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Solar and Stellar Dynamos

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Abstract. Records of the solar magnetic field extend back for millennia, and its surface properties have been observed for centuries, while helioseismology has recently revealed the Sun's internal rotation and the presence of a tachocline. Dynamo theory has developed to explain these observations, first with idealized models based on mean-field electrodynamics and, more recently, by direct numerical simulation, notably with the ASH code at Boulder. These results, which suggest that cyclic activity relies on the presence of the tachocline, and that its modulation is chaotic (rather than stochastic), will be critically reviewed. Similar theoretical approaches have been followed in order to explain the magnetic properties of other main-sequence stars, whose fields can be mapped by Zeeman-Doppler imaging. Of particular interest is the behaviour of fully convective, low-mass stars, which lack any tachocline but are nevertheless extremely active.

Keywords. magnetic fields, MHD, Sun: activity, Sun: magnetic fields, Sun: rotation, sunspots, stars: activity, stars: magnetic fields, stars: spots

1. Introduction

Half a century has passed since Parker (1955), Babcock (1961) and their predecessors set out the basic principles of the solar dynamo, and identified the key roles of differential rotation and cyclonic eddies. It is now accepted that turbulent convection in a star is able not only to amplify any pre-existing field but also to maintain both small-scale and large-scale fields indefinitely against ohmic decay. Mean field electrodynamics has been developed as a sophisticated theory, while space and ground-based observations have revealed the structure of solar and stellar magnetic fields in ever-increasing detail. Yet there is still no fully satisfactory model of the solar dynamo: mean-field dynamo theory has to rely on plausible parametrizations of the key effects, while computational models – despite massive efforts in which Juri Toomre has played a leading part– are still not able to reproduce the essential features of the solar cycle.

In this review I shall therefore consider general properties of solar and stellar dynamos and avoid detailed descriptions of specific models. First I shall summarize the main physical processes that are involved. Although it is generally accepted that strong toroidal fields are formed and stored near the base of the convection zone, where there is a steep radial gradient of angular velocity in the tachocline, there are two competing mean-field scenarios. Interface dynamos have all the action concentrated near the tachocline, but in flux-transport dynamos the poloidal fields are formed near the surface and transported down by a meridional circulation. I shall contrast what we can deduce from mean-field models with what we hope to learn from direct numerical simulations. The next section focuses on the modulation of cyclic activity in stars like the Sun, a subject that has become topical owing to the feeble start of the latest cycle, which raises the question of whether solar activity might be heading for a grand minimum like that in the seventeenth century. Section 4 is devoted to dynamos in other stars, and the final section looks ahead to the future.

2. Physics of the solar dynamo

The challenge to dynamo models is to explain the large-scale systematic features of the solar cycle: the emergence of isolated, predominantly toroidal fields (antisymmetric about the equator) in sunspots and active regions, the equatorward movement of the activity zones, the reversal of polarities from one 11-year activity cycle to the next, and the development of a weak polar field (see, for example, Solanki, Inhester & Schüssler 2006; Thomas & Weiss 2008).

2.1. *Relevant processes*

The generation of magnetic fields is ultimately governed by hydrodynamic processes in stellar convection zones. The Sun's radiative interior is surrounded by a convection zone that occupies the outer 30% by radius; in most of this region the angular velocity Ω is constant on conical surfaces and there is a balance between Coriolis and baroclinic effects that satisfies the thermal wind equation (Balbus *et al.* 2009). At the base of the convection zone there is an abrupt transition in the tachocline (Hughes, Rosner & Weiss 2007) to the almost uniformly rotating radiative zone, as shown in Figure 1. Superimposed on this pattern are zonal shear flows ("torsional oscillations") that vary with the solar cycle, appearing as a branch of enhanced rotational velocity that coincides with the activity belt, together with a subsidiary poleward branch, as depicted in Figure 2 (Howe 2009). As expected for a flow driven by the quadratic Lorentz force, the torsional oscillations have an 11-year period, half that of the underlying magnetic cycle. (It is still worth

