On the Effect of Plotting Performance by the Errors of Pointing Targets in the ARPA System

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The content of this paper was presented at the International Maritime Association of the Mediterranean, 8th International Congress on Marine Technology in Istanbul, November 1997.

The effect of different measurement points on an acquired target is theoretically investigated with respect to the ARPA plotting performance. The target is described by simple geometrical figures and the echo is simplified. The actual measurement position is generally not at the centre of the target. This may cause errors in the standard plotting results. The characteristics of these errors are discussed from a nautical viewpoint.

1. introduction. Radar-computed anti-collision (ARPA) systems were introduced two decades ago in the interest of navigational safety at sea. The ARPA facility allows the navigator to track a number of targets automatically. The standard plotting results of the individual target are graphical presentation of the velocity vector and alpha-numerical display of Closest Point of Approach (CPA), Time to Closest Point of Approach (TCPA), range, bearing, speed and course.

The acquired targets are displayed on the radar screen by echoes. The position on which the target vector and alpha-numeric ARPA plotting results are based is not regulated in any Performance Requirement. Thus, there exist different approaches among the ARPA radar manufacturers regarding the measurement (reference) point of the echo.

The following are found to be representative for the majority of present solutions: minimum distance to the echo regardless of bearing, minimum distance to the echo and the centre in azimuth, minimum distance to the echo at the centre in azimuth, and centre of gravity of the echo.

In general, the measurement points are not identical to the centre of the target (Figure 1). This is a source for errors in the calculated plotting results. Some references related to these types of errors are Qui *et al.* (1981) and Wei *et al.* (1997).

The individual measurement point can be dependent on the shape and construction of the target, its orientation and position relative to own ship, as well as the pulse length and beam width of the radar. The point referring to the centre of the echo is more sensitive to the radar dependent parameters (pulse length and beam width) than those referring to the minimum distance in a given direction.

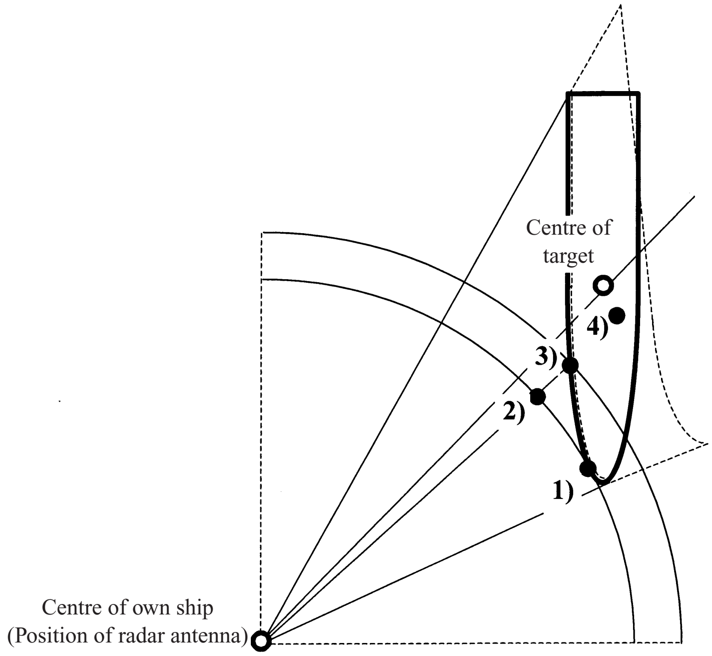

Figure 1. Definition of fundamental different measurement points which are in use by ARPA radar manufacturers to calculate the target data: (1) minimum distance regardless of bearing, (2) minimum distance and the centre in azimuth, (3) minimum distance at the centre in azimuth, and (4) centre of gravity of echo.

The shape of the echo is generally complex from an analytical viewpoint, and it normally becomes more complex with smaller radar ranges and larger target sizes. Two geometrical figures; an ellipse and a rectangle representing the fore and aft body, respectively, describe the target. The echo is strongly simplified as its radial extension is considered to be equal in all directions and as its extension towards own ship is ignored. Mathematical expressions have been derived to calculate the errors in ARPA plotting results caused by the different measurement points.

From the viewpoint of the mariner, the fundamental question is how the various measurement points affect the setting of minimum CPA and TCPA limits. The present analysis shows that a CPA limit within the length of own ship and a TCPA limit within 1 minute should compensate for the errors in question.

