Subsequently, the FBRW method is designed. Finally, the FBRW method is extended for the weight assignment in a multi-layer dataset.

3.1 FBRW for single-layer DS

Generally, data-driven attribute weight is determined based on a single-layer dataset[9,13]. Considering a single-dimensional conditional attribute c ($c \in C$) and $D = \{d_1, d_2, \cdots, d_K\}$ in DS, the distribution of sample set X ($X = \{x_1, x_2, \cdots, x_m\}$) in c with respect to D can be depicted as Fig. 1, where d_i, d_j, $d_k \in D$.

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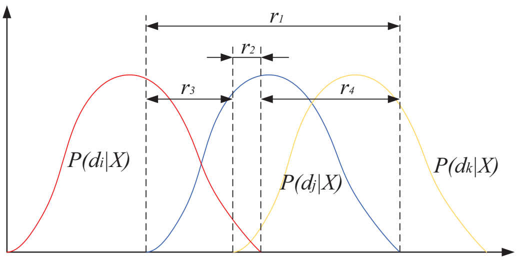

Figure 1. (Colour online) The distribution of X in c with respect to D.

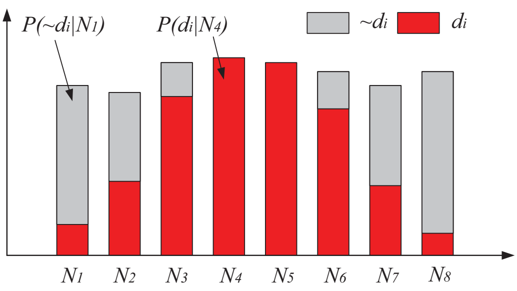

Figure 2. (Colour online) The distribution of N in c with respect to d_i.

From Fig. 1, we can see that decision making in region r_1 is bound to generate risks because of the overlaps of three probability distributions. Region r_2, in particular, will produce a riskier decision than r_3 and r_4 because there are three distributions in it.

With the help of a neighbourhood relationship, we transform the above distributions into discrete space, where the basic cells are the neighbourhood set instead of sample set. We take a set of neighbourhoods with regard to decision class d_i as an example, and the distribution of neighbourhood sets N in c is depicted as Fig. 2.

In Fig. 2, N_k ($N_k \in N$, $k = 1, 2, \cdots, 8$) is the neighbourhood of sample x_k, the red regions denote that the samples in neighbourhood N_k are classified into d_i in terms of a given metric, and the grey regions are the ones classified into other classes except d_i.

According to the above analyses and the Definition 3 of Bayes risk, the risk produced by a conditional attribute could be the error that measures the difference between the distribution of the current conditional attribute and that of the decision attribute. Therefore, we can derive the principle of attribute weight assignment in DS, as such an attribute should be assigned a greater weight whose Bayes risk with respect to decision attribute is less. Therefore, the definition of Bayes risk with respect to DS can be drawn as follows.

Definition 4. *(Bayes risk in DS) Given a decision system* $\mathrm{DS} = \{\mathrm{U}, \mathrm{C} \cup \mathrm{D}, \mathrm{V}, \mathrm{I}\}$, $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \cdots, \mathrm{x}_\mathrm{m}\}$, $\mathrm{D} = \{\mathrm{d}_1, \mathrm{d}_2, \cdots, \mathrm{d}_\mathrm{K}\}$, *for an arbitrary sample* x_i $(\mathrm{x}_\mathrm{i} \in \mathrm{U})$*, it may be divided into any decision classes of* D *with respect to an attribute* c $(\mathrm{c} \in \mathrm{C})$ *by using some metrics, but it belongs to a certain class* $\mathrm{d}_\mathrm{k}(\mathrm{d}_\mathrm{k} \in \mathrm{D})$ *in terms of the information function* I*. Therefore, the Bayes risk of* x_i *vesting in* d_k *with respect to* c *is defined as:*

$$R_c(d_k|x_i) = \sum_{j=1}^{K} \lambda_j^k(c, x_i)P(d_j|x_i), \quad \ldots (5)$$

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where $\lambda_j^k(c, x_i)$ *is a loss function that measures the loss relative to its own class* d_k *when* x_i *is classified into the possible class* d_j*, and* $P(d_j/x_i)$ *is the probability of* x_i *belonging to* d_j.

The commonly used loss function is 0–1 type, but it can not effectively evaluate the real loss of decision making. In order to improve the effectiveness of loss function, it is usually determined by experts or through a large number of statistical tests[35,36], which has been a stumbling block to the application and extension of this theory. Therefore, we propose a loss function based on Gaussian kernel in which the loss values of samples are depicted by the distribution characteristics of data.

Definition 5. *(Gaussian kernel loss function) Given a decision system* $DS = (U, C \cup D)$*,* $c \in C$*,* $D = \{d_1, d_2, \cdots, d_N\}$*, for an arbitrary sample* x_i *in* U*, its designated decision class is* d_k *and its possible class is* d_j *produced by a given metric,* $d_j, d_k \in D$*. Then, the Gaussian kernel loss function of* x_i *relative to* c *is defined as:*

$$\lambda_j^k(c, x_i) = \begin{cases} \exp\left(-\frac{(x_i-\mu_k)^2}{2\sigma_k^2}\right), & k \neq j, \\ 0, & k = j, \end{cases} \quad \ldots(6)$$

where μ_k *is the expectation of the sample set belonging to class* d_k *with respect to* c*, and* σ_k *is the corresponding standard deviation,* $k, j \in \{1, 2, \cdots, N\}$*. Usually, we take the loss function as* λ_j^k *for short.*

Remark 1. There are three aspects about the Gaussian kernel loss function: (a) the loss of the sample is 0 if it is divided into its own decision class, i.e. $k = j$; (b) the smaller the distance between the sample and the expectation, the greater the loss will be; (c) the loss is 1 if the standard deviation is 0, which means that all the data in this class are equal to each other, i.e. every datum is the expectation.

Through the above definition and remark, we can see that the Gaussian kernel loss function model could accord with the definition of loss function in Definition 4.

Definition 6. *(Probability) Given a decision system* $DS = (U, C \cup D)$*, for an arbitrary sample* $x_i \in U$*, its neighbourhood is* $N(x_i) = \{x_1, x_2, \cdots, x_m\}$*, and the corresponding decision set is* $N(d) = \{d_1, d_2, \cdots, d_p\}$*,* $N(d) \subseteq D$*. Thus, the probability of* x_i *classifying to* d_j *(*$d_j \in N(d)$*) is denoted as:*

$$P_c(d_j|x_i) = \frac{\sum_k^m \triangle(x_i^j, x_k^j)}{\sum_k^m \triangle(x_i, x_k)}, \quad \ldots(7)$$

where x^j *is the sample belonging to decision class* d_j *in terms of the information function in this DS and* $\triangle$ *is the fuzzy similarity relation between* x_i *and* x_k*. This definition can also be called fuzzy membership due to the fuzzy similarity relation, and the Bayes risk in Definition 4 can be named as Fuzzy Bayes Risk (FBR), which is more succinct than our previous result*[37].