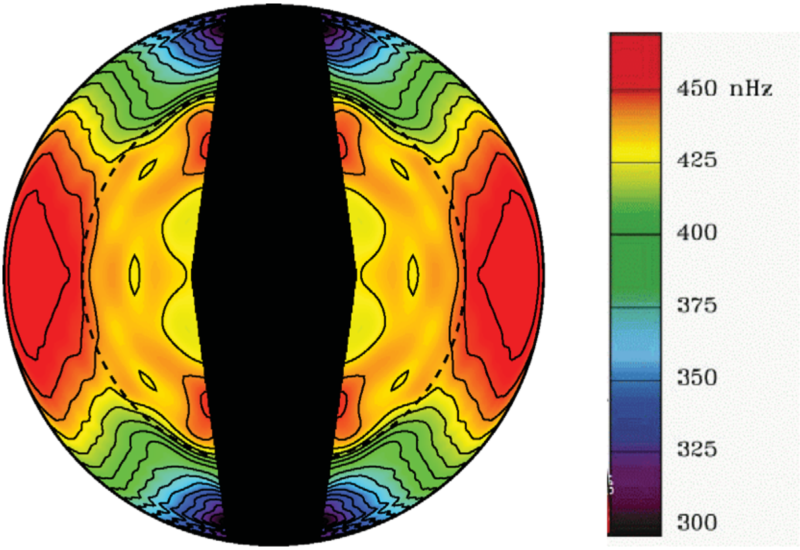

Figure 1. Solar differential rotation, as determined from the rotational splitting of acoustic p-modes (Thompson *et al.* 2003; Christensen-Dalsgaard & Thompson 2007). The dashed line denotes the base of the convection zone at a radius $r = 0.713R_\odot$. Near the surface the rotation frequency $\Omega/2\pi$ is greatest at the equator and least at the poles; in the radiative zone Ω is essentially uniform. The slender tachocline lies below the convection zone at the equator, with an estimated thickness of only $0.03R_\odot$ but is slightly prolate and thicker at the poles. (Courtesy of J. Christensen-Dalsgaard.)

emphasizing that any “oscillator” theory requires torsional oscillations with a 22-year period, which have never been seen. If the steady dipolar field in the radiative zone does protrude into the region where the dynamo operates, its only effect is to bias the cyclic field slightly towards one polarity – as appears to be the case (Boruta 1996).)

Given an initial dipolar field, such a pattern of differential rotation can readily stretch field lines to generate a toroidal field that is antisymmetric about the equator, and this process (the ω-effect of mean-field dynamo theory) is most effective at the tachocline. In a turbulent region, large-scale fields are expelled down the gradient of turbulent intensity, an effect that is enhanced at the base of the convection zone, owing to downward pumping of magnetic flux by the vigorous sinking plumes (Tobias *et al.* 2001; Weiss *et al.* 2004). Hence we expect the strong toroidal fields to be stored in the region immediately below the convection zone itself, with typical strengths of up to 10^4 G, corresponding to equipartition with the turbulent motion, although fields may be locally more intense. This is the most that differential rotation can create within the tachocline itself. Such fields will be liable to instabilities driven by magnetic buoyancy (Hughes 2007), which allow loops to enter the convection zone, where they can acted upon by cyclonic motions, as indicated in Figure 3(a). If the toroidal field were weak then different loops might be twisted through arbitrary angles, as sketched in Figure 3(b), so that their combined effects would cancel out – as demonstrated in the numerical calculations of Cattaneo & Hughes (2006, 2008). With a strong toroidal field, the loops are only slightly twisted and their contributions can combine constructively, as Parker originally suggested. It is this process that is parametrized by the α-effect in mean-field dynamo models. Field strengths will of course vary and may locally be much higher than the average. Some flux tubea, with fields an order of magnitude greater ($\sim 10^5$ G) can break out of the

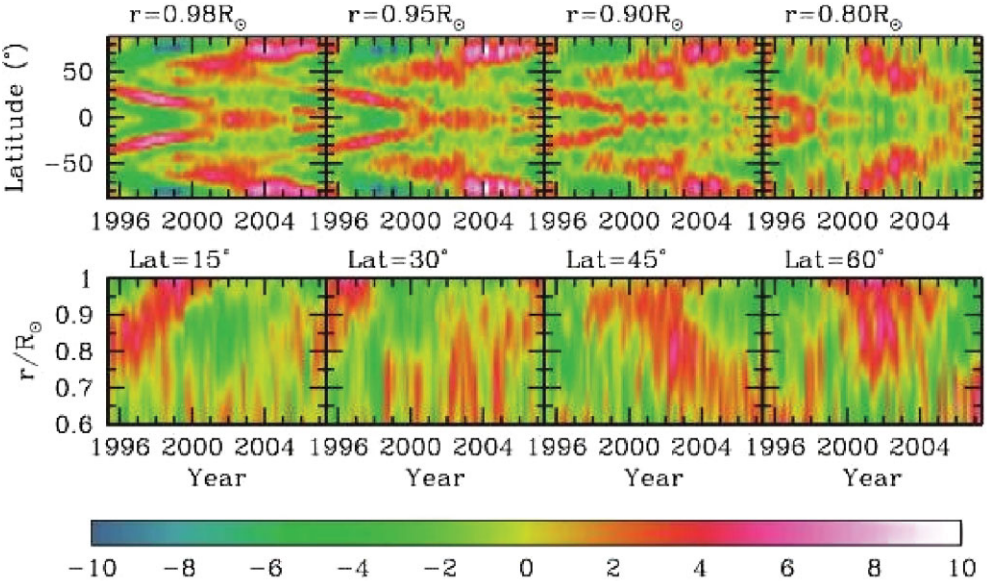

Figure 2. Zonal shear flows (“torsional oscillations”), in m s^{-1}, as functions of time at different latitudes and depths in the solar convection zone. The band of positive velocity tracks the sunspot zones at low latitudes but there is also a strong poleward branch. At low latitudes the shear flow first appears near the base of the convection zone. (From Antia, Basu & Chitre 2008, using GONG data.)

magnetic layer and, aided by magnetic buoyancy, rise unimpeded by Coriolis forces, to erupt as active regions or as sunspots at the surface.

