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VIRTUES OF BAD TIMES

Interaction Between Productivity Growth and Economic Fluctuations

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We develop a model of optimal productivity growth under demand fluctuations. We consider two alternative hypotheses. First, we assume that productivity growth is costly in terms of current production. Second, we assume that the cost of productivity improvements is independent of current production. It is shown that, in the first case, productivity improvements will be countercyclical whereas, in the second case, they will be procyclical. The model then is used to study the impact of the frequency and amplitude of fluctuations on long-run growth. The results corresponding to the first hypothesis are shown to be consistent with recent empirical work.

Keywords: Business Cycles, Endogenous Growth, Opportunity Cost, Labor Hoarding

1. INTRODUCTION

Productivity growth and the business cycle had long been recognized as closely interrelated. Yet, for several decades, the two phenomena have been investigated separately by the economic community: On one hand, business-cycle theorists would analyze detrended data and then possibly introduce the trend as exogenous to the cycle; on the other hand, growth theorists would focus on the existence and stability of a long-run deterministic growth path.

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However, the emergence in the eighties of the real-business-cycle literature,[1] emphasizing productivity shocks as a main driving force behind cyclical fluctuations, called into question the traditional division of macroeconomic theory between trends and cycles, and suggested a return to the Schumpeterian view of growth and cycles as a unified phenomenon. At the same time, the endogenous growth literature[2] allowed for a large set of factors, including government policy, to affect long-run growth.

This paper develops an analysis of the causal relationship from the business cycle to long-run growth. It focuses on the incentives for firms to implement new technologies (or, more generally, to increase productivity through various activities, including perhaps reorganization, training, and research and development) along successive phases of the business cycles. Two alternative sets of assumptions are considered. Under one set of assumptions (world 1), productivity growth has disruptive effects on current production. Under the other set of assumptions (world 2), it is costly but does not interfere with current production. The key results are as follows: In world 1, productivity-improving activities will be countercyclical. The reason is that the opportunity cost of these activities goes down in recessions by more than their return. As a result, a recession will have a positive long-run effect on total factor productivity. By contrast, in world 2, productivity-improving activities will be procyclical. Their cost does not fall in recessions but their return does. As a result, recessions are damaging to long-run productivity. More generally, the model investigates the effects of the structure of the business cycle (amplitude, frequency) on the average rate of growth.

The opportunity-cost view emphasized in this paper also has been developed by other authors, including Davis and Haltiwanger (1990) and Hall (1991). The former found that more labor reallocation is going on during recessions; they constructed a model in which it is optimal to increase the pace of labor reallocation during downturns because of a lower opportunity cost of such activities. The latter argues that recessions are appropriate times for reorganizations. Our model also is related to the independent contribution of Gali and Hammour (1991), who focus on the incentives to accumulate human capital during recessions. Other contributions [e.g., Caballero and Hammour (1992) and King and Robson (1989)] describe complementarity mechanisms whereby recessions can increase the average productivity of the economy. The former authors focus on the beneficial effect of recessions on the age structure of capital, and its implications for the cyclical behavior of job creation and destruction. They do not, however, deal with the growth effects of fluctuations, which is the main contribution of our paper. We focus both on the growth effects of any individual expansion or recession and on the effect of the overall stochastic structure of demand shocks on the average growth rate of the economy.[3]

Another strand of literature [Shleifer (1986), Stadler (1989)] insists on positive complementarities between aggregate demand and growth, either through aggregate demand externalities in the implementation of innovation or learning-by-doing mechanisms. These effects go in the same direction as our description of world 2, but the mechanisms at work are quite different.[4,5]

The paper is organized as follows: Section 2 sets up the model. Section 3 discusses the steady state. Section 4 introduces business cycles. Section 5 studies the cyclical behavior of productivity growth. Section 6 studies the effect of the frequency and amplitude of fluctuations on long-run growth. Section 7 summarizes the paper and discusses the empirical relevance of the results as well as directions for future research.

2. MODEL

2.1. Goods Market

We consider an open economy that produces a variety of exported goods indexed by i and consumes a homogeneous imported good.[6] The world price of the latter is normalized to one. The demand of any home good i at time t is given by

$$D_{it} = y_t/p_t \cdot (p_{it}/p_t)^{-\eta}, \tag{1}$$

where y_t is an index of world demand, p_{it} is good i's price, and p_t is an aggregate price index for home goods given by the usual formula.

$$p_t = \left(\int_0^{N_t} p_{it}^{1-\eta}\right)^{1/(1-\eta)}. \tag{2}$$

Equations (1) and (2) are consistent with the world consumers' instantaneous utility function being Cobb–Douglas in an aggregate CES index of home goods. In equation (2), N_t is the number of varieties produced by the country at time t. Furthermore, we assume that $\eta > 1$.

Each home good i is supplied by a monopolist. We assume that the size of each monopolist is fixed. Therefore, this is an economy in which increases in employment would be met through increases in the number of varieties produced, rather than increases in the quantity of each variety. At any given time t, each firm i is characterized by a technological level, x_{it}. Its total production is assumed to be equal to $e^{x_{it}}$. Let $v_{it} = dx_{it}/dt$ denote firm i's rate of productivity growth. This rate is a choice variable whose determination results from the trade-off between current profits, net of the cost of technology-improving activities, and the future net present value (NPV) of the firm, which increases with the technological parameter x. We consider two alternative assumptions about the cost of productivity-improving activities:

Assumption 1. To grow at rate v_{it}, the firm must sacrifice a fraction $k(v_{it})$ of its production. We assume $k' \geq 0,\ k'' > 0, k'(0) = 0$.

Assumption 2. To grow at rate v_{it}, the firm must buy a quantity $h(v_{it})$ of the imported good. We assume $h' \geq 0, h'' > 0, h'(0) = 0, h'(+\infty) = +\infty$.

Assumption 1 describes world 1 in which productivity improvements have disruptive effects on production. This may happen if, at the time when new technologies are implemented, the firm's managers and skilled workers must receive training in the new technology, and if, at the same time, the firm cannot find other workers to perform their current production tasks.[7]

By contrast, assumption 2 describes a world in which productivity growth can be bought on the market without interfering with current production tasks.

An alternative interpretation of Assumptions 1 and 2 is in terms of labor adjustment costs. In world 1, hiring and firing workers is very costly so that, to perform productivity-improving activities, it is desirable for the firm to use its current labor force and shift it away from directly productive activities. In world 2, hiring and firing costs are very low so that the firm can hire extra workers to perform productivity-improving activities.

In both worlds, the net output of firm i is given by the following formula:

$$z_{it} = e^{x_{it}} \phi_{it}, \tag{3}$$

where ϕ_{it} is defined as $1 - k(v_{it})$ in world 1 and as 1 in world 2.

