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Local temperature perturbations of the boundary layer in the regime of free viscous–inviscid interaction

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We analyse the disturbed flow in the subsonic laminar boundary layer, disturbances being generated by local heating elements, which are placed on the surface. It is exhibited that these flows are described in terms of free interaction theory for specific sizes of thermal sources. We construct the numerical solution for the case of a flat subsonic stream in the viscous asymptotic layer, in which the flow is described by nonlinear equations for vorticity, temperature and an interaction condition which provides the influence of perturbations to the pressure in the main order. The obtained solutions are compared with those for corresponding linear problems with small perturbations. It is demonstrated that strong temperature perturbations in some situations allow us to obtain the flow close to the separated flow.

Key words: boundary layer structure, boundary layer control, Navier–Stokes equations

1. Introduction

The present work continues the studies started in Lipatov (2006) and Koroteev & Lipatov (2009), which were devoted to the construction of asymptotic solutions of Navier–Stokes equations in the regions containing local heating elements which are situated on the surface of the body. In our previous work (Koroteev & Lipatov 2009), we demonstrated how this problem can be solved analytically for small temperature perturbations. The idea lying behind this approach consists of the possibility to utilize small temperature perturbations to control separation of the boundary layer and influence the delay of laminar–turbulent transition. The former question is basically related to the zero shear stress point in the stream, which can alter its location if one affects the boundary layer by some, not necessarily thermal, perturbations. The latter is related to the possibility to slow down the flow by means of the same perturbations to decrease the local Reynolds number *Re*. Potentially this can bring us to the separation which is induced by local thermal perturbations. The main internal mechanism which enables us to affect the boundary layer is alteration of the pressure induced by that of the displacement thickness in the flow due to perturbations. The source of the perturbations can be various in nature, e.g. variation of the curvature of the surface which can be in the form of small humps or irregularities. The problems

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of perturbations generated by small humps located on the surface in the bottom of the boundary layer have been the subject of studies during past several decades (Bogolepov & Neiland 1971; Smith 1974; Bogolepov & Neiland 1976).

Lipatov (2006) presented a general description of problems emerging when local heating elements are located on the surface. In the same work an important similarity between the problems in question and those of flows over the surface with small humps was demonstrated. It was, in part, shown that the local heating forms an effective hump and the outline of the flow becomes similar, to an extent, to that with physical humps on the surface. Based on these observations a classification of different regimes of flows depending on sizes (in three dimensions) of the thermal element was presented. Thus, the main difference between flows with local physical humps and those with local thermal perturbations consists in that a thermal hump is generated in the process of interaction of the stream with the local thermal element. The form of this thermal hump nonlinearly depends on the pressure and temperature perturbations.

Localization of the perturbations and their influence on the flow functions made it possible to employ asymptotic analysis based on a triple-deck structure (Neiland 1969; Stewartson & Williams 1969; Messiter 1970) to derive equations for different asymptotic regions in the vicinity of the thermal element (Lipatov 2006). Stationary solutions which are described in terms of free interaction of the boundary layer with the inviscid flow for the supersonic case corresponding to small perturbations of temperature $\Delta T \ll T \sim O(1)$ were found in Koroteev & Lipatov (2009) and were also obtained for the subsonic case in Koroteev & Lipatov (2011).

The purpose of the present paper is to extend the analysis to the more general case of nonlinear perturbations. Linear problems, corresponding to small perturbations $\Delta T \ll 1$, although being novel, present in this sense only the first step in studying flows in the boundary layer with thermal perturbations. In addition, linear analysis omits important nonlinear effects that are essential for boundary layer theory. However, we only study preseparated flows here.

2. Analysis of the problem for the viscous asymptotic sublayer

We consider a uniform subsonic flow over a flat semi-infinite plate for large Reynolds numbers Re when, however, laminar–turbulent transition is still absent. We use the notation $Re = \epsilon^{-2}$ and thus ϵ is a small parameter. The flow is assumed to be flat and stationary. On the surface of the body a heating element of size a is located which produces thermal perturbations in the stream. Temperature perturbations on the wall are described by a finite function. The size of the element is an important parameter of the problem and discussed below. We assume, unlike in Koroteev & Lipatov (2009), that thermal perturbations are not small $\Delta T \sim T \sim O(1)$.