This dynamo process relies, of course, on some form of diffusion that allows the individual loops to reconnect and so to generate a reversed poloidal field. However, the same diffusion leads to dissipation of magnetic energy and the critical question is whether the constructive effects of stretching and reconnection can overcome magnetic dissipation. The actual magnetic diffusivity within a star is very small (though much greater than the viscous diffusivity) and turbulent processes are intrinsically complicated (Tobias, Cattaneo & Boldyrev 2011). All models therefore rely on some parametrized description of turbulent diffusion, which may or may not be realistic. The least that one can require is that diffusion should be explicit, in the hope that its effects on reconnection and dissipation might be estimated and compared. This is essential, for there is a long history of over-optimistic model calculations which exhibited dynamo action that subsequently faded as the numerical procedure was refined.

Doppler measurements have revealed a poleward meridional flow in both hemispheres at the solar surface, with a peak speed of about 20 $\mathrm{m\,s^{-1}}$. Helioseismology has confirmed that this motion persists downwards for at least 15 Mm but below that depth its magnitude and direction become increasingly uncertain. If there is an equatorward flow of 1 $\mathrm{m\,s^{-1}}$ at the base of the convection zone, as has plausibly been suggested, it could significantly affect the latitudinal drift of activity with the solar cycle. In fact, the surface motion does itself vary with the activity cycle too (Basu & Antia 2010). Moreover, the latitudinal component may reverse more than once within the convection zone.

2.2. *Location of the dynamo*

Although it is widely accepted that the ω-effect is concentrated around the tachocline, where radial and latitudinal gradients of Ω have comparable effects, there are several competing locations for the α-effect (Rüdiger & Hollerbach 2004; Charbonneau 2005). In *interface* dynamos (Parker 1993), the strong toroidal fields become locally unstable to modes driven by magnetic buoyancy and, in the nonlinear regime, Coriolis effects eventually lead to the formation of a reversed poloidal field (Tobias & Weiss 2007; Jones, Thompson & Tobias 2010)i, as outlined above. In this picture, all the action is at the base of the convection zone and the flux loops that make their way to the surface and give

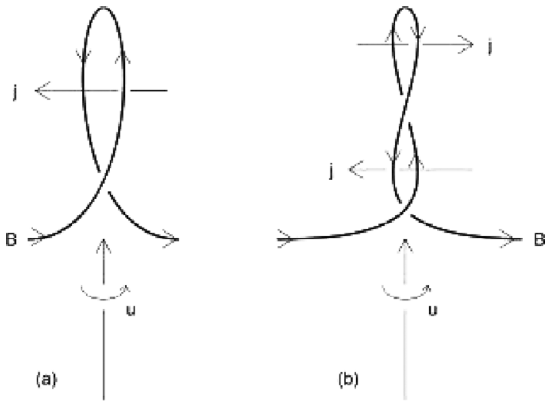

Figure 3. Sketches showing (a) twisting of a toroidal field by a cyclonic eddy to give a poloidal component, as envisaged by Parker, and (b) the consequences of repeated twisting if the toroidal field is too weak to limit the process. (From Jones, Thompson & Tobias 2010.)

rise to sunspots are just by-products, or epiphenomena (Cowling 1975), of the dynamo process.

An alternative (currently much favoured in the USA) is to assert that photospheric fields and meridional flows do indeed play an essential role in the process, as in *flux transport* dynamos. Those flux loops that emerge at low latitudes must have had 10^5 G fields in order to survive and they are observed to be slightly tilted with respect to lines of latitude, as expected from the cyclonic effect of the Coriolis force. Presumably the underlying flux tubes eventually detach themselves from the deep-seated toroidal field. As active regions decay, their fields are acted upon by the diffusive effect of supergranular convection and the erratic poleward meridional motion in such a way that reversed poloidal flux accumulates at high latitudes, whence it can be transported to the base of the convection zone by the meridional circulation.

The attraction of the flux transport model is that it relates the global dynamo to observable fields at the surface. This was a very plausible assumption when it was thought that the convection zone was shallow (Leighton 1969) but we now know that it is deep. I remain of the opinion that interface models are more reliable. It is hard to imagine how a weak poloidal field can be transported intact down to the tachocline without being tangled up or even destroyed by turbulent convection on the way; indeed, radial diffusion may be more effective than meridional transport. On the other hand, the weak polar fields, which reverse at sunspot maximum, do indeed appear to be a global feature, for they supply reliable predictions of the following activity maximum. Are they just accumulated rubble from the preceding cycle, or are they generated much deeper down?

