Relativistic and ponderomotive effects on evolution of laser beam in a non-uniform plasma channel

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Abstract

The nonlinear parabolic partial differential equation governing the evolution of the complex envelope in the slowly varying envelope approximation is solved using the variational approach. The basic nonlinear phenomena of relativistic and ponderomotive self-focusing in a plasma channel are taken into account. Self-focusing, self-phase modulation as well as self-trapping of laser beam is studied in a variety of situations. Further, in the absence of dissipation mechanisms, the stability of the beam is also studied.

Keywords: Ponderomotive; Relativistic and plasma channel; Self-focusing and self-trapping; Self-phase modulation

1. INTRODUCTION

Theoretical and experimental study of evolution of high power laser beam, as they propagate through plasmas, is an active area of research due to their potential relevance as plasma based accelerators (Sarkisov *et al.*, 1999), inertial confinement fusion (Tabak *et al.*, 1994; Regan *et al.*, 1999), and new radiation sources (Suckewer & Skinner, 1990, 1995; Benware *et al.*, 1998; Yu *et al.*, 1998; Foldes *et al.*, 1999; Fedotov *et al.*, 2000). For success of these applications, a long distance propagation of intense lasers in plasma is desirable. For example, in laser fusion, the coupling and the propagation of the laser beams in the plasma surrounding the pellet must be well controlled to produce high energy level. As laser beam propagates through plasma, many instabilities and nonlinear phenomena such as self-phase modulation, the filamentation instability, group velocity dispersion, finite pulse effects, relativistic and ponderomotive self-focusing effects, become important. This implies a deep understanding of laser plasma interaction physics to keep the nonlinear process at low level.

The self-focusing, a genuinely nonlinear basic phenomenon, which plays crucial role in beam propagation, arises due to increase of the on-axis index of refraction relative to edge of the laser beam. For example, for ponderomotive force type nonlinearity, we have expulsion of electrons

from the region of high intensity laser field. Relativistic self-focusing results from the effect of quiver motion leading to reduced local plasma frequency. The self-focusing is counter-balanced by the tendency of the beam to spread because of diffraction. In the absence of nonlinearities, the beam will spread substantially in a Rayleigh length, $R_d \sim k_0 r_0^2$. Self-focusing and filamentation are among the most dangerous nonlinear phenomena, which destroy the uniformity of overall irradiation required for direct-drive fusion experiment as well as leads to seeding and growth of hydrodynamic instabilities. Experimental as well as theoretical observations of relativistic self-focusing and ponderomotive self-channeling have been reported in a number of investigations (Borisov *et al.*, 1992, 1998; Faenov *et al.*, 2007; Chessa *et al.*, 1998; Sun *et al.*, 1987; Kurki-Suonio *et al.*, 1989; Abramyan *et al.*, 1992; Chen & Sudan, 1993; Tzeng & Mori, 1998; Monot *et al.*, 1995; Krushelnick *et al.*, 1997; Konar & Manoj, 2005; Wagner *et al.*, 1997; Chen *et al.*, 1998). The dynamics of ponderomotive channeling in underdense plasma has recently been studied experimentally (Borghesi *et al.*, 2007). Relativistic laser-plasma interaction physics has also been a focus of attention as many nonlinear processes playing key roles in the generation of new ion sources has been reported recently (Laska *et al.*, 2007; Torrisi *et al.*, 2008; Strangio *et al.*, 2007). Since ponderomotive and relativistic channeling occur together, we investigate their combined effects on the evolution of intense laser beam in non-uniform plasma channel. Plasma channels are important as they act as a duct to guide laser

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beams over long distance of propagation. Further, the novel experiments to study rich physics issues in nuclear and particle physics, atomic physics (Stoehlker *et al.*, 2003), plasma physics (Hoffmann *et al.*, 2005; Schaumann *et al.*, 2005), and applied sciences (Kuehl *et al.*, 2007; Kasperczuk *et al.*, 2008) has opened new vistas in this field and has given a multidisciplinary character to laser produced plasmas. Most general form of the field envelope in the absence of dispersion and under slowly varying envelope approximation is:

$$2\imath k \frac{\partial \psi}{\partial z} + \nabla_{\perp}^2 \psi + Q\left(|\psi|^2\right)\psi = 0. \tag{1}$$

Eq. (1) also known as the nonlinear Schrödinger equation, is a nonlinear parabolic partial differential equation type governing field envelope of laser electromagnetic beam during its propagation in a nonlinear medium. Thus, conventional separable techniques using Fourier expansions are not applicable. These higher dimensional nonlinear Schrödinger equations (NLSE) are not integrable so that they do not have special solution such as soliton solutions. However, they possess stationary solution, which are unstable on propagation. For deeper insight into the physical process, we solve Eq. (1) by using some analytical approximate models. Thus, the resulting solutions provide qualitative rather than quantitative estimates.

Some of the approximate theories used to study self-focusing process are a paraxial ray approximation (Akhmanov *et al.*, 1968; Sodha *et al.*, 1974, 1976), moment theory approach (Firth, 1977; Lam *et al.*, 1975, 1977), variational approach (Firth, 1977; Anderson & Bonnedal, 1979), and source-dependent expansion (SDE) method (Sprangle *et al.*, 2000). Each of these theories has limitations in describing completely the experimental/computer simulation results. However, the mostly used theory based on Wentzel-Krammers-Brillouin approximation and paraxial ray approach is given by Akhmanov *et al.* (1968) and developed by Sodha *et al.* (1976). Due to its mathematical simplicity, it has been extensively used to account for general features of self-focusing as witnessed by the increasing number of publications. However, this theory being local in character over-emphasizes the field closest to beam axis and lacks global pulse dynamics. Furthermore, it also predicts unphysical phase relationship (Karlsson *et al.*, 1991). It has been pointed out that paraxial ray theory is not applicable when high power laser beams are used (Subbarao *et al.*, 1998). With some partial remedies to paraxial ray theory, the moment theory of self-focusing gives results closer to computer simulations. However, this theory was not actively pursued as it lacks generalization and phase description. Another global approach is variational approach, although crude to describe the singularity formation and collapse dynamics is general in nature to describe the propagation and correctly predicts the phase.

In most of the earlier investigations on self-focusing, carrier distributions takes place on the transverse wave front of the laser beam, leading to inhomogeneity in the plane transverse to beam propagation direction, where refractive index profile is modified on wave front transverse to beam propagation direction. In real physical situation, the laser beam propagates in the underdense plasma up to critical surface, which is spatially inhomogeneous, both in longitudinal and transverse directions. In this underdense plasma, several parametric processes occur before the critical surface is approached. Several nonlinear processes occur due to self-focusing as reported in a number of recent experiments (Torrisi *et al.*, 2008; Faenov *et al.*, 2007). Moreover, $(2 + 1)$-dimensional self-focusing that is guided in both transverse directions is unstable under collapse (Berge, 1998). In an earlier investigation, two dimensional self-focusing of laser beam in an inhomogeneous laser produced plasma is studied (Gill *et al.*, 2001). The effect of inhomogeneity of scale lengths is introduced (Andreev *et al.*, 1987). Since diffraction and nonlinearities in-play takes at different spatial characteristic lengths, we have introduced $d(z)$ and $\lambda(z)$ as two scale lengths for varying diffraction and nonlinearity. We have introduced this feature phenomenologically to simulate the realistic situations. With the advancement of laser science and pulse shaping technology along with the novel idea of light controlling light, it may be possible to tailor underdense plasma in which diffraction, dispersion and nonlinearity can be controlled. The problem is akin to dispersion and nonlinearity management of laser beam propagation in nonlinear fiber optics (Ablowitz & Mussliamani, 2003; Matuszewski *et al.*, 2004; Abdullaev *et al.*, 2003). Further, we can control these parameters to prevent the collapse of the beam due to strong self-focusing.

The organization of the paper is as follows: In Section 2, a model is set up in a weakly relativistic limit starting from Maxwell's equations and hydrodynamic equations. These equations under approximate conditions lead to an evolution equation. Lagrangian for the problem is set up and variational approach is used. Section 3 is devoted to discussion, and in Section 4, we study the stability characteristics of the laser beam. In the last, Section 5, conclusions of present investigation are presented.