2. theory. For the purposes of this analysis, the target is assumed to travel in a straight line at constant speed relative to its own ship (Figure 2). A Cartesian (X, Y) coordinate system, whose origin is at the position of own ship's radar antenna, is representing a moveable plane tangential to the Earth's surface. The Y axis is pointing North (or in any other fixed direction). Another Cartesian (x, y) coordinate system is located at the centre of the target and is moving relative to own ship. The x axis is pointing in the bow direction.

The position of the target, at time t, is then given by

$$X(t) = X(0) + V_{rx}\, t$$
$$Y(t) = Y(0) + V_{ry}\, t$$

https://doi.org/10.1017/S037346339800811X Published online by Cambridge University Press

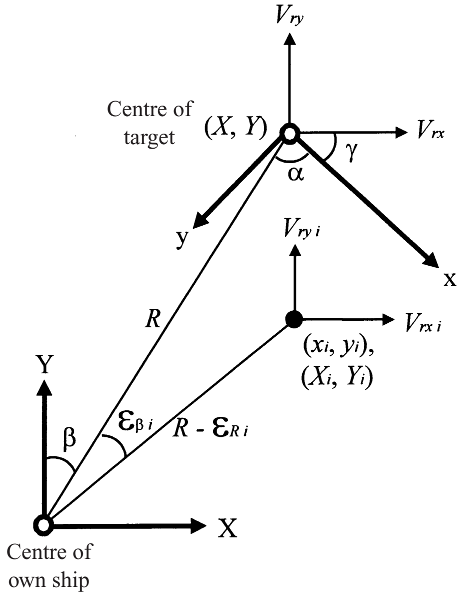

Figure 2. Definition of coordinate systems and parameters for motion of the target relative to own ship: (x_i, y_i), (X_i, Y_i), (X, Y) = positions, R = distance (range), V_r = relative velocity, α = aspect angle, β = bearing, γ = relative orientation, ϵ_{Ri} = error in range, and $\epsilon_{\beta i}$ = error in bearing. The index i represents a measurement point.

where $X(0)$, $Y(0)$ is the initial position of the centre of the target, and V_{rx}, V_{ry} are the components of the relative velocity V_r.

The relative range to the target is

$$R(t) = \sqrt{X^2(0) + Y^2(0) + V_r^2\, t^2 + 2(X(0)V_{rx} + Y(0)V_{ry})\, t}.$$

It follows from $dR/dt|_{CPA} = 0$ that the time to closest point of approach can be written as

$$TCPA = -\frac{X(0)V_{rx} + Y(0)V_{ry}}{V_r^2}$$

and the corresponding minimum distance is

$$CPA = \left|\frac{X(0)V_{ry} - Y(0)V_{rx}}{V_r}\right|.$$

The ARPA radar is processing the detected range and bearing to the echo based on a certain measurement point i which is generally not at the centre of the target. The differences in range and bearing can be considered as errors in the radar measurements.

Let ϵ_{Ri} and $\epsilon_{\beta i}$ represent the errors in range and bearing, respectively, whose mathematical expressions are given in detail in Pedersen *et al.* (1997) for the measurement points in question.

The position of measurement point i relative to own ship can be expressed as

$$X_i = (R - \epsilon_{Ri})\sin(\beta + \epsilon_{\beta i})$$
$$Y_i = (R - \epsilon_{Ri})\cos(\beta + \epsilon_{\beta i})$$

and the components of relative velocity are

$$V_{rxi} = V_{rx} + \left(\frac{dx_i}{dt}\cos\gamma - \frac{dy_i}{dt}\sin\gamma\right)$$

$$V_{ryi} = V_{ry} - \left(\frac{dx_i}{dt}\sin\gamma + \frac{dy_i}{dt}\cos\gamma\right)$$

where dx_i/dt and dy_i/dt are the velocities of measurement point i relative to the target's coordinate system, and γ is the angle of orientation between the coordinate systems. The errors in CPA and $TCPA$ are now

$$\epsilon_{CPAi} = CPA - CPA_i$$
$$\epsilon_{TCPA_i} = TCPA - TCPA_i$$

where CPA_i and $TCPA_i$ are referred to the position and velocities of measurement point i.