2.3. *Modelling stellar dynamos*

Up to now, nearly all numerical models have been axisymmetric and relied on mean-field formalisms, with processes that are arbitrarily parametrized. Field growth has to be limited by some nonlinear effect and many variants have been tried. In the example illustrated in Figure 4, the field drives an azimuthal flow that corresponds to the observed zonal shear flows. By varying parameters and adding more effects both interface and flux

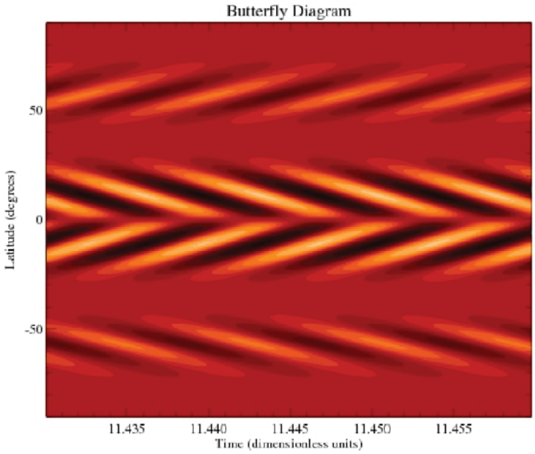

Figure 4. Butterfly diagram for a highly idealized mean-field dynamo, with both the α-effect and the ω-effect concentrated around the interface between the radiative and convective zones. Growth is limited by the nonlinear effect of the Lorentz force on differential rotation, giving rise to zonal shear flows such as are observed. Shown here is the toroidal field near the base of the convection zone, which is antisymmetric about the equator. The corresponding zonal shear flows have twice the frequency of the magnetic cycle. (From Bushby 2005.)

transport dynamo models can be adjusted to reproduce the principal observed features of the solar cycle, and many examples are available. It follows, however, that such models, though certainly instructive, are only illustrative and can have no detailed predictive power.

The obvious alternative is direct numerical simulation. The earliest attempt, by Gilman (1983) already revealed some of the difficulties: the behaviour of a nonlinear dynamo – whether fields have dipole or quadrupole symmetry, whether they are steady or cyclic, whether waves progress towards or away from the equator – is very sensitive to the parameter values that are assumed and small changes may have big effects. Although the geodynamo has been convincingly modelled (Glatzmaier and Roberts 1995; Christensen, Schmitt & Rempel 2009) there is as yet no fully convincing computational representation of the solar cycle. The greatest efforts have been put into the anelastic ASH code developed by Juri Toomre and his associates at Boulder (Miesch & Toomre 2009). The first requirement was to match the helioseismic measurements of differential rotation; that was solved by imposing a latitudinal entropy gradient at the tachocline (Miesch, Brun & Toomre 2006; Miesch *et al.* 2008). When a seed field was inserted, dynamo action was indeed found, yielding an antisymmetric toroidal field at the base of the convection zone, as shown in Figure 5 (Browning *et al.* 2006; Miesch & Toomre 2009). However, this large-scale field did not reverse and there was no indication of cyclic behaviour.

The ASH code introduces a laminar viscosity and resistivity, with a ratio (the magnetic Prandtl number) that is of order unity, as expected for turbulent diffusion. The actual coefficients are as small as accuracy allows, and get reduced as the numerical resolution is refined, in the hope that large-scale behaviour will eventually become independent of their precise numerical values. In a parallel calculation, with much lower resolution and a different numerical procedure, Ghizaru, Charbonneau & Smolarkiewicz (2010) have recently reported cyclic behaviour. The differential rotation in their model does not resemble that in the Sun. and the reversing toroidal fields are all at high latitudes. Moreover, both viscous and magnetic diffusion rely on the numerical scheme – a procedure that can lead to spurious results. There is an analogy here with studies of dynamo action

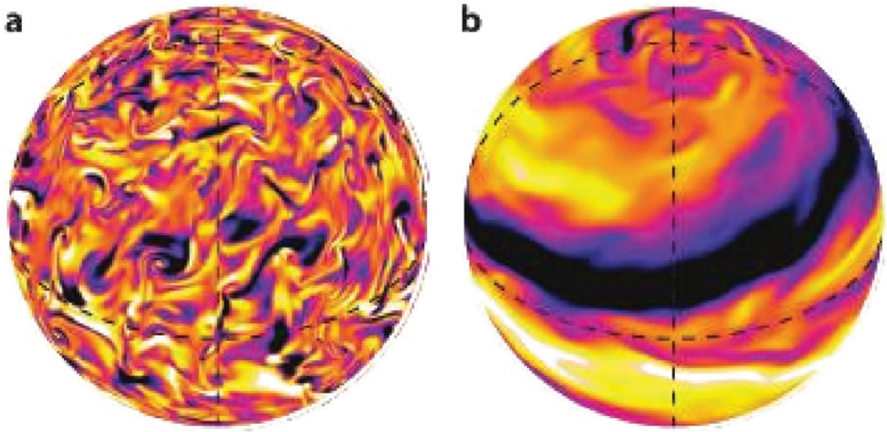

Figure 5. Simulation of dynamo action in a star like the Sun. Results obtained with the anelastic ASH code and a realistic representation of the actual differential rotation. The azimuthal field B_ϕ is shown, with positive and negative values denoted by orange-yellow and blue-black tones respectively. Snapshots (a) in the middle of the convection zone, with small-scale magnetic structures only, and (b) in the tachocline, with a dominant large-scale antisymmetric field. (From Miesch & Toomre 2009.)