2. BASIC FORMULATION

The present model is set up in a weakly relativistic limit starting from Maxwell's equations and hydrodynamic equations. Two coupled equations for density perturbation and laser beam vector potential in a preformed plasma channel (Hafizi *et al.*, 2000, 2003; Panwar & Sharma, 2009; Duda & Mori, 2000) are given as follows:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = \frac{\omega_p^2}{c^2}\left(1 + \frac{r^2}{r_{ch}^2} + \frac{\delta n}{n_o} - \frac{|\psi|^2}{2}\right)\psi, \tag{2}$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\frac{\delta n}{n_o} = c^2\nabla^2\frac{|\psi|^2}{2}, \tag{3}$$

where ψ is the vector potential normalized by $m_0 c^2/e$, m_0 and e are the rest mass and charge of electron, c is the velocity of light in vacuum. n_0 is the initial axial electron density, and δn is the perturbed electron density, respectively. r_{ch} is the effective channel radius.

The effect of inhomogeneity with two scale lengths was first introduced by Andreev *et al.* (1987) and later used by Gill *et al.* (2001) using variational approach has been introduced. We have assumed parabolic density profile and long pulse limit approximation (Liu *et al.*, 2004) along with slowly varying envelope approximation. On introducing characteristic scale length $d(z)$ of diffraction and $\lambda(z)$ for non-linearity, we obtain the following evolution equation governing the electric field envelope in collisionless plasmas as follows:

$$
\left(2ik\frac{\partial}{\partial z} + d(z)\nabla_\perp^2 - \lambda(z)\left(k_p^2 \frac{r^2}{r_{ch}^2} + \nabla_\perp^2 \frac{|\psi|^2}{2} \right.\right.
$$
$$
\left.\left. - k_p^2 \frac{|\psi|^2}{2} \right) \right)\psi(r,z) = 0.
\tag{4}
$$

We further assume $d(z) = d_0 + d_1(z)$, $\lambda(z) = \lambda_0 + \lambda_1(z)$.

Eq. (4) is a special case of Eq. (1) and is a nonlinear parabolic partial differential equation in which the second term has its origin in diffractional divergence along with diffraction management coefficient, the term within parenthesis corresponds to channel, ponderomotive self-focusing (PSF), and relativistic self-focusing (RSF), respectively. Variational approach that have rigorous basis, as applied in other fields, is used here to investigate nonlinear wave propagation. It may also be mentioned that we have assumed parabolic profile for the plasma channel. We use the procedure of Anderson and Bonnedal (1979) to reformulate Eq. (4) into a variational problem corresponding to a Lagrangian L, so as to make $\delta L/\delta z = 0$. Thus, Lagrangian L corresponding to Eq. (4) is given by:

$$
L = \upsilon k\left(\psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z} \right) - d(z)\left| \frac{\partial \psi}{\partial x} \right|^2 - d(z)\left| \frac{\partial \psi}{\partial y} \right|^2
$$
$$
+ \frac{1}{4}\left[\left(\frac{\partial |\psi|^2}{\partial x} \right)^2 + \left(\frac{\partial |\psi|^2}{\partial y} \right)^2 \right]\lambda(z)
\tag{5}
$$
$$
- \frac{x^2}{r_{ch}^2}k_p^2|\psi|^2\lambda(z) - \frac{y^2}{r_{ch}^2}k_p^2|\psi|^2\lambda(z) + k_p^2\frac{|\psi|^4}{4}\lambda(z)
$$

Thus, the solution to the variational problem

$$
\delta \iiint L\,dx\,dy\,dz = 0,
\tag{6}
$$

which also solves the nonlinear Schrödinger Eq. (4). Using the trial function as Gaussian beam of the form as:

$$
\psi(x,y,z) = \psi_0(z)\exp\left(\frac{-x^2}{a(z)^2} - \frac{y^2}{a(z)^2} + ib(z)x^2 + ib(z)y^2 + i\phi(z) \right),
\tag{7}
$$

where $a(z)$ is the beam width, $b(z)$ is the spatial chirp, and $\phi(z)$ is the phase of the laser beam. Using the ansatz, with expression for ψ as the trial function, we can perform the integration to write:

$$
\langle L\rangle = \frac{ik\pi a^2}{2}\left(\psi_0^* \frac{\partial \psi_0}{\partial z} - \psi_0 \frac{\partial \psi_0^*}{\partial z} \right) - \frac{\pi k a^4}{4}|\psi_0|^2 \frac{db}{dz}
$$
$$
- \pi k a^2 |\psi_0|^2 \frac{d\phi}{dz} - \frac{d(z)}{2}\left(b^2 + \frac{1}{a^4} \right)\pi|\psi_0|^2 a^4
\tag{8}
$$
$$
+ \lambda(z)\left(\frac{\pi|\psi_0|^4}{8} - \frac{k_p^2\pi}{8r_{ch}^2}|\psi_0|^2 a^4 + \frac{k_p^2}{16}\pi|\psi_0|^4 a^2 \right)
$$

Using the procedure of Anderson and Bonnedal (1979) and Saini and Gill, 2006) we arrive at the following equations for a, ϕ:

$$
\frac{d^2 a}{dz^2} = \frac{8}{k^2 a^3}d(z)^2 + \frac{1}{d(z)}\frac{da}{dz}\frac{d}{dz}(d(z))
$$
$$
- d(z)\lambda(z)\left(\frac{4|\psi_0|^2}{k^2 a^3} + \frac{2k_p^2 a}{k^2 r_{ch}^2} + \frac{k_p^2}{ak^2}|\psi_0|^2 \right).
\tag{9}
$$

The second term in Eq. (9) is neglected assuming $(da/dz)^2$ as slowly varying with respect to other terms.

$$
\frac{d\phi}{dz} = -\frac{d(z)}{2a^2 k} + \lambda(z)\left(\frac{|\psi_0|^2}{4ka^2} + \frac{3}{32}\frac{k_p^2}{k}|\psi_0|^2 \right).
\tag{10}
$$

After normalization, using $\eta = kz$, we get the following equations:

$$
\frac{d^2 a}{d\eta^2} = \frac{8}{k^4 a^3}d(z)^2 - d(z)\lambda(z)\left(\frac{4|\psi_0|^2}{k^4 a^3} + \frac{2k_p^2 a}{k^4 r_{ch}^2} + \frac{k_p^2}{ak^4}|\psi_0|^2 \right),
\tag{11}
$$

$$
\frac{d\phi}{d\eta} = -\frac{d(z)}{2a^2 k^2} + \lambda(z)\left(\frac{|\psi_0|^2}{4k^2 a^2} + \frac{3}{32}\frac{k_p^2}{k^2}|\psi_0|^2 \right).
\tag{12}
$$

Euler-Lagrange equations can also be algebraically manipulated to yield

$$
\pi a^2 |\psi_0|^2 = \pi A_0^2 a_0^2 = N = cons\tan t,
\tag{13}
$$

where N is the conserved quantity associated with the energy of the beam. The energy of the beam is given by:

$$
E = \int_0^\infty |\psi_0|^2 r\,dr = \frac{N}{4\pi}.
\tag{14}
$$

Rearranging Eq. (9), we get:

$$\frac{d}{dz}\left(\frac{1}{d(z)}\frac{da}{dz}\right) = -\frac{\partial U}{\partial a}, \tag{15}$$

where

$$U = \frac{4d(z)}{k^2 a^2} - \frac{N\lambda(z)}{\pi k^2 a^2}\left(\frac{1}{a^2} + \frac{k_p^2}{2} - \frac{\pi k_p^2}{N r_{ch}^2}a^4\right). \tag{16}$$

Thus, Hamiltonian $H\left(a, \frac{da}{dz}, z\right)$ of the system is given as follows:

$$H\left(a, \frac{da}{dz}, z\right) = \frac{1}{d(z)}\frac{1}{2}\left(\frac{da}{dz}\right)^2 + U(a, z). \tag{17}$$