Theorem 1. *The Bayes risk (Definition 4) defined as* $R_c(d_k|x_i) = \sum_{j=1}^K \lambda_j^k P(d_j|x_i)$ *is equivalent to* $R_c(d_k|x_i) = \lambda_{\sim k}^k(1 - P(d_k|x_i))$*, where* $d_{\sim k}$ *is the decision class except for* d_k.

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Proof. *According to [Equation (6)](), the loss function can be rewritten as* $\lambda_{\sim k}^{k} = \exp\left(-\frac{(x_i-\mu_k)^2}{2\sigma_k^2}\right)$ *if* $\mathrm{k} \neq \mathrm{j}$ *and* $\lambda_k^k = 0$ *(*$\mathrm{k} = \mathrm{j}$*). Therefore, the risk function can be written as:*

$$\begin{aligned} R_c(d_k|x_i) &= \sum_{j=1}^{K} \lambda_j^k P(d_j|x_i) \\ &= \lambda_1^k P(d_1|x_i) + \lambda_2^k P(d_2|x_i) + \cdots + \lambda_k^k P(d_k|x_i) + \cdots + \lambda_K^k P(d_K|x_i) \\ &= \lambda_{\sim k}^k P(d_1|x_i) + \lambda_{\sim k}^k P(d_2|x_i) + \cdots + \lambda_k^k P(d_k|x_i) + \cdots + \lambda_{\sim k}^k P(d_K|x_i) \\ &= \lambda_{\sim k}^k (P(d_1|x_i) + P(d_2|x_i) + \cdots + P(d_K|x_i)) + \lambda_k^k P(d_k|x_i) \\ &= \lambda_{\sim k}^k (1 - P(d_k|x_i)) + \lambda_k^k P(d_k|x_i) \\ &= \lambda_{\sim k}^k (1 - P(d_k|x_i)). \end{aligned}$$

Thus, $R_c(d_k|x_i) = \sum_{j=1}^{K} \lambda_j^k P(d_j|x_i)$ *is equivalent to* $R_c(d_k|x_i) = \lambda_{\sim k}^k (1 - P(d_k|x_i))$.

The above Theorem 1 greatly reduces the computational complexity of the Bayes risk, which will be helpful for promoting the proposed method.

Remark 2. Through the above theorem and its corresponding proof, the loss function can be rewritten as $\lambda_{\sim k}^{k} = \exp\left(-\frac{(x_i-\mu_k)^2}{2\sigma_k^2}\right)$.

Theorem 2. *The Bayes risk (Definition 4) satisfies that* $0 \leqslant \mathrm{R_c(d_k/x_i)} < 1$.

Proof. *According to Theorem 1, the Bayes risk could be written as* $R_c(d_k|x_i) = \lambda_{\sim k}^k (1 - P(d_k|x_i))$*, and* $\lambda_{\sim k}^{k} = \exp\left(-\frac{(x_i-\mu_k)^2}{2\sigma_k^2}\right)$*. The loss function satisfies that* $0 < \lambda_{\sim k}^k \leq 1$. *On the other hand, the probability satisfies that* $0 < \mathrm{P(d_k/x_i)} \leqslant 1$ *according to Definition 6. Therefore, the above theorem holds.*

According to the aforementioned definitions, we can derive the definition of attribute weight based on the proposed fuzzy Bayes risk as follows.

Definition 7. *(Bayes risk weight) Given a decision system* $\mathrm{DS} = (\mathrm{U}, \mathrm{C} \cup \mathrm{D})$*,* $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \cdots, \mathrm{x_m}\}$*, for an arbitrary conditional attribute* $\mathrm{c} \in \mathrm{C}$*, its weight based on fuzzy Bayes risk is denoted as:*

$$w_c = 1 - \overline{R}_c, \qquad \ldots (8)$$

where $\overline{R}_c = \frac{1}{m} \sum_{i=1}^{m} R_c(d_k|x_i)$*,* $\mathrm{x_i} \in \mathrm{U}$*,* $\mathrm{d_k} \in \mathrm{D}$.

It is easy to see that $0 \leqslant w_c \leqslant 1$ holds according to Theorem 2. Thus, the weight vector of *DS* is $\overline{W} = (\overline{w}_1, \overline{w}_2, \cdots, \overline{w}_n)$, where $\overline{w}_c = w_c / \sum_{c=1}^{n} w_c$.

3.2 FBRW for multi-layer DS

In a practical application, the multi-layer index system is frequently used in MADA. Generally, the weight determination of multi-layer DS depends on some subjective approaches. Nevertheless, the method proposed in this paper can easily obtain the weights of each layer conditional attributes by using a neighbourhood relation.

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Remark 3. The main difference of weight assignment between single-layer attribute set and multi-layer attribute set is the metric of fuzzy neighbourhood. Equation (2) is employed in a single-layer attribute weight assignment and that is Equation (3) in the multi-layer's case.

Remark 4. The other differences of weight determination between single-layer attribute set and multi-layer attribute set are that (a) the single attribute c in the aforementioned definitions is replaced by a subset B ($B \subseteq C$); and (b) the normalised risk of an attribute set B should be rewritten as $\overline{R}_B = \frac{1}{m \cdot n} \sum_{k=1}^{n} \sum_{i=1}^{m} R_B(d_k|x_i)$, where n is the number of conditional attributes in B.

In addition, although there are some above-mentioned differences, all the theorems still hold.

3.3 FBRW algorithm

Based on the preceding theories and definitions, the algorithm of FBRW can be designed as Algorithm 1.

Algorithm 1 FBRW algorithm

Input: the $m \times n$ raw data and the $m \times 1$ labels as $DS = (U, C \cup D)$, threshold δ

Output: weights $\overrightarrow{W}$

Normalise the raw data into [0,1].
for each class in D **do**
  Obtain the expectation and standard deviation.
end for
for each c or B in C **do** // $c : m \times 1, B : m \times k, (k \leqslant n)$
  for each x_i in U **do**
    Gather its fuzzy neighbourhood $N(x_i)$ according to Definition 2.
    Compute $P(d(x_i)|x_i)$ according to Definition 6.
    Compute $\lambda(x_i)$ according to Remark 1 and 2.
    $R(x_i) = \lambda(x_i) \cdot P(d(x_i)|x_i)$.
  end for
  $R_c = R_c + R(x_i)$ or $R_B = R_B + R(x_i)$.
end for
$\overline{R}_c = R_c/m$ or $\overline{R}_B = R_B/(m \cdot k)$.
$w_c = 1 - \overline{R}_c$ or $w_B = 1 - \overline{R}_B$
$\overline{w}_c = w_c / \sum_{c=1}^{n} w_c$ and the same as $\overline{w}_B$.
return $\overrightarrow{W}$ combined with $\overline{w}_c$ or $\overline{w}_B$

4.0 WEIGHT EVALUATION

To the best of our knowledge, there are no ideal methods to evaluate the weight results. The existing methods are mostly based on the consistency between the assigned weights and the actual situations to measure the quality of the methods. In fact, if we can obtain reliable importances of attributes by using some approaches, such as a large number of expert surveys and classification accuracy (CA) in DS, we can employ the similarity degree or correlation coefficient to evalute the rationality of the assigned weights. In statistics, the commonly used

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metrics of similarity measure are Simple Matching Coefficient (SMC), Jaccarb Coefficient (JC), Cosine Similarity (CS), Generalised Jaccarb Coefficient (GJC) and Pearson Correlation Coefficient (PCC)(34). Therein, SMC and JC are not suitable for continuous data, CS and GJC employ vector dot product, and PCC considers the covariance and standard deviation of data. All of these methods do not take account of the fluctuation of data that includes two aspects, i.e. the longitudinal direction and the transverse direction. Therefore, we propose a kind of correlation coefficient that considers both the longitudinal deviation and transverse residual.