driven by the magnetorotational instability in accretion discs, where numerical diffusion produces behaviour that depends on the resolution, and it is necessary to introduce laminar diffusivities in order to obtain meaningful results (Fromang & Papaloizou 2007; Fromang *et al.* 2007).

Small-scale convection in a plane layer, whether incompressible or stratified, stationary or rotating, is capable of maintaining magnetic fields if the magnetic Reynolds number is sufficiently large (Cattaneo 1999; Cattaneo & Hughes 2006; Vögler & Schüssler 2007; see Bushby in these Proceedings). In a star like the Sun, we should expect small-scale dynamo action at different depth-dependent scales, and fields produced on any of these scales, as well as any large-scale field, will be processed and amplified on all other scales. Hence it is not surprising that we now see magnetic flux emerging on all scales at the photosphere, as described by Title elsewhere in these Proceedings. In modelling large-scale dynamos it is implicitly assumed that the overall pattern is not critically dependent on details of the coexisting small-scale dynamos.

3. Modulation of cyclic activity

3.1. *Observations and predictions*

The level of activity varies from one activity cycle to the next, and the feeble start to the latest Cycle 24 has made such variability more topical; estimates of the amplitude of the new cycle keep falling as it progresses. Predictions of future behaviour depend critically on the length of the dataset that is used. Just as tomorrow's weather is likely to resemble today's, so the next cycle may resemble the last, or else the average of the last few cycles. A longer record shows irregular modulation on a timescale of a century, and then there is the Maunder Minimum of th seventeenth century, when sunspots almost completely disappeared (Weiss 2010).

To proceed further, we can use the proxy record supplied by the abundances of cosmogenic isotopes (^{12}C and ^{10}Be) that are created by galactic cosmic ray particles impinging on the Earth's atmosphere. Since these particles are deflected by magnetic fields in the heliosphere, the production rates of ^{12}C and ^{10}Be vary in antiphase with the solar cycle. The time-series derived from ^{10}Be abundances in polar ice cores provides a record of solar activity extending back for 9500 years that can be combined with direct measurements of neutron fluxes over the past few decades (Abreu *et al.* 2008). Figure 6 shows the last 2000 years of this record, smoothed to eliminate the basic solar cycle. It can be seen that solar activity has been unusually high for most of the last century, but that similar episodes have recurred aperiodically and some were more extreme. Taken over the entire record, values of Φ are normally distributed and we can define grand maxima and grand minima by setting levels that are only exceeded for 20% of the total time. Power spectra yield robust peaks corresponding to periods of 205 and 2300 years in both the ^{12}C and ^{10}Be records (Damon & Sonett 1991; Stuiver & Braziunas 1993; Beer 2000; Wagner *et al.* 2001).

Based on this record, we can adopt a statistical approach and estimate the life expectancy of the recent grand maximum, which has already lasted for more than 80 years, making it the third longest in the entire record. Fitting a gamma distribution to the lifetimes of the 66 grand maxima in the smoothed record, we find a further life expectancy of only 15 years. From the record there is then a 40% chance that activity will collapse into a grand minimum (Abreu *et al.* 2008; Weiss 2010). If so, we may expect a slight cooling effect on the climate, though not enough to cancel out the global warming caused by anthropogenic greenhouse gases.

3.2. *Origins of aperiodic modulation*

What causes this irregular modulation? Is it a purely stochastic effect, or is it an example of deterministic chaos? Economists have long regarded the solar cycle as an example of stochastic behaviour, and Barnes, Sargent & Tryon (1980) showed that an Autoregressive Moving Average (ARMA) model (constructed by combining a three-term recurrence relation with filtered Gaussian noise) provided a very plausible representation of solar activity (see also Brajša *et al.* 2009). If we seek a more causal description, we may suppose that the cyclic dynamo is disturbed by fluctuations in the underlying hydrodynamics of the convection zone, which we can regard as random. These might, for example, be variations in the meridional circulation, as suggested by Charbonneau & Dikpati (2000; Dikpati *et al.* 2010). The only snag is that these do have to be very substantial disturbances. Choudhuri & Karak (2010) recently used a mean-field dynamo model to simulate behaviour during the Maunder Minimum – but they did have to introduce an order-one perturbation to the system in order to achieve this. An interesting possibility is that there is hysteresis in the dynamo, as suggested by Schmitt, Schüssler & Ferriz-Mas (1996). They considered a dynamo that would only operate when the toroidal field B_T exceeds some critical value that allows instabilities driven by magnetic buoyancy to free flux tubes that are then acted on by Coriolis forces to form a poloidal field. If a stochastic perturbation causes B_T to drop below its critical value then the dynamo will switch off until another perturbation from some weaker dynamo process can switch it on again.