Confronting (3) and (1), one can see that to have $z_{it} = D_{it}$ firm i's price at time t must be equal to

$$p_{it} = y_t^{1/\eta} p_t^{(\eta-1)/\eta} e^{-x_{it}/\eta} \phi_{it}^{-1/\eta}. \tag{4}$$

2.2. Determination of Productivity Growth at the Firm Level

At each point in time, a firm's market value depends on its technological level x_{it}, the aggregate price level p_t, and the level of demand y_t. We summarize the effects of these two variables by letting the firm's market value depend on t as well as x_{it}. We assume that firms may borrow and lend at a world real interest rate given by r. As a result, their expected net present discounted value is given by the following recursive expression:

$$V_t[x_{it}] = \pi_{it}\, dt + (1 - r\, dt) E_t V_{t+dt}[x_{it} + v_{it}\, dt], \tag{5}$$

where $\pi_{it} = p_{it} z_{it}$ in world 1 and $\pi_{it} = p_{it} z_{it} - h(v_{it})$ in world 2. The firm's optimal policy then is determined by maximizing (5) with respect to v_{it}. The first-order condition is

$$\frac{-\partial \pi_{it}}{\partial v_{it}} = E_t \frac{\partial V_{t+dt}}{\partial x}. \tag{6}$$

In world 1, the first-order condition (FOC) becomes, using (4) and (3),

$$\frac{(\eta - 1)}{\eta} \pi_{it} k' \frac{v_{it}}{\phi_{it}} = E_t \frac{\partial V_{t+dt}}{\partial x}. \tag{6a}$$

In world 2, the FOC is

$$h'(v_{it}) = E_t \frac{\partial V_{t+dt}}{\partial x}. \tag{6b}$$

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We also can derive an Euler equation for the marginal value $\partial V/\partial x$ by direct derivation of (5) with respect to x_{it}:

$$\begin{aligned}\frac{\partial V_t}{\partial x} &= \frac{\partial \pi_{it}}{\partial x} + (1 - r\,dt)E_t\frac{\partial V_{t+dt}}{\partial x}\\ &= \frac{\eta-1}{\eta}p_{it}z_{it} + (1 - r\,dt)E_t\frac{\partial V_{t+dt}}{\partial x}.\end{aligned} \tag{7}$$

Later, it will be possible to eliminate $\partial V/\partial x$ between (7) and (6) and solve for the actual policy of the firm.

2.3. Entry, Exit, and Goods Market Equilibrium

The number of differentiated monopolists is determined at each point in time by entry and exit conditions. We assume a fixed entry cost equal to C. Each firm can exit the market at each instant in time, in which case it recoups a liquidation value equal to θC, $\theta \leq 1$. In this simple formulation, productivity growth affects the value of continuing to operate but not the liquidation value. In the two-state equilibria that we are going to consider, this will generate an asymmetry between expansions and recessions: Firms in expansions expect to stay in the market with probability lower than one if the economy falls into a recession. Because productivity-improving decisions do not increase their liquidation value, firms will have a lower incentive to increase x_{it}. By contrast, firms in recessions are guaranteed to continue operating if there is a shift to an expansion. We call this the exit effect. Both to improve our analytical understanding of the model and because we do not find this effect too plausible, we choose to get rid of it in the following analysis. The implications of the exit effect, however, are studied in the appendix.[8] We therefore generalize the exit value in the following way: Let x_t be the average productivity level in the economy (because we only consider symmetric equilibria with no idiosyncratic shocks, we will always have $x_{it} = x_t \,\forall i$). We assume that the liquidation value is equal to $\theta C e^{\beta x_{it}} e^{-\beta x_t}$. In symmetric equilibrium, this is precisely equal to θC. However, each individual firm will now consider that its productivity improvements not only increase its continuation value but also its liquidation value. The scope of this effect depends on β. For some value of β, an increase in x_{it} has exactly the same marginal impact on the operating value and the exit value. In that case the exit effect disappears.[9] To complete our description of entry and exit, we need to make some assumption about the productivity level of entering firms. To preserve the existence of a symmetric equilibrium, we assume that firms can enter at the average productivity level of the economy, x_t.

The existence of entry and exit costs implies that whenever there is an incipient rise in V_t above C, the number of firms N_t increases (possibly in a discrete fashion) up to the point where V_t is exactly equal to C. Similarly, an incipient drop in V_t below θC triggers exit until $V_t = \theta C$ holds. If $\theta < 1$, there is an inaction band $V_t \in [\theta C, C]$ where $dN_t/dt = 0$.

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Restricting ourselves to symmetric equilibria where $x_{it} = x_t$ for all i makes it possible to compute the aggregate price level p_t. Using (2) and (4), we get

$$p_t = y_t\, e^{-x_t} N_t^{-\eta/(\eta-1)} \phi_t^{-1}. \tag{8}$$

Equation (8) then implies that

$$p_{it} = y_t\, e^{-x_t} \big/ (N_t \phi_t). \tag{9}$$

We now drop all i indices whenever doing so does not introduce ambiguities. Before we proceed, note that (8) and (9), along with (3) and (4), yield a simple expression for profits. In world 1, we simply get

$$\pi_t = y_t / N_t. \tag{10a}$$

In world 2, the expression becomes

$$\pi_t = y_t / N_t - h(v_t). \tag{10b}$$

3. STEADY STATE

Before proceeding any further, it is worth considering the steady state of the economy. This will allow us to show that long-run changes in the level of demand y have no effect on growth. Equations (8) and (9) imply that domestic prices are inversely proportional to the economy's productivity. This result follows from the assumption that the elasticity of world demand to the domestic price level is equal to -1.[10] Thus, in a steady state, profits are constant, as well as the number of firms and the marginal value to the firm of an increase in x.

Let u be the steady-state value of $\partial V/\partial x$. Then (7) implies, along with (8), that

$$u = [(\eta - 1)/\eta] y/(rN). \tag{11}$$

Plugging this into (6a) gives

$$rk'(v)/[1 - k(v)] = 1, \tag{12a}$$

which determines the value of v in world 1. Similarly, plugging (11) into (6b) yields

$$h'(v) = ((\eta - 1)/\eta) y/(rN), \tag{12b}$$

which gives a steady-state relationship between v and N in world 2.

The model is closed by adding a free-entry condition. We assume that the economy is always on the margin of entry. As a result, $V = C$. Equation (5) therefore implies that

$$\pi = rC,$$

which, using (10), implies that

$$N = y/(rC) \tag{13a}$$

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in world 1 and

$$N = y/[rC + h(v)] \tag{13b}$$

in world 2.

The number of firms is simply equal to the ratio of world demand over each firm's total size (in terms of cost or turnover).

We are now in a position to investigate the relationship between the steady-state level of world demand, y, and long-run growth, v. Starting with world 1, it is clear from (12a) that v does not depend at all on y. Turning to world 2, we can see that plugging (13b) into (12b) yields

$$h'(v) = (\eta - 1)[rC + h(v)]/(\eta r). \tag{14}$$

The joint determination of v and N in world 2 is summarized in Figure 1. Equation (12b) is given by the VV locus, whereas equation (13b) is represented by the NN locus. Both curves are negatively sloped. To preserve the system's stability, we impose the following condition:

Assumption 3. The h function satisfies the following condition:

$$h''(v) - (\eta - 1)h'(v)/(r\eta) > 0\,\forall\, v \tag{15}$$

Assumption 3 guarantees that the comparative statics properties of the steady state remain intuitive; for example, an increase in C reduces the equilibrium number of firms, whereas a reduction in the marginal cost of productivity improvements increases the equilibrium value of v. This is equivalent to VV being flatter than NN in Figure 1.