Definition 8. *(Longitudinal correlation coefficient) Given two vectors* $X = \{x_1, x_2, \cdots, x_m\}$ *and* $Y = \{y_1, y_2, \cdots, y_m\}$*, the longitudinal similarity degree between* X *and* Y *is defined as follows:*

$$LCC = \exp(-\mathrm{std}(\bigsqcup_{i}^{m} |x_i - y_i|)), \qquad \ldots(9)$$

where $\mathrm{std}(\cdot)$ *is the operation of standard deviation,* $\bigsqcup$ *denotes the combination of the elements in it, and* $|\cdot|$ *represents the absolute value of the elements in it.*

Definition 9. *(Transverse correlation coefficient) Given two vectors* $X = \{x_1, x_2, \cdots, x_m\}$ *and* $Y = \{y_1, y_2, \cdots, y_m\}$*, the transverse similarity degree between* X *and* Y *is defined as follows:*

$$TCC = \exp(-\mathrm{std}(\bigsqcup_{i}^{m-1} |\widetilde{x}_i - \widetilde{y}_i|)), \qquad \ldots(10)$$

where $\widetilde{x}_i = x_i - x_{i+1}$ *is the residual of* x_i.

Definition 10. *(LTCC) Given two vectors* $X = \{x_1, x_2, \cdots, x_m\}$ *and* $Y = \{y_1, y_2, \cdots, y_m\}$*, their longitudinal deviation and transverse residual correlation coefficient is defined as follows:*

$$LTCC = \frac{LCC + TCC}{2}. \qquad \ldots(11)$$

It is easy to see that the following properties $0 < LCC \leqslant 1$, $0 < TCC \leqslant 1$ and $0 < LTCC \leqslant 1$ hold. Usually, the two vectors have a strong correlation if $LTCC > 0.95$, and they are completely uncorrelated if $LTCC < 0.5$.

5.0 NUMERICAL EXPERIMENTS

In this section, we carry out two parts of experiments, one of which is the comparison experiment on the proposed correlation coefficient, and the other one includes some comparison experiments to reveal the superiority of FBRW.

With regard to the reference of weight, we employ the CA in the following experiments, which can effectively measure the importance of attributes and has been widely used in feature selection and data reduction(27). Therefore, it is suitable to be the reference of attribute weight determination in DS.

Table 1
The details of the artificial data

	f_1 x	f_2 $x \pm 0.5$	f_3 $x+1$	f_4 $x-2$	f_5 $r(x)^a$	f_6 $x+0.2(i-5)^b$	f_7 $x+0.2(5-i)$
x_1	0.9949	0.4949	1.9949	−1.0051	1	0.1949	1.7949
x_2	0.9879	1.4879	1.9879	−1.0121	1	0.3879	1.5879
x_3	1.0160	0.5160	2.0160	−0.9840	1	0.6160	1.4160
x_4	1.0156	1.5156	2.0156	−0.9844	1	0.8156	1.2156
x_5	0.9568	0.4568	1.9568	−1.0432	1	0.9568	0.9568
x_6	0.9985	1.4985	1.9985	−1.0015	1	1.1985	0.7985
x_7	0.9918	0.4918	1.9918	−1.0082	1	1.3918	0.5918
x_8	1.0314	1.5314	2.0314	−0.9686	1	1.6314	0.4314
x_9	1.0547	0.5547	2.0547	−0.9453	1	1.8547	0.2547
x_{10}	1.0555	1.5555	2.0555	−0.9445	1	2.0555	0.0555

[a] $r(\cdot)$ is a rounding operation, [b] $i = 1, 2, \cdots, 10$.

Table 2
The results of comparison

	$f_1 \approx f_1$	$f_1 \approx f_2$	$f_1 \approx f_3$	$f_1 \approx f_4$	$f_1 \approx f_5$	$f_1 \approx f_6$	$f_1 \approx f_7$
CS	1.0000	0.9070	0.9999	-0.9986	0.9996	0.8822	0.8700
GJC	1.0000	0.8112	0.6685	-0.3330	0.9993	0.7726	0.7619
PCC	1.0000	0.5570	1.0000	1.0000	-	0.5708	-0.5022
LTCC	1.0000	0.6856	1.0000	1.0000	0.9745	0.8303	0.8303

With the help of Weka[2], we employ as many as ten classifier methods and 10-fold cross-validation in order to guarantee that the results are highly credible. Therein, the employed ten classification algorithms produced in Weka are C4.5(J48), REPTree, NaiveBayes, SVM(SMO), IBk, Bagging, LogitBoost, FilteredClassifier, JRip and PART, and default parameters in Weka are chosen. The weights produced by CAs are the average values of the ten methods. All of the following experiments are carried out on the same platform and compiled by Matlab.

5.1 Comparison experiments on LTCC

In this subsection, we choose CS, GJC and PCC[(34)] as the objects of comparison, and use some artificial data to illustrate the superiority of LTCC. The details of the artificial data are shown in Table 1 and the corresponding curves are shown in Fig. 3.

We take the data of f_1 as the reference and compute the similarity degrees or correlation coefficients between the others and f_1. The results are shown in Table 2.

From the results in Table 2, we can see that (a) all the methods produce the similarity degree of $f_1 \approx f_1$ is 1; (b) CS and GJC consider that $f_1 \approx f_2$ should be given a greater similarity degree, however, this is not quite consistent with the actual situation in Fig. 3; (c) if the spatial

[2] http://weka.wikispaces.com, v3.6.13

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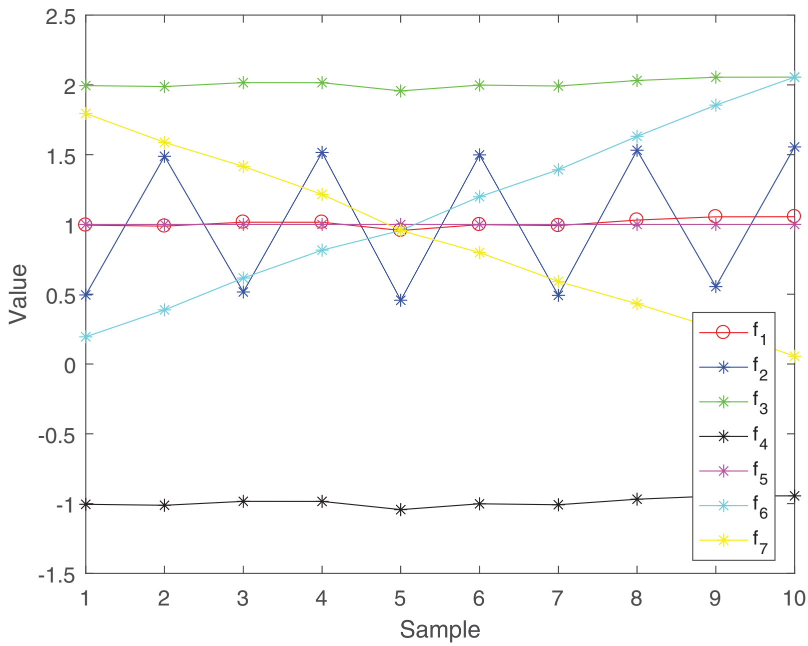