The alternative is that the modulation is deterministic. In the simplest case, the fluctuations may themselves be the product of an independent deterministic process – for example, that governing variations in the meridional circulation – and give rise to on-off

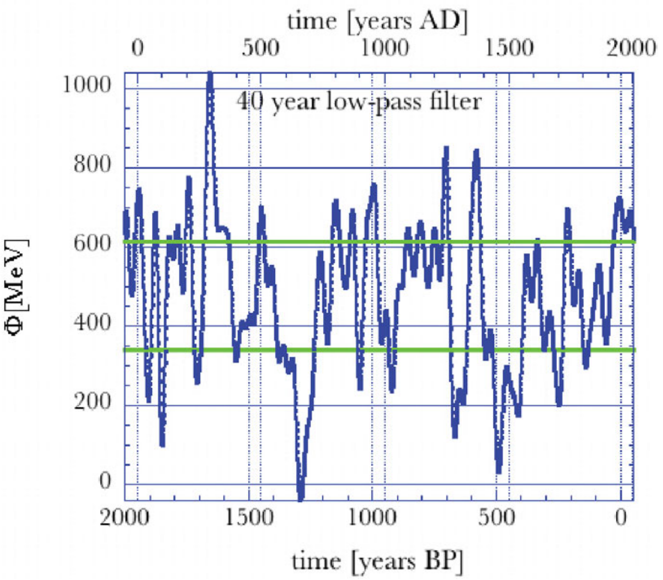

Figure 6. Variation of the solar modulation function Φ, a measure of solar activity derived from a composite record of ^{10}Be abundances in the Greenland GRIP ice core and a South Pole ice core and recent direct measurements, over the past 2000 years. The average value of Φ over the total period is 479 MeV and grand maxima and minima lie outside the horizontal lines. The recent grand maximum is now coming to an end, and both the Dalton minimum (around 1810) and the Maunder Minimum (1645–1710) are clearly visible, along with many similar episodes. Note the extremely high maximum around 300 AD, the longest in this record, which lasted for 95 years. (After Abreu *et al.* 2008.)

intermittency (Spiegel 1994). In the simplest models of deterministic switching, the amplitude of a nonlinear oscillation is controlled by the output of an independent chaotic oscillator (Platt, Spiegel & Tresser 1993). A more likely situation is that hydrodynamic processes are themselves influenced by the magnetic field, giving rise to in-out intermittency. The resulting bifurcation structure, illustrated in Figure 7, can be identified in low-order systems of nonlinear ordinary differential equations, with properties that are generic and therefore robust (Tobias, Weiss & Kirk 1995; Weiss 2010). Similar patterns appear in various mean-field dynamo models with differing nonlinearities. What appears essential for chaos is that the nonlinear coupling should introduce a time delay into the system, as first suggested by Yoshimura (1978). This is shown explicitly in the model of Jouve, Proctor & Lesur (2010), which is described elsewhere in these Proceedings.

A well-known feature of such systems is that a "ghost" of the torus survives after it is destroyed, giving rise to a peak in the power spectrum within the chaotic regime (Ott 1993; Tobias, Weiss & Kirk 1995). If modulation is stochastic, on the other hand, the spectrum should not contain such peaks; in particular, fluctuations in the meridional circulation are unlikely to generate periods longer than a few decades. The well-attested 205-year periodicity in the records of cosmogenic isotope abundances strongly suggests that the measured modulation of solar activity is indeed a product of chaotic modulation in a deterministic system.

4. Other stars

4.1. *Cool stars with deep convection zones*

Magnetic activity in lower main-sequence stars is strongly correlated with their normalized rotation rates, as measured by an inverse Rossby number (Donati & Landstreet 2009), as explained by Donati in these Proceedings. It is convenient to separate sun-like stars (spectral types F, G and K) from stars that are fully or almost fully convective (late M stars) and then to consider early-type stars separately.