When coefficient of diffraction and nonlinearity are constants, total energy is conserved and is given by:

$$H\left(a, \frac{da}{dz}, z\right) = \frac{1}{2}\left(\frac{da}{dz}\right)^2 + \frac{C}{a^2}, \tag{18}$$

where

$$C = \frac{4d_0}{k^2} - \frac{N\lambda_0}{\pi k^2}\left(\frac{1}{a^2} + \frac{k_p^2}{2} - \frac{\pi k_p^2}{N r_{ch}^2}a^4\right). \tag{19}$$

We phenomenologically introduce diffraction and nonlinearity as periodically varying functions in the form $d(z) = d_0 + d_1 Sin\Omega(z)$, $\lambda(z) = \lambda_0 + \lambda_1 Sin\Omega(z)$. Such variations are generally considered for soliton propagation in optical fiber for long propagation through diffraction and nonlinearity management. Advancement in laser technology can cater to pulse shaping of laser beams/pulses, which may be proposed for similar profiles of $d(z)$ and $\lambda(z)$. However, in laser plasma interaction, we here introduce such paradigm phenomenologically. Eq. (9) can be treated analytically by means of Kapitsa averaging method (Landau & Lifshitz, 1976). Substituting $a(z) = a_0'(z) + \rho(z)$ in Eq. (9) and separating the resulting equation into rapidly varying and slowly varying parts as:

$$\frac{d^2\rho}{dz^2} = \frac{16d_1 Sin\Omega(z)}{k^2 a_0'^3} - \frac{8Nd_1 Sin\Omega(z)}{\pi k^2 a_0'^5}$$
$$- \frac{2d_1 k_p^2 N Sin\Omega(z)}{\pi k^2 a_0'^3} - \frac{4k_p^2 d_1 Sin\Omega(z)a_0'}{k^2 r_{ch}^2}, \tag{20}$$

$$\frac{d^2 a_0'}{dz^2} = \frac{8d_0}{k^2 a_0'^3} - \frac{4Nd_0}{\pi k^2 a_0'^5} - \frac{k_p^2 Nd_0}{\pi k^2 a_0'^3} - \frac{2k_p^2 d_0 a_0'}{k^2 r_{ch}^2}$$
$$- \frac{48d_1 \overline{\rho Sin\Omega(z)}}{k^2 a_0'^4} + \frac{40d_1}{\pi k^2 a_0'^6}N\overline{\rho Sin\Omega(z)} \tag{21}$$
$$+ \frac{6d_1}{\pi k^2 a_0'^4}\overline{\rho Sin\Omega(z)}k_p^2 N - \frac{4k_p^2}{k^2 r_{ch}^2}d_1\overline{\rho Sin\Omega(z)}$$

where over-line indicates the average value. Integrating

Eq. (20) twice, we obtain:

$$\rho(z) = -\frac{16d_1 Sin\Omega(z)}{k^2 a_0'^3 \Omega^2} + \frac{8Nd_1 Sin\Omega(z)}{\pi k^2 a_0'^5 \Omega^2}$$
$$+ \frac{2d_1 k_p^2 N Sin\Omega(z)}{\pi k^2 a_0'^3 \Omega^2} + \frac{4k_p^2 d_1 Sin\Omega(z)a_0'}{k^2 r_{ch}^2 \Omega^2}. \tag{22}$$

Substituting Eq. (22) into Eq. (21), we obtain the equation of motion for the slowly varying part as:

$$\frac{d^2 a_0'}{dz^2} = \frac{8d_0}{k^2 a_0'^3} - \frac{4Nd_0}{\pi k^2 a_0'^5} - \frac{k_p^2 Nd_0}{\pi k^2 a_0'^3} - \frac{2k_p^2 d_0 a_0'}{k^2 r_{ch}^2}$$
$$+ \frac{384d_1^2}{k^4 a_0'^7 \Omega^2} - \frac{512Nd_1^2}{\pi k^4 a_0'^9 \Omega^2} - \frac{96k_p^2 Nd_1^2}{\pi k^4 a_0'^7 \Omega^2} - \frac{64k_p^2 d_1^2}{k^4 r_{ch}^2 a_o'^3 \Omega^2},$$
$$+ \frac{160d_1^2 N^2}{\pi^2 k^4 a_0'^{11}\Omega^2} + \frac{64k_p^2 d_1^2 N^2}{\pi^2 k^4 a_0'^9 \Omega^2} + \frac{64k_p^2 d_1^2 N}{\pi k^4 r_{ch}^2 a_0'^5 \Omega^2} \tag{23}$$
$$+ \frac{6k_p^4 d_1^2 N^2}{\pi^2 k^4 a_0'^7 \Omega^2} + \frac{8k_p^4 d_1^2 N}{\pi k^4 r_{ch}^2 a_0'^3 \Omega^2} - \frac{8k_p^4 d_1^2 a_0'}{k^4 r_{ch}^4 \Omega^2}$$

where

$$\frac{d^2 a_0'}{dz^2} = -\frac{\partial U}{\partial a_0'}. \tag{24}$$

Hence, the effective potential U for the system is given by:

$$U(a_0', N) = \frac{4d_0}{k^2 a_0'^2} - \frac{Nd_0}{\pi k^2 a_0'^4} - \frac{k_p^2 Nd_0}{2\pi k^2 a_0'^2} + \frac{k_p^2 d_0 a_0'^2}{k^2 r_{ch}^2}$$
$$+ \frac{64d_1^2}{k^4 a_0'^6 \Omega^2} - \frac{64Nd_1^2}{\pi k^4 a_0'^8 \Omega^2} - \frac{16k_p^2 Nd_1^2}{\pi k^4 a_0'^6 \Omega^2}$$
$$- \frac{32k_p^2 d_1^2}{k^4 r_{ch}^2 a_0'^2 \Omega^2} + \frac{16d_1^2 N^2}{\pi^2 k^4 a_0'^{10}\Omega^2} + \frac{8k_p^2 d_1^2 N^2}{\pi^2 k^4 a_0'^8 \Omega^2}. \tag{25}$$
$$+ \frac{16k_p^2 d_1^2 N}{\pi k^4 r_{ch}^2 a_o'^4 \Omega^2} + \frac{k_p^4 d_1^2 N^2}{\pi^2 k^4 a_0'^6 \Omega^2}$$
$$+ \frac{4k_p^4 d_1^2 N}{\pi k^4 r_{ch}^2 a_0'^2 \Omega^2} + \frac{4k_p^4 d_1^2 a_0'^2}{k^4 r_{ch}^4 \Omega^2}$$

3. DISCUSSION

Eq. (11) describes the beam dynamics in plasma with relativistic, ponderomotive self-channeling along with channel focusing. The Eqs. (11) and (12) are nonlinearly coupled ordinary second order differential equations governing the beam width parameter a and phase ϕ. There are several terms appearing on right-hand-side (r.h.s.) of both Eqs. (11) and (12). Further, Eq. (12) describes the longitudinal phase. Analytical solutions to these equations are not possible. We therefore seek numerical computational techniques to study beam dynamics. Before that, it is worth noting to understand the physical mechanisms and origin of various terms on the

r.h.s. of Eq. (11). The first term on the r.h.s. of Eq. (11) is responsible for diffractional divergence of the laser beam. It has its origin in the Laplacian ($\nabla_\perp^2$) appearing in the evolution Eq. (4). The other terms represent the combined effects of RSF, PSF, and self-channeling (SC). Self-focusing/defocusing of laser beam in plasma is determined by the competing mechanisms on the r.h.s. of Eq. (11). The normalized beam width parameter, $a < 1$ corresponds to self-focusing and $a > 1$ is the result of diffractional dominance over all other terms leading to defocusing of laser beam. In the absence of these terms, beam diverges due to diffraction. However, as mentioned in the Introduction, long distances of several Rayleigh lengths are prerequisite for novel applications of laser-produced plasma. In contrast to earlier research work, where relativistic and ponderomotive self-focusing using different approaches have been reported, the present investigation makes use of variational approach along with diffraction and nonlinearity management. We have considered all three mechanisms in our analysis to understand the combined role of RSF, PSF, and SC. To study the propagation characteristics, it is important to observe the evolution of beam width and self-phase modulation with distance of propagation. In order to observe the role of nonlinearity and diffraction management, we have chosen the following set of parameters for numerical computation of normalized beam width as a function of distance of propagation, η.