Figure 3. (Colour online) The curves of the artificial data.

translation is not considered, the similarity degrees of $f_1 \approx f_3$ and $f_1 \approx f_4$ should be equal to 1; (d) the data of f_5 are the same as those of f_1, which are the rounding ones of f_1's, however, there is no result generated by PCC because the standard deviation of them are 0; (e) almost all the methods do not assign $f_1 \approx f_6$ and $f_1 \approx f_7$ high similarity degrees.

Through the above analyses, we can see that by considering the factors of both longitudinal direction and transverse direction, it can evaluate the correlation between spatial vectors more reasonably. What's more, if the standard deviation of the concerned data is not 0, the PCC method can also produce satisfied results. Therefore, in order to effectively evaluate the assigned weight results, we employ the two methods, i.e. PCC and LTCC, in the following experiments.

5.2 Comparison experiments

In this subsection, we select some commonly used objective weight determination methods and some mature methods that can produce the weights of DS as comparisons to illustrate the advantage of the proposed method. Therein, the objective methods are Entropy method(9), CRITIC(14), SD(14) and CCSD(13), and the other mature ones are GRA(25), CE(24) and Neighbourhood Rough Set (NRS)(27). In the GRA model, we take the decision attribute as the optimal one to measure the importance degrees of conditional attributes, which could be considered as the weights of attributes. In the CE model, we take the conditional entropy of the decision attribute with regard to the conditional attribute as the metric to produce the weight of the conditional attribute; the smaller the conditional entropy is, the greater the weight should be assigned. In addition, we select the Supervised and Multivariate Discretization Naive Scaler (SMDNS) method as the discretisation tool who has the best performance compared with other models in the literature(31) because the discretisation method plays an important role in

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Table 3
The details of UCI data

ID	Data	Samples	Feature	Discrete	Continuous	Class
1	Blood	748	4	0	4	2
2	E. coli	336	7	2	5	8
3	Iris	150	4	0	4	3
4	Seeds	210	7	0	7	3
5	Vote	435	16	16	0	2
6	Vowel	528	10	0	10	11
7	WDBC	569	30	0	30	2
8	Wine	178	13	0	13	3

Table 4
The results of comparison experiments using PCC

Data	Entropy	CRITIC	SD	CCSD	GRA	CE	NRS	FBRW
Blood	– 0.8336	0.7665	0.0278	0.5557	– 0.5322	– 0.3677	0.7279	**0.9891**
E. coli	– 0.7523	0.6690	0.8135	0.4204	– 0.5871	0.4058	0.5838	**0.9202**
Iris	0.9817	– 0.4432	0.9955	– 0.2401	0.9834	0.6054	0.9617	**0.9973**
Seeds	0.9041	– 0.8018	0.9709	– 0.9399	– 0.6036	0.4256	0.9053	**0.9944**
Vote	0.0631	0.0065	– 0.2194	0.1375	– 0.2304	– 0.2443	–	**0.9752**
Vowel	0.3539	– 0.2432	– 0.4202	– 0.1263	0.2056	0.3521	0.3339	**0.8989**
WDBC	0.3340	– 0.1334	0.5685	– 0.4151	– 0.7614	0.6076	0.7914	**0.9414**
Wine	0.6350	0.0377	0.4810	– 0.0110	– 0.6843	0.7167	0.4570	**0.9637**

the CE model. In the NRS model, the dependencies are employed to transform into the weights of attributes, and the neighbourhood threshold is the same as that of FBRW, which is 0.2 in our method. In addition, the CA and the correlation coefficients produced by PCC and LTCC are also employed to evaluate the performance of each method, and the University of California Irvine (UCI) data in Table 3 are used in these experiments. Therein, the feature is the number of the conditional attributes in DS and the class is the number of decision categories. The comparison results are shown in Tables 4 and 5 and the bold values indicate the maximum ones.

From the results in Tables 4 and 5, we can see that the results obtained by FBRW are the best ones; in other words, almost all the correlation coefficients of each data have reached the maximum values. It is worth noting that there is no result of Vote with regard to the NRS model because there are always some overlap regions between the classes of each discrete conditional attribute and that of the decision attribute, which produces an empty positive region and a zero-weight result in the NRS model. The above problem is a main drawback of the NRS model.

6.0 EFFECTIVENESS EVALUATION OF FIGHTER

In this section, one of the practical applications of MADA, i.e. EEF, is carried out to illustrate the validity of the proposed method.

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Table 5
The results of comparison experiments using LTCC

Data	Entropy	CRITIC	SD	CCSD	GRA	CE	NRS	FBRW
Blood	0.9197	0.9131	0.9134	0.9129	0.9725	0.9799	0.8263	**0.9955**
E. coli	0.7966	0.9653	0.9646	0.9580	0.9721	0.9769	0.7662	**0.9771**
Iris	0.9620	0.8624	**0.9911**	0.8585	0.9846	0.9543	0.9194	0.9811
Seeds	0.9792	0.9011	0.9872	0.9078	0.9638	0.9836	0.9494	**0.9912**
Vote	0.9821	0.9910	0.9904	0.9913	0.9879	0.9909	–	**0.9978**
Vowel	0.9574	0.9697	0.9671	0.9709	0.9714	0.9689	0.8764	**0.9737**
WDBC	0.9847	0.9896	0.9934	0.9881	0.9941	0.9961	0.9597	**0.9982**
Wine	0.9743	0.9758	0.9849	0.9721	0.9846	0.9919	0.9261	**0.9952**

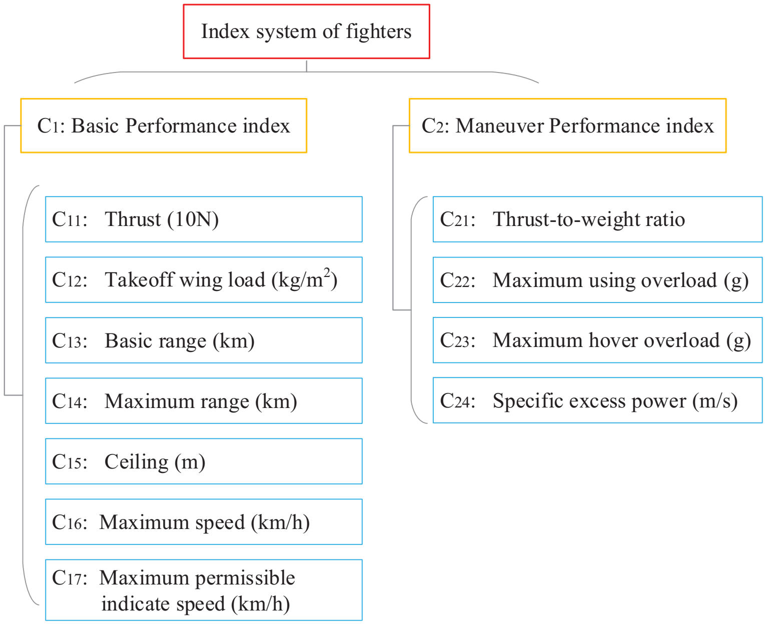

Figure 4. (Colour online) The index system of fighters.