There is a small population of slowly rotating stars like the Sun, with ages greater than 2–3 Gyr, that have been studied intensively over an interval of about 40 years and show similar cyclic activity (Baliunas *et al.* 1995; Baliunas, Sokoloff & Soon 1996). The cycle period decreases the more rapidly the star rotates; for a star of given mass, the cycle period $P \propto \Omega^{-(1+q)}$, where estimates of q vary from 0.25 to 1.0 (Noyes, Vaughan & Weiss 1984; Ossendrijver 1997; Saar & Brandenburg 1999; Saar 2002). These stars can

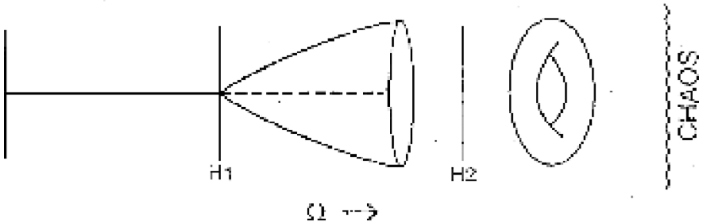

Figure 7. Bifurcation structure for transitions to chaos in large-scale magnetic activity. Although a star like the Sun actually spins down as it ages (Mestel 1999) it is appropriate to consider the effect of increasing its rotation velocity Ω from a value too low to allow the dynamo to operate. Then there are two successive Hopf bifurcations, the first leading to periodic cycles and the second to periodically modulated cycles; trajectories in phase space are attracted successively to a fixed point, to a limit cycle and to a two-torus. Then there are a series of bifurcations, involving frequency-locking and period-doubling, that lead to chaotically modulated cycles with a strange attractor in phase space.. (After Tobias *et al.* 1995.)

all be expected to possess tachoclines and to host dynamos like that in the Sun. It is to be expected that there are also other similar stars that are currently passing through grand minima and therefore relatively inactive.

Younger stars rotate much more rapidly and are correspondingly much more active, though activity appears to saturate for rotation periods less than a day. Here again, long-term measurements have revealed multiple cyclic behaviour but the corresponding periods do not follow the same pattern as the slow rotators. A key feature of these stars is the prevalence of huge spots, revealed by Zeeman-Doppler imaging, that cluster around the poles rather than at low latitudes as in the Sun (Thomas & Weiss 2008).

The Coriolis force becomes increasingly more important the more rapidly the star rotates. It is not to be expected that the pattern of rotation found in the Sun and shown in Figure 1 will apply also to these rapid rotators, nor do they need to possess a tachocline. Naively, we should expect rotation to be dominated by the Proudman-Taylor constraint, so that Ω is constant on cylindrical surfaces, at least outside the tangent cylinder that encloses the radiative zone. This structure will profoundly affect the stellar dynamo (Bushby 2003). The Boulder group have computed the convection pattern in a star rotating at three times the solar rate and attempted to model the resulting dynamo (Brown *et al.* 2008, 2010). They find that the magnetic fields form persistent wreathlike structures, with predominantly azimuthal fields that are antisymmetric about the equator, as shown in Figure 8. Further calculations, with rotation at up to $10\Omega_\odot$ are discussed by Brown in these Proceedings.

4.2. *Fully convective stars*

The presence of a tachocline, or of a radiative core, is not an essential prerequisite for the existence of a dynamo. Indeed, Dobler, Stix & Brandenburg (2006) demonstrated dynamo action in a particular model of a fully convective star. In fact, stars much smaller than the Sun, of spectral class M, have very deep convection zones and, for masses less than 0.35 $M_\odot$, are fully convective. Yet these stars are known to be magnetically very active. Zeeman-Doppler imaging of 16 M-dwarfs reveals three different patterns of magnetic activity (Donati *et al.* 2008; Morin *et al.* 2008, 2010). Stars with masses in the range $0.8 > M/M_\odot > 0.5$ have significant radiative cores and exhibit weak, non-axisymmetric fields with strong azimuthal components and an average strength of a few hundred gauss. A star with mass less than 0.5 $M_\odot$ is effectively fully convective. Those stars in the range $0.5 > M/M_\odot > 0.2$ show very strong poloidal fields that are predominantly dipolar

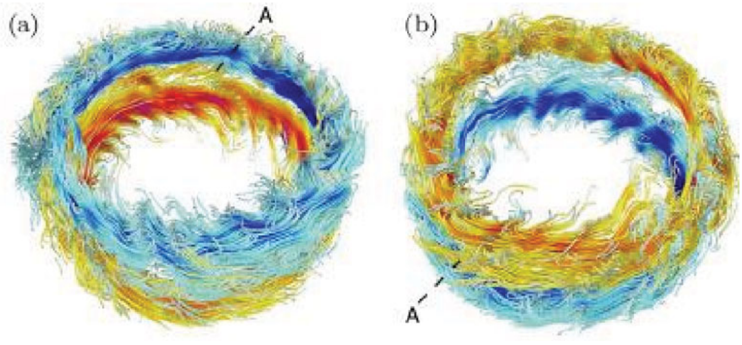

Figure 8. Magnetic wreaths in a model of the field in a rapidly rotating Sun. Colour denotes the strength and orientation of the predominant azimuthal field, and individual field lines are indicated. The two images show the two wreaths viewed from above and below the equator. The line A passes across the equator. (From Brown *et al.* 2010.)