$$a_0 = 0.002\,cm, k = 1.25 \times 10^3\,cm^{-1}, \omega_p = 0.1 \times \omega_0,$$
$$\omega_0 = 1.778 \times 10^{14}\,\frac{rad}{\sec}, k_p = \frac{\omega_p}{c}, r_{ch} \cong \frac{c}{\omega_p}, d_0 = 1,$$
$$\lambda_0 = 1, d_1 = 0, \lambda_1 = 0.$$

The results are displayed in the form of graphs for three different values of intensity parameter, $|\psi_0|^2$. In this case, apparently control parameter, $R_{d\lambda}(= d(z)/\lambda(z))$ is unity. It is observed from Figure 1 that oscillatory self-focusing takes place in all three cases of intensity. However, there is substantial increase in self-focusing with increase in $|\psi_0|^2$. It is further important to notice that beam propagates oscillatory and infinitely over several number of Rayleigh lengths (Fauser & Langhoff, 2000). On the other hand, longitudinal phase is both positive as well as negative depending on the mechanism incorporated in the beam dynamics as shown in Figure 2. However, regularized phase is always negative [not shown]. Last, we analyze the effect of management control parameter $R_{d\lambda}$. We choose the following set of parameters:

$$a_0 = 0.002\,cm, k = 1.25 \times 10^3\,cm^{-1}, \omega_p = 0.1 \times \omega_0,$$
$$\omega_0 = 1.778 \times 10^{14}\,\frac{rad}{\sec}, k_p = \frac{\omega_p}{c}, r_{ch} \cong \frac{c}{\omega_p}, \Omega = 50,$$
$$d_0 = 1, \lambda_0 = 1, d_1 = 3.5, \lambda_1 = -3.5.$$

The results of numerical computation are shown in the form of graphs. As obvious from Figure 3, there is a substantial
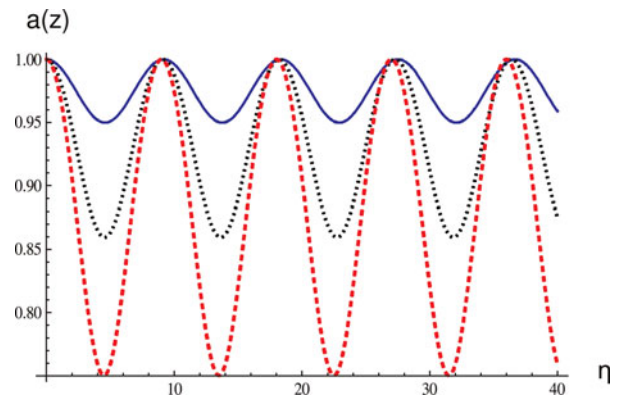

Fig. 1. (Color online) Variation of normalized beam width $a(z)$ with normalized distance of propagation η for constant diffraction and nonlinearity with the following set of parameters:

$$a_0 = 0.002\,cm, k = 1.25 \times 10^3\,cm^{-1}, \omega_p = 0.1 \times \omega_0,$$
$$\omega_0 = 1.778 \times 10^{14}\,rad/\sec, k_p = \omega_p/c, r_{ch} \cong c/\omega_p, d_0 = 1,$$
$$\lambda_0 = 1, d_1 = 0, \lambda_1 = 0.$$

Solid curve corresponds to intensity parameter $|\psi_0|^2 = 0.8$. Dotted curve corresponds to $|\psi_0|^2 = 0.99$. Dashed curve corresponds to $|\psi_0|^2 = 1.2$.

decrease in the value of $|\psi_0|^2$ for oscillatory self-focusing, which highlights the role of diffraction and nonlinearity management. The plot of longitudinal phase is shown in Figure 4. For comparative study of RSF, PSF, and CF, we first consider the case of various equilibrium radiuses when the channel term is absent. In this case, as usual, the propagation of laser beam in self-trapped mode is analyzed by putting $d^2a/dz^2 = 0$ in Eq. (9) and $a = a_0$, the initial value of beam width. The results displayed in the form of graphs are plotted in Figure 5 for the three values of $R_{d\lambda}$. The solid curve represents the case when diffraction and nonlinearity parameters are equal. We observe monotonic fall of equilibrium radius with increase in $|\psi_0|^2$. However, when
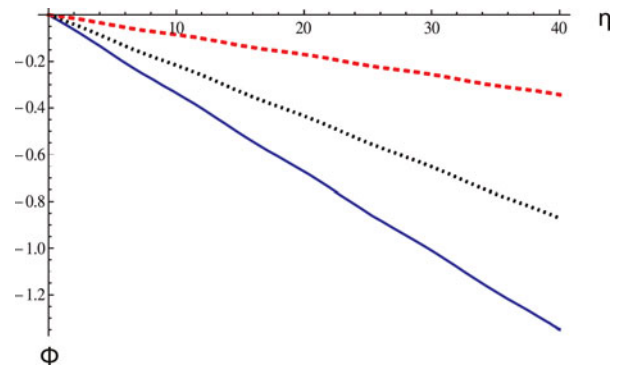

Fig. 2. (Color online) Plot of longitudinal phase ϕ *versus* normalized distance of propagation η for the same parameters as mentioned in the caption of Figure 1. Solid curve corresponds to intensity parameter $|\psi_0|^2 = 0.8$. Dotted curve corresponds to $|\psi_0|^2 = 0.99$. Dashed curve corresponds to $|\psi_0|^2 = 1.2$.
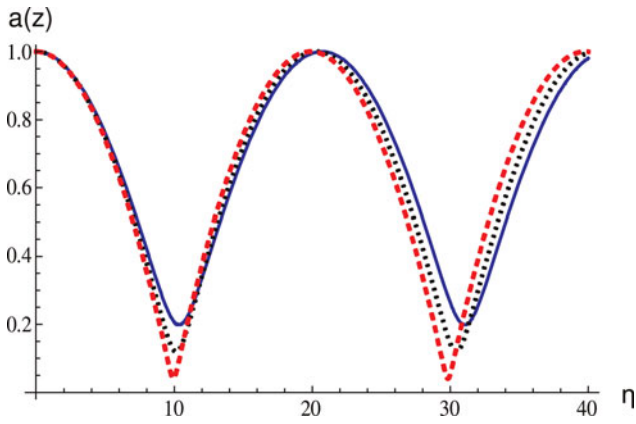
Fig. 3. (Color online) Variation of normalized beam width $a(z)$ with normalized distance of propagation η for periodically varying diffraction and nonlinearity with the same set of parameters as in the caption of Figure 1 with $d_1 = 3.5$, $\lambda_1 = -3.5$, $\Omega = 50$. Solid curve corresponds to intensity parameter $|\psi_0|^2 = 0.001$. Dotted curve corresponds to $|\psi_0|^2 = 0.05$. Dashed curve corresponds to $|\psi_0|^2 = 0.08$.

Fig. 5. (Color online) Plot of equilibrium beam width a_0 as intensity parameter $|\psi_0|^2$ when only relativistic and ponderomotive nonlinearities are present and channel term is assumed to be zero. Solid curve corresponds to $d(z)/\lambda(z) = 1$. Dotted curve corresponds to $d(z)/\lambda(z) = 0.2$. Dashed curve corresponds to $d(z)/\lambda(z) = 0.1$.

$R_{d\lambda}$ is decreased, initially faster fall in $\omega_p a_0/c$ with $|\psi_0|^2$ becomes much slower with further increase in $|\psi_0|^2$. Finally, radius becomes almost independent of $|\psi_0|^2$. Similar results are investigated in the presence of three mechanisms *viz* RSF + PSF + CF $\neq 0$ as shown in Figure 6. We study again the normalized equilibrium radius as a function of $|\psi_0|^2$ for three values of $R_{d\lambda}$. The following important observations are made: (1) Normalized equilibrium radius is much smaller than case 1. (2) Decrease in value of $R_{d\lambda}$ leads to initially faster fall in $\omega_p a_0/c$, variation of which becomes slower with increase in $|\psi_0|^2$. (3) Stabilization of $\omega_p a_0/c$ with $|\psi_0|^2$ is obtained for higher values of $|\psi_0|^2$.