6.1 Index system of fighters

The index system of fighters consists of two parts, i.e. the Basic Performance (BP) index set and the Manoeuvre Performance (MP) index set, and there are some sub-attributes in the two sets. The index system can be regarded as a multi-layer system shown in Fig. 4.

We have sorted out the index data of some typical fighters[1] shown in Tables 11–12 in the Appendix. These index data are classified into four categories according to the generational criteria[32].

6.2 Weight assignment based on FBRW

We demonstrate the calculation process of FBRW based on the above index system. For single-layer attribute weight determination, we take the basic performance index set as an example. First of all, the raw data should be normalised into the range of [0,1], where the cost normalised model $\overline{x} = \frac{\max(x)-x}{\max(x)-\min(x)}$ is used for the second attribute, i.e. take-off wing load, because the less the wing load is, the better the manoeuvreability will be in air combat. Meanwhile the income normalised model $\overline{x} = \frac{x-\min(x)}{\max(x)-\min(x)}$ is employed for the other attributes.

Firstly, we calculate the expectation μ and the standard deviation σ of each generation produced by the basic performance indexes. Secondly, we take the index value, namely thrust of MiG-9, as an example to demonstrate the following calculation process. The normalised value of MiG-9's thrust is 0 denoted as x_1, and its neighbourhoods are $N_{C_{11}}(x_1) = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_{10}, x_{13}\}$ if the δ is 0.2. Subsequently, the classification probability of x_1 can be calculated according to Equation (7), which is shown as follows:

$$P_{C_{11}}(d_1|x_1) = \frac{1 + 0.9781 + 0.9625 + 0.9372 + 0.9913 + 0.9607}{1 + \cdots + 0.9607 + 0.8300 + 0.8057 + 0.8130} = 0.7042. \quad \ldots(12)$$

Then, the loss of x_1 is obtained in terms of Definition 5 and Remark 2, which is presented as follows:

$$\lambda^{d_1}_{\sim d_1}(C_{11}, x_1) = \exp - \frac{(0 - 0.0284)^2}{2 \times 0.0229^2} = 0.4640. \quad \ldots(13)$$

After that, the risk of x_1 with regard to C_{11} can be produced as follows according to Definition 4 and Theorem 1.

$$R_{C_{11}}(d_1|x_1) = \lambda^{d_1}_{\sim d_1}(C_{11}, x_1) \cdot (1 - P_{C_{11}}(d_1|x_1)) = 0.4640 \times 0.2958 = 0.1373. \ldots(14)$$

Finally, we can obtain all the normalised risks of the conditional attributes, i.e. $\overline{R}_{C_1} = \{0.3546, 0.4739, 0.4739, 0.4387, 0.4223, 0.3176, 0.3097\}$, and the normalised weights are $\overline{W}_{C_1} = \{0.1533, 0.1250, 0.1250, 0.1333, 0.1372, 0.1621, 0.1640\}$ according to Definition 7. Moreover, the normalised weights of manoeuvre performance indexes are $\overline{W}_{C_2} = \{0.2676, 0.1828, 0.2450, 0.3046\}$.

Similarly, for the multi-layer index system, we can obtain the fuzzy Bayes risks of the two index sets which are 70.9604 and 38.8923, respectively. Then, the weights of each index set can be generated as 0.4897 and 0.5103.

In addition, if we crudely combine the weight of each index to obtain the weights of the attribute sets, which are shown as follows:

$$W_1 = \frac{\sum W_{C_1}/7}{\sum W_{C_1}/7 + \sum W_{C_2}/4} = 0.5150, \quad \ldots(15)$$

$$W_2 = \frac{\sum W_{C_2}/4}{\sum W_{C_1}/7 + \sum W_{C_2}/4} = 0.4850, \quad \ldots(16)$$

where

$$W_{C_1} = \{0.6454, 0.5261, 0.5261, 0.5613, 0.5777, 0.6824, 0.6903\}, \quad \ldots(17)$$

$$W_{C_2} = \{0.6834, 0.4669, 0.6258, 0.7778\} \quad \ldots(18)$$

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Table 6
The weights of basic performance index set

Method	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}
Entropy	0.2757	0.0503	0.1140	0.1187	0.1224	0.1155	0.2033
CRITIC	0.1173	0.2609	0.1085	0.1246	0.1329	0.1269	0.1289
SD	0.1477	0.1123	0.1123	0.1269	0.1586	0.1655	0.1768
CCSD	0.1294	0.2414	0.1363	0.1540	0.1313	0.1188	0.0888
GRA	0.1562	0.1104	0.1387	0.1441	0.1363	0.1548	0.1595
CE	0.1452	0.1646	0.1853	0.1300	0.1148	0.1300	0.1300
NRS	0.0833	0.0833	0.0833	0.1667	0	0.1667	0.4167
FBRW	0.1533	0.1250	0.1250	0.1333	0.1372	0.1621	0.1640
CA	0.2028	0.1055	0.1093	0.0954	0.1269	0.1906	0.1695

Table 7
The weights of manoeuvre performance index set

Method	C_{21}	C_{22}	C_{23}	C_{24}
Entropy	0.2431	0.1459	0.3297	0.2813
CRITIC	0.1795	0.3306	0.1886	0.3013
SD	0.2283	0.2360	0.2624	0.2733
CCSD	0.1532	0.3365	0.1633	0.3470
GRA	0.2507	0.2202	0.2508	0.2784
CE	0.2227	0.2227	0.3009	0.2536
NRS	0.4286	0	0.2143	0.3571
FBRW	0.2676	0.1828	0.2450	0.3046
CA	0.2780	0.1290	0.2393	0.3538

are the un-normalised weights, and the constant coefficients 7 and 4 are the numbers of the elements in their set.

From the above results we can see that the weights determined by FBRW conforms to the standard of the evaluation of fighters in reality, i.e. the manoeuvre performance indexes (C_2) are more important than the basic performance indexes (C_1). However, the weights obtained by the simple method reveal the opposite conclusion. This shows that the weights of datasets can not be determined by such a simple and rude method, e.g. the above method.