In the problem under consideration, Navier–Stokes equations for momentum are supplied with the equation for temperature. It turns out that by the effect produced by perturbations, the whole region in the vicinity of the heating element may be divided into three smaller regions or decks (lower, middle and upper; see figure 1) (Neiland 1969; Stewartson & Williams 1969; Messiter 1970). The sizes of these regions are proportional to powers of the parameter $Re = \epsilon^{-2}$. For the problem under consideration the detailed analysis of equations corresponding to each region was fulfilled in Lipatov (2006) and Koroteev & Lipatov (2009) and therefore is not described here. Note that the effect of perturbations is defined by scales of terms in Navier–Stokes equations. In part, assuming similar scales for convective $u\partial u/\partial x$, diffusive $\partial^2 u/\partial y^2$ and pressure gradient terms we can obtain estimates of the

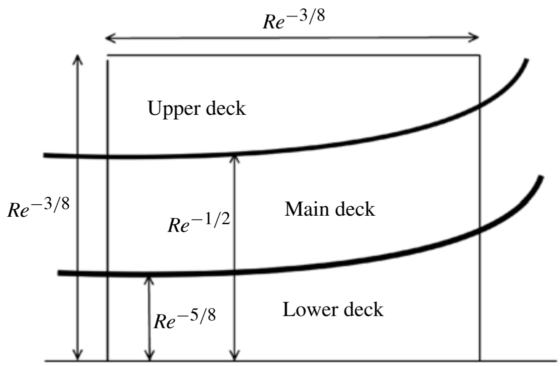

FIGURE 1. Sketch of triple deck region in the vicinity of a heating element.

size of disturbed region where corresponding terms have the main order. Nonlinear perturbations of the temperature $\Delta T \sim T$ give $\Delta\rho \sim \rho$, $\Delta u \sim u$. In the region near the wall we have in the main order $u \sim O(y/\epsilon)$, i.e. velocity grows linearly with respect to y. Then convective and dissipative terms of Navier–Stokes equations yield

$$u\frac{\partial u}{\partial x} \sim \epsilon^2 \frac{\partial^2 u}{\partial y^2}. \tag{2.1}$$

The estimate of thickness of the disturbed region, namely $\Delta y \sim O(\epsilon \Delta x^{1/3})$, follows from (2.1). We have to stress that as we deal with thermal perturbations in the above estimate, it is also assumed that $Pr \sim O(1)$. This corresponds to compressible gas flow for quite a broad range of parameters. In addition, the condition $Pr \sim O(1)$ implies that the thicknesses of diffusive and thermal layers are of the same order, thus thermal perturbations are restricted by the outer boundary of the boundary layer. Further localization of thermal perturbations requires an additional analysis.

Below we study only one possible case, namely when the pressure gradient term has the same order as dissipative and convective terms. From the asymptotic relation for these terms we find $\Delta p \sim a^{2/3}, \Delta u \sim u \sim a^{1/3}$. The main contribution to the displacement of stream lines from perturbations is given by a thin near-wall layer, while Prandtl boundary layer (main deck) is passive, thus $\Delta\delta \sim \Delta y$, where δ is the displacement thickness. Thus, all three terms of Navier–Stokes equations are of the same order only under the latter assumptions, from which we have $\Delta p \sim \Delta y/\Delta x$ and finally we obtain the desirable estimate for the length of the disturbed region $a \sim O(\epsilon^{3/4})$. Then we easily rewrite all asymptotic relations in terms of ϵ to obtain $\Delta p \sim O(\epsilon^{1/2})$, $a \sim O(\epsilon^{3/4})$, $\Delta y \sim O(\epsilon^{5/4})$. The flow under these conditions is said to be in the regime of free interaction (Neiland 1969; Stewartson & Williams 1969).

Further assumptions are related to the temperature. We take the Chapman viscosity law in the lower deck in the form $\rho_3\mu = 1$, where μ is the viscosity. Physically it is implied that the temperature before the thermal region is sufficiently low.