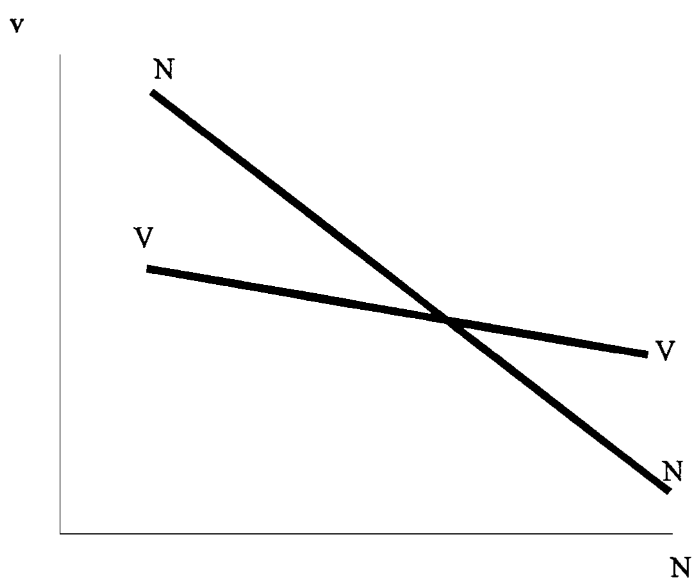

FIGURE 1. Equilibrium determination in the steady state.

Going back to equation (14), note that it determines v independently of y. Therefore, in world 2 also, demand has no steady-state effect on growth. We summarize the results in the two following propositions:

PROPOSITION 1. *Under Assumption* 1, $\partial v/\partial y = 0$.

PROPOSITION 2. *Under Assumption* 2, $\partial v/dy = 0$.

The intuition behind these neutrality results is not the same for both propositions. The result in Proposition 1 follows from the fact that, in steady state, an increase in *any individual firm's* demand has no effect on that firm's productivity growth, because it does not affect the terms of trade between production and productivity-improving activities. Because the cost of these activities is in terms of foregone production, an increase in the individual firm's demand (or price) is equivalent to an identical multiplicative shift of both the costs of productivity growth [the LHS of (6a)] and its benefits [the RHS of (6a)]. By contrast, in world 2, an increase in an *individual firm's* steady-state demand would increase v_i. Indeed, it would increase the benefits of productivity growth [the RHS of (6b)] while leaving its costs unaffected [the LHS of (6b)]. *However*, an increase in y is entirely met through an equiproportionate increase in the number of firms. Each firm's individual demand therefore is unaffected, so that v also is unchanged.

Propositions 1 and 2 imply that a *permanent* change in demand should have no effect of growth. Using this neutrality result as a benchmark, we now can concentrate on the effect of demand fluctuations.

4. INTRODUCING ECONOMIC FLUCTUATIONS

We now turn to the effect of economic fluctuations on productivity growth. We formalize economic fluctuations as a two-state stationary Markov process. The economy is either in expansion E or recession R. The corresponding levels of demand are y_E and y_R, $y_R < y_E$. Transitions from one state to another follow a Poisson process: With flow probability γ the economy drops from E to R. The reverse transition occurs with flow probability ε.[11]

We are looking at a stochastic steady state in which all variables (except x and p) are constant in each state. Let v_j and N_j, $j \in \{E, R\}$ be the constant values taken by v and N in state j. We follow the same steps as in the preceding section: First, we compute $\partial V/\partial x$ in both states using (8), (3), (4) and (7); then, we use (6) to get expressions for v_E and v_R. The precise derivations are left to the Appendix.

4.1. World 1

Let us first consider the solution of the system under Assumption 1. We determine v_E and v_R (see Appendix) by

$$d_E k' \frac{v_E}{1 - k(v_E)} = \frac{(r+\varepsilon)d_E + \gamma d_R}{r(r+\varepsilon+\gamma)}, \tag{16a}$$

$$d_R k' \frac{v_R}{1 - k(v_R)} = \frac{(r+\gamma)d_R + \varepsilon d_E}{r(r+\varepsilon+\gamma)}, \tag{17a}$$

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where $d_j = y_j/N_j$ is an index of each individual firm's demand in state j. The LHS of (16a) and (17a) is the opportunity cost of reallocating workers from the production activity to the productivity growth activity. It is proportional to d_j, the current state of demand. The higher the current demand, the higher the foregone profits from shifting to the productivity-improving activity. The RHS of (16a) and (17a) is the present discounted value of the flow of gains from such a reallocation. Because these gains are reaped over the whole future, including periods when the economy is in the other state, a weighted average of d_E and d_R intervenes.

We assume that recessions are sharp enough to induce exit. As a result, we have $V = C$ in expansions and $V = \theta C$ in recessions, with $N_R < N_E$. If π_j denotes profits in state j, then (5) implies that[12]

$$\pi_E = [r + \gamma(1-\theta)]C, \quad \textbf{(18)}$$

$$\pi_R = [r\theta + \varepsilon(\theta - 1)]C. \quad \textbf{(19)}$$

Using (10a), this can simply be rewritten as

$$d_E = [r + \gamma(1-\theta)]C, \quad \textbf{(20a)}$$

$$d_R = [r\theta + \varepsilon(\theta - 1)]C, \quad \textbf{(21a)}$$

and v_j and d_j (and therefore N_j) are clearly determined by (16a) and (17a) and (20a) and (21a).

4.2. World 2

We now solve for the equilibrium under Assumption 2. It is determined by the following equations (see Appendix):

$$h'(v_E) = \frac{\eta - 1}{\eta} \cdot \frac{(r+\varepsilon)d_E + \gamma d_R}{r(r+\varepsilon+\gamma)}, \quad \textbf{(16b)}$$

$$h'(v_R) = \frac{\eta - 1}{\eta} \cdot \frac{(r+\gamma)d_R + \varepsilon d_E}{r(r+\varepsilon+\gamma)}. \quad \textbf{(17b)}$$

Using (10b) in (18)–(19) yields

$$d_E = [r + \gamma(1-\theta)]C + h(v_E), \quad \textbf{(20b)}$$

$$d_R = [r\theta + \varepsilon(\theta - 1)]C + h(v_R). \quad \textbf{(21b)}$$

Now, d_i and v_i are jointly determined by (16b) and (17b) and (20b) and (21b).

We are now in a position to answer the main questions in which we are interested: What is the effect of recessions and expansions on productivity growth v, and what is the effect of the amplitude and frequency of fluctuations on long-run growth?

5. CYCLICAL BEHAVIOR OF PRODUCTIVITY-IMPROVING ACTIVITIES

In this section, we analyze whether v is pro- or countercyclical, that is, whether v_E is greater or less than v_R. We start with the analysis of world 1.

5.1. World 1

Under Assumption 1, productivity-improving activities tend to be countercyclical, but their overall effect crucially depends on whether entry costs are fully recouped when exit occurs. It is first possible to establish the following *neutrality* result:

PROPOSITION 3. *Suppose Assumption* 1 *holds. Assume that* $\theta = 1$. *Then,* $v_E = v_R$.