6.3 Comparison experiments

In this subsection, we also take the aforementioned weight assignment methods in Subsection 5.2 to compare and analyse the practicability of FBRW. The weight determination results of the basic performance index set and manoeuvre performance index set are shown in Tables 6-7. Therein, CA is the weight determination method based on CAs that are produced by the ten classification methods in Subsection 5.2. In addition, the PCCs and LTCCs between the weights assigned by the eight methods and those of CAs are shown in Table 8, and the values in bold are the maximum ones. Table 8 shows that the weights obtained by FBRW are closely related to the reference weights determined by CAs.

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Table 8
The comparison results of weight assignment

CC	Index	Entropy	CRITIC	SD	CCSD	GRA	CE	NRS	FBRW
PCC	BP	0.7297	−0.3525	0.7210	−0.5714	0.7197	−0.3099	0.2901	**0.8851**
PCC	MP	0.6762	−0.2648	0.5632	−0.0737	0.9828	0.2276	0.8805	**0.9993**
LTCC	BP	0.9437	0.9114	0.9708	0.9254	0.9717	0.9585	0.8889	**0.9729**
LTCC	MP	0.9222	0.8450	0.9146	0.8266	0.9263	0.9166	0.8726	**0.9552**

Table 9
The rank-order of basic performance index weights

Method	Rank-order
Entropy	$C_{11} > C_{17} > C_{15} > C_{14} > C_{16} > C_{13} > C_{12}$
CRITIC	$C_{12} > C_{15} > C_{17} > C_{16} > C_{14} > C_{11} > C_{13}$
SD	$C_{17} > C_{16} > C_{15} > C_{11} > C_{14} > C_{12} > C_{13}$
CCSD	$C_{12} > C_{14} > C_{13} > C_{15} > C_{11} > C_{16} > C_{17}$
GRA	$C_{17} > C_{11} > C_{16} > C_{14} > C_{13} > C_{15} > C_{12}$
CE	$C_{13} > C_{12} > C_{11} > C_{14} > C_{16} > C_{17} > C_{15}$
NRS	$C_{17} > C_{14} > C_{16} > C_{11} > C_{12} > C_{13} > C_{15}$
FBRW	$C_{17} > C_{16} > C_{11} > C_{15} > C_{14} > C_{12} > C_{13}$
CA	$C_{11} > C_{16} > C_{17} > C_{15} > C_{13} > C_{12} > C_{14}$

Table 10
The rank-order of manoeuvre performance index weights

Method	Rank-order
Entropy	$C_{23} > C_{24} > C_{21} > C_{22}$
CRITIC	$C_{22} > C_{24} > C_{23} > C_{21}$
SD	$C_{24} > C_{23} > C_{22} > C_{21}$
CCSD	$C_{24} > C_{22} > C_{23} > C_{21}$
GRA	$C_{24} > C_{23} > C_{21} > C_{22}$
CE	$C_{23} > C_{24} > C_{21} > C_{22}$
NRS	$C_{21} > C_{24} > C_{23} > C_{22}$
FBRW	$C_{24} > C_{21} > C_{23} > C_{22}$
CA	$C_{24} > C_{21} > C_{23} > C_{22}$

For further comparative analysis, we rank the weights determined by the nine methods in descending order, and the results are shown in Tables 9-10. In order to fully compare and explain the results, we analyse the weight assignment from two aspects, i.e. the distribution characterisation of data and the practical meanings of attributes. For simplicity, we take the basic performance index set as an example for the first aspect and the manoeuvre performance index set for the second one.

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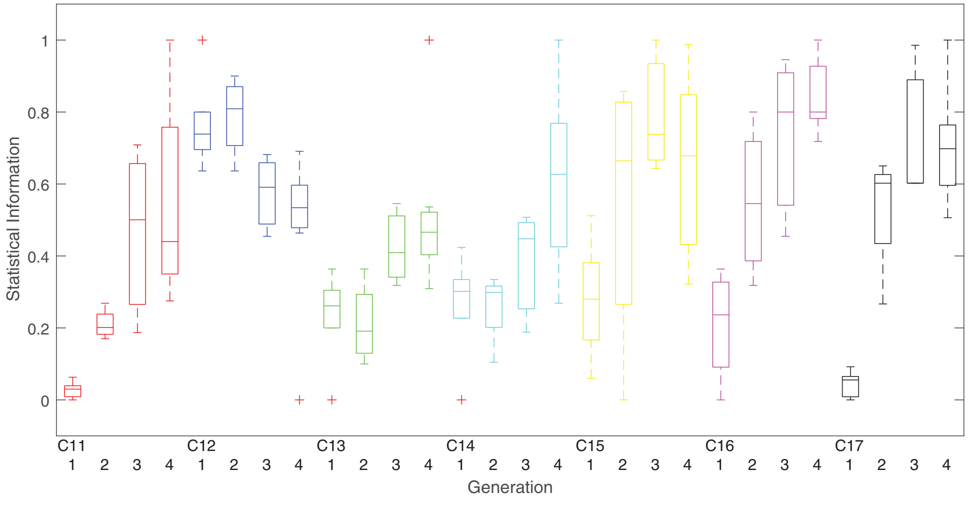

Figure 5. (Colour online) The statistical distribution of each attribute data

From the results in Table 9, we can see that there are some differences between the rank-orders of the weights obtained by the above methods. In order to analyse the reasons of the above problem more accurately and clearly, we visualise the basic performance index data through the data statistical distribution produced by the generations (shown in Fig. 5). Therein, the short horizontal lines distributed at both ends of the boxes indicate the maximum and minimum values, and the boxes mean the ranges between the 25th percentile and the 75th percentile, the lines in the boxes are the median values, and the cross symbols are the abnormal data.

From the results in Fig. 5 we can see that there are obvious discriminations in the distributions of C_{11}, C_{16} and C_{17}, and it is just the opposite for C_{12} and C_{13}. In other words, we can easily distinguish the fighters according to the distributions of C_{11}, C_{16} and C_{17} rather than those of C_{12} and C_{13}. Therefore, the attributes C_{11}, C_{16} and C_{17} should be assigned greater weights, and the weights of C_{12} and C_{13} should be less than others'.

According to the above analyses, we can see that the results (see Table 9) produced by CRITIC, CCSD and CE have lower credibility, which can also be verified by the results in Table 8. The PCCs of CRITIC, CCSD and CE are negative, and their LTCCs are less than others'.