Some interesting physics issues are observed when Eq. (25) for $U(a'_0, N)$ plays the role of effective potential for the evolution of laser beam under the combined effects of RSF, PSF,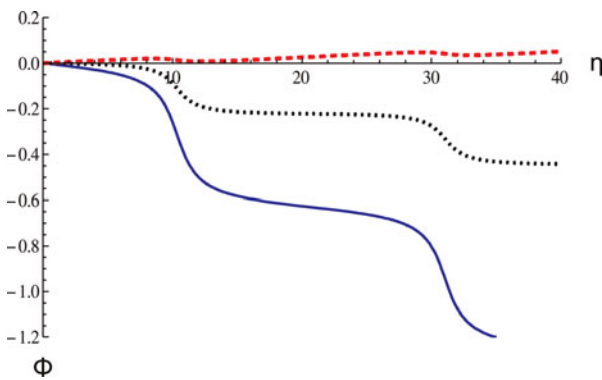 and SC. Using the analogy with a particle in a potential well, it is possible to acquire deeper physical insight into the understanding of laser beam dynamics in underdense plasma. For this purpose, we have plotted $U(a'_0, N)$ as a function of (a'_0) for different values of N and results are shown in Figures 7 to 10. We consider separate as well as combined effects of RSF, PSF, and SC. The results are correspondingly compared with those obtained by the SDE method. In Figure 7, we have chosen two values of N where only relativistic self-focusing is considered. Contrary to the earlier results (Hafizi *et al.*, 2000) where single minimum is obtained in case of relativistic and ponderomotive self-focusing, we observe the same results for relativistic self-focusing, i.e., region around the minimum is narrower. Some interesting results are obtained when both relativistic and

Fig. 4. (Color online) Plot of longitudinal phase ϕ versus normalized distance of propagation η for periodically varying diffraction and nonlinearity with the same set of parameters as in the caption of Figure 3. Solid curve corresponds to intensity parameter $|\psi_0|^2 = 0.001$. Dotted curve corresponds to $|\psi_0|^2 = 0.05$. Dashed curve corresponds to $|\psi_0|^2 = 0.08$.
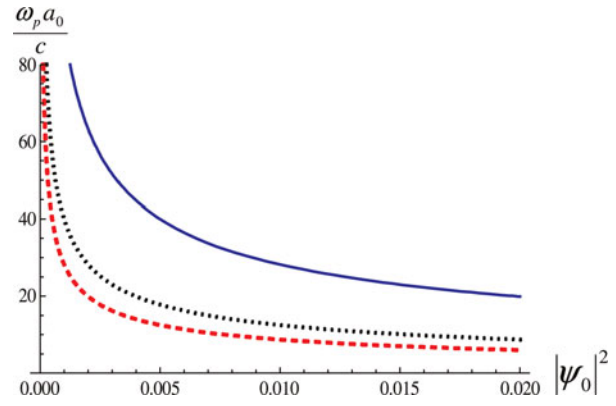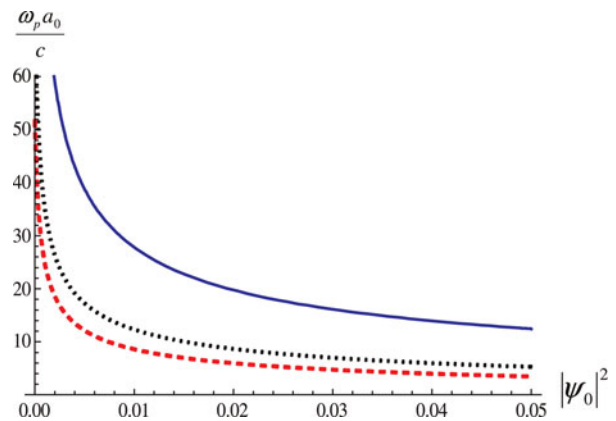

Fig. 6. (Color online) Plot of equilibrium beam width a_0 as intensity parameter $|\psi_0|^2$ when all the three mechanisms are present. Various parameters used are:

$a_0 = 0.002\,cm$, $\omega_p = 0.001 \times \omega_0$, $\omega_0 = 1.778 \times 10^{14}\,rad/\,sec$, $r_{ch} \cong c/\omega_p$. Solid curve corresponds to $d(z)/\lambda(z) = 1$. Dotted curve corresponds to $d(z)/\lambda(z) = 0.2$. Dashed curve corresponds to $d(z)/\lambda(z) = 0.1$.
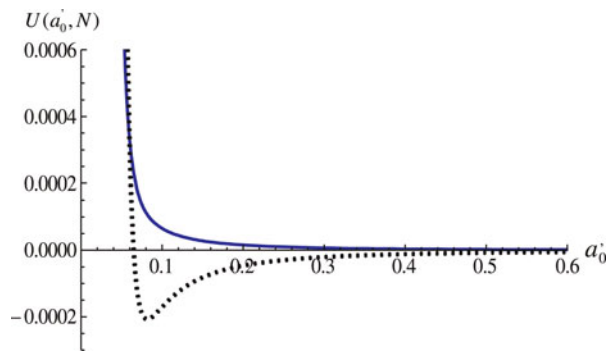
Fig. 7. (Color online) Variation of potential $U(a_0')$ versus beam width a_0' for the case when only relativistic nonlinearity is present in the case of $N = 0.7$ and $N = 1.6$. The parameters used are: $\omega_p = 0.001 \times \omega_0$, $\omega_0 = 1.778 \times 10^{14} rad/sec$, $k = 1.25 \times 10^3 cm^{-1}$. Solid curve corresponds to $N = 0.7$. Dotted curve corresponds to $N = 1.7$.

Fig. 9. (Color online) Variation of potential $U(a_0')$ versus beam width a_0' for the case when relativistic and ponderomotive nonlinearities along with channel term are present in the case of $N = 0.07$ and $N = 0.4$. The parameters used are the same as in the caption of Figure 7 along with $r_{ch} \cong c/\omega_p$. Solid curve corresponds to $N = 0.07$. Dotted curve corresponds to $N = 0.4$.

ponderomotive self-focusing are taken into account as shown in Figure 8 and following observations are made: (1) Self-focusing takes place at lower power density, (N). (2) On increasing (N), the characteristics of the potential well is affected drastically. We observe unphysical continuous focusing of the laser spot as the potential keeps on falling and no stationary state exists. Thus, catastrophic self-focusing does occur. Now, we include the combined effects of RSF, PSF, and SC, the catastrophic self-focusing can be prevented and stable self guiding of laser beam takes place at lower power density as apparently observed in Figure 9. Last, for sake of comparison, we have plotted $U(a_0', N)$ for RSF, PSF, and SC in Figure 10. The graphs clearly depicts that stable propagation at lower N is possible when all three mechanisms are considered and catastrophic self-focusing can be prevented only when plasma channel is introduced. Our results based on variational approach agree well with those Yu *et al.* (2007) who based their analysis on SDE method.
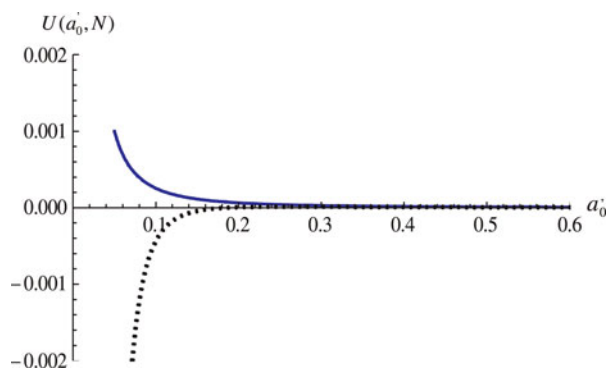
4. STABILITY CRITERION OF BEAM DYNAMICS