The flow with the above-derived scales exists in a thin layer near the wall inside the main deck (boundary layer) which is called the lower deck or a viscous sublayer. The upper deck is described by inviscid Euler equations, the middle deck by Prandtl boundary layer equations and the lower layer by some nonlinear set of equations of parabolic type (Koroteev & Lipatov 2009, 2011). The asymptotic layout of the three decks with corresponding scales is portrayed in figure 1. In addition, the equations on the lower deck are supplied with boundary and initial conditions, which are formulated below, and also by an interaction condition which not only connects functions of the lower deck to those of the upper deck, but also accomplishes the influence of the right boundary on the behaviour of functions upstream. It implies that the set of equations in the lower deck cannot be viewed as that of purely parabolic type, unlike the equations of Prandtl boundary layer theory, and gains, in a sense, the property of ellipticity.

The pressure gradient retained in the equations for each deck is not fixed but becomes self-induced, i.e. varies on account of the displacement thickness δ of the boundary layer to the outer inviscid flow, and the main contribution to the variation of the pressure is furnished by the lower deck, while the main deck merely conveys perturbations from the lower deck to the outer flow. On the other hand, the induced pressure gradient affects streamlines on the lower deck. The indicated scheme enables to construct solutions of equations in the lower deck with the interaction condition.

We can give the following representation of functions for the lower deck taking into account the asymptotic scales derived above

$$x = 1 + ax_3, \quad y = \epsilon a^{1/3} y_3, \quad u = a^{1/3} u_3 + \cdots, \tag{2.2}$$

$$v = \epsilon a^{-1/3} v_3 + \cdots, \quad \rho = \rho_3 + \cdots, \quad p = p_\infty + a^{2/3} p_3 + \cdots, \quad T = T_3 + \cdots. \tag{2.3}$$

The subscript 3 refers to the variables on the lower deck. Substituting these relations into the Navier–Stokes equations and taking $\epsilon \to 0$ gives (in the main order after the additional transform $x_b = x_3,\ y_3 = \int_0^{y_b} \rho_b^{-1}\,\mathrm{d}\eta$) the following system

$$\left.\begin{aligned} &\frac{\partial u_b}{\partial x_b} + \frac{\partial v_b}{\partial y_b} = 0, \quad u_b \frac{\partial u_b}{\partial x_b} + v_b \frac{\partial u_b}{\partial y_b} + T_b \frac{\partial p_b}{\partial x_b} = \frac{\partial^2 u_b}{\partial y_b^2}, \\ &u_b \frac{\partial T_b}{\partial x_b} + v_b \frac{\partial T_b}{\partial y_b} = \frac{\partial^2 T_b}{\partial y_b^2}. \end{aligned}\right\} \tag{2.4}$$

Here u_b, v_b are longitudinal and transverse components of velocity in the sublayer, T_b is the disturbed temperature, $p_b(x)$ is the pressure which only depends on x and consequently is constant across the decks.

The system is supplied with the following boundary conditions. On the surface the no-slip conditions are stated

$$u_b(x_b, 0) = v_b(x_b, 0) = 0. \tag{2.5}$$

We also prescribe a temperature profile on the surface:

$$T_b(x_b, 0) = T_w(x_b). \tag{2.6}$$

The boundary condition at a significant distance from the surface is presented as follows

$$u_b \to y_b + \mathscr{A}(x_b), \quad \mathscr{A}_1(x_b) = \int_0^{+\infty} (1 - T_b)\,\mathrm{d}\eta + \mathscr{A}(x_b), \tag{2.7}$$

$$T_b(x, y_b) \to 1, \quad y_b \to \infty. \tag{2.8}$$

The function $\mathscr{A}(x_b)$ describes the displacement of stream lines in the sublayer (Jobe & Burgraff 1974; Sychev *et al.* 1998). Local perturbations of the temperature generate an effective local hump (Lipatov 2006) and produce an additional displacement of stream lines. The resulting displacement is described by the function $\mathscr{A}_1(x_b)$. The temperature perturbations vanish far from the location of the heated region and $\mathscr{A}(x_b)$ tends to $\mathscr{A}_1(x_b)$ as follows from the above equation.

The conditions which relate the displacement thickness with the longitudinal velocity are supplied with the interaction condition, which, in turn, relates the displacement thickness with the pressure and thus expresses the interaction of the viscous sublayer with the outer inviscid flow.