On the other hand, for the manoeuvre performance indexes, the attribute C_{24} (specific excess power) has been recognised as the most important parameter to measure the operational effectiveness of fighters because some indexes such as stable hover performance, climb rate, longitudinal acceleration and ceiling are closely related with it[(1)]. Therefore, C_{24} should be assigned the greatest weight. The ranking results in Table 10 show that Entropy, CRITIC, CE and NRS do not put C_{24} at the leading place. The attribute C_{21} (thrust-to-weight) is considered as a relatively important index for the combat effectiveness evaluation of fighter, which will directly affect the manoeuvreability of fighter[(1)] and should also be assigned a greater weight. However, CRITIC, SD and CCSD assign C_{21} to the least important position (shown in Table 10). Additionally, the significance of C_{23} is greater than that of C_{22} in the combat effectiveness evaluation of fighter, thus, the weight of C_{23} should be greater than

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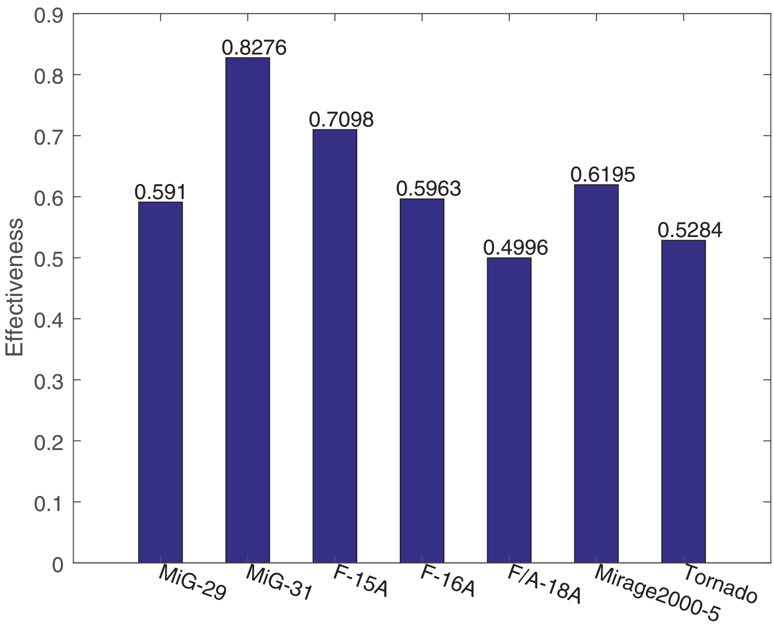

Figure 6. (Colour online) The basic effectiveness of the fighters.

C_{22}'s. Therefore, only the rank-order results produced by FBRW and CA are in line with the actual situation.

In summary, the weights assigned by FBRW are more reasonable and explanatory than others'.

6.4 Effectiveness evaluation

In this subsection, we take seven fighters, i.e. MiG-29, MiG-31, F-15A, F-16A, F/A-18A, Mirage 2000-5 and Tornado, to demonstrate the combat effectiveness evaluation of the fighters, which belong to the 4th generation. In MADA, the effectiveness evaluation is a linear weighted calculation of sample data. Therefore, the effectiveness results are depicted in Figs 6-8.

From the results in Fig. 6, we can see that MiG-31 has the maximum effectiveness, i.e. 0.8276. However, it is the opposite that the manoeuvre effectiveness of MiG-31 is the minimum one in Fig. 7. This reason can be summed up as follows. Most of the basic performances of MiG-31 are better than those of the other fighters. Its manoeuvre performances, however, are so poor that they result in the lowest manoeuvre effectiveness.

On the other hand, with the help of the reasonable weights of the two index sets, the total effectiveness is more in line with the actual situation than other sub-effectiveness (see Figs 6-7). From the results in Fig. 8, we can see that the F-15A fighter has the maximum effectiveness value, i.e. 0.7882, and Tornado has the minimum one. Therefore, we can conclude two levels of the above fighters, which is consistent with the actual situation and the results in literature[1,4,5]. One includes four fighters, i.e. F-15A, MiG-29, F-16A and Mirage 2000-5. The other three fighters, i.e. MiG-31, F/A-18A and Tornado, belong to the second level. With respect to F-15A and F-16A, F-15A is recognised as an outstanding fighter, and its performance is better than that of F-16A. Actually, F-16A

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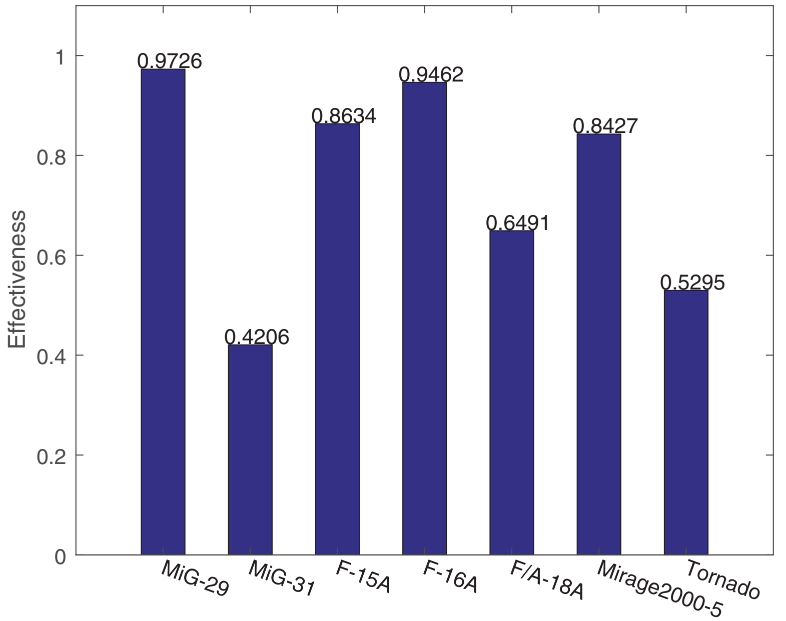

Figure 7. (Colour online) The manoeuvre effectiveness of the fighters.

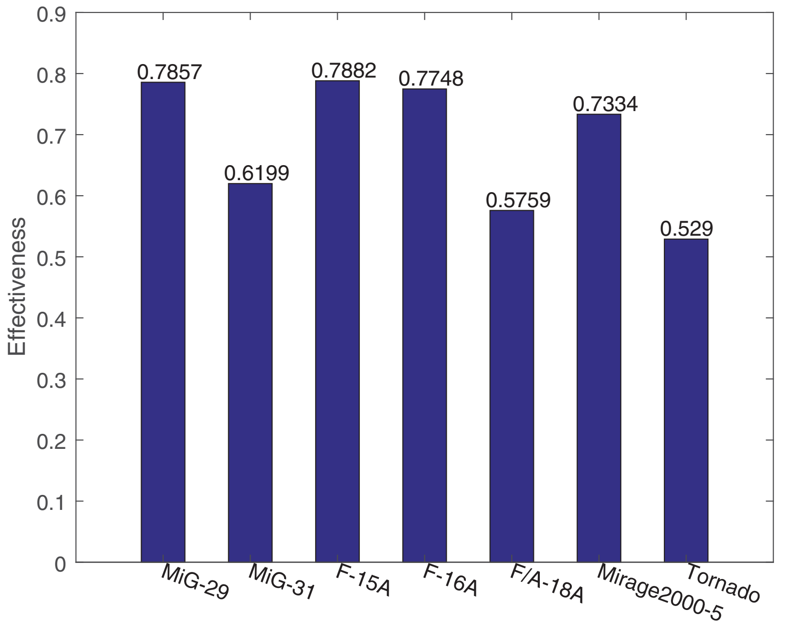

Figure 8. (Colour online) The total effectiveness of the fighters.