Proof. Note that (20a) and (21a) imply that $d_E = d_R$. The RHS of (16a) and (17a) are therefore equal. So are the LHS's as functions of the corresponding values of v. By strict monotonicity of the LHS of (16a) and (17a), the unique solution is such that $v_E = v_R$. ■

The intuition behind Proposition 3 is simple: Because entry costs are fully recouped at exit, each firm's value must be equal to C regardless of the economy's state. Therefore, fluctuations in demand are entirely met through entry. From the point of view of each individual firm, there are no fluctuations. Therefore, productivity growth remains constant along the cycle.[13]

Let us now turn to the case in which entry costs are not fully recouped in recessions.

PROPOSITION 4. *Suppose Assumption* 1 *holds and* $\theta < 1$. *Then* $v_E < v_r$.

Proof. From (20a) and (21a), one clearly has $d_E > d_R$. Now, note that (16a) and (17a) can be rewritten

$$rk' \frac{v_E}{1 - k(v_E)} = \frac{(r + \varepsilon) + \gamma d_R/d_E}{(r + \varepsilon + \gamma)}, \tag{22a}$$

$$rk' \frac{v_R}{1 - k(v_R)} = \frac{(r + \gamma) + \varepsilon d_E/d_R}{(r + \varepsilon + \gamma)}. \tag{23a}$$

Given that the LHS's are identical and increasing in v, and that the RHS of (23a) is larger than that of (22a), it is clear that $v_E < v_R$. ■

The intuition underlying Proposition 4 is also quite simple. In world 1, the opportunity cost of productivity-improving activities is given by the marginal *current* foregone revenue. It is therefore higher in expansions than in recessions. On the other hand, the gains from such activities are also higher in expansions than in recessions, but they are spread throughout the whole future. More precisely, these gains are equal to the present discounted value of future marginal revenues from an increase in log-productivity x, including those generated when the economy is in

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recession. It is therefore less cyclical than the cost.[14] As a result, v is countercyclical. Firms take advantage of a low marginal revenue in recessions to engage in the productivity-improving activity because they benefit from it once the economy has returned to the good state. This, in essence, is an *intertemporal substitution* effect.

5.2. World 2

We now turn to the cyclical behavior of v in world 2. First, let us note that, if $\theta = 1$, the same neutrality result holds as in world 1: $v_E = v_R$. The value of each individual firm is unaffected by the business cycle because it is entirely accommodated through entry and exit.

PROPOSITION 5. *Assume Assumptions* 2 *and* 3 *hold. Assume that* $\theta = 1$. *Then,* $v_E = v_R$.

Proof. Substitute (20b) and (21b) into (16b) and (17b). Then, subtract (17b) from (16b), yielding

$$g(v_E) - g(v_R) = 0,$$

where $g(v) = h'(v) - (\eta - 1)h(v)/[\eta(r + \varepsilon + \gamma)]$. Note that (15) implies that g is strictly increasing in v. One therefore must have $v_E = v_R$. ■

Assuming now that θ is strictly less than one, it is possible to show that v is procyclical.

PROPOSITION 6. *Suppose Assumptions* 2 *and* 3 *hold. Assume that* $\theta < 1$. *Then,* $v_E > v_R$.

Proof. The same steps as in the preceding proof yield

$$g(v_E) - g(v_R) = (\eta - 1)(1 - \theta)C/\eta > 0$$

Because g is increasing, clearly $v_E > v_R$. ■

The economic intuition underlying Proposition 6 is as follows: In world 2, the marginal cost of productivity-improving activities only depends on productivity growth v and is unaffected by the business cycle. By contrast, the marginal value of an increase in x is expressed as a discounted sum of future revenues, which includes states of expansion as well as recession. Because entry costs are not fully recouped upon exit, the firm's revenues are larger in expansions than in recessions. When the economy is currently in expansion, more weight is being put on the E state in the firm's present discounted value than when the economy is currently in recession. As a result, the marginal gain from increasing the speed of productivity improvements v is greater in expansions than in recessions.

We have therefore shown that, in world 1, productivity growth should be countercyclical, whereas in world 2, it should be procyclical.

We now turn to the growth effects of economic fluctuations.

6. EFFECT OF ECONOMIC FLUCTUATIONS ON LONG-RUN GROWTH

It is very easy, in this model, to compute the average growth rate of the economy as a function of v_E and v_R. Note that the economy grows at rate v_E in expansions and v_R in recessions. Note also that in stochastic steady state, it spends a fraction $\varepsilon/(\gamma+\varepsilon)$ of the time in expansions and $\gamma/(\gamma+\varepsilon)$ in recessions. As a result the average growth rate is

$$g = \frac{\gamma}{\gamma+\varepsilon} v_R + \frac{\varepsilon}{\gamma+\varepsilon} v_E. \qquad \textbf{(24)}$$

We now study, in world 1 and world 2, how the amplitude and frequency of economic fluctuations affect g. Concerning the amplitude, note that the model entirely determines v_E, v_R, d_E, and d_R regardless of y_E and y_R. That is to say, an increase in the amplitude of fluctuations y_E/y_R has no effect on the equilibrium solution. Fluctuations in demand are instead entirely met by entry and exit.

We therefore consider the effect of the amplitude of *individual* fluctuations. The relevant parameter for this experiment is θ, the fraction of the entry cost that is recouped upon exit. The corresponding comparative static analysis, however, also determines the effect of aggregate volatility on productivity growth in the following sense: If θ is low enough, there will be no exit in recessions. As a result the number of firms will remain constant over the cycle. Then it can be shown that d_E/d_R must be replaced with y_E/y_R in (22a) and (23a). In that case, an increase in y_E/y_R is equivalent to a drop in θ in the case in which exit occurs in recessions.

Concerning the effect of the frequency of fluctuations on long-run growth, it is derived from a comparative static analysis of the rate g with respect to γ and ε. The following three effects of γ and ε on g turn out to be at work:

1. A *composition* effect, apparent from equation (24): If γ increases or ε decreases, the economy will spend more time in recessions. As a result, growth will tend to increase (decrease) if $v_R > v_E (v_R < v_E)$.
2. A *return* effect that can be seen from the RHS of (16) and (17): Given that one typically has $d_E > d_R$, an increase in γ (the frequency of recessions) tends to lower the average return to productivity growth, thus pushing both v_E and v_R down. The same holds for a decrease in ε.
3. A *cost of capital* effect corresponding to the term in $[r+\gamma(1-\theta)]C$ in the RHS of (20) or $[r\theta+\varepsilon(\theta-1)]C$ in the RHS of (21). This term can be interpreted as the firm's capital costs. Because entry costs are not fully recouped at exit, firms suffer a capital loss when the economy shifts from expansion to recessions. This capital loss is more likely when γ is higher. This increases the cost of capital in expansions, thus limiting entry. Therefore, d_E must increase. This tends to speed productivity growth in both states. (A similar argument can be made for recessions: An increase in ε increases the capital gain made when the economy shifts from recession to expansion, thus reducing required profits and, therefore, d_R in recessions. This pushes down productivity growth in both states.)

In what follows, we study the effect of θ, γ, and ε on g in both worlds 1 and 2.

6.1. World 1

We start with world 1 and first consider the case of small, first-order fluctuations at the firm level, i.e, we assume $\theta = 1 - \omega$, with $\omega \ll 1$. It is then possible to establish the following result.

PROPOSITION 7. *Suppose Assumption* 1 *holds. Suppose* $\theta = 1 - \omega$, *with* $\omega \ll 1$. *Then* ω, ε, *and* γ *have no first-order effect on* g.