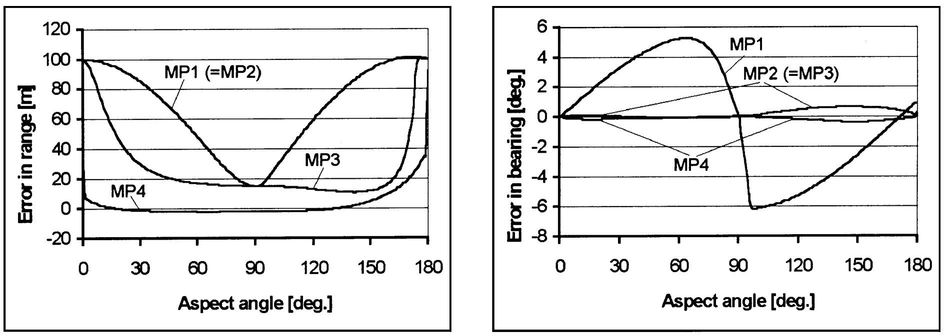

Figure 3. Error in range and bearing vs. aspect angle for the four measurement points: MP1, minimum distance to the echo regardless of bearing; MP2, minimum distance to the echo and the centre in azimuth; MP3, minimum distance to the echo at the centre in azimuth; MP4, centre of gravity of the echo. The target's size is 200 × 30 m and its distance to own ship is 0·5 nautical mile.

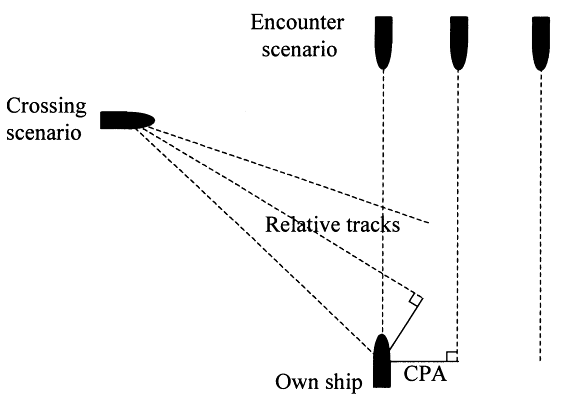

Figure 4. Scenarios considered for evaluation of the effect of different radar measurement points on the ARPA plotting performance. Own ship course is 000°. The relative speed is 20 knots. The course of the target is 180° and 090° for the encounter and crossing scenarios, respectively. The initial TCPA is 360 s, and the relative bearing to the target is 045° to port for the crossing scenario.

3. errors in arpa plotting results. An example considered to be representative for the characteristics of errors in range and bearing to variations in the aspect angle from 0° to 180° is given in Figure 3. The figure shows the difference in errors for the given measurement points. (The target considered has a length (L) of 200 m and breadth (B) of 30 m, and its range (R) to own ship is 0·5 nautical mile. The radar pulse length is set to 0·2 μs (= 30 m) which is typical for medium range radar scales.)

Operational scenarios (Figure 4) are constructed to evaluate the effect of the various measurement points on the plotting performance represented by the errors in CPA and TCPA. The scenarios considered are encounter and crossing situations with a relative speed of 20 knots. The course of own ship is 000°, and the course of the target is 180° and 090° for the encounter and crossing scenarios, respectively.

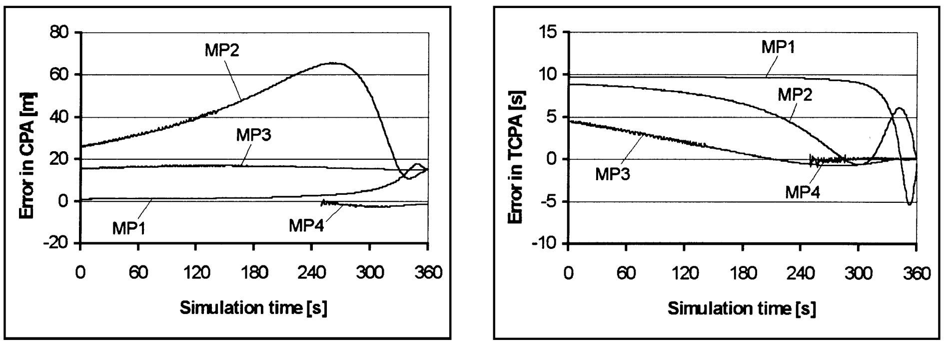

Figure 5. Encounter scenario: Error in CPA and TCPA vs. simulation time for the four measurement points: MP1, minimum distance to the echo regardless of bearing; MP2, minimum distance to the echo and the centre in azimuth; MP3, minimum distance to the echo at the centre in azimuth; MP4, centre of gravity of the echo. The initial TCPA and CPA is 360 s and $3 \times L$, respectively. ($L = 200$ m.)