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is designed to be the partner of F-15A. The three fighters, MiG-29, F-16A and Mirage 2000-5, are considered to be the ones with the same level. With regard to these three fighters, the thrust (C_{21}) and maximum speed (C_{211}) of MiG-29 are larger than those of F-16A, but its take-off wing load (C_{23}) is more than F-16A's. The power system is the "short board" of Mirage 2000-5; however, its take-off wing load and maximum speed make its combat performance comparable to that of F-16A. The design objectives of MiG-31 are high speed and strong firepower, which reduce the air combat capability of MiG-31. F/A-18A is a kind of carrier-based aircraft whose combat performance is certainly not as good as the other subgrade fighters. Compared with the other fighters, many performances of Tornado are worse than those of others, which can be seen in Tables 11-12.

7.0 DISCUSSION

Based on the above amount of experiments, we provide some extension discussions as follows.

From the results of the comparison experiments we can see that, with the help of considering some relationships, i.e. the risk of decision and the fuzzy membership, between conditional attributes and the decision attribute in DS, the weights produced by FBRW are better than others'. Nevertheless, all the compared methods have their shortcomings that can be summed up as follows. (a) The Entropy, CRITIC, SD and CCSD methods do not take account of the above relationships. (b) Data-driven weight assignment technique is usually based on the assumption of attribute independent so that it extracts the weights from the aspect of statistic. However, GRA ignores the above assumption due to considering the maximum and minimum differences between all the conditional attributes and the decision attribute, which results in some unsatisfactory weights. (c) CE is greatly affected by discretisation methods; different discretisation methods generate kinds of conditional attributes' partitions, which result in various weights. (d) The dependencies of conditional attributes with respect to the decision attribute are employed in NRS, which may generate zero weight (see Tables 4-7).

For the effectiveness evaluation of fighter, it is undeniable that fighters are optimised for certain roles, i.e. a particular aircraft may not have a high overall effectiveness but may be the most effective in a specific role. With regard to this study, it is better to gather as many fighters with the same function (role) as possible to evaluate the effectiveness of fighters. However, due to the confidentiality of fighter data, our research on effectiveness evaluation can only be based on a small number of fighters with open data, which are characterised with different roles (such as high altitude combat role, close combat role), but they are all belonging to the combat fighters. On the other hand, the index system consisting of basic performance indexes and manoeuvre performance indexes in our research is the most basic system for evaluating fighters, through which the comprehensive combat performance values of fighters can be obtained. These values can be regarded as the basic references for evaluating the performance of the fighters from the comprehensive performance perspective.

With regard to weight assignment for IS, the combination of FBRW and some clustering methods is an optional approach, where the clustering methods provide labels as the decision attribute for FBRW. Therefore, the FBRW method can be applied to both IS and DS.

8.0 CONCLUSION

In order to deal with the labelled multiple attribute decision-making issue, this paper proposes an object weight assignment method based on FBRW. Firstly, some preliminaries are presented and the FBRW method follows, where a Gaussian kernel loss function is raised to make up the

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deficiency of the Bayes risk. Subsequently, the problem of weight determination for a multi-layer attribute set is discussed. Then, a weight assignment algorithm based on FBRW is given. In order to evaluate the credibility of assigned weights, the LTCC model is designed. Finally, a large number of experiments are carried out which include the comparison experiments on LTCC, the comparison experiments of weight assignment using a UCI dataset, and the effectiveness evaluation of the fighter. The experimental results and discussions show that (a) LTCC is suitable for evaluating the assigned weights, and (b) the proposed FBRW method is not only good at dealing with single-layer or multi-layer DS, but can also be extended to cope with IS. Compared with other weight assignment methods, the weights produced by FBRW are more reasonable and closely related to those determined by CAs.

In future work, we will focus on combining the FBRW method with some clustering methods to deal with the weight assignment for IS and apply FBRW to other fields.

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APPENDIX

The index sets for EEF are shown as follows, which include the BP index set (see Table 11) and the MP index set (see Table 12).

Table 11
The BP index set of fighters

ID	Fighter	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	Generation
1	MiG-9	1,568	277	800	1,100	12,800	950	910	1
2	MiG-15	2,200	234	1,420	1,860	15,000	1,080	1,050	1
3	MiG-15bis	2,650	241	1,330	2,520	15,500	1,080	1,076	1
4	MiG-17	3,380	236	1,240	2,020	16,600	1,150	1,145	1
5	F-80C	1,820	248	1,600	2,220	13,700	1,000	932	1
6	F-86F	2,700	275	1,470	2,200	14,300	1,130	1,053	1
7	MiG-21	6,468	364	1,020	2,100	19,000	1,300	2,571	2
8	Su-7	9,310	320	1,150	1,450	19,500	1,200	2,448	2
9	F-100D	7,565	372	1,600	2,100	12300	1,125	1,591	2
10	F-104G	7,170	520	1,290	2,220	16,760	1,390	2,448	2
11	MiG-25	22,000	569	1,500	1,730	20,700	1,200	3,427	3
12	F-4E	16,000	403	2,000	2,600	17,700	1,390	2,448	3
13	Mirage F-1	6,960	460	1,700	2,800	18,500	1,470	2,448	3
14	MiG-29	16,260	389	1,480	2,000	18,000	1,400	2,877	4
15	MiG-31	30,400	666	3,000	3,300	20,600	1,500	3,464	4
16	F-15A	21,200	316	1,980	4,450	18,300	1,380	2,815	4
17	F-16A	10,800	375	1,825	3,800	18,000	1,380	2,387	4
18	F/A-18A	14,250	443	1,800	2,600	15,240	1,345	2,203	4
19	Mirage2000-5	9,500	268	1,650	3,200	18,300	1,480	2,693	4
20	Tornado	14,220	680	1,850	2,500	15,000	1,390	2,570	4

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Table 12
The MP index set of fighters

ID	Fighter	C_{21}	C_{22}	C_{23}	C_{24}	Generation
1	MiG-9	0.310	4.0	2.5	16	1
2	MiG-15	0.460	8.0	4.0	42	1
3	MiG-15bis	0.530	8.0	4.4	50	1
4	MiG-17	0.630	8.0	4.8	75	1
5	F-80C	0.340	6.5	3.0	35	1
6	F-86F	0.375	7.0	3.5	47	1
7	MiG-21	0.770	8.0	5.9	145	2
8	Su-7	0.850	8.0	5.0	150	2
9	F-100D	0.557	7.3	5.5	130	2
10	F-104G	0.760	6.0	3.0	254	2
11	MiG-25	0.630	4.5	3.0	200	3
12	F-4E	0.810	8.0	6.0	152	3
13	Mirage F-1	0.610	8.0	5.0	165	3
14	MiG-29	1.100	9.0	9.0	310	4
15	MiG-31	0.740	5.0	3.2	235	4
16	F-15A	1.190	7.3	7.3	300	4
17	F-16A	1.030	9.0	9.0	305	4
18	F/A-18A	0.870	7.0	6.0	245	4
19	Mirage2000-5	0.860	9.0	9.0	255	4
20	Tornado	0.700	7.5	5.5	180	4

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