The condition of interaction varies depending on the Mach number M in the stream. We do not provide its derivation as this question has been discussed numerous times in the literature (Stewartson 1981; Sychev *et al.* 1998). This condition can be obtained by solving Euler equations on the outer deck. In part, for the subsonic flow ($M < 1$) which we study, the condition of interaction has the form

$$p(x_b) = \frac{1}{\pi} v.p. \int_{-\infty}^{+\infty} \frac{\mathscr{A}_1'(s)}{s - x_b}\, \mathrm{d}s. \tag{2.9}$$

This Hilbert integral has a singularity point and hence is taken in the principal value sense.

3. Numerical solution of boundary problem

The numerical solution of the problem (2.4)–(2.9) is constructed on the premise of the method for computations of problems with interaction as suggested in Ruban (1976) (see also Sychev *et al.* (1998), and especially Korolev, Gajjar & Ruban (2002) for the recent progress). We modify the method by supplementing the momentum equation and the continuity equation with that for the temperature which also requires the above-mentioned changes in the boundary conditions. In addition, the interaction condition has an additional term for temperature perturbations. Further we only provide a brief description of the numerical procedure, paying attention to necessary modifications and analysing the resulting difference scheme.

First, the boundary problem is reduced to that for vorticity $\tau = \partial u/\partial y_b$. This is carried out by differentiating the momentum equation with respect to y_b and taking into account the equation of continuity. Then the system becomes

$$\left.\begin{aligned} u_b\frac{\partial \tau}{\partial x_b} + v_b\frac{\partial \tau}{\partial y_b} + \frac{\partial T_b}{\partial y_b}\frac{\partial p_b}{\partial x_b} &= \frac{\partial^2 \tau}{\partial y_b^2}, \\ u_b\frac{\partial T_b}{\partial x_b} + v_b\frac{\partial T_b}{\partial y_b} &= \frac{\partial^2 T_b}{\partial y_b^2}. \end{aligned}\right\} \tag{3.1}$$

The boundary conditions have the following form

$$\tau \to 1, \quad y_b \to +\infty, \quad x_b \to -\infty. \tag{3.2}$$

No-slip conditions (2.5) as well as those for the temperature (2.6) are retained on the wall.

Next, it is easily seen that the momentum equation yields

$$T_w(x_b)\frac{\partial p}{\partial x_b} = \frac{\partial \tau}{\partial y_b}, \quad y_b = 0. \tag{3.3}$$

From this equation we obtain using the interaction condition (2.9)

$$-T_w(x_b)\frac{\partial}{\partial x_b}\int_{-\infty}^{+\infty} \frac{\mathscr{A}_1'(s)}{s - x_b}\, \mathrm{d}s = \frac{\partial \tau}{\partial y_b}, \quad y_b = 0. \tag{3.4}$$

Localization of the effective hump implies the decay of perturbations downstream, i.e.

$$p(x_b) \to 0, \quad x_b \to +\infty. \tag{3.5}$$

The numerical method provides some freedom in the consecutive evaluation of the functions as the number of relations between them exceeds the number of functions. We describe one possible realization of the procedure which enables us to carry out the computations.

The grid is given by (x_j, y_k), $j = 1, 2 \dots N$, $k = 1, 2 \dots M$, with steps $\Delta x, \Delta y$, respectively. The first equation of the system (3.1) is approximated using central differences for convective derivatives with respect to y. Note that convective derivatives with respect to x are approximated as follows

$$\left(\frac{\partial \tau}{\partial x}\right)^*_{jk} = \begin{cases} \dfrac{3\tau_{jk} - 4\tau_{j-1k} + \tau_{j-2k}}{2\Delta x}, & u_{jk} \geqslant 0 \\ \dfrac{-\tau_{j+2k} + 4\tau_{j+1k} - 3\tau_{jk}}{2\Delta x}, & u_{jk} < 0, \end{cases} \tag{3.6}$$

thus different templates are employed depending on the sign of the longitudinal velocity. This method secures static stability of the difference scheme (Roache 1972) and turns out to allow to compute stronger nonlinearities. Two-point template for approximation of these derivatives can be used for moderate ΔT.