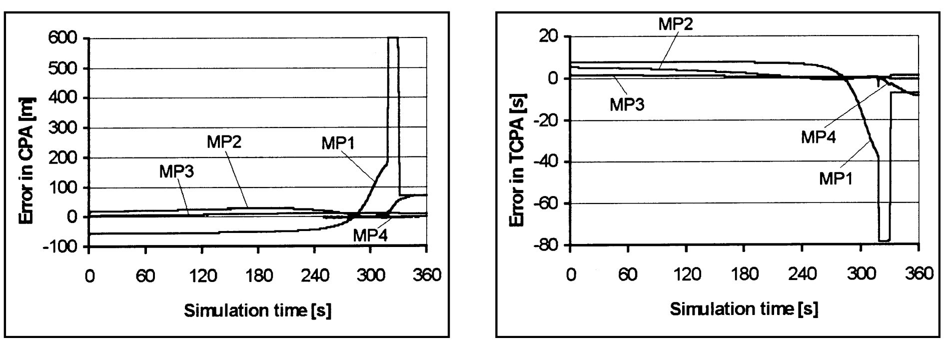

Figure 6. Crossing scenario: Error in CPA and TCPA vs. simulation time for the four measurement points: MP1, minimum distance to the echo regardless of bearing; MP2, minimum distance to the echo and the centre in azimuth; MP3, minimum distance to the echo at the centre in azimuth; MP4, centre of gravity of the echo. The initial TCPA and CPA is 360 s and $3 \times L$, respectively. ($L = 200$ m.)

https://doi.org/10.1017/S037346339800811X Published online by Cambridge University Press

The initial TCPA is set to 360 s, and the bearing to the target is 045° to port for the crossing scenario. Variations in CPA relative to the target's length are obtained by varying the initial position of the target in the encounter scenario and by varying the direction of relative velocity vector for the crossing scenario.

Figures 5 and 6 show the characteristics of errors in CPA and TCPA for the encounter and crossing scenarios, respectively, when the initial CPA is set to $3 \times L$. The errors are referred to the different points vs. simulation time.

4. discussion. $L/2$ and $B/2$ generally restrict the maximum and minimum errors in range for the measurement points referring to the minimum distance in a given direction, respectively. These limits are only slightly exceeded for some aspect angles above 90° due to the rectangular shape of the target's aft body. The error in range of the point referring to the centre of the echo is strongly dependent on the pulse length relative to the size of the target. In general, a pulse length comparable to the breadth minimizes this error except for aspect angles $\sim 0°$ and $\sim 180°$.

The maximum error in bearing is dominated by the point referring to the minimum distance regardless of bearing. $L/2$ relative to R restricts the error. (It should be emphasized that the aspect angle has a significant influence on the reflections from the target and consequently the width of the echo. This effect is not taken into account in the present analysis.)

For encounter and crossing situations $L/2$ and $(L/2)/V_r$ generally restrict the maximum errors in CPA and TCPA, respectively. However, these values can be exceeded when the aspect angle approaches 90° for the measurement point referring to the minimum distance regardless of bearing. This is because the point's rapid movement towards the target's aft end then causes the error in the relative velocity to approach the velocity value referred to the centre of the target.

For large TCPAs (far more than $L/2/V_r$) the errors in CPA and TCPA generally err on the safe side except for crossing situations where the error in CPA errs on the dangerous side for the point referring to the minimum distance regardless of bearing. This is because of the target's relative orientation to own ship. (It should be pointed out that the solution for large TCPAs is omitted for the measurement point referring to the centre of gravity because of numerical instabilities).

When setting the limit for CPA and TCPA the mariner should consider the location of own ship radar antenna (or the length of own ship). The target size is generally of interest for the points referring to the minimum distance regardless of bearing and to the centre of gravity as these measurement points may give errors that err on the dangerous side. Thus, an additional safety margin should be added.

5. conclusions. This article states the theoretical effect of different measurement points on the ARPA plotting performance. In general, a minimum CPA limit within the length of own ship and a minimum TCPA limit within 1 minute should compensate for the errors in question. The effect of aspect angles on the reflections from the echo as well as errors in own ship speed should be considered in further studies.

acknowledgements

Financial support for this research was obtained from the Science and Technology Agency of Japan through an STA Fellowship to the first author.

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key words

1. ARPA. 2. Radar. 3. Safety.