Proof. Let $f^{-1}(v) = rk'(v)/[1-k(v)]$. Then, plugging (20a)–(21a) into (16a)–(17a) and substituting into (24) yields

$$g = \frac{\varepsilon}{\gamma+\varepsilon} f\left[\frac{r}{r+\gamma(1-\theta)}\right] + \frac{\gamma}{\gamma+\varepsilon} f\left[\frac{r\theta}{r\theta+\varepsilon(\theta-1)}\right] \qquad \textbf{(25a)}$$

Assuming $\theta = 1 - \omega$, it is clear that the first-order terms from the first term exactly cancel out with those from the second term. Therefore, $g = f(1) + o(\omega)$. ■

Proposition 7 tells us that if entry costs are nearly recouped at exit, then the structure of the business cycle has no effect on long-run growth. This, despite the fact that each individual expansion and recession has a long-run impact on the level of total factor productivity. Therefore, on average, the positive effects of recessions on growth and the negative effects of expansions cancel each other.[15]

This neutrality result is due to the fact that intertemporal substitution and intertemporal aggregation exactly cancel each other when the average growth rate is computed. The effect of a recession on v, for example, is stronger when it is more temporary (ε is larger), because of the intertemporal substitution effects at work here. If ε were equal to zero, then v_R would just be equal to the steady-state value of v. Therefore, at a first-order approximation, the amplitude of fluctuations ω has a weight proportional to ε in v_R. However, v_R itself has a weight proportional to the frequency of recessions γ when these are aggregated with expansions to compute the average growth rate. Symmetrically, ω enters negatively in v_E with a weight proportional to γ, the likelihood of falling into a recession, and v_E itself has a weight proportional to ε. As a result the contribution of expansions and recessions to average growth are both symmetrical in ε and γ and exactly cancel each other.

Can we say more about the effects of θ, γ, and ε on growth? It is possible to do a second-order Taylor expansion of (25a) and get the following results.

PROPOSITION 8. *The following results hold*:

(i) *If* $f'(1) \geq -f''(1)/2$, *then* $\partial g/\partial\omega > 0$, $\partial g/\partial\gamma > 0$, $\partial g/\partial\varepsilon > 0$.
(ii) *(ii) If* $f'(1) < -f''(1)/2$, *then let* $M = rf'(1)/[-f'(1) - f''(1)/2]$.

Then,

(α) $\partial g/\partial\omega > 0$ *if and only if* $(\gamma+\varepsilon) < M$
(β) $\partial g/\partial\varepsilon > 0$ *if and only if* $\varepsilon < (\gamma M)^{1/2} - \gamma$
(γ) $\partial g/\partial\gamma > 0$ *if and only if* $\gamma < (\varepsilon M)^{1/2} - \varepsilon$.

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Proof. See Appendix. ■

Proposition 8 illustrates the point that, although the frequency and amplitude of fluctuations generally will affect the long-run rate of growth, the proper direction of these effects crucially depends on the curvature of the f function.

It is difficult to get some clean economic intuition behind Proposition 8, but it is essentially related to how reactive v_R is, relative to v_E, to changes in parameter values. This clearly depends on the curvature of the $f(\cdot)$ function which relates v, the pace of productivity improvements, to the terms of trade between productivity growth and current production, that is, the RHS's of (22a) and (23a). These terms of trade are more favorable in recessions than in expansions. If f is convex enough [case (i) in Proposition 8], then v_R will be more reactive than v_E. An increase in ε, which increases v_R, then will have a positive effect on growth (the large effect on v_R dominates the composition effect); an increase in γ, which reduces v_E, also has a positive effect on growth (the small effect on v_E is dominated by the composition effect); an increase in the amplitude $\omega = 1 - \theta$, which increases v_R and reduces v_E, also will have a positive effect on growth. If, on the contrary, f is very concave [case (ii)], the analysis reveals that the amplitude of fluctuations is still favorable for growth if their overall frequency is not too high. Similarly, the frequency of expansions (resp. recessions) is good for growth up to a limit that depends on the frequency of recessions (resp. expansions). The more concave is f, the smaller is M, and the smaller are the sets of parameter values over which the amplitude and the frequencies may be good for growth. Outside those sets, the results of case (i) simply are reversed, as is the intuition behind those results.[16]

6.2. World 2

In world 2, the composition effect and the return effect reinforce each other. One therefore would expect an increase in the frequency of recessions to be harmful for growth, whereas an increase in the frequency of expansions would be beneficial. On the other hand, the cost-of-capital effect runs counter to the composition and return effects. It never outweighs them, however, so that one can prove the following proposition.[17]

PROPOSITION 9. *Under Assumption* 2, *one has*

$$\partial v_E/\partial\gamma < 0; \qquad \partial v_R/\partial\gamma < 0; \qquad \partial v_E/\partial\varepsilon > 0; \qquad \partial v_R \partial\varepsilon > 0.$$

Proof. See Appendix. ■

COROLLARY 1. *Under Assumption* 2, *one has*

$$\partial g/\partial\gamma < 0; \qquad \partial g/\partial\varepsilon > 0$$

Proof. Use (24). ■

Contrary to what occurs in world 1, ε and γ therefore have unambiguous effects on growth; furthermore, these effects go in the conventional direction.

7. DISCUSSION

We have analyzed the interaction between productivity growth and business cycles under two alternative sets of assumptions. When the cost of productivity-enhancing activities does not depend on current production (i.e, in world 2), the results are rather conventional: A recession lowers the return to productivity growth without affecting its cost. The converse result holds for expansions.

When the cost of productivity improvements positively depends on current production (i.e, in world 1), however, this cost drops by more than its present discounted benefit in a recession. The model therefore predicts that a recession has a positive long-term effect of output. However, as we have seen in the preceding section, the *expectation* of more frequent recessions by the public also lowers the expected return to productivity growth, thus pushing v_E, and v_R, down. In case the amplitude of firm-level fluctuations $(1 - \theta)$ is small, we have established the striking result that the above two effects, along with the cost-of-capital effect, exactly cancel each other so that the structure of the business cycle has no first-order effect on long-run growth.

If the magnitude of firm-level fluctuations is larger, then γ and ε typically should have an effect on long-run growth, but the analysis has revealed that this effect is ambiguous and depends on the whole set of parameters of the model.

Therefore, one should be cautious at drawing policy implications from the conclusion that in world 1, any given recession can be beneficial for growth. In particular, it would be wrong to infer from our analysis that an increased *frequency* of recessions should enhance long-run growth.