The equation for the temperature is approximated as follows

$$u_{jk}\frac{T_{j+1k} - T_{jk}}{\Delta x} + v_{jk}\frac{T_{jk+1} - T_{jk-1}}{2\Delta y} = \frac{T_{jk-1} - 2T_{jk} + T_{jk+1}}{\Delta y^2}. \tag{3.7}$$

Once the vorticity and temperature are computed we can evaluate $\mathscr{A}_1(x_j)$ in accordance with (2.7) as follows

$$\mathscr{A}_1(x_j) = \int_{y_0}^{y_M} (\tau(x_j, y) - 1)\, \mathrm{d}y + \int_{y_0}^{y_M} (1 - T(x_j, y))\, \mathrm{d}y. \tag{3.8}$$

The interaction condition (3.4) requires special manipulations due to the singularity of the integral. There are a number of methods for computing this integral (see, e.g., Demidovich & Maron (1970), this method is also used in Bos & Ruban (2005)). First, the integral is taken on the finite interval (x_1, x_N) so that the singularity is localized on the interval $[x_i - \Delta x, x_i + \Delta x]$. Other integrals are regular and can be evaluated by a polynomial interpolation of $\mathscr{A}'(s)$. Assuming that Δx is small we take $\mathscr{A}_1'(s) \approx \mathscr{A}_1'(x_{j+1})$ for $s \in [x_j, x_{j+1}], j < i - 2$ and $\mathscr{A}_1'(s) \approx \mathscr{A}_1'(x_j), j \geqslant i + 1$. Then

$$I_{ij} = \int_{x_j}^{x_{j+1}} \frac{\mathscr{A}_1'(s)}{s - x_i}\, \mathrm{d}s = \begin{cases} \mathscr{A}_1'(x_{j+1}) \log \dfrac{x_i - x_{j+1}}{x_i - x_j}, & j < i - 2 \\ \mathscr{A}_1'(x_j) \log \dfrac{x_{j+1} - x_i}{x_j - x_i}, & j \geqslant i + 1. \end{cases} \tag{3.9}$$

The approximation of the boundary condition (3.3) has the form

$$\frac{\tau_{i1} - \tau_{i0}}{\Delta y} = -\frac{1}{\pi} T_w(x_i) \left[\mathscr{A}_1'''(x_i)\Delta x + \mathrm{D}\left(\sum_{j=2}^{i-2} I_{ij} + \sum_{j=i+1}^{N-2} I_{ij}\right)\right], \tag{3.10}$$

where we use the notation $\mathrm{D}f = (f_{i+1} - f_{i-1})/(2\Delta x)$ and $\mathscr{A}_1'''(x_i)$ is taken from the evaluation of the principal value of the singular integral.

The numerical solution is constructed by iterations with respect to $\mathscr{A}_1(x_b)$ using under relaxation (Sychev *et al.* 1998). If we know the approximation of $\mathscr{A}_1^n(x_b)$, then at each line $x = x_j$ the equation for vorticity is solved by the tridiagonal matrix method consecutively from the bottom to the top of the grid with the boundary conditions (3.10) and (3.2) and then the equation for temperature (3.7) is solved with the boundary conditions (2.6) and (2.8). Then the computations are transferred to the next line x. It allows us to fulfil computations of vorticity and temperature in the whole two-dimensional region.

With the right boundary attained, the functions u_{jk}, v_{jk} as well as $\hat{\mathscr{A}}_1(x_b)$ are recomputed on the current iteration in the whole field and the new approximation for $\mathscr{A}_1(x_b)$ is evaluated from the relaxation

$$\mathscr{A}_1^{n+1} = (1 - r)\mathscr{A}_1^n + r\hat{\mathscr{A}}_1(x_b), \tag{3.11}$$

where r is a relaxation parameter. The convergence of iterations in this problem is strongly sensitive to the values of this parameter, which, in turn, depends on ΔT, i.e. $r = r(\Delta T)$. In our computations we usually took $r = 0.02$ for temperature perturbations $\Delta T \leqslant 0.5$, while for larger perturbations r should be diminished.