Variational method that can be used in several branches of physics and mathematics, can also be applied to study the stability characteristics of the evolution of laser beam when RSF, PSF, and SC are taken into consideration. The methods of nonlinear dynamics applied to dissipative solitons (Skarka *et al.*, 1997, 1999; Skarka & Aleksic, 2006) can also be used to study stability properties in the present research work. Euler-Lagrange equations using variational approach are the starting point to establish stability criterion. The dependent variables are disturbed about their equilibrium values and method of Lyapunov's exponents (Lakshman & Rajasekar, 2003; Skarka & Aleksic, 2006) is used. Thus, for stability characteristics of the system, the following Jacobi determinant is constructed from derivatives with respect to amplitude, width, and curvature in terms of S, F, and G where

$$S = \frac{d\psi_o}{dz} = \frac{2b|\psi_o|}{k} d(z), \qquad (26)$$

Fig. 8. (Color online) Variation of potential $U(a_0')$ versus beam width a_0' for the case when both relativistic and ponderomotive nonlinearities are present in the case of $N = 0.001$ and $N = 0.3$. The parameters used are same as in the caption of Figure 7. Solid curve corresponds to $N = 0.001$. Dotted curve corresponds to $N = 0.3$.
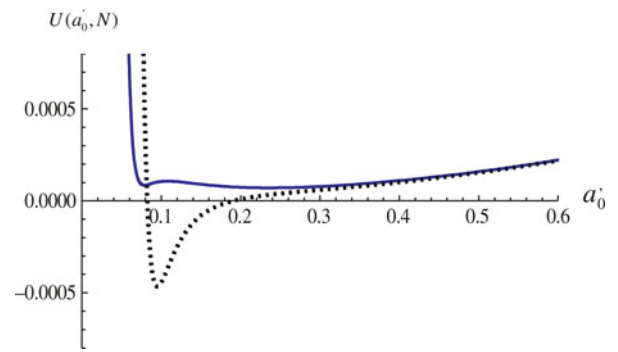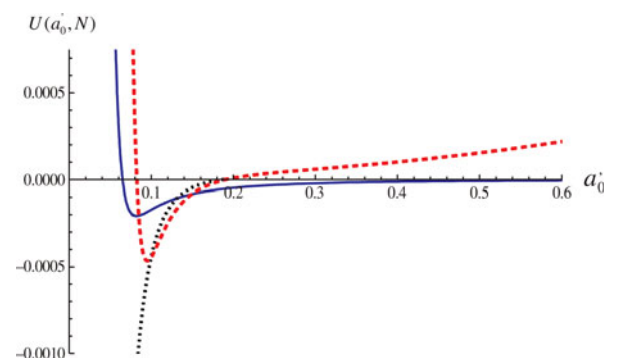

Fig. 10. (Color online) Plot of potential $U(a_0')$ *versus* beam width a_0'. Solid curve is for the case when only relativistic nonlinearity is present, dotted curve is for when both relativistic and ponderomotive nonlinearities are present and dashed curve is the case when all the three mechanisms are present. The parameters used are the same as in the caption of Figure 7. Solid curve corresponds to $N = 1.6$. Dotted curve corresponds to $N = 0.3$. Dashed curve corresponds to $N = 0.4$.

$$F = \frac{da}{dz} = \frac{4abd(z)}{k}, \tag{27}$$

$$G = \frac{db}{dz} = -\frac{2b^2 d(z)}{k} + \frac{2d(z)}{ka^4} - \lambda(z)\left(\frac{|\psi_0|^2}{ka^4} + \frac{k_p^2}{2kr_{ch}^2}\right.$$
$$\left. + \frac{k_p^2 |\psi_0|^2}{4ak^2}\right), \tag{28}$$

$$\det |J - \lambda I| = \begin{vmatrix} \frac{\partial S}{\partial \psi_0} - \lambda & \frac{\partial S}{\partial a} & \frac{\partial S}{\partial b} \\ \frac{\partial F}{\partial \psi_0} & \frac{\partial F}{\partial a} - \lambda & \frac{\partial F}{\partial b} \\ \frac{\partial G}{\partial \psi_0} & \frac{\partial G}{\partial a} & \frac{\partial G}{\partial b} - \lambda \end{vmatrix} = 0. \tag{29}$$

This leads to the following characteristic equation cubic in λ:

$$\lambda^3 + \alpha_1 \lambda^2 + \alpha_2 \lambda + \alpha_3 = 0, \tag{30}$$

where

$$\alpha_1 = -\frac{2bd(z)}{k}, \tag{31}$$

$$\alpha_2 = \frac{32d(z)^2}{k^2 a^4} - \frac{16b^2 d(z)^2}{k^2} - \frac{24A_0^2 a_0^2 \lambda(z)d(z)}{ka^6}$$
$$- 4\frac{k_p^2 A_0^2 a_0^2}{k^2 a^4}\lambda(z)d(z), \tag{32}$$

$$\alpha_3 = \frac{32b^3 d(z)^3}{k^3} + \frac{48bA_0^2 a_0^2 \lambda(z)d(z)^2}{k^2 a^6}$$
$$+ \frac{8bk_p^2 A_0^2 a_0^2 \lambda(z)d(z)^2}{k^3 a^4} - 64\frac{bd(z)^3}{k^3 a^4}. \tag{33}$$

In order to have Lyapunov's stability, Hurwitz conditions must be fulfilled. i.e., $\alpha_1 \alpha_2 - \alpha_3$ must be positive. According to the Routh-Hurwitz criterion, a necessary and sufficient condition for the stationary solutions to be stable is:

$$\alpha_1 \alpha_2 - \alpha_3 > 0. \tag{34}$$

Eq. (30) has a pair of purely imaginary roots at a critical point (Lugiato & Narducci, 1985):

$$\lambda = \pm \iota v, \ v \rangle 0. \tag{35}$$

We may substitute Eq. (35) into Eq. (30) and we get

$$v^2 - \alpha 2 = 0, \tag{36}$$

and

$$\alpha_1 v^2 - \alpha_3 = 0. \tag{37}$$

The critical condition for Hopf-bifurcation is:

$$f = \alpha_1 \alpha_2 - \alpha_3 = 0. \tag{38}$$

$f > 0$ is a necessary condition for the stationary solution to be stable, $f < 0$ is a necessary condition for the Hopf-bifurcation to emerge. It is observed that the condition $f = 0$ is satisfied for the chosen set of parameters and therefore Hopf-bifurcation, resulting from the unstable fixed point, does not come into play, leading to overall stability of the beam dynamics (Wang, 1990).

5. CONCLUSIONS

In the present investigation, we have studied the self-focusing and self-phase modulation of laser beam with relativistic, ponderomotive nonlinearity along with non-uniform plasma channel. In order to control the diffraction and nonlinearity, we have introduced two scale lengths. Equation for beam width and phase is derived using variational approach. Beam width equation is further manipulated by putting it in the form of a quasi-particle moving in an effective potential. All the cases with relativistic and ponderomotive self-focusing and relativistic and ponderomotive self-focusing along with non-uniform plasma channel are studied. It is found that by combining the effects of RSF and PSF, catastrophic self-focusing is observed. However, long propagation distance is only possible when the combined effects of three mechanisms are considered. These results agree well with the source-dependent expansion method considered in Yu et al. (2007). Further, control parameter can be used to lead optimum self-focusing. Last, stability properties of beam dynamics is studied and it is found that beam is marginally stable in the absence of any dissipation mechanism.

REFERENCES

ABDULLAEV, F.K., BAIZAKOV, B. & SALERNO, M. (2003). Stable two-dimensional dispersion-managed soliton. *Phys. Rev. E* **68**, 066605/1–4.

ABLOWITZ, M.J. & MUSSLIAMANI, Z.H. (2003). Dark and gray strong dispersion-managed solitons. *Phys. Rev. E* **67**, 025601/1–4.

ABRAMYAN, L.A., LITVAK, A.G., MIRONOV, V.A. & SERGEEV, A.M. (1992). Self-focusing and relativistic waveguiding of an ultra-short laser pulse in a plasma. *Sov. Phys. JETP* **75**, 978.

AKHMANOV, S.A., SUKHORUKOV, A.P. & KHOKHLOV, R.V. (1968). Self-focusing and diffraction of light in a nonlinear medium. *Sov. Phys. Usp* **10**, 609–636.