Now, turning to the empirical implications of the model, our model suggests that two set of tests could be implemented:

1. an analysis of the response of productivity shocks to innovations in the business cycle, both in the long run and in the short run; and
2. a study of the interaction between the time-series properties of the business cycle and the long-run rate of growth—a topic hardly tackled by the previous empirical literature.[18]

A prerequisite for such tests is to provide an adequate definition of the "business cycle." In our model, there are only demand shocks. Introducing supply shocks would tend to automatically generate higher productivity growth in expansions.[19] To test the model, we have to isolate these shocks from demand shocks. The previous empirical literature performing the first set of tests includes Bean (1990), Gali and Hammour (1991), and Saint-Paul (1993). Bean (1990) isolates demand shocks by singling out demand instruments such as government spending and monetary policy. By contrast, Gali and Hammour (1991) and Saint-Paul (1993) use a semistructural VAR technique as in Blanchard and Quah (1989). They run a VAR in the Solow residual and the unemployment rate (or, equivalently, the rate of capacity utilization) and identify demand shocks by assuming a zero contemporaneous effect of these shocks on the Solow residual. In other words, productivity-improving activities induced by aggregate demand fluctuations only show up in total factor productivity after a one-period lag.[20] A more recent paper, by Malley and Muscatelli (1996), uses the identifying assumption that demand shocks have no long-run *growth* effect on total factor productivity. Using the NBER productivity database, they find a strong negative impact of positive demand shocks on long-run productivity in almost all sectors of the economy.

The empirical results derived in these papers are supportive of our analysis of the world-1 version of the model; in particular, they are consistent with the view that productivity-improving activities must take place at the expense of directly productive activities. All authors indeed find a positive long-term effect of negative demand shocks on the Solow residual. Saint-Paul (1993) also finds a significant one-year-ahead effect for most countries. These findings call into question both the world-2 model, in which the cost of productivity-improving activities does not fall during recessions, and those models that insist on the positive externalities of expansions [e.g., the learning-by-doing model of Stadler (1989) or the aggregate-demand externality model of Shleifer (1986)].[21]

These empirical results nevertheless remain fragile because of the poor quality of aggregate data. In particular, more empirical work is needed for identifying the nature of the productivity-improving activities that take place in recessions. Gali and Hammour (1991) find an increase in within-firm training; Dellas (1993) finds similar evidence. Saint-Paul (1993) finds R&D investment to be essentially acyclical once one controls for supply shocks.

We turn now to the second set of tests, namely, about the correlation between long-run growth and the frequency and amplitude of business-cycle fluctuations. These tests are, at the same time, more demanding on the data and less on the model: Although all sorts of results may be generated by our model under various functional forms for $k(v)$ [or, equivalently, $f(\cdot)$], it is unclear whether the data can generate enough meaningful cross-sectional variations in the characteristics of the business cycle. Saint-Paul (1993), abstracting from limitation, computes the spectral density of demand shocks using his VAR coefficients for 20 OECD countries. He then computes the share of the total variance of demand-induced fluctuations in output that is due to high-frequency components, and computes the cross-country correlation of this share with long-run total factor productivity growth. Surprisingly, he finds a significant negative correlation: High-frequency components of the cycle seems to be bad for growth.

This finding, although striking, essentially relies on a peculiar definition of the business cycle based on the Gali-Hammour identifying assumption. How robust it is to alternative assumptions remains to be established. Also, this does not constitute a direct test of the above model. In particular, the stochastic process followed by the business cycle in the existing empirical literature is different from the one defined above. A more direct test would consist in estimating a two-state Markov process for the business cycle as in Hamilton (1989).[22] This, in turn, would make it possible to recover structural parameters such as ε, γ, y_R, and y_E. Using these parameters, one could calibrate the model and recover the cyclical behavior of productivity growth as well as its average value. It then would be possible to confront this calibration with the data, both in the times-series dimension and in a cross section of countries. Here may lie a valuable direction for further research.

NOTES

1. See Kydland and Prescott (1982); Long and Plosser (1983); King and Rebelo (1986).

2. See Romer (1986), Lucas (1988), Grossman and Helpman (1990), Aghion and Howitt (1991).

3. The cleansing effect studied by Caballero and Hammour (1992) could be analyzed within our model by introducing heterogeneity across firms.

4. The Shleifer model focuses on business cycles and does not deal with growth effects; the Stadler model has growth effects, but they typically are explosive, which makes it difficult to use that model for analyzing the effects of business cycles on long-run growth.

5. An early contribution to the analysis of real business cycles with endogenous growth is King and Rebelo (1986), but it focuses on technology shocks as a source of fluctuations, whereas our model is best thought of as an analysis of the impact of aggregate demand shocks.

6. Equivalently, we could consider an aggregate index of a bundle of differentiated imported goods.

7. At least with the same productivity.

8. There, it is shown that the exit effect tends to reinforce the countercyclicality of v in world 2 and to reduce its procyclicality in world 2.

9. For larger values of β, the exit effect runs in the opposite direction: It tends to increase productivity growth in expansions. The value of β equating the two marginal impacts is computed in the Appendix and can be shown to be just equal to $(\eta - 1)/\eta$ in world 1.

10. As a result, growth is entirely captured by foreigners, and does not benefit, or harm, the country.

11. Although this is a stylized, and not very common, way of formalizing the business cycle, Hamilton (1989) has shown that it can yield productive insights. Contrary to standard linear approaches, it is especially good at capturing asymmetries between recessions and expansions.

12. Note that, in expansions, (5) can be written $0 = \pi_E - (r + \gamma)V_E + \gamma V_R$, and in recessions, $0 = \pi_R - (r + \varepsilon)V_R + \varepsilon V_E$, where V_j is the value function in state j.

13. The result collapses if one reintroduces the exit effect. See the Appendix.

14. Algebraically, this is represented by the fact that the LHS of (16a) is proportional to d_E, whereas the RHS is proportional to a weighted average of d_E and d_R.

15. It can be shown that this neutrality result disappears if one reintroduces the exit effect, for example, by assuming $\beta = 0$. In that case, it can be shown that an increased amplitude of y_E/y_R is necessarily bad for growth because the likelihood of exit is greater, thus lowering the incentives for productivity growth. An increased frequency (a higher γ or a higher ε) may be either good or bad for growth depending on whether $\theta y_E/y_R$ is small or large.

16. These sets, however, are never empty, regardless of the curvature of $f(\cdot)$. This is because when γ and ε are small enough, v_E is close to v_R, and the contribution of the curvature of $f(\cdot)$ becomes a third-order effect. What dominates then is the effect of ε, γ, and ω on the terms of trade, and the results then are similar to what would happen if $f(\cdot)$ were linear, i.e., to case (i).

17. We do not study the effect of θ here because it does not provide a pure increase in the amplitude of firm-level demand fluctuations. The reason is that whereas C disappears from the determination of v_E and v_R in world 1, it does not in world 2 [compare, for example, (22) and (20a)]. As a result, a drop in θ is associated with a drop in the firm's value on *average over* the business cycle, which has a depressing effect on both v_E and v_R. This is essentially a *level*, not an amplitude, effect, which would be similar to a drop in C in the steady state.

18. See Saint-Paul (1993) for a more detailed discussion of these issues.

19. It is one reason why the model is not incompatible with the observed procyclicality of the Solow residual, the other being that some of the productivity-improving activities (such as reorganization) may not be measured in the national accounts, a point emphasized by Bean (1990). A recent contribution by Burnside et al. (1993) analyzes labor hoarding along the business cycle and concludes that it is a

significant source of procyclicality of the Solow residual. It is fair to say that when these two sources of procyclicality are removed, there is no evidence left of a procyclical Solow residual. Indeed, under the identifying assumptions made by Gali and Hammour (1991) and Saint-Paul (1993), the evidence points to a countercyclical residual, in accordance with the world-1 version of the model.