4. Results of computations

The key functions to evaluate are distributions of pressure and shear stress on the surface. The self-induced pressure is constant across the layers and thus its values, computed in the sublayer, remain the same in the inviscid flow for the fixed x_b. The shear stress, in turn, is important for localization of putative points of zero shear stress, indicating possible separation of the boundary layer.

In figure 2 we show the pressure in the viscous sublayer for heat humps which are given by

$$T_w(x_b) = \begin{cases} 1 + h, & |x_b| \leqslant a/2 \\ 1, & |x_b| > a/2. \end{cases} \tag{4.1}$$

Here $h = \Delta T$ is the amplitude of perturbations of the temperature and a is the size of the heated region. In real vehicles, we can take typically $Re = 10^7$. Let the characteristic size of a wing be $\sim$10 m. Then, the size of the interaction region is $L \sim 10\ \text{m} \times Re^{-3/8} \sim 2.5$ cm. This size is even comparable with microelectromechanical systems (MEMSs), which were employed for the control of separation. The size can be increased by means of additional energy release. Such a thermal element physically corresponds to a region of length a to which a constant temperature is conducted. This form of temperature distribution is simpler for analytic study and in the same time enables to represent adequately qualitative behaviour of functions in the stream, although it presents, to a certain extent, an idealization on the edges of the thermal region. Therefore, we also carried out computations for smoother types of thermal 'humps'. For (4.1) computations were carried out for a broad range of amplitudes h as is indicated in figure 2.

From figure 2 it is also seen that when $\Delta T \to 0$, the solution becomes close to that of the linear problem (Koroteev & Lipatov 2011), the latter corresponding to small temperature perturbations. The linear approximation obviously will be inapplicable for $\Delta T \sim 1$. Nonlinear perturbations $\Delta T \sim O(1)$ produce noticeable deviations from the pressure for the linear case both inside the interaction region and on the edge which

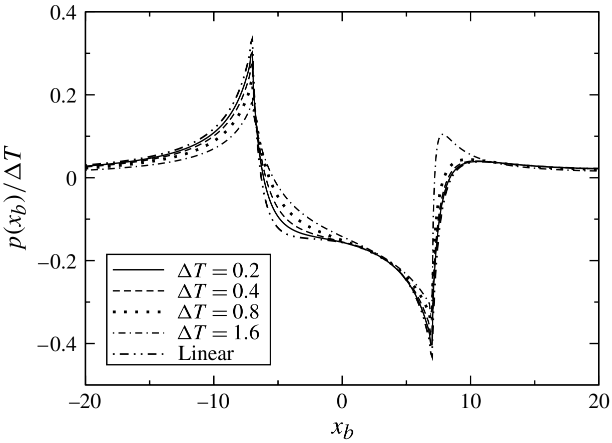

FIGURE 2. Scaled pressure in the viscous sublayer obtained from the numerical solution for temperature perturbations (4.1) with various amplitudes ΔT. A curve is also presented for the linear subsonic problem corresponding to small perturbations (Koroteev & Lipatov 2011) demonstrating closeness of the nonlinear solutions with small perturbations to the solution of the linear problem. For these computations we take $\Delta x = 0.1$, $\Delta y = 0.1$ and $a = 7$.

is also confirmed by figure 3 where a jump for the convective part of the thickness displacement on the right boundary of the thermal element is observed.

Two mechanisms providing displacement of streamlines in the vicinity of the heating element are alteration of the pressure and of the temperature, which produces the decrease in density. The contribution from these effects is shown in figure 3. Despite some dominance of the temperature term, both terms are seen to be of the same order. However, it is also seen that both functions in general have different signs in the region of interaction thus compensating for each other. This results in the reduced effect of perturbations to the boundary layer.

In all of the figures it is assumed that the flow moves from the left to the right. When approaching the heated region the positive pressure gradient produces additional deceleration of the stream. The curves for the pressure are portrayed with scaling $p(x_b)/\Delta T$ on y axis (figure 2). This scaling allows us to demonstrate that the observed nonlinear effects are not reproduced by the linear regime. The first effect is presented by a sharper drop of the pressure over the thermal element, decreasing the amplitude of the pressure on the upstream edge of the thermal region. The second peculiarity emerges near the downstream edge of the thermal element exhibiting an additional local maximum of the pressure.