and axisymmetric, with rms strengths of up to 2 kG (Reiners, Basri & Browning 2009). However, those M-dwarfs with $M/M_\odot < 0.2$ can display both strong poloidal and weak toroidal patterns, possibly even switching between them. Browning (2008) modelled dynamo action in a 0.3 $M_\odot$ M dwarf, rotating at the same rate as the Sun. He found that strong magnetic fields were generated, and that differential rotation was almost entirely quenched, leaving only a slight equatorial acceleration. The total magnetic energy was then comparable to the total kinetic energy in the uniformly rotating frame. When azimuthally averaged, the toroidal field gradually developed an antisymmetric structure, corresponding to a predominantly dipolar external field. Browning's more recent calculations, for stars rotating up to 10 times faster, are described in his contribution to these Proceedings.

4.3. *Upper main-sequence stars*

In stars more massive than the Sun the surface temperature is sufficiently high that hydrogen is already ionized and there is only a shallow convective layer caused by ionization of helium. However, the transition from energy generation by the *pp*-chain to the temperature-sensitive CNO-cycle leads to the appearance of a convective core. We naturally expect such a core, in a star rotating sufficiently rapidly, to act as a large-scale dynamo and to generate a magnetic field – though that field may not be visible at the surface of the star. Brun, Browning & Toomre (2005) attempted to model dynamo action in the core of an A-type star, with mass 2 $M_\odot$ and $1 \leqslant \Omega/\Omega_\odot \leqslant 4$. Similar dynamos must be responsible for the magnetic activity observed in some massive O and B type stars.

The best known magnetic stars are slow rotators of types A and B that exhibit peculiar anomalies in the abundances of certain elements, along with strong magnetic fields that vary periodically as the stars rotate. It is generally cosidered that these are fossil fields, predominantly dipolar but with an axis inclined to the rotation vector (Mestel 1999; Donati & Landstreet 2009). For such a field to be stable and to survive within a radiative zone it must have a mixed poloidal-toroidal structure (Tayler 1973; Wright 1973; Spruit 2002). Featherstone *et al.* (2009) have modelled the interaction between such a fossil field and a core dynamo in an Ap star, giving rise to complex magnetic topologies.

5. What next?

Dynamo theory, as applied to stars, is at present caught between two stages of development. The mean field approach has illuminated the main features of cyclic behaviour in the Sun – but it involves ad hoc assumptions, and there is no obvious way of establishing a unique model. Surely the time has now come to turn to direct numerical simulation to resolve these issues. Yet, despite prodigious efforts (led by Juri Toomre and his group at Boulder) there is still no convincing computational model of the solar cycle. Progress has been slow but steady, and it is reassuring that the latest calculations can now replicate the observed pattern of differential rotation. The next stage must be to reproduce the detailed behaviour of magnetic fields around the base of the convection zone and in the tachocline. That will require high radial resolution, accompanied by a corresponding reduction in the local values of viscous and magnetic diffusivities. For such a calculation to be credible it remains essential that diffusive processes should be treated explicitly, on the optimistic assumption that the large-scale dynamo (like linear fast dynamos) does not depend critically on fine details of the magnetic and kinetic energy cascades. It is extremely important therefore that this massive computational project should continue - -and there is surely a grand future ahead for Juri and his colleagues.

Acknowledgments

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Discussion

Brandenburg: The number of spotless days during the current minimum is still not as low as it was in 1908. Are there other reasons suggesting that the current minimum might still be deeper than in 1908?

Weiss: Current predictions are that the next sunspot maximum will be similar to that in 1908. We cannot predict what will happen next. As a theoretician, I hope for a grand minimum, which could tell us something new about the solar dynamo – but I expect that our colleagues who observe the solar atmosphere would prefer a resumption of vigorous activity.

Gough: You commented that not all the stars have spots aligned with the rotation axis in such a way that one should expect alignment. Yet most of the Doppler images one sees of the stars have two spots whose axes lie almost in the equatorial plane, not even inclined to mid latitudes. Please could you comment on this?

Weiss: It depends which stars one looks at. Rapidly rotating late-type stars tend to have polar spots (unlike the slowly rotating Sun). The Ap stars are typically oblique rotators, with a magnetic axis strongly inclined to the rotation axis. Mestel has argued that dynamical processes in a rotating system act to promote this obliquity.

Vasil: Is there any historical evidence for a grand minimum starting right after a grand maximum ?

Weiss: Well, yes: there are a good many examples of a rapid descent from a grand maximum to a grand minimum (and vice versa) in the smoothed ^{10}Be record. The Maunder Minimum actually started quite abruptly (though it was not preceded by a grand maximum). Hevelius commented in 1668 that, while in 1643–1644 there were 100 to 200 spots visible in one hemisphere during a single year, “for a good many years recently, ten or more, I am certain that absolutely nothing of great significance (apart from some rather unimportant and small spots) has been observed either by us or by others”.