20. Saint-Paul (1993) also has tried an alternative identification assumption that yields similar results.

21. It is often argued [see King (1993), for example] that increased restructuring during recessions has no long-run effect on growth because it is purely due to changes in the timing of such activities. However, the existence of a long-run effect rules out such an interpretation.

22. See also Acemoglu and Scott (1992).

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APPENDIX

A.1. DERIVATION OF (16) AND (17)

Let $u_j = \partial V/\partial x$ in state j. Plugging (8), (3), and (4) into (7) for $j = R$, we get, in both worlds,

$$u_R = \frac{(\eta - 1)d_R/\eta + \varepsilon u_E}{r + \varepsilon}, \tag{A.1}$$

where $d_j = y_j/N_j$ is an index of each individual firm's demand level in state j.

Using now the same equations for state E, one gets

$$u_E = \frac{(\eta - 1)d_E/\eta + \gamma[qu_R + (1-q)\theta\beta C]}{r + \gamma}, \tag{A.2}$$

where $q = N_R/N_E$ is the probability that the firm continues to operate in recession, and $\theta\beta C$ is the marginal effect of an increase in x_i on the firm's liquidation value. As argued above, we want to get rid of the exit effect by assuming the same marginal impact of x on the exit value and the continuation value, i.e., $u_R = \theta\beta C$. Therefore, (A.2) becomes

$$u_E = \frac{(\eta - 1)x_E/\eta + \gamma u_R}{r + \gamma}. \tag{A.3}$$

Solving (A.3) and (A.1) in terms of u_E and u_R yields

$$u_E = \frac{\eta - 1}{\eta}\frac{(r+\varepsilon)x_E + \gamma x_R}{r(r+\varepsilon+\gamma)}, \tag{A.4}$$

$$u_R = \frac{\eta - 1}{\eta}\frac{(r+\gamma)x_R + \varepsilon x_E}{r(r+\varepsilon+\gamma)}. \tag{A.5}$$

Equations (16) and (17) then are obtained by replacing the RHS of (6) with the RHS of (A.4) and (A.5) for both states E and R and both worlds 1 and 2.

A.2. COMPUTING THE VALUE OF β SUCH THAT THERE IS NO EXIT EFFECT

To eliminate the exit effect, we must have $u_R = \theta\beta C$. Plugging this into (A.5) and using (20a) and (21a) to substitute for d_E and d_R allows us to get the corresponding value of β in world 1. Computations yield

$$\beta = (\eta - 1)/\eta. \tag{A.6a}$$

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Using (20b) and (21b) allows us to follow the same steps for world 2. There, one cannot get a closed-form solution for β because of the appearance of $h(v_E)$ and $h(v_R)$ in (20b) and (21b). Equation (A.6a) then becomes

$$\beta = (\eta - 1)/\eta + \left(\frac{\eta - 1}{\eta}\right)\left(\frac{1}{\theta C}\right)\left[\frac{(r+\gamma)h(v_R) + \varepsilon h(v_E)}{r(r+\varepsilon+\gamma)}\right]. \quad \textbf{(A.6b)}$$

A.3. CONSEQUENCES OF EXIT EFFECT

We now analyze the consequences of the exit effect by assuming $\beta = 0$. Using (A.1) and (A.2), we may recover the counterparts to (A.4) and (A.5):

$$u_E = \left(\frac{\eta - 1}{\eta}\right)\left[\frac{(r+\varepsilon)\,d_E + \gamma q d_R}{(r+\varepsilon)(r+\gamma) - \varepsilon\gamma q}\right], \quad \textbf{(A.7)}$$

$$u_R = \left(\frac{\eta - 1}{\eta}\right)\left[\frac{(r+\gamma)\,d_R + \varepsilon d_E}{(r+\varepsilon)(r+\gamma) - \varepsilon\gamma q}\right], \quad \textbf{(A.8)}$$

where $q = N_R/N_E$. In world 1, $u_i = d_i k'(v_i)/[1 - k(v_i)]$, $i \in \{E, R\}$. Clearly, (A7) and (A8) then imply that v_E is lower than v_R even if $\theta = 1$.

In world 2, $u_i = h'(v_i)$. Let us first consider the case in which $\theta = 1$. Subtracting (A.8) from (A.7), using (20b) and (21b) yields

$$h'(v_E) - h'(v_R) = (\eta - 1)/(\eta D)[\gamma(q-1)rC + rh(v_E) - rh(v_R) + \gamma(q-1)h(v_R)], \quad \textbf{(A.9)}$$

where $D = (r+\varepsilon)(r+\gamma) - \varepsilon\gamma q$; Equation (A.9) may be rewritten

$$\tilde{g}(v_E) - \tilde{g}(v_R) = (\eta - 1)/(\eta D)\gamma(q-1)[rC + h(v_R)], \quad \textbf{(A.10)}$$

where $\tilde{g}(v) = h'(v) - r(\eta-1)h(v)/(\eta D)$. Now, it is enough to note that the RHS of (A.10) is less than zero and that (15) implies that $\tilde{g}$ is strictly increasing in v to conclude that

$$v_E < v_R.$$

Therefore, in world 2, the exit effect also tends to make v countercyclical. When $\theta < 1$, this countercyclicality effect is balanced by the procyclicality effect analyzed in Section 5. The net effect, obviously, depends on whether $1 - \theta$ is large or small compared to $1 - q$. More specifically, following the same steps as above when $\theta < 1$ gives

$$\tilde{g}(v_E) - \tilde{g}(v_R) = (\eta - 1)/(\eta D)[\gamma(q-1)[rC + (r+\varepsilon)(\theta - 1)C + h(v_R)] + r(r+\gamma+\varepsilon)(1-\theta)C]. \quad \textbf{(A.11)}$$

Procyclicality of v will be restored if the RHS is greater than zero. Rearranging terms and noting that $q = N_R/N_E = (y_R d_E)/(y_E d_R)$ yields the following condition:

$$\gamma[rC + h(v_R) - \rho(rC + h(v_E)] < (1-\theta)C[(r+\varepsilon)(r+\gamma) + \gamma(r+\gamma\rho)],$$

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where $\rho = y_R/y_E$. Intuitively, this condition is more likely to be satisfied if θ and γ are small. A low θ increases the procyclicality of the returns to productivity growth, whereas a low γ makes the exit effect less important.

A.4. PROOF OF PROPOSITION 8

To prove Proposition 8, we just have to do a second-order Taylor expansion on (25a). Let $\omega = 1 - \theta \ll 1$. After some tedious but elementary computations, we get

$$g = f(1) + [f'(1) + f''(1)/2][\omega^2 \varepsilon\gamma/r^2] + [f'(1)]\{\gamma\varepsilon\omega^2/[r(\gamma+\varepsilon)]\} + o(\omega^2)$$

The claims of the proposition then derive from this formula and the observation that $f'(1) > 0$. To get claims (α), (β), and (γ), just derive the above formula with respect to ω, ε, and γ, respectively.