Addressing the shear stress we note that there are two points in the flow where the shear stress drops noticeably, namely boundaries of the heated region upstream and downstream (figure 4). The declination at the leading edge of the thermal element is, however, sufficiently weak to produce crucial deviation of the shear stress while the back edge demonstrates quite small values for the shear stress potentially close to the preseparated regime. In the transient region between these points and over the heating element the flow is accelerated by means of the negative pressure gradient, which simultaneously increases the shear stress. Finally, perturbations of the pressure decay behind the heated region and the flow again slows down. The amplitude of perturbations is influenced by ΔT (figures 4 and 2), which, thus, has to be considered

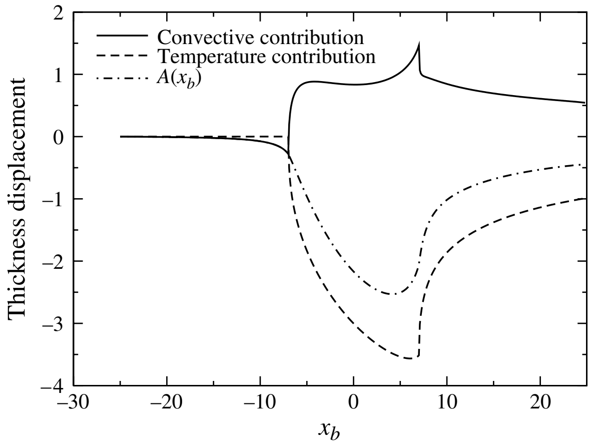

FIGURE 3. Displacement thickness $A(x_b)$ in the vicinity of the thermal region incorporates two terms: one emerging from the alteration of the pressure or convective term and the term related to the increase of temperature. The curves are presented for $\Delta T = 1.6$.

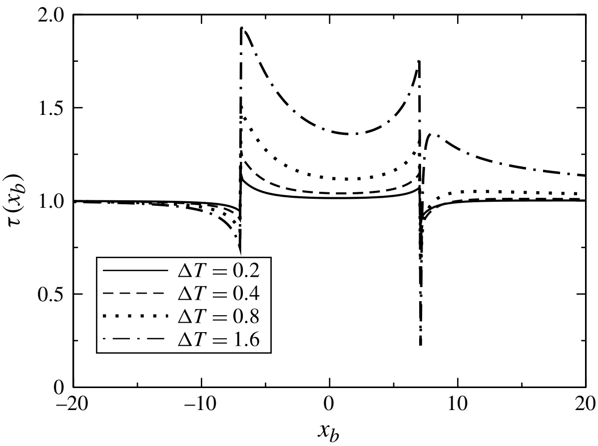

FIGURE 4. Shear stress in the lower part of the boundary layer.

as a crucial governing parameter of the problem. Our computations accomplished for smoother types of thermal regions show (supplementary figures are available at journals.cambridge.org/flm) that, in general, the same effects are observed for different thermal elements but discontinuity of temperature in the form (4.1) and that of its first derivative has more influence on the drop of the shear stress at the back edge of the thermal irregularity. For example we fulfilled computations assuming that the temperature varies according to some parabolic profile and Gaussian-like profile. The results for the former are presented as supplementary materials for the paper. In this case, the declination of shear stress in the vicinity of the back part of the thermal element is essential while in the latter case it vanishes because of the continuity of the profile and due to the above-mentioned compensation between two terms of (2.7).

We also carried out the analysis of the numerical solution with respect to the grid. It was shown for a range of grid sizes that the solution remains stable (supplementary figures).

5. Discussion

The problem studied above, which includes thermal effects in the boundary layer, demonstrated noticeable deviations from previously studied linear flows. The solution was obtained for strongly nonlinear amplitudes of perturbations $\Delta T > 1$ when the linear approximation becomes incorrect.

In the region of free interaction which corresponds to equal orders of convective, dissipative and pressure gradient terms of Navier–Stokes equations there are two effects which may produce the deviation of streamlines in the boundary layer. We saw, in part, that nonlinear thermal perturbations for some types of temperature distribution on the wall produce flows close to separated ones. The flow in the separated region constitutes a different problem which will be described elsewhere.