ANDERSON, D. & BONNEDAL, M. (1979). Variational approach to nonlinear self-focusing of Gaussian laser beams. *Phys. Fluids* **22**, 105–109.

ANDREEV, A.A., EROKHIN, N.S., SUTYAGIN, A.N. & FADEEV, A.P. (1987). Self-focusing of a laser beam in an expanding plasma which is inhomogeneous in two dimensions. *Soviet J. Plasma Phys.* **13**, 608–613.

BENWARE, B.R., MACCHIETTO, C.D., MORENO, C.H. & ROCCA, J.J. (1998). Demonstration of a high average power tabletop soft X-ray laser. *Phys. Rev. Lett.* **81**, 5804–5807.

BERGE, L. (1998). Wave collapse in physics: principles and applications to light and plasma waves. *Physics Rpt.* **303**, 259–370.

BORGHESI, M., KAR, S., ROMAGNANI, L., TONCIAN, T., ANTICI, P., AUDEBERT, P., BRAMBRINK, E., CECCHERINI, F., CECCHETTI, C.A., FUCHS, J., GALIMBERTI, M., GIZZI, L.A., GRISMAYER, T., LYSEIKINA, T., JUNG, R., MACCHI, A., MORA, P., OSTERHOLTZ, J., SCHIAVI, A. & WILLI, O. (2007). Impulsive electric fields driven by high-intensity laser matter interactions. *Laser Part. Beams* **25**, 161–167.

BORISOV, A.B., BOROVSKIY, A.V., KOROBKIN, V.V., PROKHOROV, A.M., SHIRYAEV, O.B., SHI, X.M., LUK, T.S., MCPHERSON, A., SOLEM, J.C., BOYER, K. & RHODES, C.K. (1992). Observation of relativistic and charge-displacement self-channeling of intense subpicosecond ultraviolet (248 nm) radiation in plasmas. *Phys. Rev. Lett.* **68**, 2309–2312.

BORISOV, A.B., LONGWORTH, J.W., BOYER, K. & RHODES, C.K. (1998). Stable relativistic/charge-displacement channels in ultrahigh power density (10^{21} Watt/cm^2) plasmas. *Proc. Natl. Acad. Sci.* **95**, 7854–7859.

CHEN, S.Y., SARKISOV, G.S., MAKSIMCHUK, A., WAGNER, R. & UMSTADTER, D. (1998). Evolution of a plasma waveguide created during relativistic-ponderomotive self-channeling of an intense laser pulse. *Phys. Rev. Lett.* **80**, 2610–2613.

CHEN, X.L. & SUDAN, R.N. (1993). Necessary and sufficient conditions for self-focusing of short ultraintense laser pulse in underdense plasma. *Phys. Rev. Lett.* **70**, 2082–2085.

CHESSA, P., MORA, P. & ANTONSEN, T.M. (1998). Numerical simulation of short laser pulse relativistic self-focusing in underdense plasma. *Phys. Plasma* **5**, 3451–3458.

DUDA, B.J. & MORI, W.B. (2000). Variational principle approach to short-pulse laser-plasma interactions in three dimensions. *Phys. Rev. E* **61**, 1925–1939.

FAENOV, A.Y., MAGUNOV, A.I., PIKUZ, T.A., SKOBELEV, I.Y., GASILOV, S.V., STAGIRA, S., CALEGARI, F., NISOLI, M., DE SILVESTRI, S., POLETTO, L., VILLORESI, P. & ANDREEV, A.A. (2007). X-ray spectroscopy observation of fast ions generation in plasma produced by short low-contrast laser pulse irradiation of solid targets. *Laser Part. Beams* **25**, 267–275.

FAUSER, C. & LANGHOFF, H. (2000). Focussing of laser beams by means of a z-pinch formed plasma guiding system. *Appl. Phy. B* **71**, 607–609.

FEDOTOV, A.B., NAUMOV, A.N., SILIN, V.P., URYUPIN, S.A., ZHELTIKOV, A.M., TARASEVITCH, A.P. & VON DER LINDE, D. (2000). Third-harmonic generation in a laser-pre-excited gas: the role of excited-state neutrals. *Phys. Lett. A* **271**, 407–412.

FIRTH, W.J. (1977). Propagation of laser beams through inhomogeneous media. *Opt. Commun.* **22**, 226–230.

FOLDES, I.B., BAKOS, J.S., BAKONYI, Z., NAGY, T. & SZATMARI, S. (1999). Harmonic generation in plasmas of different density gradients. *Phys. Lett. A* **258**, 312–316.

GILL, T.S., SAINI, N.P. & KAUL, S.S. (2001). Two-dimensional self-focusing of a laser beam in an inhomogeneous laser-produced plasma. *J. Plasma Phys.* **66**, 39–51.

HAFIZI, B., TING, A., HUBBARD, R.F., SPRANGLE, P. & PENANO, J.R. (2003). Relativistic effects on intense laser beam propagation in plasma channels. *Phys. Plasma* **10**, 1483–1492.

HAFIZI, B., TING, A., SPRANGLE, P. & HUBBARD, R.F. (2000). Relativistic focusing and ponderomotive channeling of intense laser beams. *Phys. Rev. E* **62**, 4120–4125.

HOFFMANN, D.H.H., BLAZEVIC, A., NI, P., ROSMEJ, O., ROTH, M., TAHIR, N., TAUSCHWITZ, A., UDREA, S., VARENTSOV, D., WEYRICH, K. & MARON, Y. (2005). Present and future perspective for high energy density physics with intense heavy ions and laser beams. *Laser Part. Beams* **23**, 395–395.

KARLSSON, M., ANDERSON, D., DESAIX, M. & LISAK, M. (1991). Dynamic effects of Kerr nonlinearity and spatial diffraction on self-phase modulation of optical pulses. *Opt. Lett.* **16**, 1373–1375.

KASPERCZUK, A., PISARCZYK, T., KALAL, M., MARTINKOVA, M., ULLSCHMIED, J., KROUSKY, E., MASEK, K., PFEIFER, M., ROHLENA, K., SKALA, J. & PISARCZYK, P. (2008). PALS laser energy transfer into solid targets and its dependence on the lens focal point position with respect to the target surface. *Laser Part. Beams* **26**, 189–196.

KONAR, S. & MANOJ, M. (2005). Effect of higher order nonlinearities on induced focusing and on the conversion of circular Gaussian laser beams into elliptic Gaussian laser beam. *J. Opt. A: Pure Appl. Opt.* **7**, 576.

KRUSHELNICK, K., TING, A., MOORE, C.I., BURRIS, H.R., ESAREY, E., SPRANGLE, P. & BAINE, M. (1997). Plasma channel formation and guiding during high intensity short pulse laser plasma experiments. *Phys. Rev. Lett.* **78**, 4047–4050.

KUEHL, TH., URSESCU, D., BAGNOUD, V., JAVORKOVA, D., ROSMEJ, O., CASSOU, K., KAZAMIAS, S., KLISNICK, A., ROS, D., NICKLES, P., ZIELBAUER, B., DUNN, J., NEUMAYER, P., PERT, G. & THE PHELIX TEAM. (2007). Optimization of non thermal incidence, transient pumped plasma X-ray laser for laser spectroscopy and plasma diagnostics at the facility for antiproton and ion research (FAIR). *Laser Part. Beams* **25**, 93–97.

KURKI-SUONIO, T., MORRISON, P.J. & TAJIMA, T. (1989). Self-focusing of an optical beam in a plasma. *Phys. Rev. A* **40**, 3230–3239.

LAKSHMAN, M. & RAJASEKAR, S. (2003). *Nonlinear Dynamics*. New York: Springer Verlag.

LAM, J.F., LIPPMANN, B. & TAPPERT, F. (1975). Moment theory of self-trapped laser beams with nonlinear saturation. *Opt. Commun.* **15**, 419–421.

LAM, J.F., LIPPMANN, B. & TAPPERT, F. (1977). Self-trapped laser beams in plasma. *Phys. Fluids* **20**, 1176–1179.