A.5. PROOF OF PROPOSITION 9

In world 2, the equilibrium boils down to the solution to a system of four equations: (16b), (17b), (20b), and (21b). They can be rewritten in the following form:

$$v_E = \varphi_E(d_E, d_R; \gamma, \varepsilon), \tag{16b}$$

$$v_R = \varphi_R(d_E, d_R; \gamma, \varepsilon), \tag{17b}$$

$$d_E = \delta_E(v_E; \gamma), \tag{20b}$$

$$d_R = \delta_R(v_R; \varepsilon). \tag{21b}$$

We focus on the comparative statics with respect to γ. First, note that

$$\partial\varphi_E/\partial\gamma + \left(\frac{\partial\varphi_E}{\partial d_E}\right)\left(\frac{\partial\delta_E}{\partial\gamma}\right) < 0 \tag{A.12}$$

To see this, consider the RHS of (16b) and call it R_E. We have

$$\frac{\partial R_E}{\partial\gamma} = (r+\varepsilon)(d_R - d_E)/(r+\varepsilon+\gamma)^2, \tag{A.13}$$

$$\frac{\partial R_E}{\partial d_E} = \frac{(r+\varepsilon)}{(r+\varepsilon+\gamma)}. \tag{A.14}$$

Using (20b), we get

$$\frac{\partial\delta_E}{\partial\gamma} = (1-\theta)C. \tag{A.15}$$

Subtracting (21b) from (20b), we get

$$d_R - d_E < -C(r+\varepsilon+\gamma)(1-\theta). \tag{A.16}$$

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This is because $v_E > v_R$. Now, multiplying (A.14) by (A.15), adding it to (A.13), and substituting in (A.16) yields

$$\frac{\partial R_E}{\partial \gamma} + \left(\frac{\partial R_E}{\partial d_E}\right)\left(\frac{\partial \delta_E}{\partial \gamma}\right) < 0.$$

Given that v_E is an increasing function of R_E, (A.12) follows.

Similarly, we can show that

$$\frac{\partial \varphi_R}{\partial \gamma} + \left(\frac{\partial \varphi_R}{\partial d_E}\right)\left(\frac{\partial \delta_E}{\partial \gamma}\right) < 0 \qquad \textbf{(A.17)}$$

To see this, note again that if R_R denotes the RHS of (17b), then

$$\frac{\partial R_R}{\partial \gamma} = -\varepsilon \frac{(d_E - d_R)}{(r+\varepsilon+\gamma)^2} < -\varepsilon C \frac{(1-\theta)}{(r+\varepsilon+\gamma)} \qquad \textbf{(A.18)}$$

and

$$\frac{\partial R_R}{\partial d_E} = \frac{\varepsilon}{(r+\varepsilon+\gamma)} \qquad \textbf{(A.19)}$$

Multiplying (A.19) by (A.15) and adding it to (A.18), we obtain (A.17).

Now, the net effect of a increase in γ on v_E and v_R can be obtained by full differentiation of the system, which can be reduced to the following two-equation system:

$$\begin{bmatrix} a_{EE} & a_{ER} \\ a_{RE} & a_{RR} \end{bmatrix} \begin{bmatrix} \dfrac{dv_E}{d\gamma} \\ \dfrac{dv_R}{d\gamma} \end{bmatrix} = \begin{bmatrix} m_E \\ m_R \end{bmatrix},$$

where

$$a_{EE} = 1 - \frac{\partial \varphi_E}{\partial d_E}\frac{\partial \delta_E}{\partial v_E},$$

$$a_{RR} = 1 - \frac{\partial \varphi_R}{\partial d_R}\frac{\partial \delta_R}{\partial v_R},$$

$$a_{ER} = -\frac{\partial \varphi_E}{\partial d_R}\frac{\partial \delta_R}{\partial v_R},$$

$$a_{RE} = -\frac{\partial \varphi_R}{\partial d_E}\frac{\partial \delta_E}{\partial v_E},$$

$$m_E = \frac{\partial \varphi_E}{\partial \gamma} + \frac{\partial \varphi_E}{\partial d_E}\frac{\partial \delta_E}{\partial \gamma},$$

$$m_R = \frac{\partial \varphi_R}{\partial \gamma} + \frac{\partial \varphi_R}{\partial d_E}\frac{\partial \delta_E}{\partial \gamma}.$$

Clearly, $a_{ER} < 0$ and $a_{RE} < 0$, and we already know that $m_E < 0$ and $m_R < 0$. Furthermore,

$$a_{EE} = 1 - \frac{h'(v_E)}{h''(v_E)}[(\eta - 1)/\eta]\frac{(r+\varepsilon)}{r(r+\varepsilon+\gamma)} > 0.$$

where the inequality follows from 15. Similarly, one has $a_{RR} > 0$. Applying the Cramer formulas to the above system, we then see that

$$\frac{dv_E}{d\gamma} = \frac{(a_{RR}m_E - a_{ER}m_R)}{D},$$
$$\frac{dv_R}{d\gamma} = \frac{(a_{EE}m_R - a_{RE}m_E)}{D}.$$

Given the signs of the parameters, the numerators are always negative. The denominator is the determinant of the system:

$$\begin{aligned} D &= a_{EE}a_{RR} - a_{ER}a_{RE} \\ &= \left(1 - \frac{\partial\varphi_E}{\partial d_E}\frac{\partial\delta_E}{\partial v_E}\right)\left(1 - \frac{\partial\varphi_R}{\partial d_R}\frac{\partial\delta_R}{\partial v_R}\right) - \left(\frac{\partial\varphi_E}{\partial d_R}\frac{\partial\delta_R}{\partial v_R}\right)\left(\frac{\partial\varphi_R}{\partial d_E}\frac{\partial\delta_E}{\partial v_E}\right) \\ &= \left[1 - \frac{r+\varepsilon}{r+\varepsilon+\gamma}\frac{\eta-1}{r\eta}\frac{h'(v_E)}{h''(v_E)}\right]\left[1 - \frac{r+\gamma}{r+\varepsilon+\gamma}\frac{\eta-1}{r\eta}\frac{h'(v_R)}{h''(v_R)}\right] \\ &\quad - \frac{\gamma\varepsilon}{(r+\gamma+\varepsilon)^2}\left(\frac{\eta-1}{r\eta}\right)^2\frac{h'(v_E)h'(v_R)}{h''(v_E)h''(v_R)}. \end{aligned}$$

Let

$$z_j = \frac{\eta-1}{r\eta}\frac{h'(v_j)}{h''(v_j)}, \qquad j \in \{E, R\}.$$

Then (15) implies $z_j < 1$.

Furthermore, the above formula can be rewritten

$$\begin{aligned} D &= \left(1 - \frac{r+\varepsilon}{r+\varepsilon+\gamma}z_E\right)\left(1 - \frac{r+\gamma}{r+\varepsilon+\gamma}z_R\right) - \frac{\gamma\varepsilon}{(r+\gamma+\varepsilon)^2}z_E z_R \\ &= 1 - (r+\varepsilon)z_E/(r+\gamma+\varepsilon) - (r+\gamma)z_R/(r+\gamma+\varepsilon) + r z_E z_R/(r+\gamma+\varepsilon) \\ &= [\varepsilon(1-z_E) + \gamma(1-z_R) + r(1-z_E)(1-z_R)]/(r+\varepsilon+\gamma) > 0. \end{aligned}$$

Hence $D > 0$, from which $dv_E/d\gamma < 0$ and $dv_R/d\gamma < 0$ follow. Exactly the same steps may be undertaken, *mutatis mutandis*, to prove that $dv_E/d\varepsilon > 0$ and $dv_R/d\varepsilon > 0$.