The problem allows different regimes of the flows to be described in terms of the triple-deck theory. As we mentioned in the introduction, in Lipatov (2006) an analogy of flows with local thermal elements and flows with small humps on the surface of the body was established. It should be expected that similar effects are produced by local perturbations of the boundary layer such as suction and blow-in. In particular, non-stationary perturbations of this kind may bring to intensified impulse transmission, which, in turn, influences the possibility for the boundary layer to resist the separation. This was demonstrated for the case of non-stationary blowing in Seifert *et al.* (1993). We expect a similar effect when employing different methods of control, in part, variation of the temperature near the possible region of separation.

The results can be generalized to three-dimensional flows, which are of course of more interest for applications. On the other hand, another direction is the generalization of these results to non-stationary flows where it is essential to study nonlinear perturbations and their propagation. The stationary solutions obtained here constitute the basis for further studies of non-stationary boundary layers with thermal effects.

Supplementary data

Supplementary data are available at journals.cambridge.org/flm.

REFERENCES

BOGOLEPOV, V. V. & NEILAND, V. Y. 1971 Supersonic flow over small humps located on the surface. *Trans. TsAGI* **1363**.

BOGOLEPOV, V. V. & NEILAND, V. Y. 1976 On local perturbations of viscous supersonic flows. In *Aeromechanics*, pp. 104–118. Nauka (in Russian).

BOS, S. H. & RUBAN, A. I. 2005 A semi-direct method for calculating flows with viscous-inviscid interaction. *Phil. Trans. R. Soc. Lond.* A **358** (1777).

DEMIDOVICH, B. P. & MARON, I. A. 1970 *Computational Mathematics*. MIR.

JOBE, C. E. & BURGRAFF, O. R. 1974 The numerical solution of the asymptotic equations of trailing edge flow. *Proc. R. Soc. Lond.* A **340**, 91–111.

KOROLEV, G. L., GAJJAR, J. S. B. & RUBAN, A. I. 2002 Once again on the supersonic flow separation near a corner. *J. Fluid Mech.* **463**, 173–199.

KOROTEEV, M. V. & LIPATOV, I. I. 2009 Supersonic boundary layer in regions with small temperature perturbations on the wall. *SIAM J. Appl. Maths* **70** (4), 1139–1156.

KOROTEEV, M. V. & LIPATOV, I. I. 2012 Stationary subsonic boundary layer in the regions of local heating of surface. *App. Math. Mech.* (in press).

LIPATOV, I. I. 2006 Disturbed boundary layer flow with local time-dependent surface heating. *Fluid Dyn.* **41** (5), 55–65.

MESSITER, A. F. 1970 Boundary layer near the trailing edge of a flat plane. *SIAM J. Appl. Maths* **18** (#1), 241–257.

NEILAND, V. Y. 1969 To the theory of laminar separation of boundary layer in supersonic flow. *Izv. Akad. Nauk SSSR Mekh. Zhidk. Gaza* **4**, 53–57 (in Russian).

ROACHE, P. J. 1972 *Computational Fluid Dynamics*. Hermosa.

RUBAN, A. I. 1976 To the theory of laminar separation of fluid from the point of zero turn of solid surface. *Trans. TsAGI* **7** (4), 18–28 (in Russian).

SEIFERT, A., BACHART, T., KOSS, D., SHEPSHELOVITCH, M. & WYGNANSKI, I. 1993 Oscillatory blowing: a tool to delay boundary layer separation. *AIAA J.* **31**, 2052–2060.

SMITH, F. T. 1974 Laminar flow over a small hump on a flat plate. *J. Fluid Mech.* **57** (4), 803–824.

STEWARTSON, K. 1981 D'Alembert's paradox. *SIAM Rev.* **23** (3), 308–343.

STEWARTSON, K. & WILLIAMS, P. G. 1969 Self-induced separation. *Proc. R. Soc.Ser.* A **312**, 181–206.

SYCHEV, V. V., RUBAN, A. I., SYCHEV, VIC. V. & KOROLEV, G. I. 1998 *Asymptotic Theory of Separated Flows*. Cambridge University Press.