LANDAU, L.D. & LIFSHITZ, E.M. (1976). *Mechanics*. Oxford: Pergammon Press.

LASKA, L., BADZIAK, J., BOODY, F.P., GAMMINO, S., JUNGWIRTH, K., KRASA, J., KROUSKY, E., PARYS, P., PFEIFER, M., ROHLENA, K., RYC, L., SKALA, J., TORRISI, L., ULLSCHMIED, J. & WOLOWSKI, J. (2007). Factor influencing parameters of laser ion sources. *Laser Part. Beams* **25**, 199–205.

LIU, M., GUO, H., ZHOU, B., LI, W., LI, B. & WU, G. (2004). Effects of relativistic and channel focusing on an intense laser beam propagating in a plasma channel: variational analysis. *Phys. Lett. A* **333**, 478–484.

LUGIATO, L.A. & NARDUCCI, L.M. (1985). Single-mode and multimode instabilities in lasers and related optical systems. *Phys. Rev. A* **32**, 1576–1587.

MATUSZEWSKI, M., TRIPPENBACH, M., MALOMED, B.A., INFELD, E. & SKORUPSKI, A.A. (2004). Two-dimensional dispersion-managed light bullets in Kerr media. *Phys. Rev. E* **70**, 016603/1–6.

MONOT, P., AUGUSTE, T., GIBBON, P., JAKOBER, F., MAINFRAY, G., DULIEU, A., LOUIS-JACQUET, M., MALKA, G. & MIQUEL, J.L. (1995). Experimental demonstration of relativistic self-channeling of a multiterawatt laser pulse in an underdense plasma. *Phys. Rev. Lett.* **74**, 2953–2956.

PANWAR, A. & SHARMA, A.K. (2009). Self-phase modulation of a laser in self created plasma channel. *Laser Part. Beams* **27**, 249–253.

REGAN, S.P., BRADLEY, D.K., CHIROKIKH, A.V., CRAXTON, R.S., MEYERHOFER, D.D., SEKA, W., SHORT, R.W., SIMON, A., TOWN, R.P.J. & YAAKOBI, B. (1999). Laser-plasma interactions in long-scale-length plasmas under direct-drive National Ignition Facility conditions. *Phys. Plasmas* **6**, 2072–2080.

SAINI, N.P. & GILL, T.S. (2006). Self-focusing and self-phase modulation of an elliptic Gaussian laser beam in collisionless magnetoplasma. *Laser Part. Beams* **24**, 447–453.

SARKISOV, G.S., BYCHENKOV, V.YU., NOVIKOV, V.N., TIKHONCHUK, V.T., MAKSIMCHUK, A., CHEN, S.-Y., WAGNER, R., MOUROU, G. & UMSTADTER, D. (1999). Self-focusing, channel formation, and high-energy ion generation in interaction of an intense short laser pulse with a He jet. *Phys. Rev. E* **59**, 7042–7054.

SCHAUMANN, G., SCHOLLMEIER, M.S., RODRIGUEZ-PRIETO, G., BLAZEVIC, A., BRAMBRINK, E., GEISSEL, M., KOROSTIY, S., PIRZADEH, P., ROTH, M., ROSMEJ, F.B., FAENOV, A.Y., PIKUZ, T.A., TSIGUTKIN, K., MARON, Y., TAHIR, N.A. & HOFFMANN, D.H.H. (2005). High energy heavy ion jets emerging from laser plasma generated by long pulse laser beams from the NHELIX laser system at GSI. *Laser Part. Beams* **23**, 503–512.

SKARKA, V. & ALEKSIC, N.B. (2006). Stability criterion for dissipative soliton solutions of one-, two-, and three dimensional complex cubic quintic Ginzburg-Landau equations. *Phys. Rev. Lett.* **96**, 013903/1–4.

SKARKA, V., BEREZHIANI, V.I. & MIKLASZEWSKI, R. (1997). Spatiotemporal soliton propagation in saturating nonlinear optical media. *Phys. Rev. E* **56**, 1080–1087.

SKARKA, V., BEREZHIANI, V.I. & MIKLASZEWSKI, R. (1999). Generation of light spatiotemporal solitons from asymmetric pulses in saturating nonlinear media. *Phys. Rev. E* **59**, 1270–1273.

SODHA, M.S., GHATAK, A.K. & TRIPATHI, V.K. (1974). *Self-Focusing of Laser Beams in Dielectric Plasma and Semi Conductors.* New York: Tata McGraw-Hill.

SODHA, M.S., GHATAK, A.K. & TRIPATHI, V.K. (1976). Self-focusing of laser beams in plasmas and semiconductors. *Prog. Opt.* **13**, 171–265.

SPRANGLE, P., HAFIZI, B. & PENANO, J.R. (2000). Laser pulse modulation instabilities in plasma channels. *Phys. Rev. E* **61**, 4381–4393.

STOEHLKER, T., BACKE, H., BEYER, H., BOSCH, F., BRAEUNING-DEMIAN, A., HAGMAN, S., IONESCU, D., JUNGMANN, K., KLUGE, H.-J., KOZHUHAROV, C., KUEHL, TH., LISEN, D., MANN, R., MOKLER, P. & QUINT, W. (2003). Status and perspectives of atomic physics research at GSI: The new GSI accelerator project. *Nucl. Instr. Meth. B* **205**, 156.

STRANGIO, C., CARUSO, A., NEELY, D., ANDREOLI, P.L., ANZALONE, R., CLARKE, R., CRISTOFARI, G., DELPRETE, E., DI GIORGIO, G., MURPHY, C., RICCI, C., STEVENS, R. & TOLLEY, M. (2007). Production of multi-Mev per nucleon ions in the controlled amount of matter mode (CAM) by using casually isolated targets. *Laser Part. Beams* **25**, 85–91.

SUBBARAO, D., UMA, R. & SINGH, H. (1998). Paraxial theory of self-focusing of cylindrical laser beams. I. ABCD laws. *Phys. Plasmas* **5**, 3440–3450.

SUCKEWER, S. & SKINNER, C.H. (1990). Soft X-ray lasers and their applications. *Sci.* **247**, 1553.

SUCKEWER, S. & SKINNER, C.H. (1995). Comments. *At. Mol. Phys.* **30**, 331.

SUN, G.Z., OTT, E., LEE, Y.C. & GUZDAR, P. (1987). Self-focusing of short intense pulses in plasmas. *Phys. Fluids* **30**, 526–532.

TABAK, M., HAMMER, J., GLINSKY, M.E., KRUER, W.L., WILKS, S.C., WOODWORTH, J., CAMPBELL, E.M., PERRY, M.D. & MASON, R.J. (1994). Ignition and high gain with ultrapowerful lasers. *Phys. Plasmas* **1**, 1626–1634.

TORRISI, L., MARGARONE, D., LASKA, L., KRASA, J., VELYHAN, A., PFEIFER, M., ULLSCHMIED, J. & RYC, L. (2008). Self-focusing effects in Au-target induced by high power pulsed laser at PALS. *Laser Part. Beams* **26**, 379–387.

TZENG, K.C. & MORI, W.B. (1998). Suppression of electron ponderomotive blowout and relativistic self-focusing by the occurence of Raman scattering and plasma heating. *Phys. Rev. Lett.* **81**, 104–107.

WAGNER, R., CHEN, S.Y., MAKSIMUCHUK, A. & UMSTADTER, D. (1997). Electron acceleration by a laser wakefield in a relativistically self-guided channel. *Phys. Rev. Lett.* **78**, 3125–3128.

WANG, N.Q. (1990). Chaotic behaviour in an electron-beam plasma. *Phys. Lett. A* **145**, 29–32.

YU, C., WEI, Y., CHUN, W.H., HAN, X. & WEI, T.Y. (2007). Intense laser beam guiding in self-induced electron cavitation channel in underdense plasmas. *Chin. Phys. Soc.* **16**, 456–462.

YU, W., YU, M.Y., ZHANG, J. & XU, Z. (1998). Harmonic generation by relativistic electrons during irradiance of a solid target by a short-pulse ultraintense laser. *Phys. Rev. E* **57**, 2531–2534.